GRAVITY AND THE DECAY OF THE FALSE VACUUM ☆

Stephen PARKE
Stanford Linear Accelerator Center, Stanford University, Stanford, CA 94305, USA

Received 19 April 1982
Revised manuscript received 15 October 1982

I have calculated, at zero temperature and using the thin-wall approximation, the exponential suppression factor in the rate of decay of the false vacuum per unit volume for a real scalar field. The effects of classical gravity are included. Both the false and the true vacua have arbitrary cosmological constants.

To understand the importance of gravity in the decay of the false vacuum, I have calculated the bubble nucleation rate for a single scalar field decaying from the false vacuum to the true vacuum including the effects of gravity. This calculation has a large cosmological constant for both the false and true vacua and is performed at zero temperature. Also, to obtain an analytic result, the potential for the scalar field is assumed to allow the use of the thin-wall approximation.

Since the work of Coleman [1] on the decay of the false vacuum there has been considerable interest in the application of this theory to cosmology. Guth's [2] inflationary universe scenario as well as the latter modifications of Linde [3], and Albrecht and Steinhardt [4] have been particularly important in this regard. However as Hut and Klinkhamer [5], and Hawking and Moss [6] have pointed out classical gravity can have an important effect on the phase transitions in the early universe. Also, recently Weinberg [7] has argued that in supersymmetric models the action of gravity in the decay of the false vacuum can solve the vacuum ambiguity common to these models. Therefore, there is considerable interest at present in understanding the effects of gravity in the decay of the false vacuum.

Coleman and DeLuccia [8] pioneered the inclusion of gravity in the theory of the decay of the false vacuum. In their work either the false or the true vacuum is assumed to have zero cosmological constant. In this paper I relax this condition so that both the false and the true vacuum can have non-zero cosmological constant. This zero temperature, thin-wall calculation is a direct extension of the work of Coleman and DeLuccia and I will follow their paper except for a few minor changes in notation. For a single scalar field including gravitation, the action is given by

$$S = \int d^4x \sqrt{-g}\,[\tfrac{1}{2} g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - U(\phi) - (16\pi G)^{-1}R]\,, \tag{1}$$

where R is the curvature scalar. In this theory, a cosmological term need not be included explicitly as adding a constant to $U(\phi)$ is equivalent to adding such a term. The potential $U(\phi)$ is chosen so as to have both a false vacuum at ϕ_f, $U_f \equiv U(\phi_f)$ and a true vacuum at ϕ_t, $U_t \equiv U(\phi_t)$. The barrier between the two minima is assumed to be large, so that the thin-wall approximation is applicable, at least in a flat space–time.

The decay of the false vacuum proceeds by the nucleation of bubbles of true vacuum in the false vacuum. The bubble nucleation rate per unit volume equals $A \exp[-B/\hbar](1 + O(\hbar))$, where B is the action for the bounce. The bounce, ϕ_b, is the solution of the euclidean equations of motion with minimum action, which traverses the barrier which separates the true and the false vacua.

☆ Work supported by the Department of Energy under contract DE-AC03-76SF00515.

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I will assume like Coleman and DeLuccia that the solution of minimum action is invariant under four-dimensional rotations. If this is false, then this calculation puts only a lower bound on the bubble nucleation rate. The most general rotationally invariant euclidean metric is

$$ds^2 = d\xi^2 + \rho^2(\xi)\,d\Omega^2\ , \tag{2}$$

where ξ is the radial coordinate and ρ is the radius of curvature. Also the ϕ field is now only a function of ξ. With this symmetry the euclidean equations of motion are just the Coleman–DeLuccia equations

$$\phi'' + (3\rho'/\rho)\phi' = dU/d\phi\ , \tag{3}$$

$$\rho'^2 = 1 + \tfrac{1}{3}\kappa\rho^2(\tfrac{1}{2}\phi'^2 - U)\ , \tag{4}$$

where $\kappa = 8\pi G$ and prime denotes $d/d\xi$.

The euclidean action for a solution of these equations is

$$S_e = 4\pi^2 \int d\xi[\rho^3 U - 3\rho/\kappa] + \text{surface terms}\ . \tag{5}$$

The bounce action,

$$B \equiv S_e(\phi_b) - S_e(\phi_f)\ , \tag{6}$$

can be divided into three regions in the thin-wall approximation. Outside the wall, $B_{\text{outside}} = 0$. The contribution from the wall is given by

$$B_{\text{wall}} = 2\pi^2\bar{\rho}^3 S_1\ , \tag{7}$$

where $\bar{\rho}$ is the radius of curvature of the bubble wall and

$$S_1 = 2\int d\xi[U_0(\phi_b) - U_0(\phi_f)]\ . \tag{8}$$

The potential $U_0(\phi) = U(\phi) + O(U_f - U_t)$ and is chosen such that $U_0(\phi_f) = U_0(\phi_t)$ and $dU_0/d\phi$ vanishes at both ϕ_f and ϕ_t. The contribution from inside the bubble is

$$B_{\text{inside}} = (12\pi^2/\kappa) \times \{U_t^{-1}[(1 - \tfrac{1}{3}\kappa U_t\bar{\rho}^2)^{3/2} - 1] - (t \to f)\}\ . \tag{9}$$

To find the critical bubble size, B has to be extremized with respect to $\bar{\rho}$. At the extremum

$$\bar{\rho}^2 = \bar{\rho}_0^2/[1 + 2(\bar{\rho}_0/2\lambda)^2 + (\bar{\rho}_0/2\Lambda)^4]\ , \tag{10}$$

where $\bar{\rho}_0 = 3S_1/(U_f - U_t)$ is the critical bubble size without gravity,

$$\lambda^2 = [\kappa(U_f + U_t)/3]^{-1}\ , \tag{11}$$

and

$$\Lambda^2 = [\kappa(U_f - U_t)/3]^{-1}\ . \tag{12}$$

The bounce action for the critical size bubble is

$$B = B_0 r[(\bar{\rho}_0/2\Lambda)^2, \Lambda^2/\lambda^2]\ , \tag{13}$$

where

$$B_0 = 27\pi^2 S_1^4/2(U_f - U_t)^3 \tag{14}$$

is the critical action without gravity and the function r is given by

$$r(x,y) = \frac{2[(1+xy) - (1+2xy+x^2)^{1/2}]}{x^2(y^2-1)(1+2xy+x^2)^{1/2}}\ . \tag{15}$$

The Coleman–DeLuccia calculations are the limits Λ^2/λ^2 go to ± 1. For these limits eq. (10) and (15) reduce to

$$\bar{\rho} = \bar{\rho}_0/[1 \pm (\bar{\rho}_0/2\Lambda)^2]\ , \tag{16}$$

and

$$r[(\bar{\rho}_0/2\Lambda)^2, \pm 1] = 1/[1 \pm (\bar{\rho}_0/2\Lambda)^2]^2\ , \tag{17}$$

which are the results they obtained.

Fig. 1 is a plot of B/B_0 for various values of $\bar{\rho}_0/2\Lambda$ and Λ^2/λ^2. For $\lambda^2 > 0$, the following limits are worth noting:

(i) For $\lambda^2/\Lambda^2 \ll (\bar{\rho}_0/2\Lambda)^2 \ll \Lambda^2/\lambda^2$, then

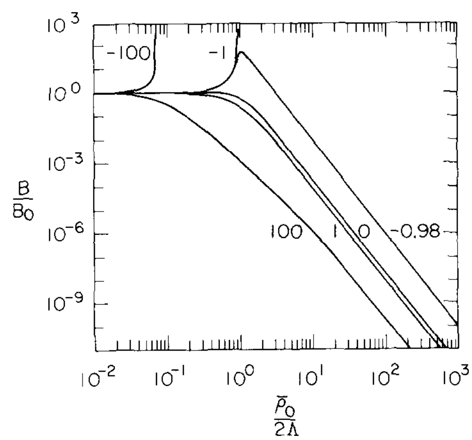

Fig. 1. The ratio B/B_0 ($= r$) as a function of $(\bar{\rho}_0/2\Lambda)$. The numbers next to each line refer to the value of Λ^2/λ^2 for that line.

$$r[(\bar{\rho}_0/2\Lambda)^2, \Lambda^2/\lambda^2] = (2^{1/2}\lambda^3/\Lambda^3)(\bar{\rho}_0/2\Lambda)^{-3} . \quad (18)$$

(ii) For $(\bar{\rho}_0/2\Lambda)^2 \gg 1$ and $(\bar{\rho}_0/2\Lambda)^2 \gg \Lambda^2/\lambda^2$, then

$$r[(\bar{\rho}_0/2\Lambda)^2, \Lambda^2/\lambda^2] = [2\lambda^2/(\lambda^2+\Lambda^2)](\bar{\rho}_0/2\Lambda)^{-4} . \quad (19)$$

Therefore, for $\Lambda^2/\lambda^2 > 1$, the falloff crosses over from a cubic to a quartic power around

$$(\bar{\rho}_0/2\Lambda)\cdot\lambda/\Lambda = 1 . \quad (20)$$

Also, for positive λ^2, $B \leqslant B_0$.

For $\lambda^2 < 0$, the following two regions are worth considering separately. First, if $\Lambda^2/\lambda^2 \leqslant -1$, that is $U_f \leqslant 0$, then the effect of gravity is to lower the bubble nucleation rate. This stabilizing influence on the false vacuum becomes complete when

$$(\bar{\rho}_0/2\Lambda)^2 \geqslant -\Lambda^2/\lambda^2 - (\Lambda^4/\lambda^4 - 1)^{1/2} , \quad (21)$$

that is, when

$$S_1\sqrt{6\pi G} \leqslant \sqrt{-U_t} - \sqrt{-U_f} , \quad (22)$$

and the false vacuum becomes a stable vacuum. This phenomenon was observed by Coleman and DeLuccia and the analysis here shows it continues for $\Lambda^2/\lambda^2 \leqslant -1$. Second, if $-1 < \Lambda^2/\lambda^2 < 0$, that is, $0 < U_f < |U_t|$, this is the transition region between $\lambda^2 > 0$ for which gravity stimulates the decay of the false vacuum and $\lambda^2 \leqslant -\Lambda^2$, for which gravity stabilizes the false vacuum. In this transition region as gravity becomes important, the effect is first to make the false vacuum more stable but as gravity becomes strong gravity stimulates the decay of the false vacuum. This can be seen in fig. 1 for the case $\Lambda^2/\lambda^2 = -0.98$.

For $U_f > 0$,

$$\bar{\rho}^2 \leqslant 2\lambda^2\Lambda^2/(\lambda^2 + \Lambda^2) = [\kappa U_f/3]^{-1} . \quad (23)$$

That is, the critical bubble size in the presence of gravity is always smaller or equal to the scale factor for the false vacuum de Sitter space universe regardless of the size of the critical bubble in flat space–time. This allows the bounce solution to fit into the euclidean de Sitter space false vacuum.

The range of validity of these results is that, first, for the semi-classical approach to be reliable $B \gg 1$. Second, the thin-wall approximation is only a good approximation when the thickness of the wall is small compared to $\bar{\rho}_0$, $|\lambda|$ and Λ.

I would like to thank the members of the SLAC theory group and Sidney Coleman for stimulating discussions.

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