Section 7: Electrothermal Propulsion

AE435

Spring 2018

2 Resistojets

Contents

Consider a small tube of diameter, D, and length, L. One end is connected to a propellant reservoir through a valve, and the other is attached to a nozzle. The tube is being heated by resistive Joule heating. A current, I, is being driven through the tube by a voltage/potential, V. Propellant is entering the tube with a constant mass flow rate, $\dot{m}$.

$$R = \frac{V}{I} \sim 1\Omega$$

Figure 9:

We seek a model to predict the temperature change of the liquid between the entrance of the tube and the exit.

Figure 10:

Simple model, $h = C_p T$

The Ohmic heating of the tube over a length dx is:

$$I^2 R \frac{\mathrm{d}x}{L} \tag{7.65}$$

The heating of the propellant over a length dx is:

$$\dot{m} C_p \mathrm{d}T \tag{7.66}$$

Equating these yields:

$$\frac{\mathrm{d}T}{\mathrm{d}x} = \frac{I^2 R}{\dot{m} C_p L} = \text{ constant} \tag{7.67}$$

Integrating (7.67) from $0 \to x$,

$$T_{out} - T_{in} = \frac{I^2 R}{\dot{m} C_p} \frac{x}{L} \tag{7.68}$$

Control the change in temperature with mass flow rate, $\dot{m}$, and power input, $P = I^2 R = I V$.

$$\frac{P}{\dot{m}} \quad \text{or} \quad \left[\frac{W}{kg/s}\right] \quad \text{or} \quad \left[\frac{J}{kg}\right]$$

We can also analyze the heat transfer between the wall and the fluid/propellant.

$$\mathrm{d}\dot{Q} = \mathfrak{H} A \Delta T = \mathfrak{H} (\pi D \mathrm{d}x)(T_{wall} - T) \tag{7.69}$$

Where $\mathfrak{H}$ is the heat transfer coefficient, $\left[\frac{W}{m^2 K}\right]$

The heat put into the system as a result of the joule heating on the tube we are applying is...

$$\mathrm{d}\dot{Q} = \frac{I^2 R}{L} \mathrm{d}x \qquad \text{(Ohmic heating in dx, Equation 7.65)}$$

Such that

$$(T_{wall} - T) = \Delta T_w = \frac{I^2 R}{(\pi D L) \mathfrak{H}} = \text{constant} \tag{7.70}$$

If we wanted to plot these we get...

Figure 11:

The ratio of convective heat transfer to conductive heat transfer is given by the Nusselt number:

Nusselt Number

$$N_{u_D} = \frac{\mathfrak{H}\, D}{k_f} \tag{7.71}$$

Where

$$\mathfrak{H} = \text{Convective Heat Transfer Coefficient} \qquad \left[\frac{W}{m^2\, K}\right]$$

$$D = \text{Diameter} \qquad [m]$$

$$k_f = \text{Thermal Conductivity of the Fluid} \qquad \left[\frac{W}{m\, K}\right]$$

For Laminar flow, convection with uniform surface heat flux for a circular tube yields:

$$N_{u_D} \cong 4.36$$