评分细则：

1. $f(x,y)=\frac{n!}{(i-1)!(j-i-1)!(n-j)!}(F(x))^{i-1}(F(y)-F(x))^{j-i-1}(1-F(y))^{n-j}f(x)f(y)$ +2

$F(x|\lambda)=1-e^{-\lambda x}$ +1

则 $f(x,y)=\lambda^2 n(n-1)(e^{-\lambda x}-e^{-\lambda y})^{n-2}e^{-\lambda(x+y)}1\{0\le x\le y\}$ +1

令 $R=X_{(n)}-X_{(1)}\ge 0$，$Z=X_{(1)}\ge 0$ $\Longrightarrow$ $\frac{\partial(X_{(1)},X_{(n)})}{\partial(R,Z)}=-1$ +2(变换)

则 $f(r,z)=\lambda^2 n(n-1)(e^{-\lambda z}-e^{-\lambda(r+z)})^{n-2}e^{-\lambda(r+2z)}1\{r\ge 0, z\ge 0\}$ +1(代入)

则 $f(r)=\int f(r,z)\,dz=\int_0^\infty \lambda^2 n(n-1)e^{-\lambda(n-2)z}e^{-2\lambda z}dz\,(1-e^{-\lambda r})^{n-2}e^{-\lambda r}$ +2(计算)

$=\lambda(n-1)e^{-\lambda r}(1-e^{-\lambda r})^{n-2}1\{r\ge 0\}$ +1(结果)

2. (1). 设 $X_i=\begin{cases}1 & \text{是废品}\\ 0 & \text{otherwise}\end{cases}$ +1(模型转化) 则 $f(\vec{x}|p)=(1-p)^n\exp\left(\sum_{i=1}^n X_i\log\frac{p}{1-p}\right)$ +1(表达式)

是指数族，+1 令 $\theta=\log\frac{p}{1-p}$ 知自然参数空间有内点 +1

故 $T=\sum_{i=1}^n X_i$ 完全 $\Longrightarrow$ $\frac{1}{n}\sum_{i=1}^n X_i$ 亦完全.

由因子分解定理 $\Longrightarrow$ T 为充分 $\Longrightarrow$ $\frac{1}{n}T$ 充分. +1

再由 $\frac{1}{n}ET=p$ 知无偏性 +2

(2). $T\sim B(n,p)$ $P(\text{接收})=P(T\le 2)$ +2

则估计 $\hat{p}=\frac{1}{n}\sum_{i=1}^n X_i$ $\hat{P}(\text{接收})=(1-\hat{p})^n+n\hat{p}(1-\hat{p})^{n-1}+\frac{n(n-1)}{2}\hat{p}^2(1-\hat{p})^{n-2}$ +1

3. $f(\vec{x}|\theta,\mu)=\theta^{-n}\exp\left(-\frac{1}{\theta}\sum_{i=1}^{n}X_i+\frac{n\mu}{\theta}\right)\mathbb{1}\{X_{(1)}\geq\mu\}:=L(\theta,\mu|\vec{x})$ 不写扣1分. +2

θ固定时是 $\mu\uparrow$ +1(理由1分) $\Longrightarrow \hat{\mu}_{MLE}=X_{(1)}$. +2

$l(\theta,\mu|\vec{x})=-n\log\theta-\frac{1}{\theta}\sum_{i=1}^{n}X_i+\frac{1}{\theta}n\mu.$

$\frac{\partial l}{\partial\theta}=-\frac{n}{\theta}+\frac{1}{\theta^2}\sum_{i=1}^{n}X_i-\frac{n\mu}{\theta^2}=0$ +1

$\Longrightarrow \hat{\theta}=\bar{X}-\hat{\mu}_{MLE}$ +1 $\bar{X}=\frac{1}{n}\sum_{i=1}^{n}X_i.$

$\frac{\partial^2 l}{\partial\theta^2}=\frac{n}{\theta^2}-\frac{2}{\theta^3}\sum_{i=1}^{n}X_i+\frac{2n\mu}{\theta^3}$ +1

$\frac{\partial^2 l}{\partial\theta^2}\Big|_{\hat{\theta}}=\frac{-n}{(\bar{X}-\hat{\mu})^2}<0$ +1 $\Longrightarrow$ 确为极大值点

综上 $(\hat{\theta},\hat{\mu})_{MLE}=(\bar{X}-X_{(1)},X_{(1)})$

+1