Rod Hitting Ball Lab

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1 Objective

To determine the distance the racquetball will land from the point of launch as well as the angle the launcher will rise.

2 Data

Quantity	Value	Unit
L (rod length)	38·1	cm
l_{cm} (distance from axis to rod-mass COM)	29·9	cm
l_{top} (distance from axis to top of rod)	2·5	cm
l_{weight} (distance from bottom of rod to top of weight)	2	cm
H (final track height above ground)	98	cm
h_i (initial track height)	9	cm
h_f (final track height)	12·4	cm
R (ball radius)	2·85	cm
M (ball mass)	81·55	g
m (total rod & weight mass)	101·7	g
m_{rod} (rod mass)	26·54	g
m_{weight} (weight mass)	75·16	g
θ_i (initial rod angle)	60	°
ϕ (track angle wrt horizontal)	1	°

3 Calculations

3.1 MOI of rod-mass

Let I_{rm} be the MOI of the rod-mass about its hinge. The distance d from the axis to the weight's COM $= L - l_{top} - \frac{l_{weight}}{2}$.

$$I_{rm} = \frac{1}{12}m_{rod}L^2 + m_{rod}(\frac{L}{2} - l_{top})^2 + m_{weight}d^2.$$

Thus, $I_{rm} \approx 0.01\, kgm^2$.

3.2 Angular speed of rod on collision

From energy conservation:

$$mgl_{cm}(1 - cos\,\theta_i) = \frac{1}{2}I_{rm}\omega_i^2.$$

Thus, $\omega_i \approx 5.46\, rad/s$.

3.3 Angular momentum

Let counterclockwise be positive by convention. We have:

$$L_{rm} = I_{rm}\omega_i - Ndt = I_{rm}\omega_f,$$

$$L_b = -NRt = -MRv - I_b v/R,$$

where N is the normal force between the ball and rod-mass, t is the collision time, $I_b = \frac{2}{5}MR^2$ is the MOI of the ball about its COM, v is the linear velocity of the ball's COM, and d is the same as above.

Here, the angular momentum of the rod-mass L_{rm} is taken wrt the hinge; the angular momentum of the ball L_b is taken wrt its contact point on the track. This eliminates any friction terms that would otherwise be present in our expressions.

Combining these equations, it follows that

$$I_{rm}\omega_i = I_{rm}\omega_f + Mvd + I_b vd/R^2.$$

3.4 Coefficient of Restitution

We require another equation to actually solve for ω_f. From a table of CORs, we will estimate that the collision has a COR of $e = 0.7$. This means that:

$$e = \frac{v_{tf} - v}{0 - v_{ti}} = 0.7,$$

where v_{ti} and v_{tf} are the initial and final linear velocities of the rod-mass contact point with the ball, respectively, and v is the ball's translational velocity.

Since the rod-mass contact point is a distance d from the hinge, $v_{ti} = \omega_i d$ and $v_{tf} = \omega_f d$. Combining, we get that

$$v = d(0.7\omega_i + \omega_f).$$

3.5 Ball velocity (right after collision)

Substituting the COR expression into angular momentum conservation, we get that $w_f \approx 0.1\, rad/s$ and $v \approx 1.36\, m/s$.

3.6 Final rod angle

We reuse the energy conservation expression and find that $\omega_f \approx 0.1\, rad/s$ corresponds to $\theta_f \approx 1°$.

3.7 Ball velocity (before bump)

By Newton's Second Law:

$$M\frac{dv}{dt} = -F_D - Mgsin\,\phi,$$

where F_D is the drag force and $Mgsin\,\phi$ is component of gravity parallel to the track.

The drag force $F_D = \frac{1}{2}\rho v^2 C_D A$. The density of air at room temperature ρ is $\approx 1.2\, kg/m^3$, the drag coefficient C_D of the ball is ≈ 0.5, and the cross-sectional area A of the ball is πR^2. Thus, $F_D \approx 0.0008v^2$.

Solving the differential equation with the condition $v(0) = 1.36$ and $x(0) = 0$:

$$v(t) = 4.18tan(.315 - .04t),$$

$$x(t) = 105ln(cos(.315 - .04t)) + 5.3.$$

The distance from the collision point to the point is $\approx 1\, m$; setting $x(t) = 1$ indicates that it takes $\approx 0.77\, s$ for the ball to travel this distance. Therefore, its translational velocity right before the bump $v_{bi} \approx 1.22\, m/s$.

3.8 Ball velocity (after bump)

We will neglect drag here because the bump was relatively short. However, a significant fraction of kinetic energy is transformed into gravitational potential energy. To simplify calculations, the total kinetic energy K of the ball is $K = K_{linear} + K_{rotation} = \frac{1}{2}Mv^2 + \frac{1}{2}I_b(\frac{v}{R})^2 = \frac{7}{10}Mv^2$ for an arbitrary translational velocity v.

By energy conservation,

$$K_i = K_f + Mg(h_f - h_i)$$

since the ball goes up a height of $h_f - h_i$ during the bump. Solving for the translational velocity right after the bump, we get that $v_{bf} \approx 1.0\,m/s$.

3.9 Ball velocity (right before projectile motion)

Now, we will account for drag and the incline once again. Solving the differential equation again with the new conditions $v(0) = 1.0$ and $x(0) = 0$:

$$v(t) = 4.18tan(.235 - .04t),$$

$$x(t) = 105ln(cos(.235 - .04t)) + 2.9.$$

We will estimate that the second portion of the track is also $\approx 1\,m$. Finally, we get that the translational velocity right before projectile motion $u \approx 0.8\,m/s$.

3.10 Ball Travel Distance

The travel time of the ball t is given by

$$-H = -\frac{1}{2}gt^2 \implies t \approx 0.45\,s.$$

Thus, the ball will travel a distance $ut \approx 36\,cm$ off the table.