Section 4: Kinetic Theory

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3 Velocity Distribution Function

Not all particles move with the same velocity. Also, the velocity of a particle doesn't remain the same over time. We need a statistical way to describe this; this the velocity distribution function.

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3.1 Mass Distribution Function

To illustrate this idea of distribution function, consider mass density.

Figure 4

Consider a gas of N particles with mass m in a volume V.

The density is:

$$a \tag{1}$$

If the gas is nonuniform, the density in a differnetial volume

$$a$$

At a position vector

$$a$$

is

$$a \tag{2}$$

Note that this assumes that is large enough to contain a large number of particles. Since the particle mass doesn't change,

$$a$$

Which we can write as:

$$a \tag{3}$$

The function gives the number of particles per unit volume as a function of position; a.k.a. a "position distribution function"

The number of particles in the differential volume is

$$a$$

So the mass within that differential volume is

$$a$$

We can define a "normalized distribution function" as

$$a \tag{4}$$

So that the total number of particles in is

$$a \tag{5}$$

This normalized distribution function can be interpreted as a **probability density function**, that is, the probability that a given randomly-chosen particle will be in .

Integrating over the entire volume,

$$a$$

So the probability that a particle within V is within V is 100%.

We can generalize this idea to state:

A distribution function gives the concentration of some quantity per unit "volume" as a function of position in some kind of "space".

3.2 Velocity Distribution Function

Now consider particle with velocity

We can define a differential volume in this velocity space

Define local point density such that the number of particles within velocity range:

$$a$$

we would write that as . This is the "velocity distribution function". Like the position distribution function you have to multiply by a volume to get a real quantity (the number of particles).

Define a normalized velocity distribution function

$$a \tag{6}$$

This is the probability that a particle will be within the specific velocity range. The number of particles within is

$$a \tag{7}$$

In terms of number density

where the integral over all possible velocities is:

$$a$$

$$a \tag{8}$$

The velocity distribution of particles is important for determining average quantities. For instance, if we have some quantity Q that depends on velocity

The mean or expectation value of Q is then:

$$a \tag{9}$$

3.3 Maxwellian Velocity Distribution Function

A gas at equilibrium has a special velocity distribution function. Called the Maxwellian velocity distribution.

Basic idea is that:

- Stationary velocity distribution
- Collisions deplete and add to population at same rate
- Thus, no net change.

Collision dynamics with simple billiard-ball model leads to:

This is Maxwellian VDF.

$$a \tag{10}$$

where

$$a$$

We can break this into components along each axis

$$a$$

The probability that a particle will be within the velocity space

$$a$$

The Maxwellian VDF looks like this: Largest probability is at this corresponds to particles moving perpendicular to the with We can transform the Maxwellian VDF into a Maxwellian SPEED distribution function.

$$a \tag{11}$$

Note that the most probable SPEED is not zero.

$$a \tag{12}$$

Also, the mean speed is not the same as the mostprobable-speed.

$$a \tag{13}$$

Finally, the mean squared speed is

$$a$$

Which works out to give the root-mean-square speed.

$$a \tag{14}$$