

Towards improved sample complexity using a quantum model based SAC

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Reminder: Utility of quantum control

- Control: System = Controllable part + Not controllable part

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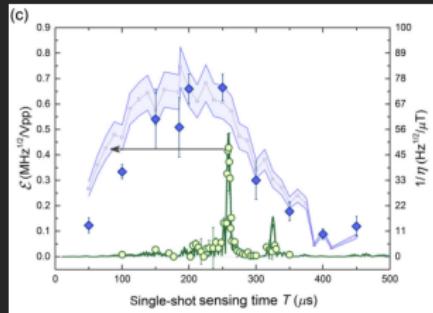
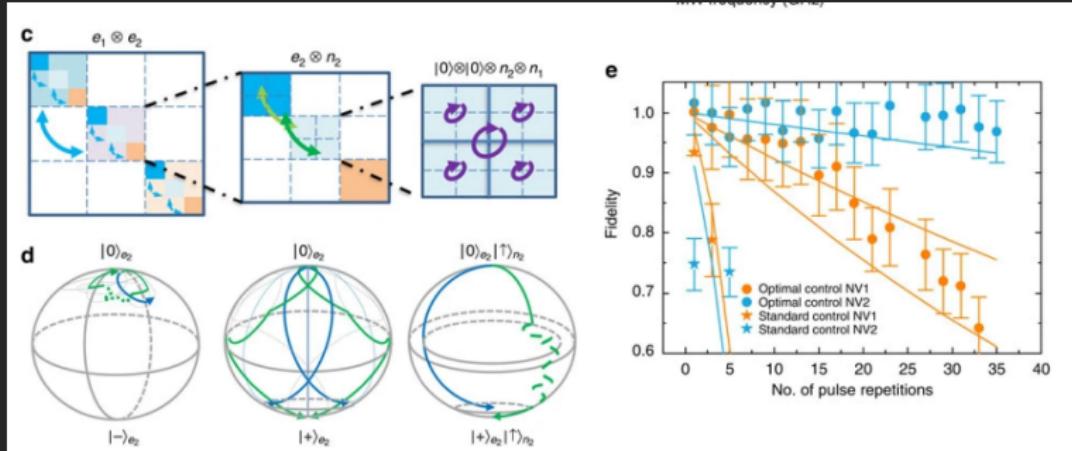
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 - metrology applications

Diamond spins $\uparrow\uparrow\downarrow$



1

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¹F. Dolde et. al. (Nature 2014)

²F. Poggiali et. al. (PRX 2018)

Definitions

- Constraint 1: Transmon H (control structure H_u fixed)
(Magesan et. al. 2020)

$$H(t)/\hbar = H_0 + \sum_{u \in \mathcal{C}} u(t) H_u \quad (1)$$

$$= \sum_{j=1}^2 \omega_j \hat{b}_j^\dagger \hat{b}_j + \frac{\delta_j}{2} \hat{b}_j^\dagger \hat{b}_j (\hat{b}_j^\dagger \hat{b}_j - 1) \quad (2)$$

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- Formal Goal:

$$\Delta^* = \arg \max_{\Delta=[\Delta_1, \dots, \Delta_m], m \leq N} \mathcal{F}(E_\Delta, V) \quad (3)$$

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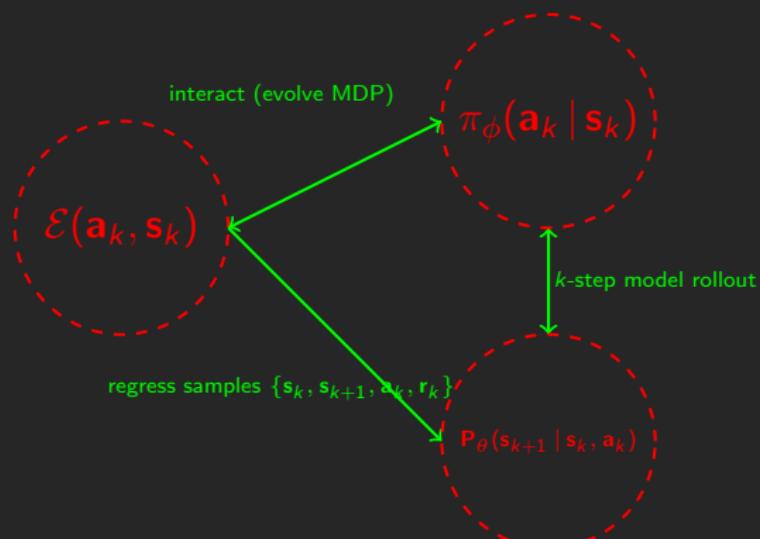
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- So, far only two things interacting: policy $\pi(\mathbf{a}_k | \mathbf{s}_k)$ and $\mathcal{E}(\mathbf{a}_k, \mathbf{s}_k)$

Model addition

- Suppose the agent is now able to store and interact with an internal representation of \mathcal{E} . Let's denote that with
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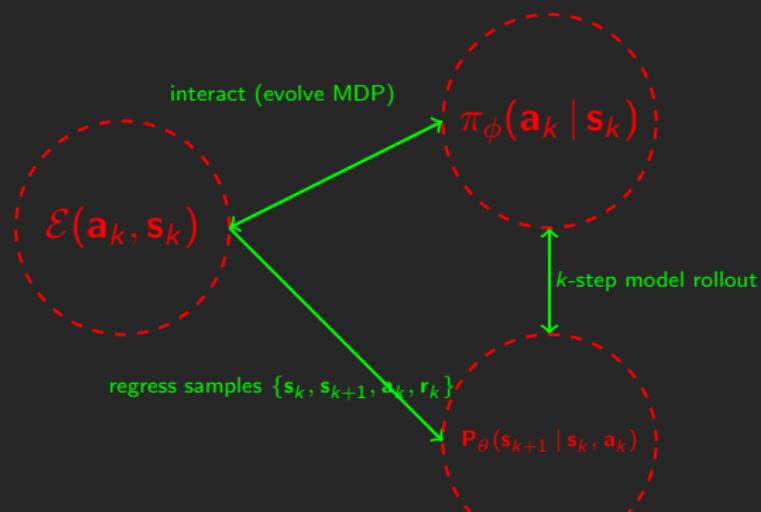
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- A simple picture
- Have a choice to go function approximation route for model or ODE solver

Why RL?

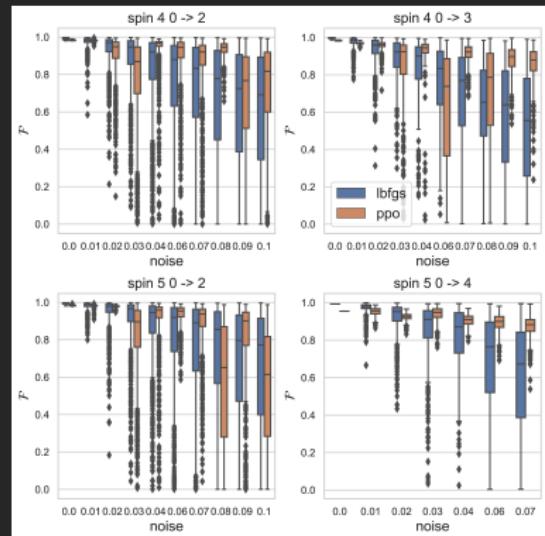
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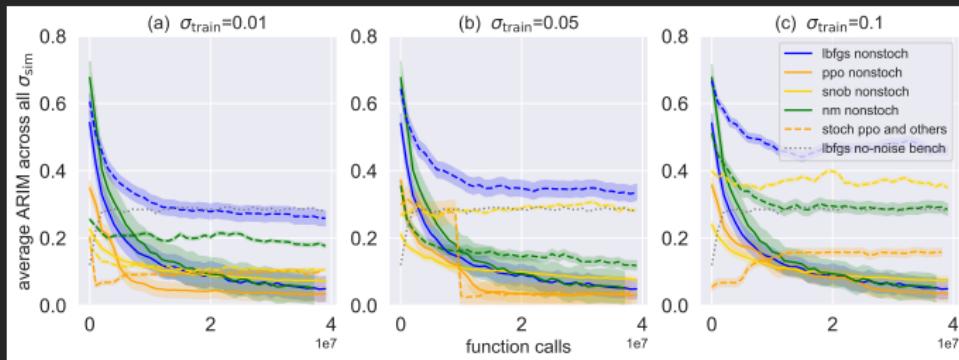
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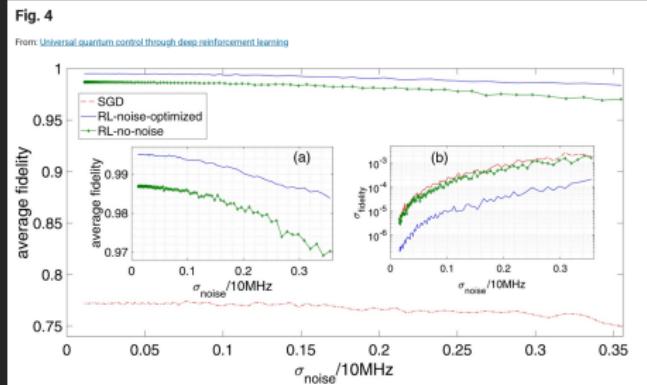
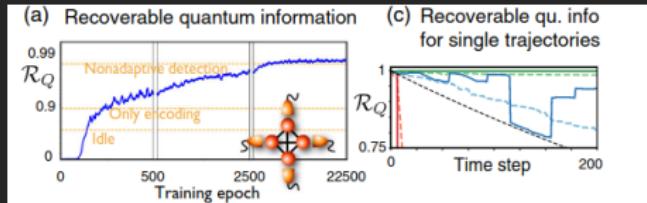
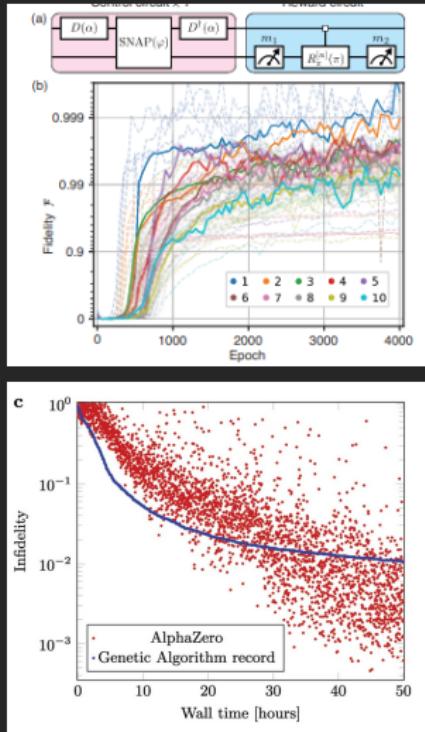
Why RL?

- Black box optimization (we don't need a model)
 - Note not adaptive!
 - Controller solution is a point in the solution space and not a function.
 - It does not *react* to perturbations

Why RL?

- Several other results in the past 3-4 years

Examples



cite³

³ TL: Sivak... (PRX 2022) TR: Fösel... (PRX 2018) BL: Dalgaard... (Nature 2020) BR: Niu... (Nature 2018)

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- at the moment constraint is a δ -perfect Hamiltonian where δ has to be small

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- ➍ optimal convergence to π^* after repeated Bellman iterations for the tabular case ensures asymptotic performance reaches that of other Model-free methods like DDPG, PPO

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- Generalization bound on the model $\mathbf{P}_\theta(\mathbf{s}_{k+1} | \mathbf{s}_k, \mathbf{a}_k)$ error ϵ_m yields the lower bound on k -branched rollouts (Thm 4.2.)

$$J(\pi) \geq J(\pi)_{\text{branch}} - 2r_{\max} \left[\frac{\gamma^{k+1}\epsilon_\pi}{(1-\gamma^2) + \frac{\gamma^k+2}{(1-\gamma)}}\epsilon_\pi + \frac{k}{1-\gamma}(\epsilon_m + 2\epsilon_\pi) \right] \quad (7)$$

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$$\mathbf{s}_{t:t+T} = \arg \max_{\mathbf{a}_{t:t+T}} \mathbb{E}_{f_\theta} \left[\sum_{i=t}^{t+T} \mathbf{r}(\mathbf{s}_i, \mathbf{a}_i) \right] \quad (8)$$

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- CEM is an importance sampling approximator. In it the cross entropy is maximized by the MLE for most natural exponential families which \mathcal{N} and a randomization over various guesses converges to the optimizer of (8)

Alternative model has a baked in ansatz

- We guess a set of Hamiltonians $\{H\}_B$ close to the true Hamiltonian H^* generating the unitary dynamics and use an ODE solver (standard or neural). This becomes our model $P_\theta(s_{k+1} | s_k, a_k)$ where $\theta = \{H\}_B$.

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- We use the $SU(N)$ parameterization:
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- We found that state regression has a better transmon-dependent loss landscape than predicted fidelity regression

Results: Step 1: With $\mathbf{P}_\theta(\mathbf{s}_{k+1} | \mathbf{s}_k, \mathbf{a}_k)$ being perfect, how many real samples does it take?

Target gate is the CNOT

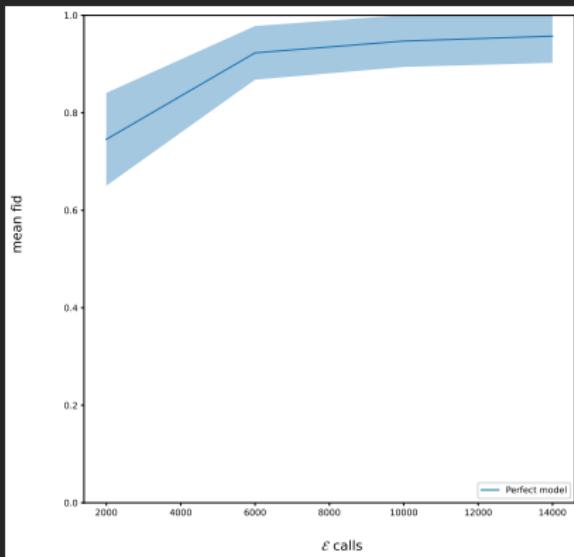


Figure: Sanity test and optimal lower bound (baseline) for the MBSAC

Understanding the exploitation-exploration tradeoff for the quantum control setting in the perfect model setting

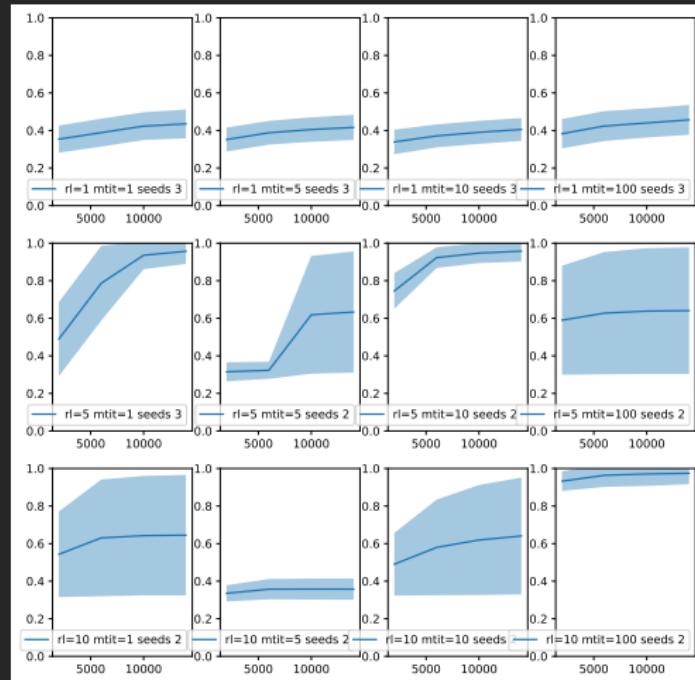


Figure: branch rollout length (rl) k (model explore) tradeoffs w.r.t. training per k model train iterations (mtit)

Step 2: How good is the δ -perfect model? What is the effect of δ on sample complexity?

- Maybe not the smartest way to pick δ but we choose it to be partially adversarial

$$\Lambda = \|F(H^*(\Delta_i)) - F(H_2(\Delta_i))\| \approx \delta \quad \forall i \quad (9)$$

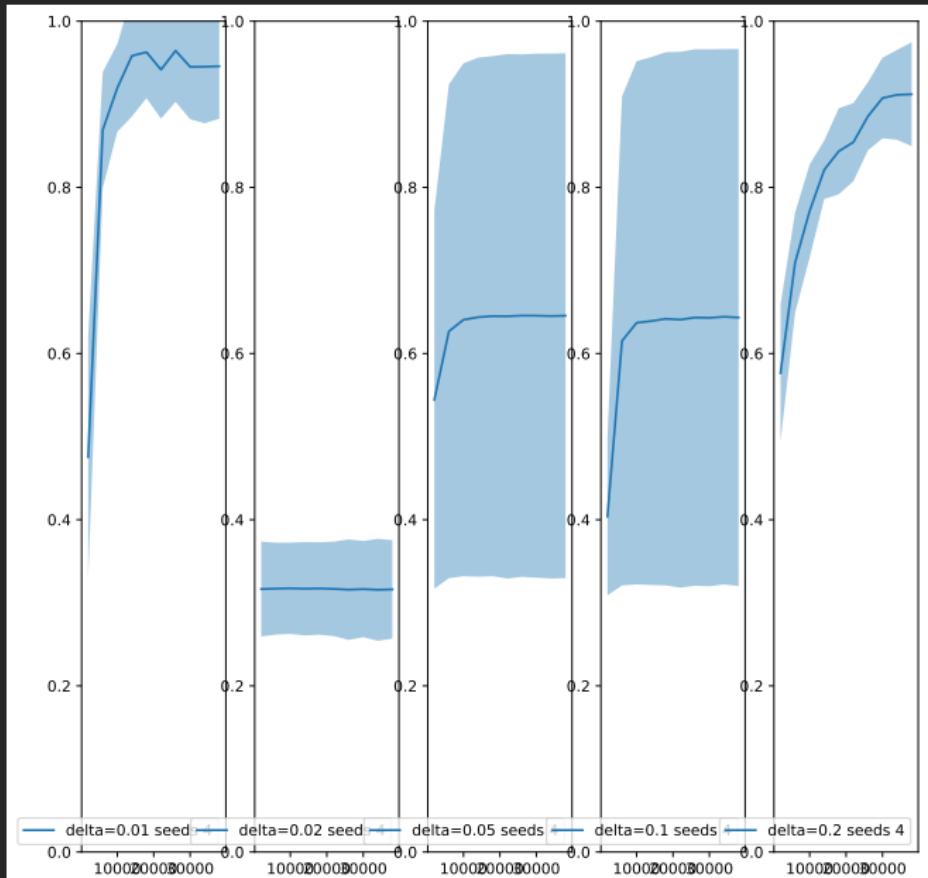
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- We minimize Λ w.r.t. H_2 .

...



Step 3: Now train and contrast

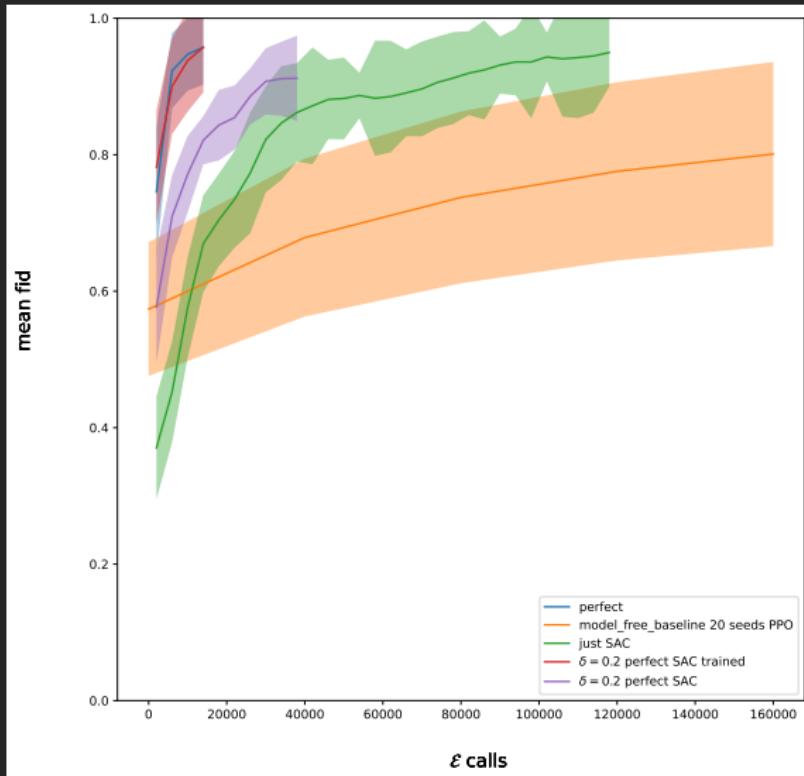


Figure: Various

Improving the definition of δ -perfection

Use Burgath et. al. (2022) upper bound

$$\|U^* - U_2\| \leq \left\| \int_0^T ds H^*(s) - H_2(s) \right\| \left(1 + \left\| \int_0^T ds H^*(s) \right\| + \left\| \int_0^T ds H_2(s) \right\| \right) \quad (10)$$

set $\|U^* - U_2\|$ as fidelity difference δ ? and solve the above relaxed optimization problem?

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- Some of above with Noisy Lindblad dynamics and shot coarse-graining