Local Regression

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Zasada podobna do KNN. Wybieramy określoną ilość najbliższych punktów i na ich podstawie wyliczamy ważoną regresję najmniejszych kwadratów dla punktu x_0

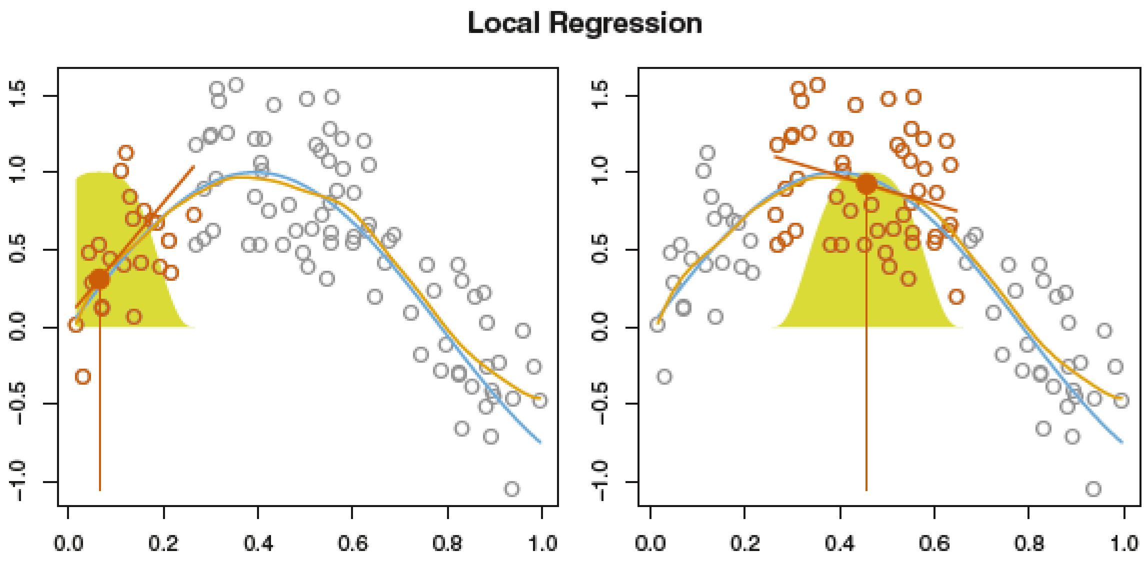


FIGURE 7.9. *Local regression illustrated on some simulated data, where the blue curve represents $f(x)$ from which the data were generated, and the light orange curve corresponds to the local regression estimate $\hat{f}(x)$. The orange colored points are local to the target point x_0, represented by the orange vertical line. The yellow bell-shape superimposed on the plot indicates weights assigned to each point, decreasing to zero with distance from the target point. The fit $\hat{f}(x_0)$ at x_0 is obtained by fitting a weighted linear regression (orange line segment), and using the fitted value at x_0 (orange solid dot) as the estimate $\hat{f}(x_0)$.*

Im mniejsza liczba s , tym regresja jest bardziej lokalna – nasze dopasowanie będzie bardziej dopasowane,
Duża liczba s będzie skutkowała dopasowaniem regresji lokalnej do całego zbioru danych.

Wybrać najlepszą s możemy przy pomocy cross-validation.

Gdy zbiór danych posiada wiele zmiennych, można zastosować takie uogólnienie, gdzie regresja jest globalna dla niektórych zmiennych, a lokalna dla innych

Algorithm 7.1 *Local Regression At* $X = x_0$

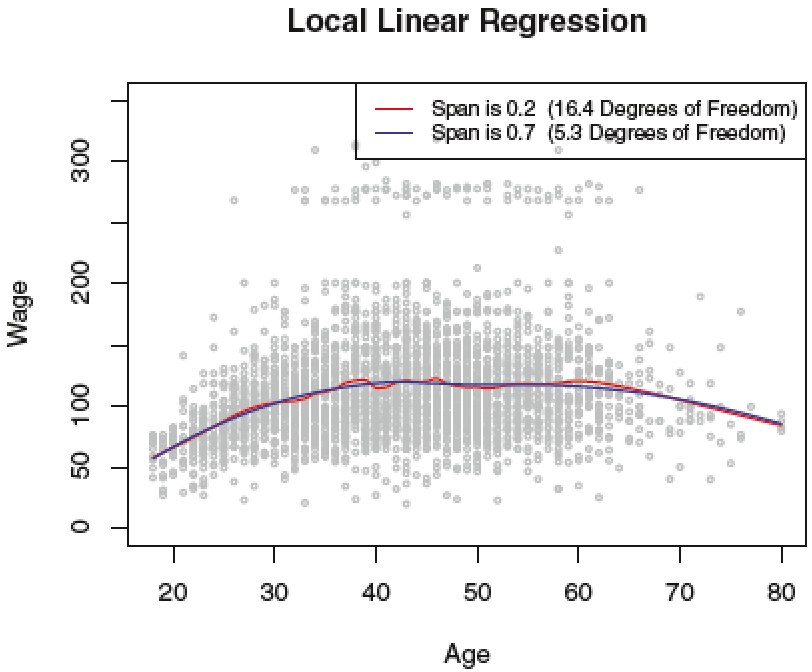
1. Gather the fraction $s = k/n$ of training points whose x_i are closest to x_0.

2. Assign a weight $K_{i0} = K(x_i, x_0)$ to each point in this neighborhood, so that the point furthest from x_0 has weight zero, and the closest has the highest weight. All but these k nearest neighbors get weight zero.

3. Fit a *weighted least squares regression* of the y_i on the x_i using the aforementioned weights, by finding $\hat{\beta}_0$ and $\hat{\beta}_1$ that minimize

$$\sum_{i=1}^{n} K_{i0}(y_i - \beta_0 - \beta_1 x_i)^2. \tag{7.14}$$

4. The fitted value at x_0 is given by $\hat{f}(x_0) = \hat{\beta}_0 + \hat{\beta}_1 x_0$.



Generalized Additive Models

- Metody wymienione w tym rozdziale były tylko dla regresji z jednym predyktorem,
- Przy wielu zmiennych tworzy się GAMy,
- Są one rozszerzeniem wszystkich opisanych wcześniej metod.

GAMy dla regresji

$$y_i = \beta_0 + \beta_1 x_{i1} + \beta_2 x_{i2} + \cdots + \beta_p x_{ip} + \epsilon_i$$

$$\begin{aligned} y_i &= \beta_0 + \sum_{j=1}^{p} f_j(x_{ij}) + \epsilon_i \\ &= \beta_0 + f_1(x_{i1}) + f_2(x_{i2}) + \cdots + f_p(x_{ip}) + \epsilon_i. \end{aligned}$$

$$\texttt{wage} = \beta_0 + f_1(\texttt{year}) + f_2(\texttt{age}) + f_3(\texttt{education}) + \epsilon$$

Additive jest w nazwie dlatego, że obliczamy osobne wartości dla każdej zmiennej i na końcu je dodajemy.

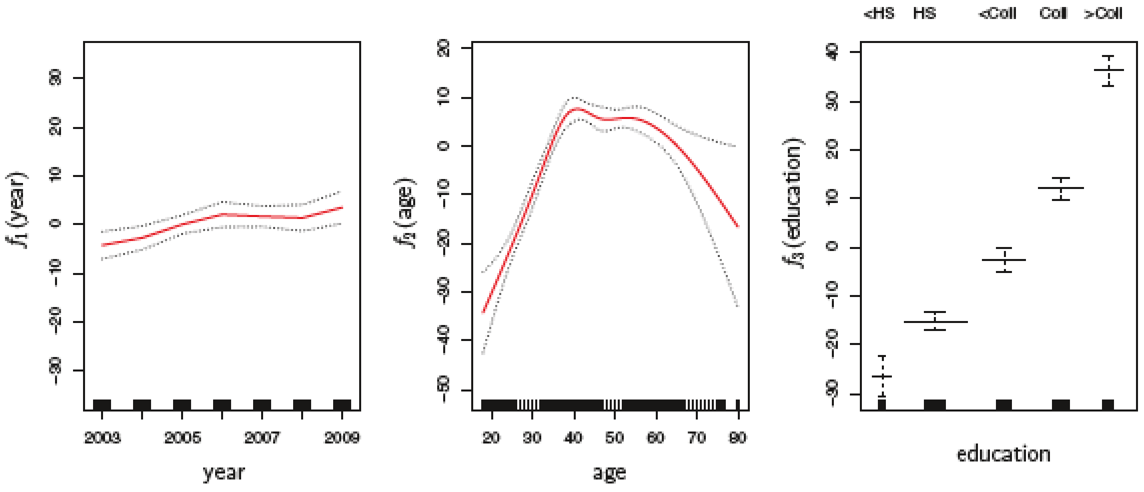


FIGURE 7.11. *For the* `Wage` *data, plots of the relationship between each feature and the response,* `wage`*, in the fitted model (7.16). Each plot displays the fitted function and pointwise standard errors. The first two functions are natural splines in* `year` *and* `age`*, with four and five degrees of freedom, respectively. The third function is a step function, fit to the qualitative variable* `education`.

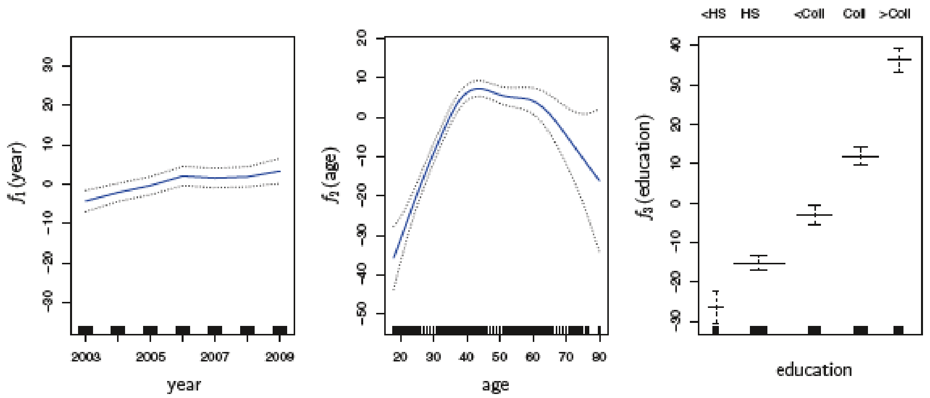


FIGURE 7.12. *Details are as in Figure 7.11, but now f_1 and f_2 are smoothing splines with four and five degrees of freedom, respectively.*

W GAMach możemy używać wszystkich poznanych wcześniej metod: regresji wielomianowej, lokalnej itd.

GAMy dla klasyfikacji

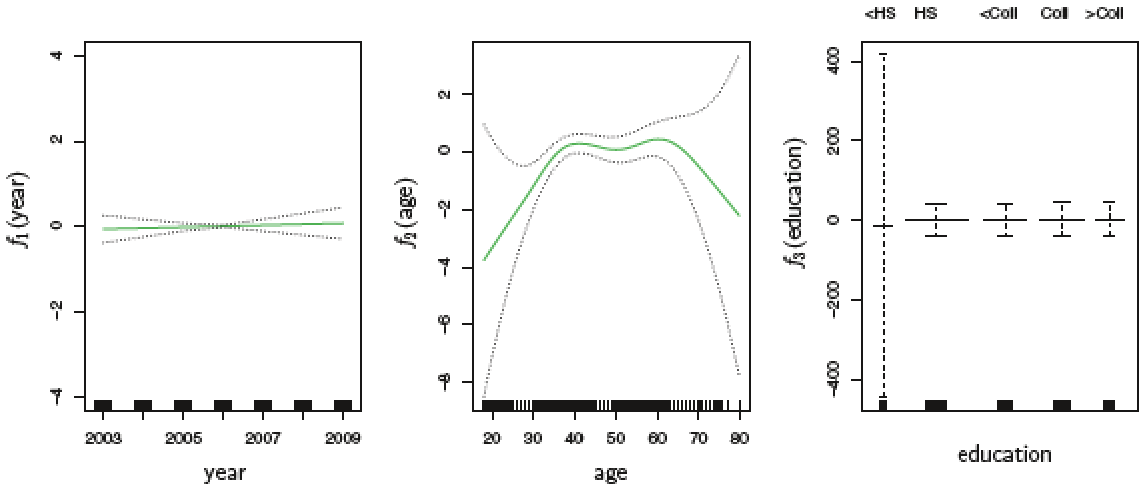


FIGURE 7.13. *For the* `Wage` *data, the logistic regression GAM given in (7.19) is fit to the binary response* `I(wage>250)`. *Each plot displays the fitted function and pointwise standard errors. The first function is linear in* `year`, *the second function a smoothing spline with five degrees of freedom in* `age`, *and the third a step function for* `education`. *There are very wide standard errors for the first level* `<HS` *of* `education`.

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_p X_p.$$

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + f_1(X_1) + f_2(X_2) + \cdots + f_p(X_p).$$

$$\log\left(\frac{p(X)}{1-p(X)}\right) = \beta_0 + \beta_1 \times \texttt{year} + f_2(\texttt{age}) + f_3(\texttt{education}),$$

$$p(X) = \Pr(\texttt{wage} > 250 | \texttt{year}, \texttt{age}, \texttt{education}).$$

Zalety i wady GAMów

- GAMs allow us to fit a non-linear f_j to each X_j, so that we can automatically model non-linear relationships that standard linear regression will miss. This means that we do not need to manually try out many different transformations on each variable individually.
- The non-linear fits can potentially make more accurate predictions for the response Y.
- Because the model is additive, we can still examine the effect of each X_j on Y individually while holding all of the other variables fixed. Hence if we are interested in inference, GAMs provide a useful representation.
- The smoothness of the function f_j for the variable X_j can be summarized via degrees of freedom.
- The main limitation of GAMs is that the model is restricted to be additive. With many variables, important interactions can be missed. However, as with linear regression, we can manually add interaction terms to the GAM model by including additional predictors of the form $X_j \times X_k$. In addition we can add low-dimensional interaction functions of the form $f_{jk}(X_j, X_k)$ into the model; such terms can be fit using two-dimensional smoothers such as local regression, or two-dimensional splines (not covered here).