1. $EX = \sum_{x=1}^{\infty} x\theta(1-\theta)^{x-1} = \theta\sum_{x=1}^{\infty} x(1-\theta)^{x-1} = \frac{1}{\theta}$ —— 均值 +1

$EX^2 = \sum_{x=1}^{\infty} x^2\theta(1-\theta)^{x-1} = \theta(1-\theta)\sum_{x=2}^{+\infty} x(x-1)(1-\theta)^{x-2} + \theta\sum_{x=1}^{\infty} x(1-\theta)^{x-1}$

$= \frac{2-\theta}{\theta^2}$.

$\Rightarrow Var(X) = EX^2-(EX)^2 = \underline{\frac{1-\theta}{\theta^2}} < +\infty$ 方差 +1

$\Rightarrow \int\sum_x \frac{\partial}{\partial\theta} P_\theta(X=x) = E[\frac{\partial}{\partial\theta}\log f(x|\theta)] = 0$

+1

$\underline{\frac{\partial \log f(x|\theta)}{\partial\theta}} = \frac{\partial}{\partial\theta}\log\theta + (x-1)\log(1-\theta) = \frac{1}{1-\theta}(X-EX)$.

$\Rightarrow I(\theta) = \underline{E_\theta[(\frac{\partial}{\partial\theta}\log f(x|\theta))^2]}$ +1 $= \frac{1}{(1-\theta)^2}Var(X) = \underline{\frac{1}{\theta^2(1-\theta)}}$ +1

指明统计量 +1 无偏 +1

令 $\underline{\hat{\tau} = \bar{X}}$. 由 $\underline{E_\theta\hat{\tau} = \frac{1}{\theta}}$ 可知. $\hat{\tau}$ 为 $\tau(\theta)$ 的一个无偏估计.

$C\text{-}R下界 = \frac{|\frac{d}{d\theta}\tau(\theta)|^2}{\underline{nI(\theta)}} = \underline{\frac{1-\theta}{n\theta^2}}$ +1 +1

由 $\underline{Var(\hat{\tau}) = \frac{1}{n}Var(X) = \frac{1-\theta}{n\theta^2}} = C\text{-}R下界$.

可知. $\underline{e_n(\hat{\tau}) = 1}$. 有效估计 —— 或 +1

2. $\Theta_0 = \{(\theta, \mu) \mid \theta = \theta_0, \mu \in \mathbb{R}\}$ $\Theta = \{(\theta, \mu) \mid \theta > 0, \mu \in \mathbb{R}\}$ +1.

$L(\theta, \mu) = f(\vec{x} \mid \theta, \mu) = \frac{1}{\theta^n} \cdot e^{-\frac{1}{\theta}\sum_{i=1}^n x_i} \cdot e^{\frac{n\mu}{\theta}} \cdot I(\mu \le X_{(1)})$ +1

−0.5

on Θ_0, L 随 μ 单增. 故 $\sup_{\Theta_0} L(\theta, \mu) = L(\theta_0, X_{(1)})$

on Θ, $\forall \theta$, $\mu = X_{(1)}$ 时 L 取 max. $\hat{\mu}_{MLE} = X_{(1)}$

$\frac{\partial \log L}{\partial \theta} = -\frac{n}{\theta} + \frac{1}{\theta^2}\left(\sum_{i=1}^n x_i - nX_{(1)}\right) = 0$. 解得 $\hat{\theta}^* = \frac{1}{n}\sum_{i=1}^n X_i - X_{(1)} \ge 0$

$\frac{\partial^2 \log L}{\partial \theta^2} = \frac{n}{\theta^2} - \frac{2}{\theta^3}\left(\sum_{i=1}^n X_i - nX_{(1)}\right) = -\frac{n}{\theta^2} < 0$. 故 $\hat{\theta}_{MLE} = \bar{X} - X_{(1)}$

+1 +1 +2

$\lambda(\vec{x}) = \frac{\sup_{\Theta_0} L(\theta, \mu)}{\sup_{\Theta} L(\theta, \mu)} = \frac{L(\theta_0, X_{(1)})}{L(\bar{X} - X_{(1)}, X_{(1)})} = \left(\frac{\bar{X} - X_{(1)}}{\theta_0}\right)^n \cdot \exp\left\{-\frac{\left(\sum_{i=1}^n X_i - nX_{(1)}\right)}{\theta_0} + n\right\}$

公式 +3 +1

扫描全能王 创建

3. 令 $\theta = -\lambda$. +1 $T = \sum_{i=1}^{n} X_i$ 为 θ 充分统计量. +1

$\dfrac{g(t\mid\theta_2)}{g(t\mid\theta_1)} = \left(\dfrac{\theta_2}{\theta_1}\right)^n \cdot e^{t(\theta_2-\theta_1)}$ +1 非降 MLR +1

$H_0: \theta = -1 \leftrightarrow H_1: \theta < -1$ 的 UMPT 等价于 +1 $H_0: \theta \geq -1 \leftrightarrow H_1: \theta < -1$

由 K-R 定理, UMPT 为 $T < t_0$ 时拒绝 H_0 +1

$\alpha = 0.1 = P_{\theta_0}(T < t_0) = P_{\lambda=1}(T < t_0) = P_{\lambda=1}(2T < 2t_0)$, $2T \sim \chi^2_{2n}$ +2

$\Rightarrow t_0 = \frac{1}{2}\chi^2_{2n}(0.9)$ +1 +1