Objective : Transform image coordinates to rover coordinates

I = Image Frame
R = Rover Frame

Final Frame Relationship
The Rover frame is located in the center of the width (# of image cols divided by 2) and at the bottom the image (# of image rows).

Let ${}^{I}\overrightarrow{P}_{RORG}$ represent a vector in the Image Frame that extends from the origin of the image frame to the origin of the Rover Frame.

$$ {}^{I}\overrightarrow{P}_{RORG} = (image\ cols/2)\hat{X}_I + (image\ rows)\hat{Y}_I $$

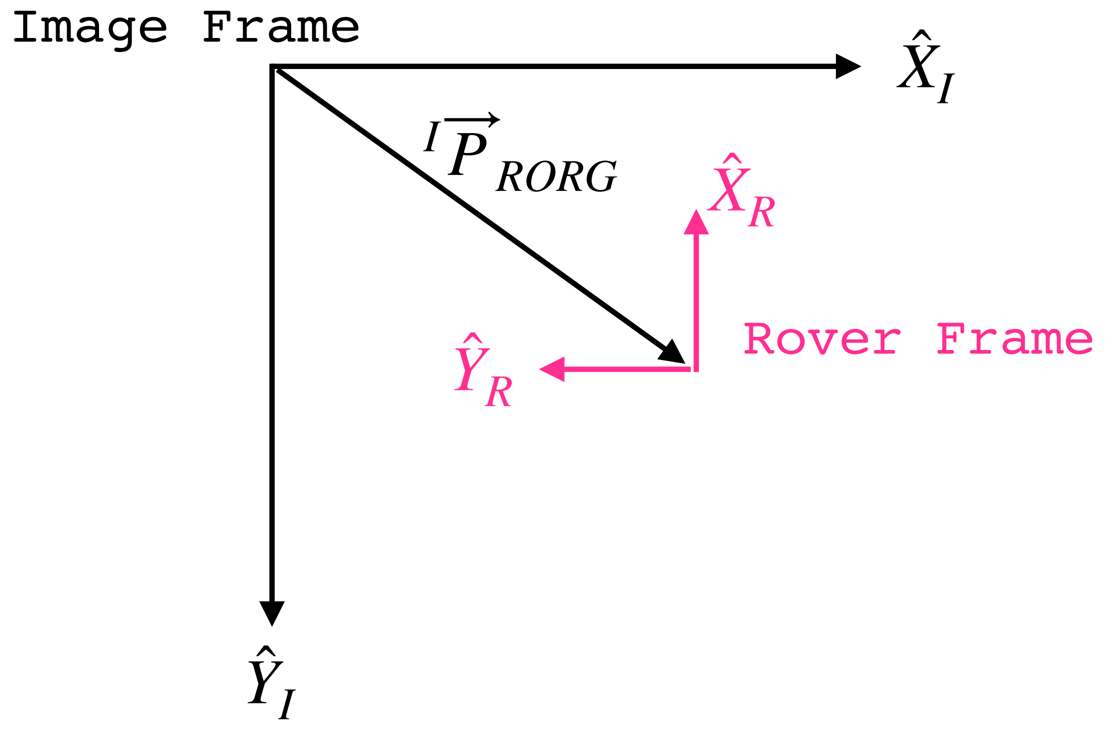
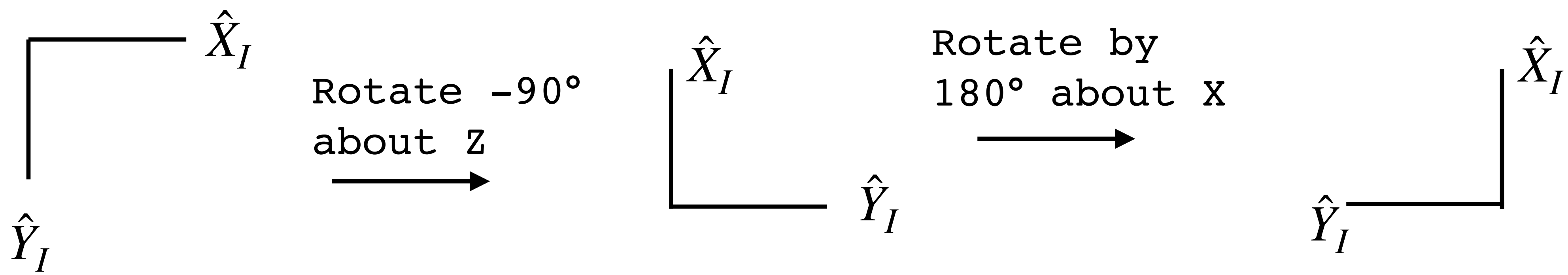


Image To Rover Transformation

The Rover Frame can be achieved by rotating the image frame -90° about the z-axis. Note for right hand system z is into page. Then rotate the result by 180° about X.



We now have the coordinate axes in the correct orientation and need to account for the translation of the Rover Frame origin.

The transformation from the Image Frame to the Rover Frame can be written:

$$ {}^{R}\overrightarrow{P} = {}^{R}_{I}T\,{}^{I}\overrightarrow{P} $$

which can be interpreted as:

$$ {}^{R}\overrightarrow{P} = a \text{ vector } P \text{ in Rover Frame} $$

$$ {}^{R}_{I}T = \text{Transformation from } I \text{ to } R $$

$$ {}^{I}\overrightarrow{P} = \text{vector } P \text{ in Image Frame} $$

$$ {}^{R}_{I}T = \begin{bmatrix} {}^{R}_{I}R & & & {}^{I}P^{T}_{RORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} $$

(${}^{I}\overrightarrow{P}_{RORG}$ *is shown on previous page.*)

$$
{}^{R}_{I}T = \begin{bmatrix} {}^{R}_{I}R & & & {}^{I}P^{T}_{RORG} \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{R}_{I}R = Rotation\ \ Matrix
$$

The rotation matix represents both of the rotations described above:

$$
{}^{R}_{I}R = R_z(-90^\circ)\ R_x(180^\circ)
$$

$$
= \begin{bmatrix} c(-90) & -s(-90) & 0 \\ s(-90) & c(-90) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(180) & -s(180) \\ 0 & s(180) & c(180) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}
$$

$$
{}^{R}_{I}R = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}
$$

So…

$$\begin{bmatrix} {}^R_I R & & & {}^I P^T_{RORG} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & cols/2 \\ -1 & 0 & 0 & rows \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and.. ${}^R\overrightarrow{P} = {}^R_I T\, {}^I\overrightarrow{P}$

$$= \begin{bmatrix} 0 & -1 & 0 & cols/2 \\ -1 & 0 & 0 & rows \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^I P_x \\ {}^I P_y \\ {}^I P_z \\ 1 \end{bmatrix}$$

Resulting in:

$${}^R P_x = -{}^I P_y + cols/2$$

$${}^R P_y = -{}^I P_x + rows$$