International Journal of Modern Physics D
Vol. 14, No. 6 (2005) 1063–1073

THE FATE OF THE FALSE VACUUM IN EINSTEIN GRAVITY THEORY WITH NONMINIMALLY-COUPLED SCALAR FIELD

WONWOO LEE* and CHUL H. LEE†

Department of Physics, Hanyang University, 133-791, Seoul, Korea
**warrior@ihanyang.ac.kr*
†chulhoon@hanyang.ac.kr

Received 29 October 2004
Communicated by IJMPD Editorial Office

The decay of false vacuum via the true vacuum bubble nucleation has been explored in Einstein theories of gravity with nonminimally-coupled scalar field using Coleman–De Luccia's semiclassical instanton approximation. In this case the false vacuum decay rates and the radius of the bubbles in Coleman's thin-wall approximation have been computed analytically and numerically with several values of nonminimal coupling constant and compared with the standard result obtained by Coleman–De Luccia in the context of scalar field minimally-coupled to Einstein gravity.

Keywords: False vacuum; bubble nucleation; nonminimally-coupled scalar field.

1. Introduction

The decay of false vacuum via the true vacuum bubble nucleation was studied within the thin-wall approximation by Coleman.[1] As for the effect of gravity, the bubble nucleation with self gravity was also discussed by Coleman and De Luccia[2] and extended by Parke[3] to arbitrary vacuum energy densities in the zero-temperature limit. Their works have been applied to old inflation[4] and open inflation.[5] As a matter of fact, the standard approach to the calculation of bubble nucleation rates during the first order phase transition is based on the work of Langer.[6]

The nonminimal coupling of the scalar field to the Ricci curvature is discussed in many cosmological scenarios such as inflation[7] and quintessence.[8] Here we study the process of bubble nucleation in the theory containing the nonminimal coupling term. Let us consider the Lagrangian density for Einstein gravity theory with a nonminimally coupled scalar field

$$\mathcal{L} = \frac{R}{2\kappa} - \frac{1}{2}g^{\mu\nu}\partial_\mu\Phi\partial_\nu\Phi - \frac{1}{2}\xi R\Phi^2 - U(\Phi)\,, \tag{1}$$

where $\kappa \equiv 8\pi G$, $U(\Phi)$ is the scalar field potential, R denotes the Ricci curvature of spacetime and the term $-\xi R\Phi^2/2$ describes the nonminimal coupling of the field Φ to the Ricci curvature. The corresponding equation satisfied by the scalar field is

$$\frac{1}{\sqrt{g}}\partial_\mu[\sqrt{g}g^{\mu\nu}\partial_\nu\Phi] - \xi R\Phi - \frac{\partial U}{\partial\Phi} = 0\,, \tag{2}$$

which includes a coupling term $\xi R\Phi$ between the field Φ and the Ricci curvature of spacetime.[9]

In this paper, we show how the standard results obtained by Coleman–De Luccia and Parke should be modified when the nonminimal coupling term is considered and how that term has influence on the bubble radius and the bubble nucleation rates, especially. Our results have been computed analytically and numerically with several values of nonminimal coupling constant without the thin-wall approximation.

2. The Bubble Nucleation Rate for Einstein Theory with Nonminimally Coupled Scalar Field

Let us consider the case where $U(\Phi)$ in Eq. (1) has the form

$$U(\Phi) = \frac{\lambda}{8}(\Phi^2 - b^2)^2 + \frac{\epsilon}{2b}(\Phi \pm b)\,. \tag{3}$$

U has two minima, $U(\Phi_-)$ corresponding to the true vacuum which is the absolute minimum of the potential and $U(\Phi_+)$ to the false vacuum which is a local minimum. We consider the nonminimal coupling parameter ξ to be both positive and negative constant.

The Einstein equations are

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu}\,, \tag{4}$$

where $R_{\mu\nu}$ is the Ricci tensor and $T_{\mu\nu}$ is the matter energy momentum tensor,[10]

$$T_{\mu\nu} = \frac{1}{1-\xi\Phi^2\kappa}\left[\partial_\mu\Phi\partial_\nu\Phi - g_{\mu\nu}\left(\frac{1}{2}g^{\alpha\beta}\partial_\alpha\Phi\partial_\beta\Phi + U(\Phi)\right)\right]. \tag{5}$$

Note that compared to the nonminimal coupling case, the curvature scalar is slightly modified to be

$$R = \frac{\kappa[4U(\Phi) + \partial_\mu\Phi\partial^\mu\Phi]}{(1-\xi\Phi^2\kappa)}\,. \tag{6}$$

In this work, we consider the cases where $\frac{\mu^2\kappa}{8\pi}$ and $\frac{\epsilon}{\mu^4}$ are small quantities and we approximate each quantity to the first order of these parameters.

$O(4)$ symmetric bubbles have the minimum Euclidean action in the absence of gravity,[11] but this result has not been successfully extended to that case in the presence of gravity yet, although $O(4)$ symmetry seems to be a reasonable assumption. In this paper we assume the $O(4)$ symmetry for both Φ and the spacetime metric

$g_{\mu\nu}$ in a similar manner to Coleman and De Luccia. The most general rotationally invariant Euclidean metric is

$$ds^2 = d\eta^2 + \rho^2(\eta)\{d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta\, d\phi^2)\}\,. \tag{7}$$

Then Φ is a function of η only and one has $R = -6(\rho'^2 + \rho\rho'' - 1)/\rho^2$. In this case the Euclidean Lagrangian density becomes

$$\mathcal{L}_E = \frac{1}{2}\Phi'^2 + \frac{1}{2}\xi R\Phi^2 + U + \frac{3}{\kappa}\left(\frac{\rho''}{\rho} + \frac{\rho'^2}{\rho^2} - \frac{1}{\rho^2}\right), \tag{8}$$

where the prime denotes the differentiation with respect to η. The Euclidean field equations for Φ and ρ turn out to be

$$\Phi'' + \frac{3\rho'}{\rho}\Phi' - \xi R\Phi = \frac{dU}{d\Phi}\,, \tag{9}$$

$$\rho'^2 = 1 + \frac{\kappa\rho^2}{3(1-\xi\Phi^2\kappa)}\left(\frac{1}{2}\Phi'^2 - U\right) \tag{10}$$

respectively. The boundary conditions for the bounce are

$$\lim_{\eta\to\infty}\Phi(\eta) = \Phi_+\,, \qquad \left.\frac{d\Phi}{d\eta}\right|_{\eta=0} = 0\,. \tag{11}$$

Multiplying Eq. (9) by $\frac{d\Phi}{d\eta}$ and rearranging the terms, one obtains

$$\frac{d}{d\eta}\left[\frac{1}{2}\Phi'^2 - U\right] = -\frac{3\rho'}{\rho}\Phi'^2 + \xi R\Phi\Phi'\,. \tag{12}$$

Here the quantity in the square parentheses can be interpreted as the total energy of the particle with the potential energy $-U$, the first term on the right hand side as the dissipation rate of the total energy and the second term as the extra source of the power.

A bounce is a solution of the Euclidean field equation satisfying appropriate boundary conditions. The nucleation rate of a true vacuum bubble in the sea of the false vacuum can be obtained[1] by

$$\Gamma/V = Ae^{-B/\hbar}\,, \tag{13}$$

where B is the Euclidean action of the bounce

$$B = \int d\tau d^3x \mathcal{L}_E^{(b)}\,. \tag{14}$$

If there are several bounces the main contribution comes from the one with the minimum Euclidean action. The effects of the rest of the bounces can be absorbed in A. The bounce with the minimum Euclidean action is assumed to have the highest symmetry. Accordingly, Coleman and De Luccia considered the solution with $O(4)$ symmetry. In this work we have been interested in computing B.

The form of ρ in the bubble wall region may be obtained by numerically solving Eqs. (9) and (10) simultaneously. The bubble nucleation rate is calculated by $\Gamma/V = Ae^{-B/\hbar}$ where B is the difference between the Euclidean action of the bounce and that of the false vacuum state,

$$B = S_E^{(b)} - S_E^{(\mathrm{f.v.})} \,. \tag{15}$$

Thus, the Euclidean action is given by

$$\begin{aligned} S_E &= 2\pi^2 \int_0^\infty d\eta \biggl[\rho^3 \left(\frac{1}{2}(\Phi')^2 + U(\Phi) \right) \\ &\qquad + \frac{3}{\kappa}(\rho\rho'^2 + \rho^2\rho'' - \rho) - 3\xi\Phi^2(\rho\rho'^2 + \rho^2\rho'' - \rho) \biggr] \\ &= 4\pi^2 \int_0^\infty d\eta \left[\rho^3 U(\Phi) - \frac{3\rho}{\kappa} + 3\rho\xi\Phi^2 + 3\xi\rho^2\rho'\Phi\Phi' \right] . \end{aligned} \tag{16}$$

Here we eliminate the second-derivative term by integration by parts and use the Euclidean field equation to eliminate ρ'. The third term in Eq. (16) will be vanished because of $\rho' \to 0$ in the wall and $\Phi' \to 0$ both inside and outside the wall in the thin-wall approximation, respectively.

Now we shall use the thin-wall approximation scheme to evaluate B. Outside the wall,

$$B_{\mathrm{out}} = S_E(\Phi_+) - S_E(\Phi_+) = 0 \,. \tag{17}$$

In the wall, we can replace ρ by $\bar{\rho}$ and Eq. (9) can be modified

$$\frac{d^2\Phi}{d\eta^2} \simeq \frac{dU}{d\Phi} + \xi R \Phi \,. \tag{18}$$

Multiplying Eq. (9) by $\frac{d\Phi}{d\eta}$ and integrating over η yield

$$\left(\frac{d\Phi}{d\eta} \right)^2 = 2[U(\Phi) - U(\Phi_+)] + \xi R(\Phi^2 - \Phi_+^2) \,, \tag{19}$$

where Ricci scalar R is a function of ρ only in the wall. Then, the B in the wall

$$\begin{aligned} B_{\mathrm{wall}} &= 4\pi^2\bar{\rho}^3 \int \left[(U(\Phi) - U(\Phi_+)) + \frac{3}{\bar{\rho}^2}\xi(\Phi^2 - \Phi_+^2) \right] d\eta \\ &= 2\pi^2\bar{\rho}^3 \int_{\Phi_-}^{\Phi_+} \sqrt{2[U(\Phi) - U(\Phi_+)] - \frac{6}{\bar{\rho}^2}\xi(\Phi_+^2 - \Phi^2)}\, d\Phi \\ &\simeq 2\pi^2\bar{\rho}^3 \left[\int_{\Phi_-}^{\Phi_+} \sqrt{2[U(\Phi) - U(\Phi_+)]}\, d\Phi - C\xi \right] = 2\pi^2\bar{\rho}^3 S \,, \end{aligned} \tag{20}$$

where $S = S_1 - C\xi$, $S_1 = \int_{\Phi_-}^{\Phi_+} \sqrt{2[U(\Phi) - U(\Phi_+)]}d\Phi$ and $C = \frac{12b}{\sqrt{\lambda}\bar{\rho}^2}$. The C reflects the small correction of surface energy density. Here we approximate the quantity in the square root to the first order in the small quantities. To evaluate the Euclidean action inside the wall, we use $d\rho = d\eta\left[1 - \frac{\kappa\rho^2 U(\Phi_\mp)}{3(1-\xi\Phi_\mp^2\kappa)}\right]^{1/2}$, then

$$B_{\rm in} = \frac{12\pi^2}{\kappa^2}\left[\frac{(1-\xi\Phi_-^2\kappa)\left\{\left[1 - \frac{\kappa\bar{\rho}^2 U(\Phi_-)}{3(1-\xi\Phi_-^2\kappa)}\right]^{3/2} - 1\right\}}{U(\Phi_-)}\right.$$
$$\left. - \frac{(1-\xi\Phi_+^2\kappa)\left\{\left[1 - \frac{\kappa\bar{\rho}^2 U(\Phi_+)}{3(1-\xi\Phi_+^2\kappa)}\right]^{3/2} - 1\right\}}{U(\Phi_+)}\right]. \tag{21}$$

B is then given by

$$B = B_{\rm in} + B_{\rm wall} + B_{\rm out}\,. \tag{22}$$

We consider the following two simple cases; (Case 1) a scalar field originally in the false vacuum state of a positive energy density decaying into the true vacuum state of zero energy density ($U(\Phi_+) = \epsilon$, $U(\Phi_-) = 0$) (Fig. 1) and (Case 2) the false vacuum state with zero energy density decaying into the true vacuum state with negative energy density ($U(\Phi_+) = 0$, $U(\Phi_-) = -\epsilon$) (Fig. 2), in these cases $U(\Phi)$ has the form in Eq. (3).

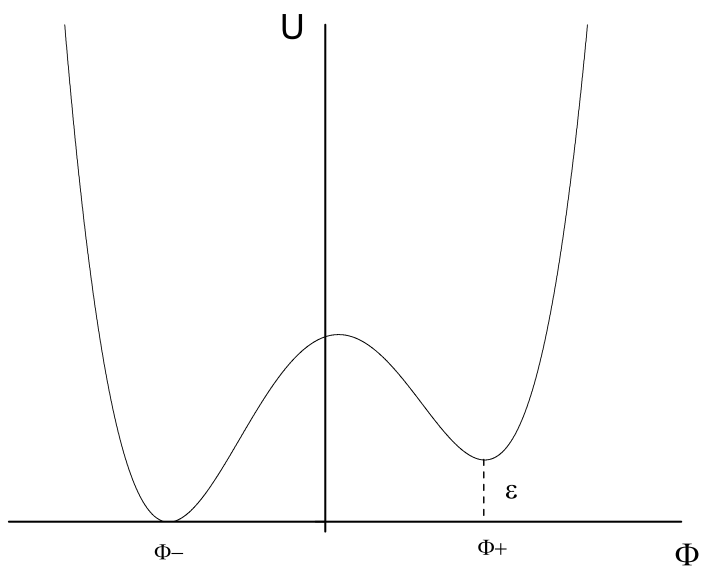

Fig. 1. Potential with two minima, $U(\phi_+) = \epsilon$ and $U(\phi_-) = 0$.

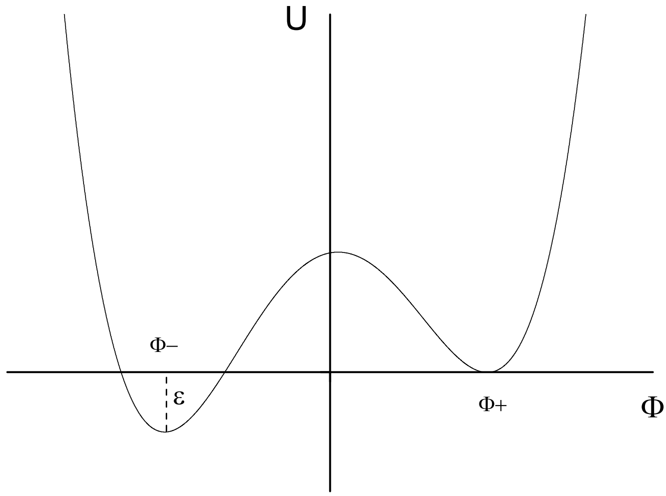

Fig. 2. Potential with two minima, $U(\phi_+) = 0$ and $U(\phi_-) = -\epsilon$.

In case 1, $\bar{\rho}_n$ is obtained by a variational computation

$$\bar{\rho}_n = \frac{\bar{\rho}_o}{\left[1 + \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]} - \bar{\rho}_o \xi \left[\frac{3C}{\bar{\rho}_o \epsilon \left[1 + \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]} + b^2 \kappa \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right], \tag{23}$$

where $\bar{\rho}_o = 3S_1/\epsilon$ is the bubble radius in the absence of gravity and $\Lambda = (\kappa\epsilon/3)^{-1/2}$. Here we approximate $\frac{\Phi_+^2 \kappa}{8\pi}$ and $\frac{\Phi_-^2 \kappa}{8\pi}$ to $\frac{b^2 \kappa}{8\pi}$ which is the first order term and $b^2 = \mu^2/\lambda$. At this point,

$$B_n = \frac{B_o}{\left[1 + \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]^2} - 2B_o \xi \left[\frac{6C}{\bar{\rho}_o \epsilon \left[1 + \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]^2} + b^2 \kappa \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right], \tag{24}$$

where $B_o = 27\pi^2 S_1^4 / 2\epsilon^3$ is the decay coefficient in the absence of gravity.

In case 2, the results are

$$\bar{\rho}_n = \frac{\bar{\rho}_o}{\left[1 - \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]} - \bar{\rho}_o \xi \left[\frac{3C}{\bar{\rho}_o \epsilon \left[1 - \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]} - b^2 \kappa \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right], \tag{25}$$

and

$$B_n = \frac{B_o}{\left[1 - \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]^2} - 2B_o \xi \left[\frac{6C}{\bar{\rho}_o \epsilon \left[1 - \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right]^2} - b^2 \kappa \left(\frac{\bar{\rho}_o}{2\Lambda}\right)^2\right], \tag{26}$$

In both case 1 and case 2, we see that the positive nonminimal coupling constant ξ makes materialization of the bubble more likely, that is, it diminishes B and makes the radius of the bubble when it materializes smaller. For the negative nonminimal coupling constant ξ, it makes materialization of the bubble less likely and makes the radius of the bubble at its moment of materialization bigger. The first term in each case is the standard result obtained by Coleman–De Luccia.[2]

In Ref. 3 Parke considered that both the false and true vacuum have arbitrary constants and his calculation is performed at zero temperature. To obtain an analytic result, the potential for the scalar field is assumed to allow the use of the thin-wall approximation. In our case, the modified results are

$$\bar{\rho}'_n = \frac{\bar{\rho}'_o}{\triangle} - \bar{\rho}'_o\xi\left[\frac{C\Lambda'^2\kappa}{\bar{\rho}'_o\triangle} + \left(b^2\kappa - \frac{2C\Lambda'^2\kappa}{\bar{\rho}'_o}\right)\left(\left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2 + \left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\right)\right], \tag{27}$$

where $\bar{\rho}'_o = 3S_1/(U(\Phi_+) - U(\Phi_-))$ is the critical bubble size without gravity, $\lambda' = [3/\kappa(U(\Phi_+) + U(\Phi_-))]^{1/2}$ and $\Lambda' = [3/\kappa(U(\Phi_+) - U(\Phi_-))]^{1/2}$, then

$$B'_n = \frac{2B_o\left[\left\{1 + \left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2\right\} - \triangle\right]}{\left[\left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\left\{\left(\frac{\Lambda'}{\lambda'}\right)^2 - 1\right\}\triangle\right]} - 2B_o\xi\left[\frac{4C\Lambda'^2\kappa\left[\left\{1 + \left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2\right\} - \triangle\right]}{\bar{\rho}'_o\left[\left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\left\{\left(\frac{\Lambda'}{\lambda'}\right)^2 - 1\right\}\triangle\right]}\right]$$
$$- 2B_o\xi\frac{\left(b^2\kappa - \frac{2C\Lambda'^2\kappa}{\bar{\rho}'_o}\right)\left(\left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2 + \left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\right)\left[\left\{1 + \left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2\right\} - \triangle\right]}{\left[\left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\left\{\left(\frac{\Lambda'}{\lambda'}\right)^2 - 1\right\}\right]}, \tag{28}$$

where $\triangle = \left[1 + 2\left(\frac{\bar{\rho}'_o}{2\lambda'}\right)^2 + \left(\frac{\bar{\rho}'_o}{2\Lambda'}\right)^4\right]^{1/2}$. The first term in each case is the result obtained by Parke.[3]

3. Numerical Calculation

In this section, we have computed numerically Eq. (9) with several values of non-minimal coupling constant for two simple cases. In the computation we can change to dimensionless variables for the numerical calculation

$$\frac{\lambda U(\Phi)}{\mu^4} = \tilde{U}(\tilde{\Phi}), \quad \frac{\lambda\Phi^2}{\mu^2} = \tilde{\Phi}^2, \quad \frac{\lambda\epsilon}{\mu^4} = \tilde{\epsilon}. \tag{29}$$

These variables give

$$\tilde{U}(\tilde{\Phi}) = \frac{1}{8}(\tilde{\Phi}^2 - 1)^2 + \frac{\tilde{\epsilon}}{2}(\tilde{\Phi} \pm 1), \tag{30}$$

and the Euclidean field equations for Φ and ρ become

$$\tilde{\Phi}'' + \frac{3\tilde{\rho}'}{\tilde{\rho}}\Phi' - \xi\tilde{R}\tilde{\Phi} = \frac{d\tilde{U}}{d\tilde{\Phi}}, \tag{31}$$

$$\tilde{\rho}'^2 = 1 + \frac{\tilde{\kappa}\tilde{\rho}^2}{3(1 - \xi\tilde{\Phi}^2\tilde{\kappa})}\left(\frac{1}{2}\tilde{\Phi}'^2 - \tilde{U}\right) \tag{32}$$

respectively, where $\tilde{R}$ is R/μ^2, $\tilde{\rho}$ is $\rho\mu$ and $\tilde{\kappa}$ is $\frac{\mu^2}{\lambda}\kappa$. The boundary conditions also become

$$\lim_{\tilde{\eta}\to\infty}\tilde{\Phi}(\tilde{\eta}) = \tilde{\Phi}_+, \qquad \left.\frac{d\tilde{\Phi}}{d\tilde{\eta}}\right|_{\tilde{\eta}=0} = 0, \tag{33}$$

where $\tilde{\eta}$ is $\eta\mu$.

We have computed three cases of bubble profiles corresponding to $\xi = 0$, $\xi = \pm 1/6$ and $\xi = \pm 1.4$, respectively. And here we take $\tilde{\epsilon} = 0.11$ and $\tilde{\kappa} = 0.03$.

In each figure, every curve indicates bubble profiles which are corresponding to dashed curve for $\xi = 0$, solid curve for $\xi = \pm 1/6$ and dotted curve for $\xi = \pm 1.4$, respectively. Figure 3 shows bubble profiles for case 1 with positive ξ. Figure 4 shows bubble profiles for case 1 with negative ξ. Figure 5 shows bubble profiles for case 2 with positive ξ. Figure 6 shows bubble profiles for case 2 with negative ξ. We see that the bubble radius is diminished as ξ is increased and the bubble radius is increased as ξ is decreased. In addition, we see that the bubble radius is influenced by ξ in case 1 more than in case 2.

4. Conclusions

In this paper we have explored the decay of false vacuum via the true vacuum bubble nucleation in Einstein theories of gravity with nonminimally-coupled scalar field using Coleman–De Luccia's semiclassical instanton approximation. We have especially focused on the bubble radius and the bubble nucleation rates. Our results have been computed analytically and numerically with several values of nonminimal coupling constant.

In both case 1 ($U(\Phi_+) > 0$, $U(\Phi_-) = 0$) and case 2 ($U(\Phi_+) = 0$, $U(\Phi_-) < 0$), we see that the positive nonminimal coupling constant ξ makes materialization of the bubble more likely, that is, it diminishes B and makes the bubble radius at its moment of materialization smaller. For the negative nonminimal coupling constant ξ, it makes materialization of the bubble less likely and makes the bubble radius at its moment of materialization bigger.

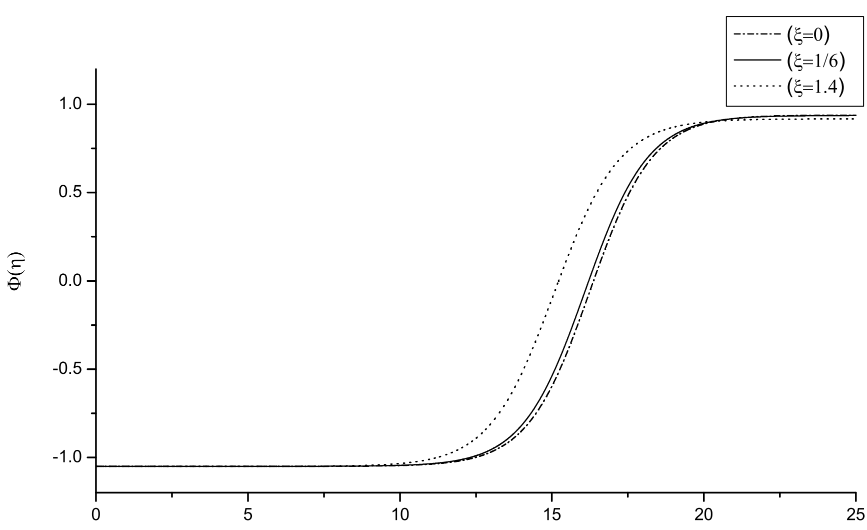

Fig. 3. Bubble profiles with respect to each positive ξ. The three curves are (a) Dashed curve: $\xi = 0$, (b) Solid curve: $\xi = 1/6$ (c) Dotted curve: $\xi = 1.4$.

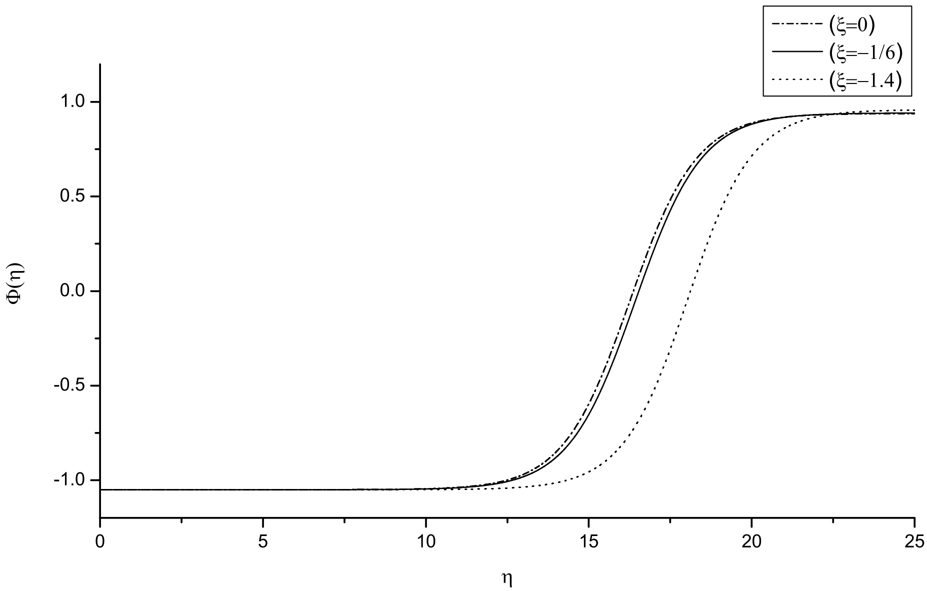

Fig. 4. Bubble profiles with respect to each negative ξ. The three curves are (a) Dashed curve: $\xi = 0$, (b) Solid curve: $\xi = -1/6$ (c) Dotted curve: $\xi = -1.4$.

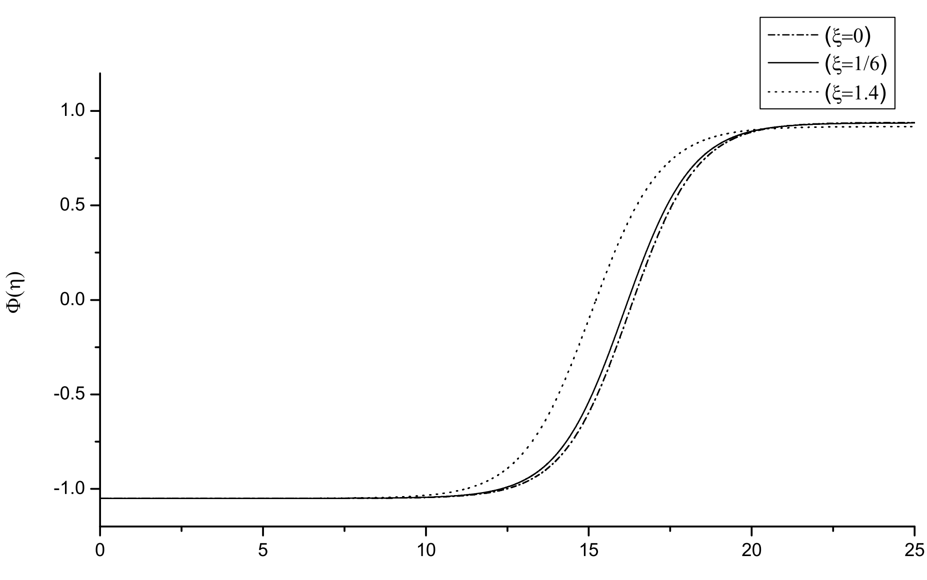

Fig. 5. Bubble profiles with respect to each positive ξ. The three curves are (a) Dashed curve: $\xi = 0$, (b) Solid curve: $\xi = 1/6$ (c) Dotted curve: $\xi = 1.4$.

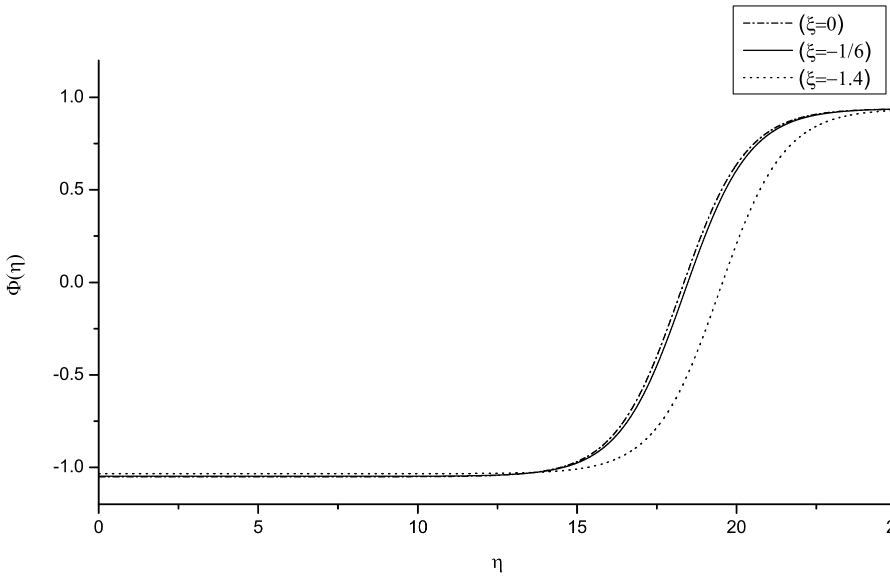

Fig. 6. Bubble profiles with respect to each negative ξ. The three curves are (a) Dashed curve: $\xi = 0$, (b) Solid curve: $\xi = -1/6$ (c) Dotted curve: $\xi = -1.4$.

In Section 3, we have computed numerically Eq. (9) with several values of non-minimal coupling constant for two simple cases. We see that the bubble radius is diminished as ξ is increased and the bubble radius is increased as ξ is decreased. Furthermore, we see that the bubble radius is influenced by ξ in case 1 more than in case 2.

Acknowledgments

We thank Prof. Hyun Kyu Lee and Prof. Yongsung Yoon for discussions and kind comments. We also thank Research Prof. Hongsu Kim for helpful remarks and proposing the further work. This work was supported by grant No. R01-1999-00020 from the Korea Science and Engineering Foundation.

References

1. S. Coleman, *Phys. Rev.* **D15**, 2929 (1977); C. Callan and S. Coleman, *Phys. Rev.* **D16**, 1762 (1977).
2. S. Coleman and F. De Luccia, *Phys. Rev.* **D21**, 3305 (1980).
3. S. Parke, *Phys. Lett.* **B121**, 313 (1983).
4. A. H. Guth, *Phys. Rev.* **D23**, 337 (1981).
5. M. Bucher, A. S. Goldhaber and N. Turok, *Phys. Rev.* **D52**, 3314 (1995); A. D. Linde and A. Mezhlumian, *Phys. Rev.* **D52**, 6789 (1995).
6. J. Langer, *Ann. Phys.* **41**, 108 (1967); *ibid.* **54**, 258 (1969).

7. L. Abbott, *Nucl. Phys.* **B185**, 233 (1981); T. Futamase and K. Maeda, *Phys. Rev.* **D39**, 399 (1989); R. Fakir and W. G. Unruh, *Phys. Rev.* **D41**, 1783 (1990); V. Faraoni, *Phys. Rev.* **D53**, 6813 (1996).
8. F. Perrotta, C. Baccigalupi and S. Matarrese, *Phys. Rev.* **D61**, 023507 (1999).
9. C. G. Callan, S. Coleman and R. Jackiw, *Ann. Phys.* (*N.Y.*) **59**, 42 (1970).
10. C. W. Misner, K. S. Thorne and J. A. Wheeler, *Gravitation* (Freeman, San Francisco, 1973).
11. S. Coleman, V. Glaser and A. Martin, *Commun. Math. Phys.* **D58**, 211 (1978).