Rod Hitting Ball Lab

Jason Guo

December 3, 2017

1 Data

Quantity	Value	Unit
L (rod length)	0·381	m
l_{cm} (distance from axis to rod/mass COM)	0·299	m
l_{top} (distance from axis to top of rod)	0·025	m
l_{weight} (distance from bottom of rod to top of weight)	0·02	m
H (final track height above ground)	0·98	m
h_i (initial track height)	0·09	m
h_f (final track height)	0·124	m
R (ball radius)	0·0285	m
M (ball mass)	0·08155	kg
m (total rod & weight mass)	0·1017	kg
m_{rod} (rod mass)	0·02654	kg
m_{weight} (weight mass)	0·07516	kg
θ_i (initial rod angle)	60	$^\circ$

2 Calculations

2.1 MOI of rod and mass

Let the MOI of the rod and mass $:= I_{rm}$. Let the distance from the axis to the weight's COM $:= d = L - l_{top} - \frac{l_{weight}}{2}$.

$$I_{rm} = \frac{1}{12} m_{rod} L^2 + m_{rod}(\frac{L}{2} - l_{top})^2 + m_{weight} d^2.$$

Thus, $I_{rm} \approx 0.01\, kgm^2$.

2.2 Angular speed of rod on collision

From energy conservation, we have

$$mgl_{cm}(1 - cos\,\theta_i) = \frac{1}{2}I_{rm}\omega_i^2.$$

Thus $\omega_i \approx 5.46\,rad/s$.

2.3 Angular momentum

Taking counterclockwise to be positive, we have

$$L = I_{rm}\omega_i = I_{rm}\omega_f + Mvd - I_b\frac{v}{R}$$

where $I_b = \frac{2}{5}MR^2$ is the MOI of the ball about its COM, v is the linear velocity of the ball's COM, and d is the same as above.

We must make an assumption about the nature of the collision; namely, that it is perfectly inelastic. Then, we have $v_{contact} = \omega_f d = v$, where v_{tip} is the linear velocity of the rod's contact point with the ball. This contact point is a distance d from the axis.

Substituting into the angular momentum expression, we get $v \approx 0.97\,m/s$ and $\omega_f \approx 2.8\,rad/s$.

3 Results

3.1 Final rod angle

We reuse the energy conservation expression and find that $\omega_f \approx 2.8\,rad/s$ corresponds to $\theta_f \approx 30°$.

3.2 Ball Travel Distance

To simplify calculations, the total kinetic energy K of the ball is $K = K_{linear} + K_{rotation} = \frac{1}{2}Mv^2 + \frac{1}{2}I_b(\frac{v}{R})^2 = \frac{7}{10}Mv^2$ for an arbitrary ball COM linear velocity v. By energy conservation,

$$K_i = K_f + Mg(h_f - h_i)$$

since the ball goes up a height of $h_f - h_i$ before leaving the track. Let u denote the ball COM's linear velocity right before leaving the track. Then, $u \approx 0.68\,m/s$. The travel time of the ball t is given by

$$-H = -\frac{1}{2}gt^2 \implies t \approx 0.45\,s.$$

Thus, the ball will travel a distance $ut \approx 0.306\,m \approx 31\,cm$ off the table.