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Decomposition Rate of Multi-Fungal Species and Its Interfering Factors

Summary

Fungi play a very important role in the cycle of ecosystem as decomposer which can supply raw materials to producers. In this paper, we use logistic curve, Volterra model and chemical kinetics to study the decomposition rate of multi-species community under different climate conditions. By using Matlab, we constructed the Single Species Decomposition Rate Model, the Multi-Species Decomposition Rate Model and the model of total decomposition rate of fungi under comprehensive weather and competition. Take five species of fungi as example, the change of the number of each species caused by competition and the change of the total decomposition rate with temperature under five kinds of climate can be calculated. Finally, some reasonable suggestions are given based on the actual situation.

For Problem 1, we construct the Single Species Decomposition Rate Model by combining logistic curve with the relationship between decomposition rate, growth rate and density, and calculate the curves of decomposition rate of five specific fungi with density without considering the interaction by Matlab.

For Problem 2 and 3, we first establish the autonomous system of ordinary differential equations of different fungi under competition by analogy with Volterra equation, and construct the Competition Model. The logarithm numerical solution of the equation is obtained by Matlab, and the change of the number of each species with time can be obtained, so as to predict the dominant and disadvantaged species of the community in the short and long term.

For Problem 4, we first use the model of enzyme-catalyzed reaction in chemical kinetics to determine the coefficient $I(T, m)$, which is affected by the mass of decomposed products and temperature. Combined with the humidity and the moisture niche width of each species, and considering the two models above, we construct the Complete Multi-Species Decomposition Rate Model in the ecosystem. Finally, we use Matlab to calculate and the decomposition rate of community with temperature under five kinds of climate was plotted. The dominant species were determined by observing the decomposition rate of each species in different climate.

For Problem 5, we combine modeling results above to change the number of fungal species under specific climate, plot the curves of decomposition rate of communities with the numbers of fungal species with temperature through Matlab, and conclude that species diversity is of great significance for the healthy and stable development of ecosystem.

Finally, we analyze the sensitivity of the model, and evaluate its advantages and disadvantages. Meanwhile, we make a statement about the possible development and perfection for future researchers based on our model.

The characteristic of this paper lies in the comprehensive use of mathematics, chemistry and ecology knowledge to construct the actual problems and the results are in good agreement with the actual situation, which is embodied in: Our method integrates the research of some ecological problems and makes reasonable extrapolation; in the idea of model construction, we have widely considered the influence and degree of various parameters on the model and the method we use in sensitivity analysis.

Keywords: decomposition rate of fungi; logistic curve; Volterra model; chemical kinetics; Matlab

Contents

1 Introduction

1.1 Background

Decomposer plays a very important role in the cycle of ecosystem, which can decompose complex organic compounds which contain carbon, nitrogen, phosphorus and so on into simple inorganic substances so that producers can reuse them and release energy. A very important part in this process is the decomposition of plant materials and wood fibers into carbon dioxide, water, and inorganic salts by fungi. [1] The corrosion decomposition of plant materials and material fibers by fungi are affected by numerous factors. Temperature and the amount of decomposed products can affect the progress of the reaction, while humidity can affect the process to different degrees according to different fungi. Meanwhile, the growth rate, the density of different fungi and the interaction relationship between various fungi can also affect the decomposition. Studying the above factors has important implications for the physical cycle of an ecosystem as well as for the healthy development of the system.

1.2 Problem Restatement

In order to describe the decomposition rate of fungi under different conditions and the relationship between them, we need to solve the following problems.

- Problem 1: By introducing relevant variables, establish a model to express the ability of single species of fungi to decompose ground litter and wood fibers.
- Problem 2: Establish a model to represent the competition among various fungi, and link the decomposition ability of various fungi to establish the decomposition rate model of multiple fungi.
- Problem 3: At the same time, the dynamic characteristics of the interaction between fungi can be expressed and predicted in different time periods.
- Problem 4: Establish a model that can express the influence of climate conditions and different environmental conditions on the decomposition rate of fungi. Predict the relative advantages and disadvantages for each species and their relationship under different kinds of climate.
- Problem 5: Point out the influence of fungal community diversity on the overall decomposition efficiency, and predict the significance of biodiversity to the environment when it changes.

Finally, analyze the environmental sensitivity and give an evaluation.

1.3 Our Work

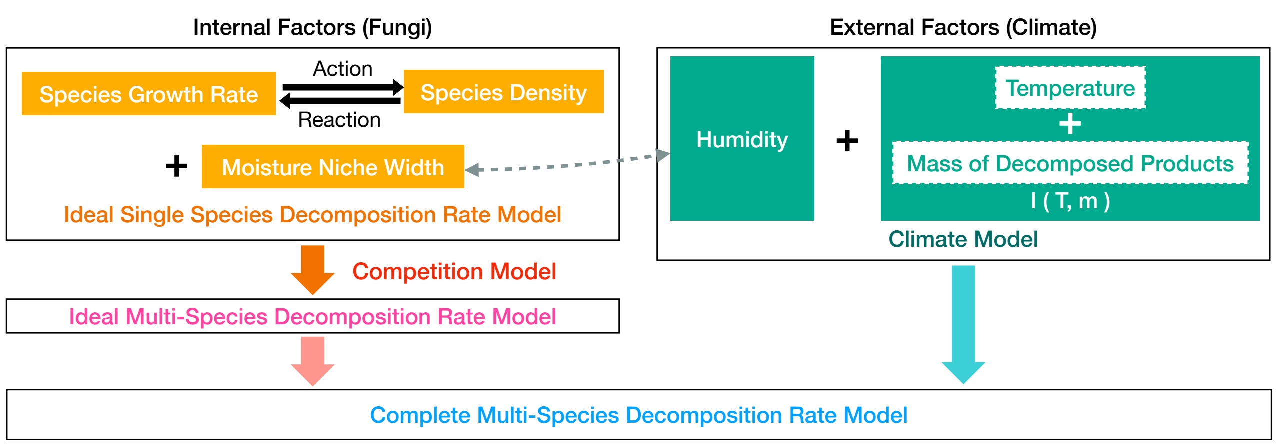


Figure 1 Solving strategies

To solve the problem, here are the steps we follow.

- Under the condition of no climate impact, we only consider the internal factors of a single species, and use relevant ecological and mathematical knowledge to build the model.
- Introduce a competition model to connect different kinds of fungi and build the ideal multi-species model.
- Consider external factors and build a model based on ecology and chemistry. Then put all the models together to obtain the complete model.
- Analysis the model we have constructed through sensitivity, strengths and weaknesses.

2 Assumptions and Notations

2.1 Assumptions

- Assuming that there is no matter other than ground litter and wood fibers in the growing soil and the initial value of decomposed products in each unit volume of soil is uniform.
- Assuming that when a single species of fungi exists in the soil alone and the environmental factors do not change, the mycelial elongation rate of the fungus remains constant.
- Assuming that there is only competition among different kinds of fungi in the soil.
- Assuming that the decrease of the growth rate of various fungi has a linear relationship with the number of competitors of other fungi when there is competition among them.
- Assuming that the soil water content in the model is totally expressed by precipitation.
- Assuming that only temperature, humidity and the mass of decomposed products are considered in the model of environment.

2.2 Notations

Table 1 Notations

Symbol	Description
R	total decomposition rate
R_{fun}	decomposition rate of fungi without considering climate
R_0	the maximum decomposition rate of a single species
REM	the total decomposition rate of 50% fungi which do not reach the fastest decomposition rate
i, j	the kind of fungi
r	growth rate of fungi
ρ	density of fungi
θ	the coefficient of moisture niche width of fungi
x	the number of each species of fungi
ξ	population growth potential index
K	environmental capacity of fungi
S	area of the investigated land
μ	intrinsic reproduction coefficient of fungus
λ_{ij}	the competition factor of fungi i caused by the introduction of fungus $j (j \neq i)$
I	temperature(T) and decomposed product(m) factor
k	reaction rate constant
α	reaction order

(Continued)

Symbol	Description
E_a	reaction activation energy
J	moisture niche width
$\hat{R}$	humidity(l) factor

3 Model 1: Ideal Single Species Decomposition Rate Model

3.1 Model Construction

We used the amount of decomposed products change per unit time to characterize the decomposition rate of fungi, that is

$$R = R(t) = -\frac{\mathrm{d}m}{\mathrm{d}t} \tag{1}$$

where the negative sign indicates the decrease of ground litter and woody fibers. From the nature of fungi, the decomposition rate is related to the growth rate, the number in unit space, that is, the density and the moisture niche width. Among them, with the growth of fungi, its density will increase, and the increase of density will slow down the growth rate of fungi [2], while both are related to time.

$$R_{fun}(t) = R_{fun}(r, \rho) \qquad r = r(t) \qquad \rho = \rho(t) \tag{2}$$

The decomposition rate is positively related to the growth rate as $R(t) \propto r(t)$ and as the density increases, the decomposition rate will decline along the negative index as $R(t) \propto e^{-\rho(t)}$. For a particular species, we take θ as a constant. Therefore, $R(t)$ is expressed as

$$R_{fun}(t) = \theta \cdot r(t) \cdot e^{-a\rho(t)} \quad (a > 0, \theta > 0) \tag{3}$$

where a is decomposition restraining coefficient.

Next, we model the relationship between r and ρ. Based on the positive and negative correlation mentioned above and the analogy with the S-type logistic curve of the relationship between population growth rate and population density described by Felchester in 1833 [2], we consider that r and ρ should also satisfy this relationship, that is

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\xi x(K - x)}{K} \tag{4}$$

$$x(t) = \frac{CKe^{\xi t}}{1 + CKe^{\xi t}} \tag{5}$$

where $C = \dfrac{x(t=0)}{K - x(t=0)} > 0$ is a constant. Due to $r = \dfrac{\mathrm{d}x}{\mathrm{d}t}$ and $\rho = \dfrac{x}{S}$, then

$$r = \frac{\xi S}{K}\rho(K - \rho S) \tag{6}$$

It is obvious that when $\rho = \dfrac{K}{2S}$, the maximum growth rate is obtained.

Therefore, take (6) into (3), we have

$$R_{fun}(t) = \frac{\xi S\theta}{K}\rho(K - \rho S) \cdot e^{-a\rho} \quad (a > 0) \tag{7}$$

3.2 Solution of Problem 1

There are two zero points of (7), which are 0 and $\frac{K}{S}$, and we can see that $\lim_{\rho \to 0} R_{fun} = \lim_{\rho \to \infty} R_{fun} = 0$. This result is in line with the biological intuition, which shows that when the density is 0, the decomposition rate is 0, and when the density is far beyond the environmental capacity density, the fungi is hard to decompose the substrate due to fierce intraspecific competition (which is different from competition between different species).

Unexpectedly, we observed that the decomposition rate presents a negative trend when the species exist, that is, the substrate mass increases continuously in the presence of fungi, which seems not intuitive. However, this situation only occurs when the species density exceeds the environmental accommodation density. This situation can not be realized for a long time in nature, because once it occurs, the density will decrease sharply due to the limitation of environmental carrying capacity, which will be lower than the environmental accommodation density and goes back to the interval $[0, K/S]$. The result is that the corrosion rate is always non negative, so we believe that in our model, we can assume that ρ is always in the interval $[0, K/S]$ so that the decomposition rate is always a non negative real number.

Based on the analysis above, the general figure between $R_{fun} \sim \rho$ is shown in Figure 2, where

$$\rho^* = \frac{(aK + 2S) - \sqrt{4S^2 + a^2K^2}}{2aS} \tag{8}$$

in which R_{fun} is maximum.

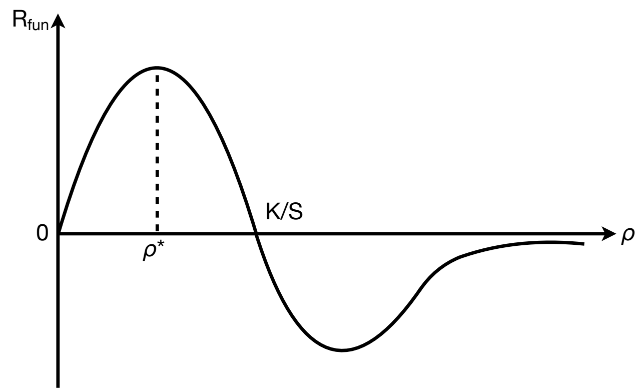


Figure 2 General figure

3.3 Result

In our model, we choose the following five species of fungi and the basic parameters of each species are given in the following table. [3–5]

Table 2 The basic parameters(relative value) of five species

i	**Scientific Name**	ξ	θ	K
1	yeasts	1.200	2	61
2	Aspergillus niger	0.737	5	53
3	Lactarius aurantiacus	2.225	1	66
4	Lactarius volemus Fr.	0.961	4	39
5	Lactarius hatsudake	1.130	3	57

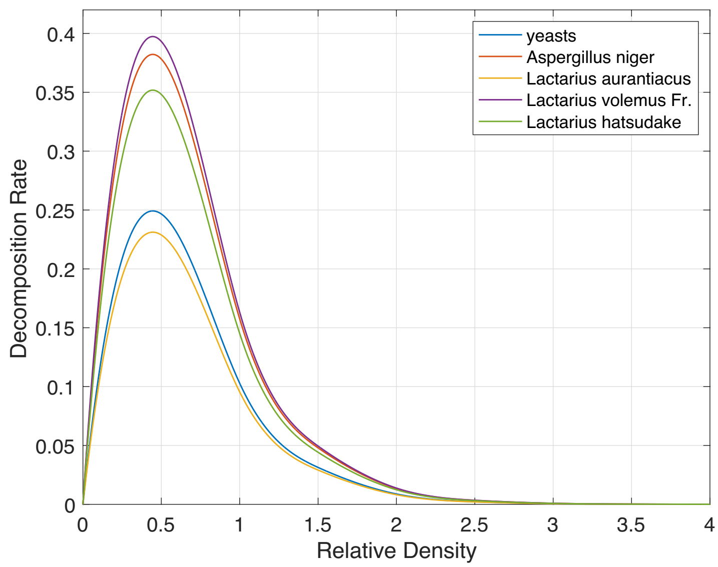


Figure 3 Relationship between the decomposition rate and relative density of each species of fungi

In this way, we can make the relationship between the decomposition rate and density of five fungi when they exist alone through Matlab based on the analysis from 3.1.As we can see, Figure 3 "seems" different from Figure 2. Because according to the derivative of the image, we cannot know whether the function is higher convex or lower convex in the vicinity of K/S. Therefore, the lower convex tends to 0 again when it passes through the horizontal axis, which is the case for all of these five fungi. However, when ρ has reached a relatively large value at K/S, R_{fun} is extremely close to 0 considering the influence from $e^{-a\rho}$, so it is difficult to distinguish the image passing through the second zero point, which leads to this "illusion".

4 Model 2: Ideal Multi-Species Decomposition Rate Model

4.1 Model Construction

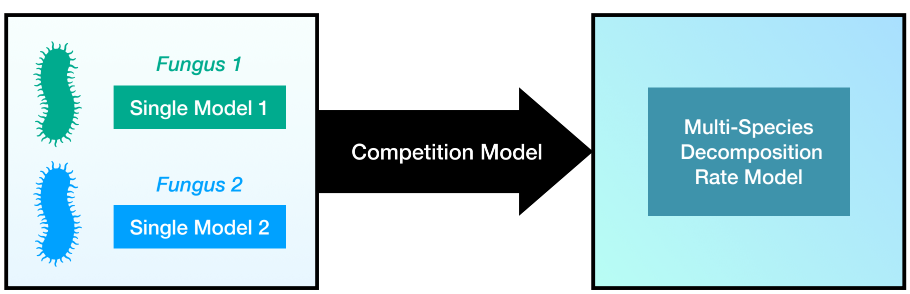


Figure 4 Construction idea of Multi-Species Decomposition Rate Model

4.1.1 Competition Model

On the basis of the single species decomposition rate model, we introduce more fungi, in which these species compete with each other for resources and space. We define the intrinsic reproduction rate of fungi as the inherent reproduction rate under the condition of no external natural enemies and sufficient food. Therefore, based on x_i, μ_i and λ_{ij}, we can get the following autonomous systems of ordinary differential equation by analogy with the Volterra equations [6] between species with predation relationship.

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = x_1\left(\mu_1 - \lambda_{12}x_2 - \lambda_{13}x_3 - \cdots - \lambda_{1n}x_n\right) \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = x_2\left(\mu_2 - \lambda_{21}x_1 - \lambda_{23}x_3 - \cdots - \lambda_{2n}x_n\right) \\ \qquad\qquad \vdots \\ \dfrac{\mathrm{d}x_n}{\mathrm{d}t} = x_n\left(\mu_n - \lambda_{n1}x_1 - \lambda_{n2}x_2 - \cdots - \lambda_{n,n-1}x_{n-1}\right) \end{cases} \tag{9}$$

Transform (9) into the form of summation is

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = x_i\left[\mu_i - \sum_{j=1(j\neq i)}^{n}\left(\lambda_{ij}x_j\right)\right] \tag{10}$$

and the form of matrix is

$$\frac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{x}^{\mathrm{T}}\boldsymbol{\mu} - \boldsymbol{x}^{\mathrm{T}}\Psi\boldsymbol{x} \tag{11}$$

where $\boldsymbol{x} = [x_1, x_2 \cdots x_n]^{\mathrm{T}}$, $\boldsymbol{\mu} = [\mu_1, \mu_2 \cdots \mu_n]^{\mathrm{T}}$, and

$$\Psi = \begin{bmatrix} 0 & \lambda_{12} & \cdots & \lambda_{1n} \\ \lambda_{21} & 0 & \cdots & \lambda_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_{n1} & \lambda_{n2} & \cdots & 0 \end{bmatrix}$$

Therefore, we can express the relationship between x_i in the form of differential equation, and finally get a function about x_i. For the system with n species of fungi, if we plot the function, we will get an $(n+1)-$dimensional image. So we will solve the binary system in detail first.

For the two species system, we have

$$\begin{cases} \dfrac{\mathrm{d}x_1}{\mathrm{d}t} = x_1\left(\mu_1 - \lambda_{12}x_2\right) \\ \dfrac{\mathrm{d}x_2}{\mathrm{d}t} = x_2\left(\mu_2 - \lambda_{21}x_1\right) \end{cases} \tag{12}$$

By eliminating t, we can get the relation between x_1 and x_2.

$$\frac{\mathrm{d}x_2}{\mathrm{d}x_1} = \frac{x_2(\mu_2 - \lambda_{21}x_1)}{x_1(\mu_1 - \lambda_{12}x_2)} \quad \Rightarrow \quad \left(\frac{\mu_1}{x_2} - \lambda_{12}\right)\mathrm{d}x_2 = \left(\frac{\mu_2}{x_1} - \lambda_{21}\right)\mathrm{d}x_1 \tag{13}$$

Integral, and define

$$H(x_1, x_2) = h = \lambda_{12}x_2 - \lambda_{21}x_1 + \mu_2 \ln x_1 - \mu_1 \ln x_2 \tag{14}$$

which is a constant. Therefore, it can be considered that the relationship between x_1 and x_2 is related to the release of the two species at the initial time.

4.1.2 Ideal Multi-Species Decomposition Rate Model

Base on (9), we can get the numerical solution $x_i = x_i(t)$ by Matlab. Divided by the area, the density of species i is $\rho_i(t) = \dfrac{x_i(t)}{S}$. Therefore, by combining the density and (7), in the competition model with n

fungi, we only need to stack the decomposition rates of different fungi to calculate the total rate in this model (Do not consider the environment temporarily).

$$R_{fun}(t) = \sum_{i=1}^{n} R_{fun,i}(t) = \sum_{i=1}^{n} \frac{\xi_i S\theta_i}{K_i}\rho_i(t)(K_i - \rho_i(t)S) \cdot e^{-a\rho_i(t)} \tag{15}$$

4.2 Solution and Result of Problem 2

We use (9) to calculate the decomposition rate of total decomposed products under competitive conditions. Specifically, we choose 5 species to establish the following matrixes [7]

$$\boldsymbol{x} = [x_1, x_2, x_3, x_4, x_5]^{\mathrm{T}} \qquad \boldsymbol{\mu} = [1, 0.5, 1.2, 0.6, 0.8]^{\mathrm{T}} \qquad \Psi = \begin{bmatrix} 0 & 0.1 & 0.02 & 0.08 & 0.01 \\ 0.02 & 0 & 0.03 & 0.01 & 0.2 \\ 0.06 & 0.08 & 0 & 0.15 & 0.06 \\ 0.01 & 0.05 & 0.25 & 0 & 0.06 \\ 0.09 & 0.05 & 0.1 & 0.16 & 0 \end{bmatrix}$$

and solve the numerical solution of the equation group by the numerical solution analysis method. The image of the number of species of five fungal species changing with time and the results of fitting five curves to the equation with the power of 6 are as follows.

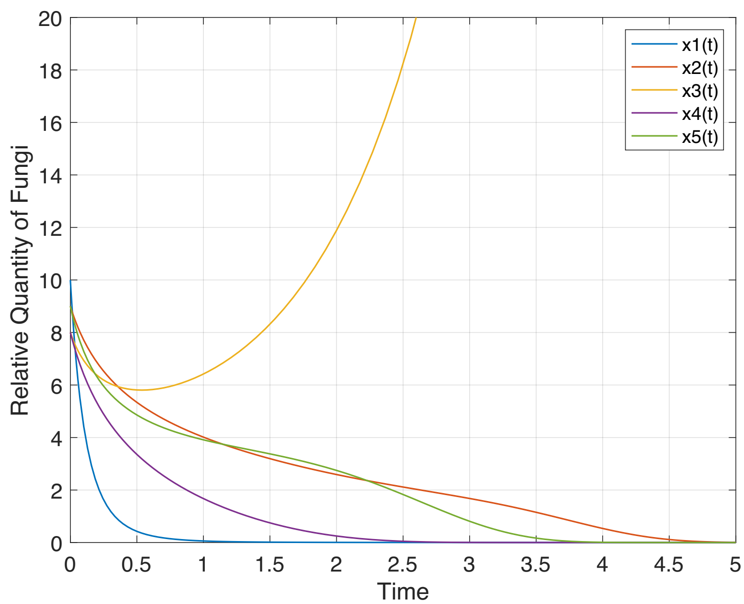


Figure 5 A schematic diagram of the statistical distribution of the number of fungi with different decomposition rates

Table 3 Results of five curves

i	Result of $x_i(t)$
1	$x_1(t) = 0.0958t^6 - 1.57t^5 + 10t^4 - 31.1t^3 + 48.3t^2 - 34.5t + 8.49$
2	$x_2(t) = 0.0172t^6 - 0.278t^5 + 1.78t^4 - 5.86t^3 + 10.5t^2 - 11t + 8.83$
3	$x_3(t) = 0.0932t^6 - 0.983t^5 + 4.73t^4 - 11.3t^3 + 16t^2 - 9.99t + 7.89$
4	$x_4(t) = 0.0198t^6 - 0.328t^5 + 2.14t^4 - 7.08t^3 + 13t^2 - 14t + 7.81$
5	$x_5(t) = 0.0193t^6 - 0.364t^5 + 2.66t^4 - 9.35t^3 + 16.2t^2 - 14t + 8.73$

So far, we have obtained the total corrosion rate curves without considering the competition of multiple species under the climate conditions.

On the other hand, we are more concerned about the competitive relationship between two species because it is much more easier for the two species to draw a phase diagram to describe the periodic characteristics of the competitive model. Specifically, we give two specific coefficients [7] (the matrix P represents the competitive ability of the two species is close and Q represents there is a great gap between their competitive strength) of (14) as follows

$$P = \begin{bmatrix} \mu_1 & \lambda_{12} \\ \lambda_{21} & \mu_2 \end{bmatrix} = \begin{bmatrix} 4096 & 889 \\ 1296 & 3321 \end{bmatrix} \qquad Q = \begin{bmatrix} \mu_3 & \lambda_{34} \\ \lambda_{43} & \mu_4 \end{bmatrix} = \begin{bmatrix} 2048 & 89 \\ 899 & 1331 \end{bmatrix}$$

With these coefficients, we can draw the diagram of H and the contour section diagrams (i.e. the phase trajectory diagrams) as shown.

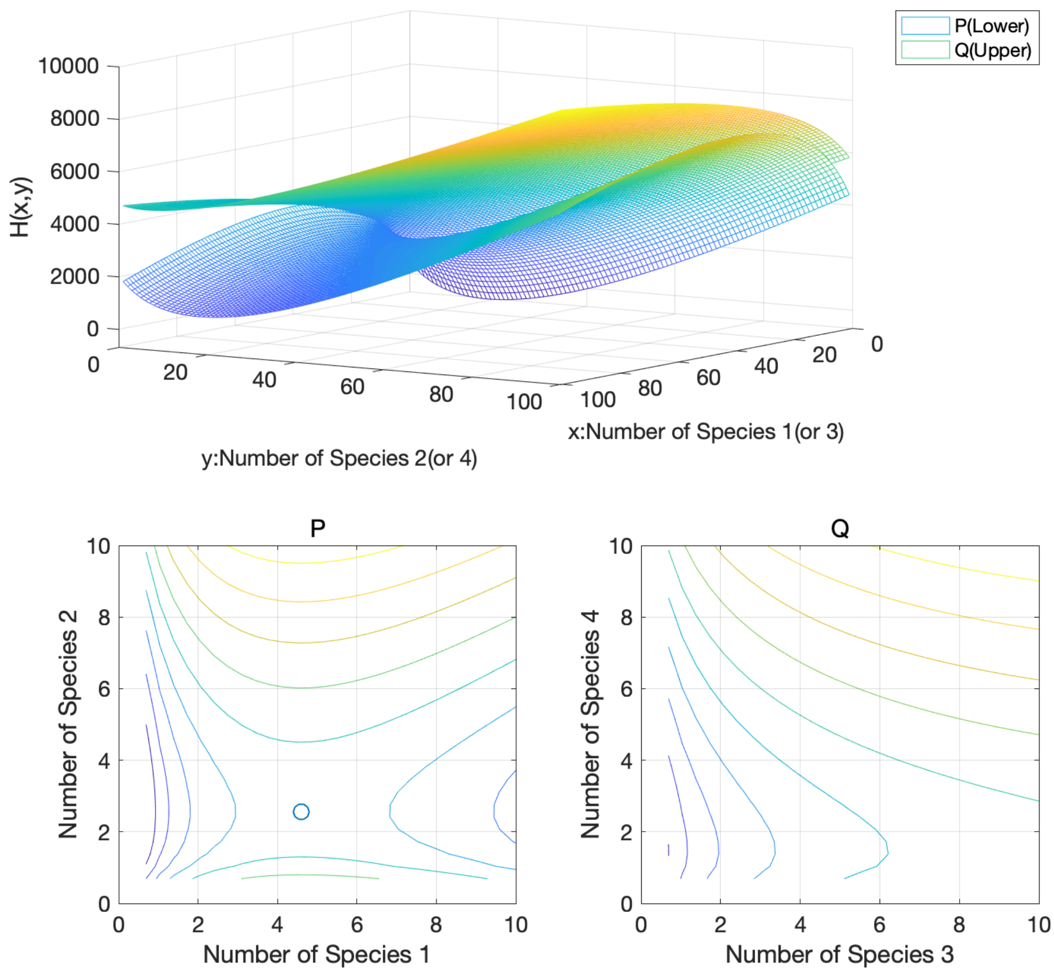


Figure 6 The result of competition between two species of fungi

Under the condition of P, we can clearly find out that the trajectory description is stable with the saddle point (the blue circle of Figure 6)

$$(\overline{x_1}, \overline{x_2}) = \left(\frac{4096}{889}, \frac{3321}{1296}\right) = (4.6074, 2.56725)$$

Combined with (12), we can get the conclusion of quantity change as shown in the table below.

Table 4 The quantity change of Figure P

Condition	Conclusion	Condition	Conclusion
$x_1 > \overline{x_1}$	$x_2 \downarrow$	$x_2 > \overline{x_2}$	$x_1 \downarrow$
$x_1 < \overline{x_1}$	$x_2 \uparrow$	$x_2 < \overline{x_2}$	$x_1 \uparrow$

which is in line with the actual laws of biology. On this basis, we notice that λ_{12} is similar to λ_{21}, while $\lambda_{43} >> \lambda_{34}$. It shows that the existence of Species 4 has little influence on Species 3, so it can be predicted theoretically that Species 3 should be the dominant species compared with Species 4, while Species 1 and 2 are more comparable than 3 and 4.

From Figure 6, we can see that although the saddle point between Species 3 and 4 (23.011, 1.4805) has deviated to the outside (this is precisely because of the great disparity in the competition). When x_3 is extremely large, x_4 decreases sharply and even has the possibility of extinction, but Species 3 still keeps growing. This situation is very easy to meet because the horizontal coordinate of the saddle point is very small. Therefore, Species 3 is the dominant species between 3 and 4, which is in line with our prediction.

4.3 Result of Problem 3

Based on Figure 5, we can draw the following conclusions.

- When $t \to \infty$, x_3 is much larger than x_i of other related species, while $x_i (i = 1, 2, 4, 5)$ is close to zero, which indicates that Aspergillus niger (Species 3) is more likely to become the dominant species.
- When t is around 0.5, Species 3 gets minimum. Therefore, in the short term, Species 3 may not be dominant, or even be at a competitive disadvantage due to the initial value of each species.
- In conclusion, from the perspective of dominant species combination, we suggest that species excepted 3 are more likely to combine to achieve dominant.

5 Model 3: Complete Multi-Species Decomposition Rate Model

5.1 Model Construction

There are three main climatic factors in the fungi decomposition model-temperature, humidity and mass of decomposed products. For the decomposition reaction of fungi [8]

$$\text{DP(decomposed products)} \xrightarrow[\text{enzyme}]{\text{fungi}} H_2O + \text{IS(inorganic salt)} \tag{16}$$

and based on chemical reaction thermodynamics and kinetics, the reaction rate is influenced by temperature and mass of decomposed products. Meanwhile, because different fungi have different niche widths, when the precipitation changes, humidity will also affect the decomposition rate of decomposed products. [9] So we divide the climate model into two parts.

5.1.1 Temperature and Decomposed Products Model

In chemical reaction kinetics, the reaction rate is related to the amount of reactants and the reaction order.

$$I = km^{\alpha} \tag{17}$$

And k is related to T as Arrhenius equation [10,11]

$$\frac{\mathrm{d}\ln k}{\mathrm{d}T} = \frac{E_a}{RT^2} \quad \Leftrightarrow \quad k = Ae^{-\frac{E_a}{RT}} \tag{18}$$

where gas constant $R = 8.314\mathrm{J/(mol\cdot K)}$ is a constant and k is usually measured by experiments. In a certain temperature range, E_a can also be regarded as a constant. So I can be given as

$$I(T, m) = Ae^{-\frac{E_a}{RT}} m^{\alpha} \tag{19}$$

Since this is an enzyme catalyzed reaction and the content of enzymes in organisms is usually stable, α is related to the concentration of substrate and satisfies the following general rules by Michaelis and Menten. [10,11]

- When m is very small, the reaction is a first order reaction and $\alpha = 1$.
- When m is very large, the reaction is a zero order reaction and $\alpha = 0$.

5.1.2 Humidity Model

We first define the maximum decomposition rate of a single species as

$$R_0 = \sup_{t>0} R(t) \tag{20}$$

For humidity, we use precipitation to characterize. For different fungi, they have different moisture niche widths. According to the definition of moisture niche width, when $l \in J$

$$\hat{R} = 50\% R_0 + REM \tag{21}$$

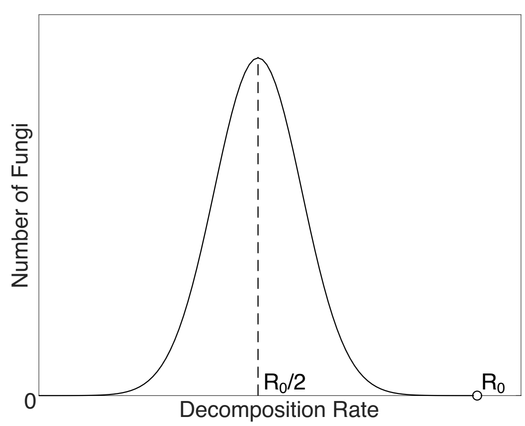


Figure 7 A schematic diagram of the statistical distribution of the number of fungi with different decomposition rates

According to the principle of statistics, we consider that these 50% fungi meet the normal distribution, so the expected decomposition rate of them is $\frac{1}{2}R_0$. Then $\hat{R} = 50\%R_0 + 25\%R_0 = 75\%R_0$. When $l \notin J$, it can be considered that the whole community satisfies the normal distribution, so its expectation is $\frac{1}{2}R_0$.

$$\hat{R} = \begin{cases} 75\%R_0 & (l \in J) \\ 50\%R_0 & (l \notin J) \end{cases} \tag{22}$$

5.1.3 Complete Model

Based on 5.1.1 and 5.1.2, we consider that for a single community of fungi, the complete model can be regarded as

$$R = I(T, m)\hat{R} \tag{23}$$

where $\hat{R}$ is related to humidity.

Now considering n fungal communities with competitive relationship under the influence of climate, we can determine $I(T, m)$ for the same decomposition reaction. But for the humidity caused by the same precipitation, different kinds of fungi have different moisture niche widths. Assuming that $l \in J_1, J_2, \cdots, J_\nu (\nu < n)$ and $l \notin J_{\nu+1}, J_{\nu+2}, \cdots, J_n$, the complete model of multi-species can be regarded as

$$R = I(T, m)\left[\sum_{i=1}^{\nu}\hat{R}_i + \sum_{j=\nu+1}^{n}\hat{R}_j\right] = I(T, m)\left[\sum_{i=1}^{\nu}0.75R_{0,i} + \sum_{j=\nu+1}^{n}0.5R_{0,j}\right] \tag{24}$$

where $R_{0,i}$ and $R_{0,j}$ can be obtained by (20).

5.2 Solution and Result of Problem 4

5.2.1 Complete Result of Decomposition Rate in Competition Considering Climate

Based on the results in 4.2 and the relevant data in Table 2, we can get the relationship between the decomposition rate of 5 species of fungi and time in the competition.And R_0 of each species are

$$R_{0,1} = 0.2797 \quad R_{0,2} = 0.4291 \quad R_{0,3} = 1.729 \times 10^{-7} \quad R_{0,4} = 0.4467 \quad R_{0,5} = 0.2444$$

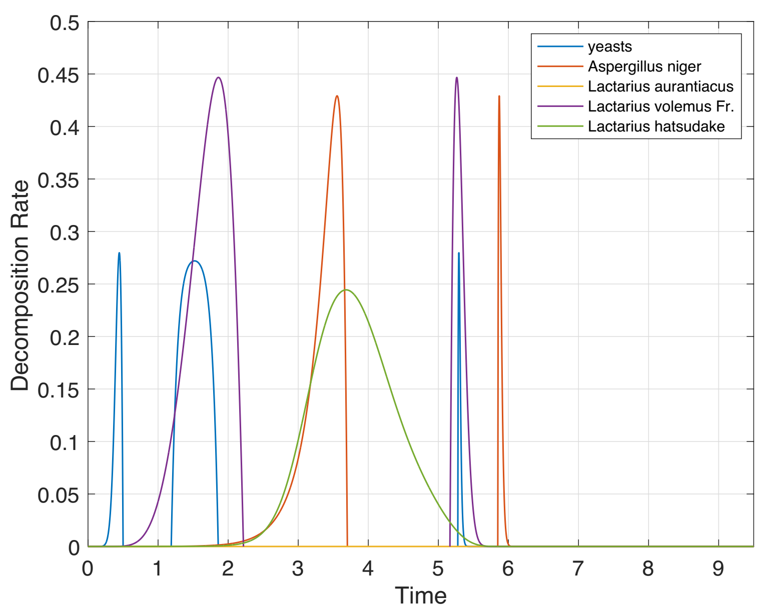


Figure 8 The relationship between the decomposition rate of 5 species and time in the competition

On the other hand, introducing the influence of environment, we consider that the precipitation from arid climate to tropical rainforest climate meets the following conditions:

- the precipitation of arid remains the least, and $l_{\text{arid}} \notin J_i (i = 1, 2, 3, 4, 5)$.
- $l_{\text{semi-arid}} \in J_i (i = 4, 5)$ and $l_{\text{semi-arid}} \notin J_j (j = 1, 2, 3)$.

- $l_{\text{temperate}} \in J_i (i = 1, 2, 3, 4, 5)$.
- $l_{\text{arboreal}} \in J_i (i = 3, 4, 5)$ and $l_{\text{semi-arid}} \notin J_j (i = 1, 2)$.
- the precipitation of tropical rain forest the most, and $l_{\text{tropical rain forest}} \in J_i (i = 5)$, $l_{\text{tropical rain forest}} \notin J_j (i = 1, 2, 3, 4)$.

In terms of m, we determine $\alpha = 1$ in arid and semi-arid climate, and the rest are calculated $\alpha = 0$, with $m_{\text{arid}} = 0.902\%$ and $m_{\text{semi-arid}} = 1.882\%$. [12]

Since the reaction rate of the enzyme-catalyzed reaction becomes two to three times larger for every 10°C increase over a certain temperature range [11], we assume that the relative value $k(298.15\text{K}) = 1$, $k(308.15\text{K}) = 2$ and $k(288.15\text{K}) = 0.5$. So the relative value of E_a can be calculated as 52.946kJ/mol. And based on (24), we can obtained the relationship between R and T of different kinds of climate.

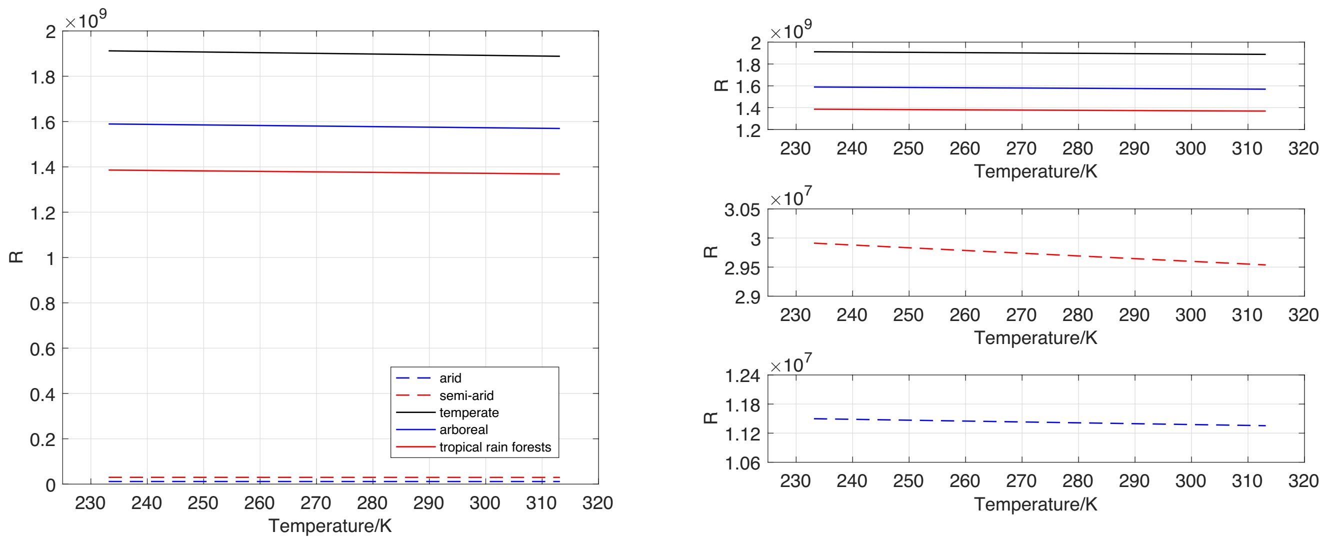


Figure 9 The relationship between R and T of different kind of climate

5.2.2 Prediction of Dominant Species and Combination in Different Climates

Based on the model above, we calculate the index of different climates through the conditions between l and J mentioned in 5.2.1. We predicted the dominant species according to the value of the decomposition rate of each fungus for spoilage, and determined the dominant combination according to the relative value (proximity) of the decomposition rate of each species. Here are the results.

Table 5 Prediction of dominant species and dominant combination in different climates

Climates	Arid	Semi-Arid	Temperate	Arboreal	Tropical
Y	0.2215	0.2215	0.3323	0.2215	0.2215
AN	0	0	0	0	0
LA	0.1322	0.1322	0.1983	0.1983	0.1322
LV	0.1261	0.1892	0.1892	0.1892	0.1892
LH	0.1389	0.2084	0.2084	0.2084	0.2039
DS	Y	V, LH	V	V, LH	V, LH
DC	LA, LV, LH	V, LH	LA, LV, LH	LA, LV, LH	LV, LH

* Y-yeasts, AN-Aspergillus niger, LA-Lactarius aurantiacus, LV-Lactarius volemus Fr., LH-Lactarius hatsudake, DS-Dominant Species, DC-Dominant Combination.

5.3 Solution and Result of Problem 5

We only take the temperate zone as example, and other kinds of climate are similar. We plot one species ($R_{0,4}$), three species ($R_{0,1}, R_{0,2}, R_{0,5}$) and five species ($R_{0,1}, R_{0,2}, R_{0,3}, R_{0,4}, R_{0,5}$) of fungi under the temperate climate.

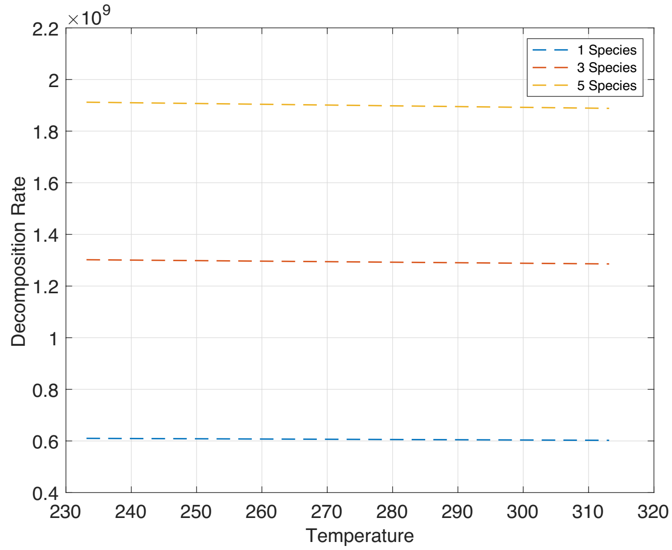
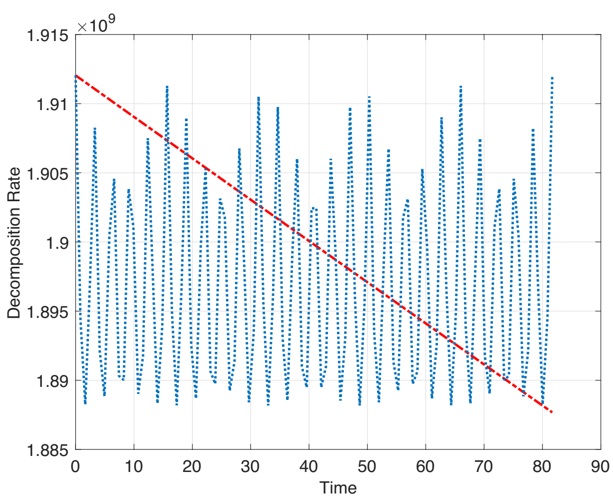


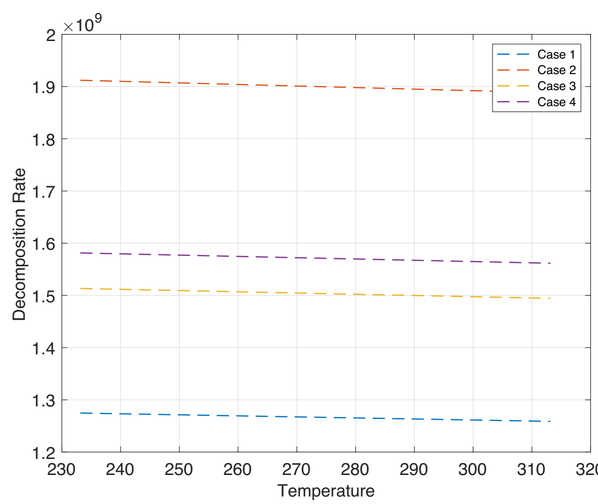
Figure 10 The influence of the number of species (biodiversity)

According to the curves, we can clearly see that there is a certain gap between the values of the three curves. This shows that the more species there are, the faster to decompose, and the more efficient of decomposition ecosystem, which is also consistent with biological intuition.

6 Sensitivity Analysis



(a) Sensitivity of Temperature



(b) Sensitivity of Humidity

Figure 11 The sensitivity analysis

6.1 Sensitivity of Temperature

In order to represent the fluctuation of temperature, we assume that temperature is a function of time, and the fluctuation of temperature is represented by the change of time. For trigonometric function is a specific periodic fluctuation function, temperature and time were triangulated, which indicates a drastic temperature change of the studied soil. When the T changes in the whole cycle in [233.15K, 313.15K], select $T(t) = 80\,|\sin t\,| + 233.15$. In order to ensure that $T(t)$ still changes in the above interval in the same time range, the simple linear relationship is selected $T(t) = t + 233.15$, where t belongs to $[0, 26\pi]$. Therfore, the sensitivity curves of decomposition rate to temperature are obtained in Figure 10 (a). As we can see, the blue curve fluctuates violently.

6.2 Sensitivity of Humidity

Having theorized that the moisture niche width for each species contains or does not contain the precipitation when there is a dramatic change in humidity, we take a few typical examples and shown in Figure 10 (b), where the 4 cases are plotted by different optional parameters. As we can see, there are gaps between different curves.

7 Model Evaluation

7.1 Strength and Weakness

7.1.1 Strength

- The usage of the logistic curve to establish the model of single species decomposition rate, which increases the fitting degree and accuracy between the model and the actuality.
- The usage of the Volterra model to study the competition between fungi more accurately reflects the influence of competition and makes the model more accurate.
- The combination between ecology and chemistry. We consider the decomposition reaction mechanism and add the relationship between reaction rate constant and temperature to modify the model to make the result more practical.
- The high sensitivity of temperature which can reflect the accuracy.

7.1.2 Weakness

- Some complex environmental conditions (such as the types of organic compounds and complex weather factors) are appropriately simplified and idealized.
- The corresponding changes of the model according to the change of humidity are not smooth and continuous and the sensitivity of humidity is not as good as temperature.
- The model can not directly show the change trend of the number of each species over time, and the dominant species reflected by the decomposition rate will have a certain deviation from the real quantity statistics.

7.2 Future Work

In this model, we've discussed the relationship between the decomposition rate of five species of fungi and the internal and external factors. In the future work, it is feasible to study more environmental variables, fungal species and other survival relationships among different species. This will be of great significance to our understanding of the importance of biodiversity and the necessity to protect the ecological environment.

Decomposition Rate of Fungi and Its Interfering Factor

Ecosystems are composed of producers, consumers, decomposers, and abiotic substances. Among them, **decomposers** can decompose organic compounds from animals and plant remains into inorganic compounds, and different decomposers have different **decomposition rate**s under different conditions. In this section, we will introduce the decomposition rate of a representative decomposer, **fungi** and their interfering factors.

What is Decomposition Rate?

Fungi can decompose a wide range of substances such as ground litter and wood fibers, and we refer to these substances as **decomposed product**s. As the fungus decomposes, the decomposed products will become less gradually, so we can use the amount of decomposed products m in unit time t to indicate the decomposition rate R.

$$R = -\frac{\mathrm{d}m}{\mathrm{d}t}$$

Interfering Factors of Decomposition Rate

There are two main parts of factors and the details of each part has been shown as follows.

Internal Factors	External Factors
Species Growth Rate	Temperature
Species Density	Humidity
Moisture Niche Width	Mass of Decomposed Products

- **Internal Factors**
  - **Species Growth Rate(r) & Density(ρ):** Felchester presents a **logistic curve** between the relationship of population size and density in 1833. In the initial state, the growth rate increases continuously due to the small number of individuals, which leads to an increase in density. But after growth to a certain extent, the increase in density slows the growth rate so that the number of individuals tends to a stable value, which we call the **environmental capacity**. Since this curve is shaped as if the letter "S", it is also called S-type curve.
  - **Moisture Niche Width(J):** It indicates the degree of tolerance of fungi to water. Mathematically, it means the difference from the maximum to the minimum of moisture level, that is, half of a fungal community can grow fastest.

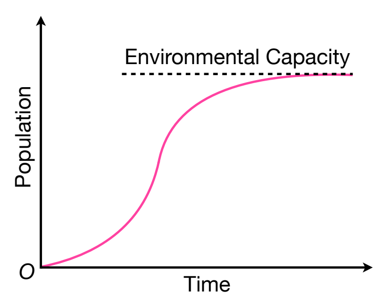


Fig.1 The logistic curve

- **External Factors**
  - **Humidity(l):** For the system of fungi and soil, we can express it in terms of precipitation. For different fungi, they have different moisture niche widths. After research, we define **humidity factor($\hat{R}$)** as the effect of humidity on R, which is

$$\hat{R} = \begin{cases} 75\% R_0 & (l \in J) \\ 50\% R_0 & (l \notin J) \end{cases}$$

where R_0 is the maximum of R without considering humidity.

- **Temperature(T) & Mass of Decomposed Products(m):** Since decomposition is essentially an **enzyme catalyzed reaction** within the fungal body,

$$\text{DP(decomposed products)} \xrightarrow[\text{enzyme}]{\text{fungi}} \text{H}_2\text{O} + \text{IS(inorganic salt)}$$

the reaction rate is only related to m and

$$\text{reaction rate} = km^{a}$$

where k is called the reaction rate constant and can be calculated by Arrhenius Equation

$$\frac{\mathrm{d}\ln k}{\mathrm{d}T} = \frac{E_a}{RT^2} \quad \Leftrightarrow \quad k = Ae^{-\frac{E_a}{RT}}$$

In a certain temperature range, E_a can be regarded as a constant.
For the enzyme catalyzed reaction, α is related to the concentration of substrate and satisfied the general rules by Michaels and Menten.

  * When m is very small, the reaction is a first order reaction and $\alpha = 1$.
  * When m is very large, the reaction is a zero order reaction and $\alpha = 0$.

Besides, different fungi will **compete** with each other for resources and space, while external factors (such as **climate**) play an important role as well. Based on the factors above, we can model the profile of change in decomposition rate of fungi with the following equation according to research.

$$R = km^{\alpha}\left[\sum_{i=1}^{\nu} 0.75R_{0,i} + \sum_{j=\nu+1}^{n} 0.5R_{0,j}\right]$$

where $l \in J_1, J_2, \cdots, J_\nu (\nu < n)$ and $l \notin J_{\nu+1}, J_{\nu+2}, \cdots, J_n$.

Example

To describe the factors better, we choose five species of fungi (yeasts, Aspergillus niger, Lactarius aurantiacus, Lactarius volemus Fr. and Lactarius hatsudake) and five climates (arid, semi-arid, temperature, arboreal and tropical rain forests). We can see from Fig.2 that the species are on one another's length (where Lactarius aurantiacus goes extinct after a period of time). And from Fig.3, we can see the decomposition rate under different climates, which was slow under arid and semi-arid due to little decomposed products, while in the other three climates where the decomposed products are sufficient, the humidity of the rainforest was too large to cause many species' $l \notin J$ resulting in decomposition rates slower than temperate. So we can conclude that the ecosystem is in **dynamic equilibrium** and tends to a balance in fluctuation under **the selection of environment**.

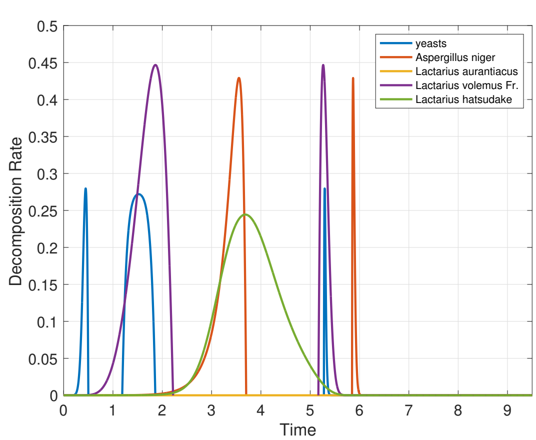


Fig.2 The competition curve

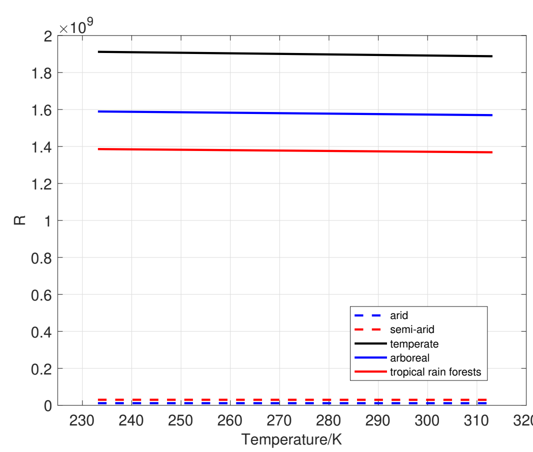


Fig.3 The climate curve

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Appendix

Listing 1 Code for the relationship between R_{fun} and ρ of five species of fungi by Matlab

```
clear
clc
rho=linspace(0,50);
a=3.14;
theta1=2;theta2=5;theta3=1;theta4=4;theta5=3;   %the coefficient of
    moisture niche width of fungi
ksi1=1.2;ksi2=0.737;ksi3=2.225;ksi4=0.961;ksi5=1.13;   %population growth
    potential index
k1=61;k2=53;k3=66;k4=39;k5=57;  %environmental capacity of fungi
R1=ksi1*theta1.*rho.*(k1-rho).*exp(-a.*rho)/k1;
R2=ksi2*theta2.*rho.*(k2-rho).*exp(-a.*rho)/k2;
R3=ksi3*theta3.*rho.*(k3-rho).*exp(-a.*rho)/k3;
R4=ksi4*theta4.*rho.*(k4-rho).*exp(-a.*rho)/k4;
R5=ksi5*theta5.*rho.*(k5-rho).*exp(-a.*rho)/k5;
xx=0:0.001:4;
RR1=spline(rho,R1,xx);
RR2=spline(rho,R2,xx);
RR3=spline(rho,R3,xx);
RR4=spline(rho,R4,xx);
RR5=spline(rho,R5,xx);  %cubic spline interpolation
hold on;grid on;box on;
plot(xx,RR1,'linewidth',1);
plot(xx,RR2,'linewidth',1);
plot(xx,RR3,'linewidth',1);
plot(xx,RR4,'linewidth',1);
plot(xx,RR5,'linewidth',1);
axis([0 4,0 0.42]);
legend('yeasts','Aspergillus niger','Lactarius aurantiacus','Lactarius
    volemus Fr.','Lactarius hatsudake', 'FontSize',12);
xlabel('Relative Density', 'FontSize',14);
ylabel('Decomposition Rate', 'FontSize',14);
set(gca,'XTick',[0:0.5:4] , 'FontSize',14);
set(gca,'YTick',[0:0.05:0.42] , 'FontSize',14);
```

Listing 2 The numerical solution analysis method by Matlab

```
function [dx] = rigid(t,x)
dx=zeros(5,1);
mu1=1;mu2=0.5;mu3=1.2;mu4=0.6;mu5=0.8;lambda12=1;lambda13=0.02;lambda14
    =0.08;lambda15=0.01;lambda21=0.02;lambda23=0.03;lambda24=0.01;lambda25
    =0.2;lambda31=0.06;lambda32=0.08;lambda34=0.15;lambda35=0.06;lambda41
    =0.01;lambda42=0.05;lambda43=0.25;lambda45=0.06;lambda51=0.09;lambda52
    =0.05;lambda53=0.1;lambda54=0.16;
dx(1)=x(1)*(mu1-lambda12*x(2)-lambda13*x(3)-lambda14*x(4)-lambda15*x(5));
dx(2)=x(2)*(mu2-lambda21*x(1)-lambda23*x(3)-lambda24*x(4)-lambda25*x(5));
dx(3)=x(3)*(mu3-lambda31*x(1)-lambda32*x(2)-lambda34*x(4)-lambda35*x(5));
```

```
dx(4)=x(4)*(mu4-lambda41*x(1)-lambda42*x(2)-lambda43*x(3)-lambda45*x(5));
dx(5)=x(5)*(mu5-lambda51*x(1)-lambda52*x(2)-lambda53*x(3)-lambda54*x(4));
end

clear
clc
options=odeset('RelTol',1e-4,'AbsTol',[1e-4,1e-4,1e-4,1e-4,1e-4]);
[t,x]=ode15s(@rigid,[0,5],[10,9,8,8,9]);
plot(t,x(:,1),'-',t,x(:,2),'-',t,x(:,3),'-',t,x(:,4),'-',t,x(:,5),'-','linewidth',1);
legend('x1(t)','x2(t)','x3(t)','x4(t)','x5(t)','FontSize',12);
hold on;grid on;box on;
axis([0 5,0 20]);
xlabel('Time','FontSize',14);
ylabel('Relative Quantity of Fungi','FontSize',14);
set(gca,'XTick',[0:0.5:5] , 'FontSize',14);
set(gca,'YTick',[0:2:20] , 'FontSize',14);
```

Listing 3 Code for the relationship between two species of fungi with competition

```
clear
clc
%Plot the 3D figures
subplot(2,2,[1,2]);
x=linspace(1,10);y=linspace(1,10);
[X,Y]=meshgrid(x,y);
Z=1296*Y-889*X+4096*log(X)-3321*log(Y);
mesh(Z)
hold on;grid on;
Z=899*Y-89*X+2048*log(X)-1331*log(Y);
mesh(Z);
xlabel('x:Number of Species 1(or 3)','FontSize',14);
ylabel('y:Number of Species 2(or 4)','FontSize',14);
zlabel('H(x,y)','FontSize',14);
set(gca,'XTick',[0:20:100] , 'FontSize',14);
set(gca,'YTick',[0:20:100] , 'FontSize',14);
set(gca,'ZTick',[0:2000:10000] , 'FontSize',14);
legend('P(Lower)','Q(Upper)','FontSize',12);
%Plot the 2D figures
subplot(2,2,3);
syms x1 x2;
f=1296*x2-889*x1+4096*log(x1)-3321*log(x2);
ezcontour(f,[0,10],30);
hold on;grid on; box on;
a=4096/889;b=3321/1296;  %The transverse and longitudinal of saddle point
plot(a,b,'o','MarkerSize',10);
xlabel('Number of Species 1','FontSize',14);
ylabel('Number of Species 2','FontSize',14);
title('P','FontSize',14);
set(gca,'XTick',[0:2:10] , 'FontSize',14);
```

```
set(gca,'YTick',[0:2:10] , 'FontSize',14);
subplot(2,2,4);
syms y1 y2;
f=899*y2-89*y1+2048*log(y1)-1331*log(y2);
ezcontour(f,[0,10],30);
hold on; grid on; box on;
xlabel('Number of Species 3','FontSize',14);
ylabel('Number of Species 4','FontSize',14);
title('Q','FontSize',14);
set(gca,'XTick',[0:2:10] , 'FontSize',14);
set(gca,'YTick',[0:2:10] , 'FontSize',14);
```

Listing 4 Code for the relationship between the decomposition rate of 5 species of fungi and time in the competition by Matlab

```
clear
clc
a=3.14;
theta1=2;theta2=5;theta3=1;theta4=4;theta5=3;
ksi1=1.2;ksi2=0.737;ksi3=2.225;ksi4=0.961;ksi5=1.13;
k1=61;k2=53;k3=66;k4=39;k5=57;
t=[0:0.001:9.5];
rho1=0.0958*t.^6-1.57*t.^5+10*t.^4-31.1*t.^3+48.3*t.^2-34.5*t+8.49;
rho2=0.0172*t.^6-0.278*t.^5+1.78*t.^4-5.86*t.^3+10.5*t.^2-11*t+8.83;
rho3=0.0932*t.^6-0.983*t.^5+4.73*t.^4-11.3*t.^3+16*t.^2-9.99*t+7.89;
rho4=0.0198*t.^6-0.328*t.^5+2.14*t.^4-7.08*t.^3+13*t.^2-14*t+7.81;
rho5=0.0193*t.^6-0.364*t.^5+2.66*t.^4-9.35*t.^3+16.2*t.^2-14*t+8.73;
R1=ksi1*theta1.*rho1.*(k1-rho1).*exp(-a.*rho1)/k1;
R2=ksi2*theta2.*rho2.*(k2-rho2).*exp(-a.*rho2)/k2;
R3=ksi3*theta3.*rho3.*(k3-rho3).*exp(-a.*rho3)/k3;
R4=ksi4*theta4.*rho4.*(k4-rho4).*exp(-a.*rho4)/k4;
R5=ksi5*theta5.*rho5.*(k5-rho5).*exp(-a.*rho5)/k5;
hold on;grid on;box on;
plot(t,R1,'-','linewidth',1);plot(t,R2,'-','linewidth',1);plot(t,R3,'-','
    linewidth',1);plot(t,R4,'-','linewidth',1);plot(t,R5,'-','linewidth',1);
axis([0,9.5,0,0.5])
xlabel('Time', 'FontSize',14);ylabel('Decomposition Rate', 'FontSize',14);
set(gca,'XTick',[0:1:9.5] , 'FontSize',14);
set(gca,'YTick',[0:0.05:0.5] , 'FontSize',14);
legend('yeasts','Aspergillus niger','Lactarius aurantiacus','Lactarius
    volemus Fr.','Lactarius hatsudake', 'FontSize',10);
```

Listing 5 Code for the relationship between R and T of different kinds of climate by Matlab

```
clear
clc
R01=0.2797;R02=0.4291;R03=1.729e-07;R04=0.4467;R05=0.2444;
Rcli1=0.5*(R01+R02+R03+R04+R05);
Rcli2=0.75*(R04+R05)+0.5*(R01+R02+R03);
Rcli3=0.75*(R01+R02+R03+R04+R05);
Rcli4=0.75*(R03+R04+R05)+0.5*(R01+R02);
```

```
Rcli5=0.75*R05+0.5*(R01+R02+R03+R04);
T=linspace(233.15,313.15);
k=exp(52945.916/(8.314*298.15))*exp(-52945.916.\(8.314.*T));
I1=k.*0.00902;I2=k.*0.01882;I3=k;I4=k;I5=k;
R1=I1.*Rcli1;R2=I2.*Rcli2;R3=I3.*Rcli3;R4=I4.*Rcli4;R5=I5.*Rcli5;
%Plot
subplot(3,2,[1,3,5]);hold on; grid on; box on;
plot(T,R1,'b--','linewidth',1);plot(T,R2,'r--','linewidth',1);
plot(T,R3,'k-','linewidth',1);plot(T,R4,'b-','linewidth',1);
plot(T,R5,'r-','linewidth',1);
legend('arid','semi-arid','temperate','arboreal','tropical rain forests', 'FontSize',10);
axis([225,320,0,20e8]);
set(gca,'XTick',[230:10:320],'FontSize',14);
set(gca,'YTick',[0:2e8:20e8],'FontSize',14);
xlabel('Temperature/K', 'FontSize',14);ylabel('R', 'FontSize',14);
subplot(3,2,6);
hold on; grid on; box on;
plot(T,R1,'b--','linewidth',1);
axis([225,320,1.06e7,1.24e7]);
set(gca,'XTick',[230:10:320],'FontSize',14);
set(gca,'YTick',[1.06e7:0.06e07:1.24e7],'FontSize',14);
xlabel('Temperature/K', 'FontSize',14);ylabel('R', 'FontSize',14);
subplot(3,2,4);
hold on; grid on; box on;
plot(T,R2,'r--','linewidth',1);
axis([225,320,2.9e7,3.05e7]);
set(gca,'XTick',[230:10:320],'FontSize',14);
set(gca,'YTick',[2.9e7:0.05e7:3.05e7],'FontSize',14);
xlabel('Temperature/K', 'FontSize',14);ylabel('R', 'FontSize',14);
subplot(3,2,2);hold on; grid on; box on;
plot(T,R3,'k-','linewidth',1);plot(T,R4,'b-','linewidth',1);
plot(T,R5,'r-','linewidth',1);
axis([225,320,1.2e9,2e9]);
set(gca,'XTick',[230:10:320],'FontSize',14);
set(gca,'YTick',[1.2e9:0.2e9:2e9],'FontSize',14);
xlabel('Temperature/K', 'FontSize',14);ylabel('R', 'FontSize',14);
```

Listing 6 Code for the influence of biodiversity by Matlab

```
clear
clc
R01=0.2797;R02=0.4291;R03=0;R04=0.4467;R05=0.2444;
T=linspace(233.15,313.15);
k=exp(52945.91608/(8.314*298.15))*exp(-52945.91608.\(8.314.*T));
R1=0.75*R04.*k;
R2=0.75*(R01+R02+R05).*k;
R3=0.75*(R01+R02+R03+R04+R05).*k;
hold on;grid on;box on;
plot(T,R1,'--','linewidth',1);plot(T,R2,'--','linewidth',1);plot(T,R3,'--',
```

```
    'linewidth',1);
axis([230,320,0.4e9,2.2e9]);
set(gca,'XTick',[230:10:321],'FontSize',14);
set(gca,'YTick',[0.4e9:0.2e9:2.2e9],'FontSize',14);
xlabel('Temperature', 'FontSize',14);
ylabel('Decomposition Rate', 'FontSize',14);
legend('1 Species','3 Species','5 Species','FontSize',10);
```

Listing 7 Code for sensitivity analysis of the model to temperature by Matlab

```
clear
clc
R01=0.2797;R02=0.4291;R03=1.729e-07;R04=0.4467;R05=0.2444;
Rcli=0.75*(R01+R02+R03+R04+R05);
t=linspace(0,26*pi);
T=t+233.15;
Tchange=80.*abs(sin(t))+233.15;
k=exp(52945.916/(8.314*298.15))*exp(-52945.916.\(8.314.*T));
kchange=exp(52945.916/(8.314*298.15))*exp(-52945.916.\(8.314.*Tchange));
I=k;
Ichange=kchange;
R=I.*Rcli;
Rchange=Ichange.*Rcli;
plot(t,R,'-.r','linewidth',2);
hold on;grid on;box on;
plot(t,Rchange,':','linewidth',2);
set(gca,'XTick',[0:10:90],'FontSize',14);
set(gca,'YTick',[1.885e9:0.005e9:1.915e9],'FontSize',14);
xlabel('Time', 'FontSize',14);ylabel('Decomposition Rate', 'FontSize',14);
```

Listing 8 Code for sensitivity analysis of the model to humidity by Matlab

```
clear
clc
R01=0.2797;R02=0.4291;R03=0;R04=0.4467;R05=0.2444;
T=linspace(233.15,313.15);
k=exp(52945.91608/(8.314*298.15))*exp(-52945.91608.\(8.314.*T));
R1=0.5*(R01+R02+R03+R04+R05).*k;
R2=0.75*(R01+R02+R03+R04+R05).*k;
R3=0.75*(R01+R05).*k+0.5*(R02+R04).*k;
R4=0.75*(R02+R05).*k+0.5*(R01+R04).*k;
hold on;grid on;box on;
plot(T,R1,'--','linewidth',1);plot(T,R2,'--','linewidth',1);
plot(T,R3,'--','linewidth',1);plot(T,R4,'--','linewidth',1);
set(gca,'XTick',[230:10:321],'FontSize',14);
set(gca,'YTick',[1.2e9:0.1e9:2e9],'FontSize',14);
xlabel('Temperature', 'FontSize',14);
ylabel('Decomposition Rate', 'FontSize',14);
legend('Case 1','Case 2','Case 3','Case 4','FontSize',10);
```