

# Difficulty Control for Blockchain Systems

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## Abstract

## TODO

*Keywords:* Blockchain, Decentralized consensus, Peer-to-peer networks, Proof-of-Work

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## 1. Introduction

Blockchain systems have attracted a growing amount of interest from various communities after publication of Bitcoin whitepaper [1] in 2008. Bitcoin security relies on the distributed protocol that maintains the blockchain, called mining, in which network nodes tries to solve computational puzzle. Other blockchain systems may relies on different computational puzzles [?] or even on virtual mining [?], while all of them use some algorithm that changes puzzle difficulty dynamically. This algorithm for retargeting the difficulty is required to make blockchain system predictable and fix latency between blocks.

Fixed latency between blocks is important for several reasons. Too often blocks leads to situation, when for a lot of miners block propagation time become bigger, then latency between blocks. This leads to significant increasing of number of blockchain forks that complicates the consensus[2] and reduce effective hash rate in blockchain system. On the other hand, increasing of the latency between blocks leads to decreasing of the network throughput[3] and may be critical for high-loaded blockchain systems like bitcoin, where blocks are already 70% full today[4]. Increasing latency by 50% in bitcoin network will

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mean, that some transactions will be never included into blockchain. Moreover this will lead to infinite growth of unconfirmed transactions pool, meaning it is likely that most bitcoin transactions will not even relay, much less confirm.

Most of blockchain systems relies on quite a naive difficulty retargeting algorithm, that assumes, that total computational power, involved in mining process, don't change over time. Using more complicated retargeting algorithms with incorrect assumptions[5] may lead to incorrect time interval between blocks even for simple case of constant hash rate as, for example, observed in Nxt, where observed mean time between blocks is 2 times bigger, then expected in whitepaper[6]. Moreover, too often difficulty recalculation leads to wide distribution of time intervals between blocks and makes blockchain system unpredictable[5]. Varying network computational power makes this algorithms inefficient for difficulty recalculation, e.g. continuous growth of computational power leads to decreasing mean latency between blocks and average block time in Bitcoin network is 1.07 times lower, then expected. Noteworthy, that exponential growth of computational power, which is the situation observed in practice in accordance with Moores law[7], is the absolutely worst case (regarding the maximal block rate) possible for Bitcoins difficulty retargeting algorithm[8]. On the other side, target recalculation algorithm should be easy enough and use integer arithmetic for all computational steps, since all nodes in the peer-to-peer network have to agree absolutely on the calculated difficulty.

Original Bitcoin white-paper, states that the security of the system is guaranteed as long as there is no attacker in possession of half or more of the total computational power used to maintain the system [1]. Most of the models used in the literature to discuss double-spending attacks assume that mining difficulty is constant [? ]. However difficulty is not constant, and can be manipulated by the attacker. The Difficulty Raising Attack, introduced in [9], enables the attacker to discard n-depth block, for any n and any attacker hash power, with probability 1 if he<sup>1</sup> is willing to wait enough time. The fact that there is no way

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<sup>1</sup>For simplicity we choose to adapt the male form. This was chosen by flipping a coin.

to determine whether a block have been computed on its declared time or not, have been used as part of other attacks [10, 11]. In section 2 we introduce new way of attack for blochcking systems, that manipulates difficulty for decreasing effective hash rate, required for block generation.

New researches of difficulty control proposes better functions for difficulty recalculation. For example paper [8] introduces target recalculation function, designed to work perfectly not just for constant hash rate but also if the hash rate grows exponentially (with a constant but unknown rate). Since it's good for situation, observed in practice in Bitcoin network, there are still a lot of open questions for future research. Is it possible to create such a function, suitable for random fluctuations in the hash rate? Is it possible to create such a function, simple enough to use integer arithmetic for all computational steps? Is it possible to create such a function, stable for attackers manipulations?

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## 2. Bitcoin Mining

First, let us briefly describe how the mining process works. The original concept is described in section 4 of the Bitcoin whitepaper [1], and discussed in plenty of papers [8, 12, 13]. We refer to the basic unit of mining work in Bitcoin as a scratch-off puzzle (SOP). Miner generates candidates to the SOP by iterating thought *nonse* and calculating SHA-256 hash of a blocks header. For a block to be valid it must value hash value less than the current *target* that is which is usually expressed in terms of the network difficulty  $D$  as  $\frac{1}{D}$ . Thus, each hash attempt yields a valid block with probability  $\frac{1}{D}$  and block frequency is  $\frac{R}{D}$  where  $R$  is effective network hash rate.

Every  $M$  blocks ( $M = 2016$  for Bitcoin) difficulty is recalculated as

$$D_{i+1} = D_i \cdot \frac{MT}{S_m} \quad (1)$$

where  $T$  is expected time interval between blocks and  $S_m$  is actual time, it took to generate  $M$  blocks. Real time ( *9min20sec*) interval is less, then desired time

interval  $T = 10min$  because of continuous growth of network computational power. Difficulty recalculation interval  $M = 2016$  has been chosen in such a way to recalculate difficulty every 2 weeks, while for real network it is a bit smaller. This time interval is big enough to see increasing computational power of the network: right after target recalculation block time is close to desired 10 minutes, whereas at the end of *epoch* it's less than 9 minutes (see figure 1).

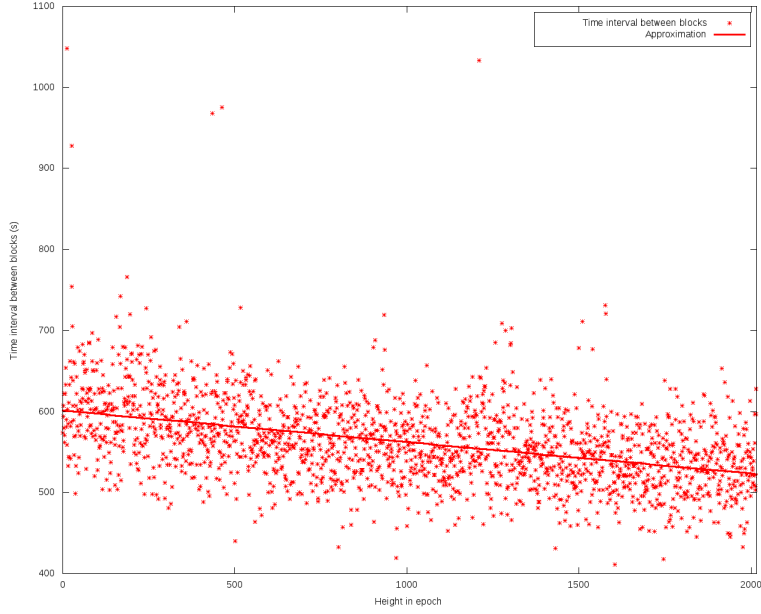


Figure 1: Average block time between difficulty recalculation

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### 3. The Difficulty Manipulating Attack

On this section we show a new way of manipulating difficulty with profit to an attacker. The main idea is very simple. The attacker may disable his mining, and wait for the next difficulty recalculation. New difficulty will be lower, cause it don't include computational power of the attacker, and he turns on his mining again. After next difficulty recalculation (which will include the attacker computational power) he turns off his mining again, and wait for next difficulty

recalculation. Such a simple algorithm allows him to mine during epochs with low difficulty, and don't mine during epoch with the low one, that will decrease computational resources expended for block generation. Of course in such a situation the attacker will create less blocks, that in situation, when he mine all the time. However, lows of economy dictate that the cost of computational resources invested into mining should be around the expected reward, and attackers profit, gained from every block may overbalance his wastes caused by decreasing number of created blocks. Moreover, in a situation with a lot cryptocurrency forks he is able to mine other forks in epoches with big difficulty and gain more blocks with more profit from each, that in situation, when he mine single cryptocurrency permanently.

#### **TODO subsection?**

Let's calculate attacker profit in circumstances of Bitcoin difficulty recalculation function and constant hash rate (which is natural for this function). Assume the situation when the network has some hash rate  $R_0$  provided by honest miners and the attacker has additional hash rate  $R_a = R_0 \cdot p$  that it can turn on or off according to its purposes. Usual miner with such a power will mine all the time with difficulty  $D_0 = (R_0 + R_a) \cdot T$  and will mine  $\frac{M \cdot R_a}{R_0 + R_a}$  blocks per epoch ( $M$  blocks) in average spending  $M \cdot R_a \cdot T$  computational power for them. The attacker will start mining when difficulty  $D_0 = R_0 \cdot T$  is calculated from honest miners  $R_0$  only. He will mine  $\frac{M \cdot R_a}{2(R_0 + R_a)}$  blocks per epoch in average spending  $\frac{M \cdot T \cdot R_a \cdot R_0}{2(R_0 + R_a)}$  computational power for them. Consequently usual miner will spend  $(R_0 + R_a)T$  computational power per block, whereas attacker will just spend  $R_0T$  computational power per block. The cost of computational resources invested into mining should be around the expected reward. If  $B$  is block reward and  $C$  is hash calculation cost, the usual miner profit is

$$\left(\frac{M \cdot R_a}{R_0 + R_a}\right) \cdot B - M \cdot R_a \cdot T \cdot C = M \cdot R_a \cdot \left(\frac{B}{R_0 + R_a} - TC\right) \quad (2)$$

per epoch. At the same time attackers profit is

$$\frac{M \cdot R_a}{2 \cdot (R_0 + R_a)} \cdot (B - T \cdot R_0 \cdot C) \quad (3)$$

and attack become profitable when

$$\frac{B}{C} < T \cdot (R_0 + 2 \cdot R_a). \quad (4)$$

Note, that if attacker is able to switch between different forks his epoch profit is twice bigger and is bigger then regular miner profit at any  $B$  and  $C$ .

Remarkable, that under such an attack mean time between blocks will be

$$T_a = \frac{T}{2} \left( \frac{R_0 + R_a}{R_0} + \frac{R_0}{R_0 + R_a} \right) = T \left( 1 + \frac{p^2}{2(1+p)} \right) \quad (5)$$

which is strictly bigger then desired time  $T$ . Please notice that we regard  $p$  as the ratio between the hash-powers of the attacker and the honest network so it changes from 0 to  $\infty$ .

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#### 4. An Improved Difficulty Control

The difficulty-update algorithm employed by Bitcoin works as designed: If the hash rate is constant, it yields the desired block rate. However it does not achieve the desired block rate in other situations and vulnerable to attack, described in 3. In this section we are going to propose an alternative difficulty-update algorithm that works better, then Bitcoin's one.

First, let us describe properties of *ideal* difficulty update algorithm:

1. It should be resistant to known types of difficulty-update attacks.
2. It should lead to desired block rate for random fluctuations in the hash rate.
3. (Optional) It should be simple enough to use integer arithmetic for all computational steps.

Security is the most important feature of blockchain systems and should be regarded with the highest priority. Incorrect block rate is not considered a big problem in the Bitcoin community, but it may be important for more advanced

applications of blockchain systems. Implementation of the *ideal* difficulty update algorithm in subclass of integer programming is desired for different platforms compatibility. This rule is not required, because, as mentioned in [8], it's possible to include non-integer algorithm parameters as part of the block, but it provides another way of difficulty manipulating to an attacker.

## 5. Simulations

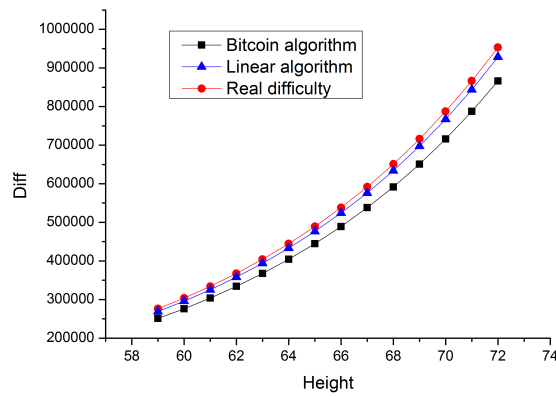


Figure 2: Real difficulty (red) and difficulties calculated from bitcoin (black) and linear (blue) algorithms in situation of exponential hash rate growth

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## 6. Conclusions

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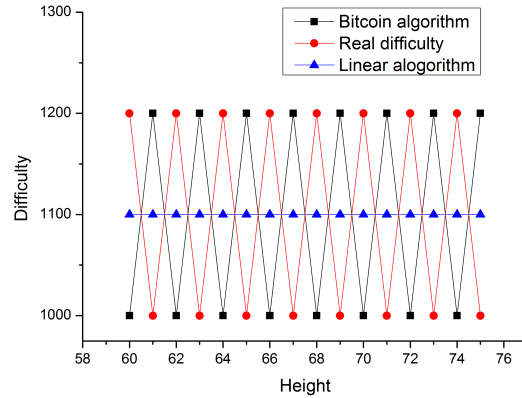


Figure 3: Real difficulty (red) and difficulties calculated from bitcoin (black) and linear (blue) algorithms in situation of attack, described in section ??sec:attack)

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