Section 9: Electrostatic Propulsion

AE435

Spring 2018

3 Electrospray/Colloid Thrusters

Contents

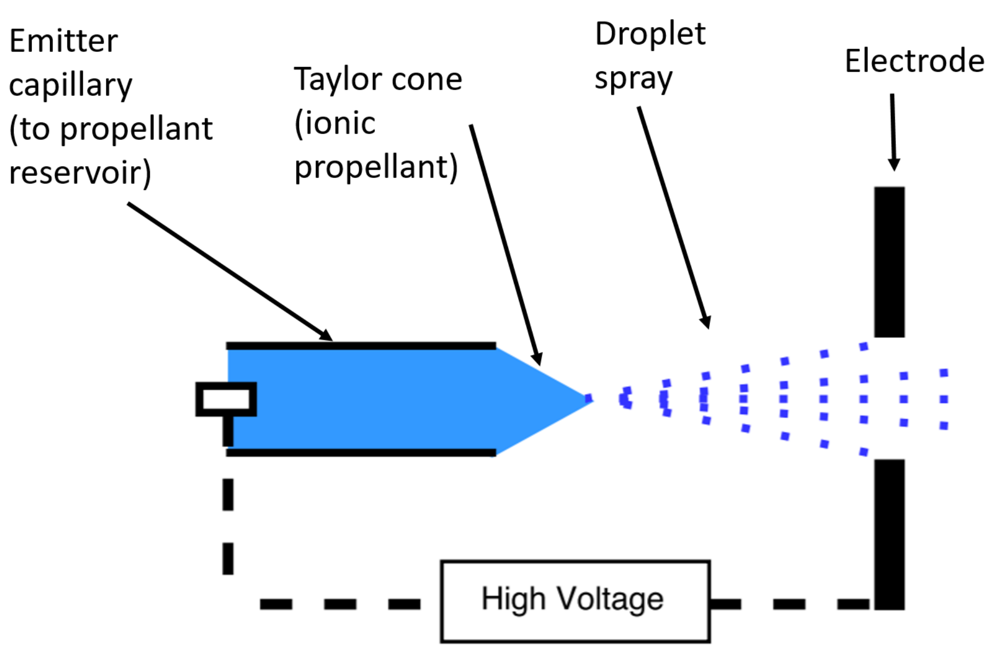
Electrospray/colloid propulsion different from ion thruster and HET, but still electrostatic propulsion. Main difference = liquid propellant, not noble gas (xenon).

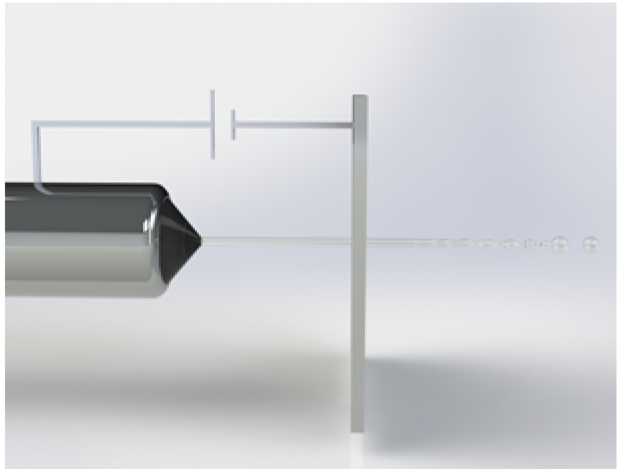
In electrospray/colloid propulsion charged particles are extracted from a liquid and accelerated to high speed. These particles are generally not individual atoms (not Xe+ or Li+), but are typically molecular species (e.g., 1-Ethyl-3-methylimidazolium a.k.a. Emim+ or BF4- tetrafluoroborate) and cluster/droplets of those species (e.g., many hundreds of molecules bonded together within a droplet, which likely has charge-state ¿1)

Three main types of technologies, based on the type of liquids and type of emission:

1. Colloid thruster - accelerates and expels charged droplets, and uses solvents such as glycerol and formamide as propellant
2. Field emission electric propulsion (FEEP) - uses liquid metals (Cs and In) to emit positively charged metal ions
3. Ionic liquid ion sources (ILIS) - room-temperature molten salts (i.e., ”ionic liquids”) used to create ion beams (individual molecular ion emission) or mixtures of ions with droplets. No need to ionize, free charges already present in the liquid.

Electrospray well-known in chemistry and materials analysis community. Electrospray ionization used in mass spectrum analysis to study liquids. Ions/droplets extracted from liquid and mass spectrum analyzed, usually for bio-related materials, e.g., proteins, prescription drugs.





3.0.1 Motivation for Electrospray Propulsion

High thrust-density.

Remember we are converting electric potential energy to kinetic energy.

$$q\,V_{acc} = \frac{1}{2}Mv^2 \quad \rightarrow \quad V_{accel} = \frac{Mv^2}{2q} \tag{9.79}$$

Thrust can be written as ...

$$T = \dot{m}_i\,v_i = \frac{I_b M}{e}\sqrt{\frac{2eV}{M}}$$

And with the beam current that is space charge limited, running the electrospray at it maximum possible current...

$$I_{b,scl} = \frac{4}{9}\,\varepsilon_o\sqrt{\frac{2e}{M}}\,\frac{V^{\frac{3}{2}}}{d^2}A$$

Then we can write thrust per unit area as...

$$\frac{T}{A} = \frac{8}{9}\,\varepsilon_o\left(\frac{V}{d}\right)^2 \tag{9.80}$$

If we want higher thrust density, we have to have a higher voltage soooo we look at 9.79. If we want higher voltage we can have higher M or higher v, exit velocity which is related to ISP which is related to the delta V. So we can think in equation 9.79, v is set by whatever mission we want to do. We must increase M/q then, this is all we can change. In other words....

Also remember the mission delta-v will dictate the optimum particle velocity, v, so v = constant for given mission. To get higher thrust-density (more thrust in smaller package) want higher voltage per (9.80). Then per (9.79) higher voltage requires higher mass-to-charge (m/q) ratio. Hence why we want singly-charged xenon in ion thruster (131.1 amu, or why early ion thrusters used mercury

201 amu, and why Hall thrusters have experimented with Bismuth 209 amu, higher thrust density). So instead of emitting atomic ion, emit polyatomic ion (e.g., Emim+ 146 amu/q for single ion, or droplet with thousands of polyatomic molecules, m/q huge!, for example droplet m/q values of 1024 amu/q)

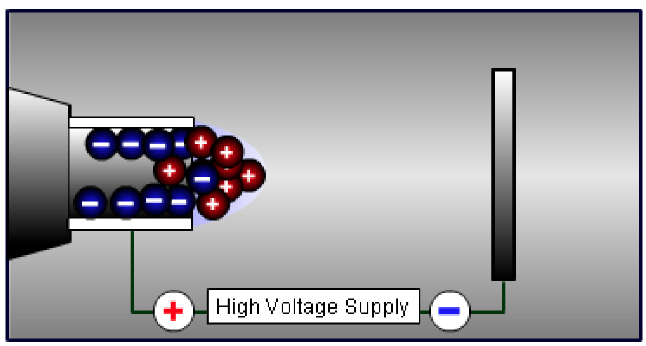
Also, higher accelerating voltage reduces cost-of-ion voltage losses, as we saw in (9.35):

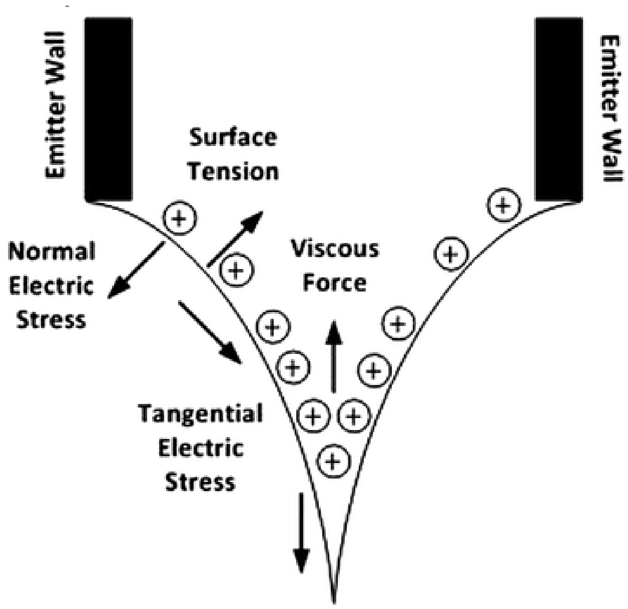
$$\eta = R = \frac{V_b}{V_s + |V_a|} = \frac{V}{V + V_{loss}} \tag{9.81}$$

Would like V_a equal zero. (remember, accelerator grid voltage is below space potential, doesn't help accelerate the ions, below space potential to keep electrons from backstreaming).

So, higher m/q means higher voltage, less voltage loss (better efficiency), and higher thrust density. BUT, don't want the voltage too high (No M/q to large)! For example, for mission requiring 1000 sec Isp, an m/q of 1024 amu/q means 100kV accelerating voltage! Yikes!

So the potential benefits are high-thrust density, low thrust, small packaging/volume, and these types of propulsion have been explored mainly most recently for small/CubeSats.





3.1 Liquid Surface Physics

We are interested in how a liquid surface responds to an applied strong normal electric field, E_o.

First consider the interface between vacuum and a conductive liquid (liquid with free charges, ions in it).

Figure:

Gauss' Law $\rightarrow$

$$\sigma = \varepsilon_o \, E_o \tag{9.82}$$

$$\int\int_S \vec{E} \cdot \mathrm{d}\vec{A} = \frac{Q}{\varepsilon_o} = \frac{\sigma \, A}{\varepsilon_o}$$

But what if it's a dielectric liquid, with no free charges? Will get polarization. The molecules of the liquid are now polarized (i.e., modeled as dipoles), and we need to account for the electric field that will be due to these dipoles, the net Efield. That is, the Displacement field? Eqn. (2.34). Now:

Figure:

$$\nabla \cdot \vec{D} = \rho_{free} = 0$$

$$\vec{D} = \varepsilon \, \vec{E} = \varepsilon_r \, \varepsilon_o \, \vec{E}$$

where Relative permittivity (i.e., dielectric constant), we called it K in (2.42), or ε_r. So...

$$\varepsilon \, \varepsilon_o \, E_o - \varepsilon \, \varepsilon_i \, E_i = \sigma_{free} = 0$$

$$\varepsilon_o \, E_o - \varepsilon \, \varepsilon_i \, E_i = \sigma_{free} = 0 \tag{9.83}$$

Also:

$$\nabla \cdot \vec{E} = \frac{\rho_{free}}{\varepsilon_o} = \frac{1}{\varepsilon_o}\,\rho_{free} + \frac{1}{\varepsilon_o}\,\rho_p$$

$$\rho_{free} = 0 \rightarrow$$

$$\int\!\!\int \vec{E} \cdot \mathrm{d}\vec{A} = \frac{1}{\varepsilon_o}\,\rho_p = \frac{\sigma_p\,A}{\varepsilon_o} \tag{9.84}$$

$$\rightarrow \varepsilon_o\,E_o - \varepsilon\,\varepsilon_i\,E_i = \sigma_p$$

where polarization (or dipole) charge density is σ_p

Combine (9.83) and (9.84) to show

$$\sigma_p = (1 - \frac{1}{\varepsilon})\,\varepsilon_o\,E_o \tag{9.85}$$

$$E_i = \frac{1}{\varepsilon}\,E_o \tag{9.86}$$

Note, if the dielectric constant (relative permitivitty) is high, then (9.85) is the same as (9.82). The relationship between surface charge density and electric field is same for dielectric liquid and conductive liquid (if $\varepsilon \gg 1$). $\varepsilon \sim 80$ for typical electrospray liquids.

3.1.1 Relaxation time

The relaxation time is the characteristic time for charges or dipoles within the liquid to respond/adjust to a change in the applied electric field.

Conductive liquid with conductivity K, (Si/m). Efield is suddenly applied, so charges within the liquid begin to move toward surface, building up a surface charge density.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = k\,E_i \tag{9.87}$$

Relationship between applied field and the field in the liquid is (9.83) with free charge density not = 0, so:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} = k\left(\frac{\varepsilon_o\,E_o}{\varepsilon_o\,\varepsilon} - \frac{\sigma}{\varepsilon_o\,\varepsilon}\right)$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}t} - \frac{k\sigma}{\varepsilon_o\varepsilon} = \frac{k\,E_o}{\varepsilon} \tag{9.88}$$

Soln. of which for $\sigma(0) = 0$ is:

$$\sigma(t) = \varepsilon_o \, E_o \left(1 - \exp\left(-\frac{t}{T}\right)\right) \tag{9.89}$$

where relaxation time of charges in the liquid,

$$T = \frac{\varepsilon \, \varepsilon_o}{k} \tag{9.90}$$

Figure: Relaxation Time of Charges in Propellant

3.1.2 Stability of the Surface

Consider a surface with a small perturbation (a small ripple) in presence of applied field. The local Efield is concentrated at the peaks of the ripple, intensifying the surface charge density and corresponding Electric field at those peaks.

Figure: Electric Field at Peaks

The electric field is pulling the fluid up with force per unit area (pressure):

$$P_{\text{Efield}} = \frac{F}{A} = \frac{1}{2}\sigma E = \frac{1}{2}\varepsilon_o \, E^2 \tag{9.91}$$

The electric field pulling the fluid is counteracted by the surface tension of the fluid. But as the Efield and force at the ripple increases, the process can become unstable. So, we need to determine the Efield at the surface (and corresponding force due to the Efield from (9.91)), determine the surface tension restoring force, and see when does Electric force overcome surface tension and cause instability.

3.1.3 Efield at Surface

Assume sinusoidal surface ripple. Outside the fluid, the electric potential obeys Laplace eqn., with the potential going to zero at the surface. This potential near the vicinity of the surface can be written as superposition of applied field and a small perturbation.

$$a \tag{9.92}$$

The small perturbation varies sinusoidal in x-direction, and decays exponentially in y-direction. Then Electric field at surface is:

$$a \tag{9.93}$$

for and on the crests of the ripple, where the Efield is strongest Finally, recognize we are interested in the change in the Electric force (pressure) on the charged liquid surface due to the small ripple/perturbation. That is, with (9.91) and (9.93):

$$a \tag{9.94}$$

3.1.4 Counteracting Surface Tension

Force (or pressure) due to surface tension is:

Force Due to Surface Tension:

$$\frac{F}{A} = P_{\text{surface tension}} = \gamma \left(\frac{1}{R_x} + \frac{1}{R_y} \right) \tag{9.95}$$

Where

- R_x and R_y are the radii of curvature of the surface along the x and y-directions for a two dimensional surface (would be x and z-directions in the ripple figure above)
- γ Is the surface tension of the fluid (a fluid property)

For a cylindrically symmetric surface, this becomes:

$$P_{\text{surface tension}} = \gamma \frac{2}{R_c} \tag{9.96}$$

This surface curvature is related to the second derivative of the surface of the fluid.

$$\frac{1}{R_c} \approx \left|\frac{\partial^2 y}{\partial x^2}\right|$$

We need a relationship between x and y at the surface. Recognize that the surface is an equipotential line with zero potential at the surface (9.92) $\rightarrow$

$$a \tag{9.97}$$

Then the surface tension storing force is:

3.1.5 Instability

Instability results if the force on the surface due to electric field overcomes the restoring force of the surface tension. That is (9.94) > (9.97):

$$a \tag{9.98}$$

Wavelength of the ripple where So for long wavelength ripples, a weaker electric field can cause instability. Similarly, liquids with lower surface tension require weaker electric field to induce instability. In electrospray we are generally working with capillary (small tubes with diameter D), that have a corresponding longest wavelength of 2D, thus:

$$a \tag{9.99}$$

Consider a new multi-mode propellant consisting of HAN and [Emim][EtSO4] with surface tension of 45 mN/m being electrosprayed from a D = 100 micron capillary. The required Efield is 1.26×10^7 V/m very high! (parallel plates separated by 5mm would require potential of 63kV!) But this analysis assumed uniform applied E across large surface of liquid. In reality, the Efield is applied between a downstream extractor plate, and the tip of a needle/emitter. The Efield concentrates at the tip of the needle/emitter where the fluid surface is. So, in reality lower voltages $\sim$1-3kV are possible to create these large Efields for stability. A more detailed analysis of this extractor-needle/emitter geometry follows.

3.2 Taylor Cone

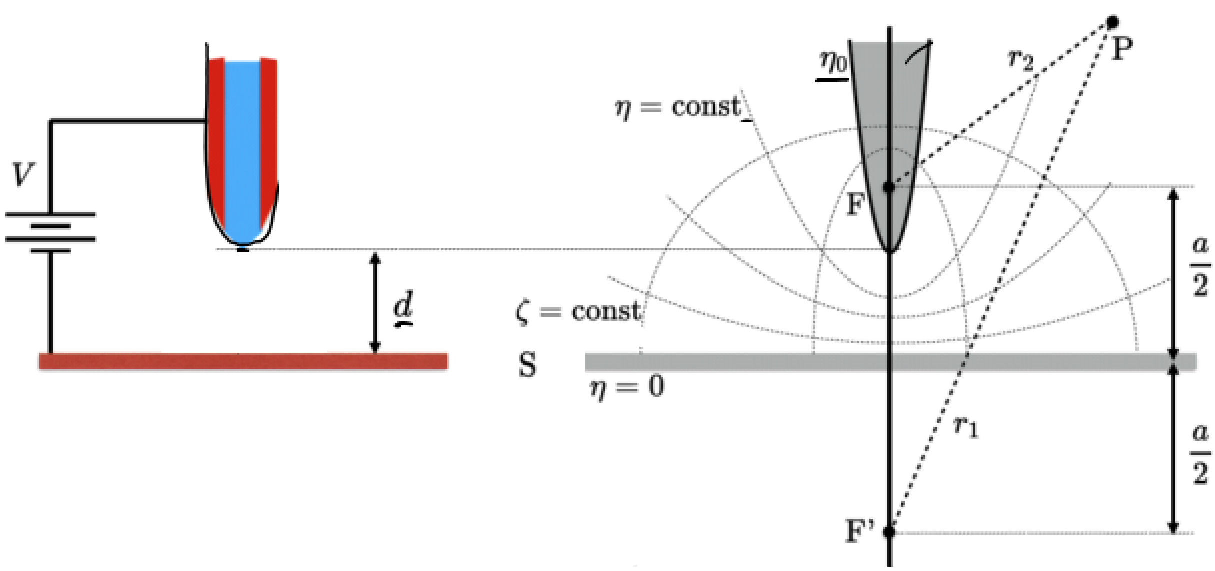
3.2.1 Electric Potential Profile/Structure

Consider an emitter-extractor setup. What would we expect the potential profile to look like?

Figure: Electric Potential Field Lines

3.2.2 Voltage to Cause Instability

To better model the Efield/electric potential structure in a emitter-extractor geometry, we use Prolate Spheroidal Coordinates.



Here

$$\eta = \frac{r_1 - r_2}{a} \quad , \quad \xi = \frac{r_1 + r_2}{a} \tag{9.100}$$

and conversion from cartesian-to-prolate coordinates is:

$$r_1 = \sqrt{x^2 + y^2 + \left(z^2 + \frac{a}{2}\right)^2}$$

$$r_2 = \sqrt{x^2 + y^2 + \left(z^2 - \frac{a}{2}\right)^2} \tag{9.101}$$

- Lines of constant η confocal hyperboloids (with foci at F, F')
- Lines of constant ξ are confocal ellipsoids (with focis at F, F')
- "confocal" means having the same foci.
- S is plane of symmetry (η=0)
- We choose a surface (η=η_o) to represent the surface of the capillary with liquid tip.

$$\eta_o = \frac{2d}{a}$$

Why use such a weird coordinate system?

Because the η surfaces are approximately equipotential surfaces. That is, each η surface corresponds to a different electric potential, so we can use this geometry to solve for the electric potential and therefore electric field within this geometry.

If the potential on the emitter/liquid (η=η_o) is assumed to be V (volts), and the S plane (η=0) is ground (V=0). Relation for η_o in this coordinate system is:

$$\eta_o = \frac{2d}{a} \quad \text{or} \quad \eta = \frac{2z}{a} \tag{9.102a}$$

where d and a are the dimensions in the figure above. Then the potential on lines of constant η within the gap is:

$$\phi(\eta) = V\,\frac{\tanh^{-1}\eta}{\tanh^{-1}\eta_o} \tag{9.102b}$$

The resulting Efield at the tip is:

Electric Field at the Tip

$$E_{\text{tip}} = \frac{-\frac{2V}{R_c}}{\ln\left(\frac{4d}{R_c}\right)} \tag{9.103}$$

assuming d≫Rc Where

$$\begin{aligned} R_c &= \text{Radius of Curvature of the Tip, } \approx \text{ radius of needle} \\ V &= \text{Capillary/Emitter Voltage} \\ d &= \text{Gap Distance} \end{aligned}$$

Finally, for the liquid-vacuum/gas interface to become unstable, we require electric pressure (traction or force per unit area, that is, pressure created by electric field force) on the surface to be greater than surface tension (pressure of surface tension).

$$\frac{1}{2}\varepsilon_o E^2 > \gamma \frac{2}{R_c}$$

which with (9.103) we find that the voltage on the emitter/needle (with grounded extractor plane S) is:

$$V > \sqrt{\frac{R_c \gamma}{\varepsilon_o}} \ln\left(\frac{4d}{R_c}\right) \tag{9.104}$$

where

R_c is radius of curvature of the surface, which is approximately the radius of the capillary/hypodermic needle emitter.

Again considering a new multi-mode propellant consisting of HAN and [Emim][EtSO4] with surface tension of 45 mN/m being γ electro-sprayed from a 100 μm capillary with extractor located 5 mm downstream. Rc = 100/2=50 μm, d = 5 mm, we get starting voltage must be greater than 3020 Volts. If extractor ground plane is only 2.5mm downstream, this becomes 2209 V.

For a 50micron capillary with extractor 1mm downstream → 1800V. Our experiments with this exact geometry required 1800-2000V.

3.2.3 Resulting Conical Surface Structure - Taylor Cone

Once the electric field and corresponding force overcomes the surface tension, we expect the shape of the liquid-vacuum interface to change from spherical to conical:

Figure: Conical Taylor Cone

This new conical surface shape was investigated and explained analytically by G.I. Taylor in 1965, hence it's typically referred to as a "Taylor" cone or Taylor's cone.

In this new conical geometry, we still have the requirement that the electric force on the surface (in normal direction!) must be balanced by the surface tension.

So we need to explore the surface tension and electric field/force in this new conical geometry.

Surface tension (9.95) requires we know the radius of curvature of the surface. Along the generator of the cone (the line that forms its surface), the curvature is zero. But perpendicular to the generator it's non-zero. We can find this curvature perpendicular to the generator using Meusnier's Theorem:

PROPOSITION 2 (Meusnier). *All curves lying on a surface* S *and having at a given point* p $\in$ S *the same tangent line have at this point the same normal curvatures.*

The above proposition allows us to speak of the *normal curvature along a given direction at p*. It is convenient to use the following terminology. Given a unit vector $v \in T_p(S)$, the intersection of S with the plane containing v and $N(p)$ is called the *normal section* of S at p along v (Fig. 3-9). In a neighborhood of p, a normal section of S at p is a regular plane curve on S whose normal vector n at p is $\pm N(p)$ or zero; its curvature is therefore equal to the absolute value of the normal curvature along v at p. With this terminology, the above

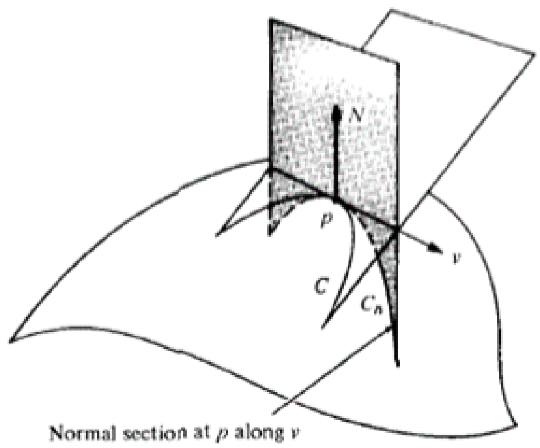


Figure 3-9. Meusnier theorem: C and C_n have the same normal curvature at p along v.

$$R = R_c \cos\theta$$
$$R = r\sin\theta$$

so

$$\frac{1}{R_c} = \frac{\cos\theta}{R} = \frac{\cos\theta}{r\sin\theta} = \frac{\cot\theta}{r} \tag{9.105}$$

$$\frac{1}{2}\varepsilon_o E_n^2 = \gamma\frac{1}{R_c} = \gamma\frac{\cot\theta}{r}$$

$$E_n = \sqrt{\frac{2\gamma\cot\theta}{\varepsilon_o r}} \tag{9.106}$$

What type of electric potential profile will give us an Efield that varies as (9.106), that is, falls off as $\sqrt{\frac{1}{r}}$?

Well, to calculate the potential outside the cone, we would use Laplace Eqn. (2.22) because there is no charge outside the cone. And the cone itself would be an equipotential surface. Further, we may choose to work in spherical coordinates (r, θ, ϕ).

So we would solve Laplace Eqn. (2.22) in spherical coordinates. This equation admits solutions that are Legendre functions of either the first (P_n) or second (Q_n) kind. When n is a positive integer these are referred to as "Legendre polynominals".

$$\phi \propto P_n(\cos\theta)r^n \qquad \text{and/or} \qquad \phi \propto Q_n(\cos\theta)r^n$$

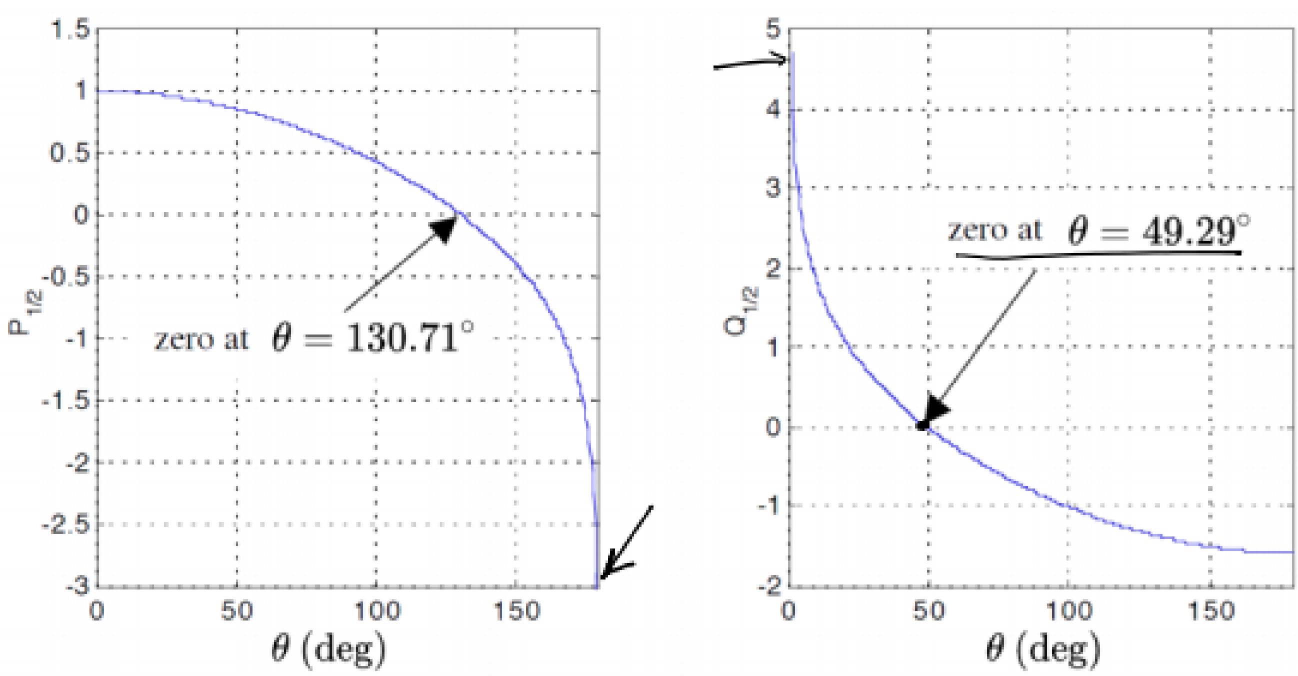
Want electric field, so

$$E_n = E_\theta = -\frac{\partial \phi}{\partial \theta} \propto \frac{\partial}{\partial \theta} Q_n \sin\theta r^{n-1}$$

And we know Efield must have a $\frac{1}{\sqrt{r}}$ dependence, so $n = \frac{1}{2}$.

Unfortunately, these are Legendre functions of "fractional order".

$$\phi \propto P_{\frac{1}{2}}(\cos\theta)r^{\frac{1}{2}} \qquad \text{or} \qquad \phi \propto Q_{\frac{1}{2}}(\cos\theta)r^{\frac{1}{2}}$$



The Legendre function of the 1st kind with fractional order 1/2 ($P_{\frac{1}{2}}$) has a singularity at pi (180°). This is outside the cone in free space where we are interested. The potential cannot go to infinity there... not good, can't choose this one. But the Legendre function of the 2nd kind with fractional order 1/2 ($Q_{\frac{1}{2}}$) has a singularity at 0 (0°). This is inside the cone, where our Laplace solution would no longer be valid (of course, it's inside the liquid where there are charges!)...so OK. Choose this one.

So we want

$$Q_{\frac{1}{2}} \cos\theta$$

which has a zero (potential equals zero when $Q_{\frac{1}{2}} = 0$, this is the liquid surface!) for an angle of 49.29°. So the liquid surface must have an angle of 49.29°.

This is the angle of emission cone, or "Taylor" cone.

But what happens at the tip of the cone?

As we move to the tip, $r \to 0$, what happens to the electric field (9.106)? Clearly something must give? $E_n \to \infty$.

A jet will issue from the cone's tip, giving rise to a flow rate and, since the surface being ejected is charged, a net current is also emitted. "Cone-jet" mode.