

Section 2: Electromagnetics

AE435

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In this section, we will review the basics of charge, electricity, magnetism, and Maxwell equations.

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1 Electrostatics

1.1 Coulomb's Law

Coulomb's Law is the measure of force between charges.

Case 1: Two Particles

Consider two charges q_1 and q_2 located at \vec{r}_1 and \vec{r}_2 .

Figure 1

The force on q_1 due to q_2 is

Coulomb Force

$$\vec{F}_{12} = c \left(\frac{q_1 q_2}{r_{12}^2} \right) \frac{\vec{r}_{12}}{r_{12}} \quad \sim \quad \frac{1}{r^2} \quad (1)$$

Where:

$$\vec{r}_{12} = \vec{r}_1 - \vec{r}_2 \quad \text{The Sum of Position Vectors}$$

$$r_{12} = |\vec{r}_{12}| \quad \text{The Magnitude of } \vec{r}_{12}$$

$$\begin{aligned} c &= \text{Coulomb's Constant} \\ &= 8.9875 \times 10^9 = \frac{1}{4\pi\epsilon_0} \left[\frac{Nm^2}{c^2} \right] \end{aligned}$$

$$\begin{aligned} \epsilon_0 &= \text{Permittivity of Free Space} \\ &= 8.854 \times 10^{-12} \left[\frac{c^2}{Nm^2} \right] \end{aligned}$$

Coulomb force scales with the square of the distance as shown by $\sim \frac{1}{r^2}$ in Equation 1.

Case 2: Many Particles - Coulomb Law**Definition 1. Principle of superposition:**

Attraction between any pair can be calculated with Equation 1, regardless of the number of particles in the ensemble.

So, let \vec{r}_i be location of test particle q_i . If we have N charged particles, the force on q_i is the linear superposition of the individual forces,

Multiple Particle Coulomb Force

$$\vec{F}_{ij} = \frac{q_i}{4\pi\epsilon_0} \sum_{j \neq i}^N q_j \left(\frac{\vec{r}_{ij}}{r_{ij}^3} \right) \quad (2)$$

Where:

$$\vec{r}_{ij} = \vec{r}_i - \vec{r}_j \quad \text{The vector from test particle } q_i \text{ to field particle } q_j$$

Case 3: Continuum Generalizations

Let us define

- **Volume Charge Density** $[\frac{C}{m^3}]$

$$\rho_e = \lim_{\Delta V \rightarrow 0} \frac{\Delta q}{\Delta V} \quad (3)$$

- **Surface Charge Density** $[\frac{C}{m^2}]$

$$\sigma_e = \lim_{\Delta S \rightarrow 0} \frac{\Delta q}{\Delta S} \quad (4)$$

With these, the force acting on charge q_o due to distributed charge sources are:

Continuum Charge Coulomb Force

$$\vec{F}_{q_o} = \frac{q_o}{4\pi\epsilon_0} \left[\int_V \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \rho_e(\vec{r}') d\vec{V} + \int_S \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \sigma_e(\vec{r}') d\vec{S} \right] \quad (5)$$

Where:

\vec{r}' = The location within V or location on S

\vec{r} = The location of q_o

σ_e and ρ_e = Functions of position \vec{r}'

1.2 Electric Field

$$\vec{E} = \lim_{q_0 \rightarrow 0} \frac{F_{q_0}}{q_0} \quad (6)$$

The force acting on a specific charge q_0 from a collection of other charges per unit charge, as the specific charge tends to zero.

We set $q_0 \rightarrow 0$ so its presence does not influence the ambient charge.

Adding Equation 2 and Equation 5 then dividing thru by q results in:

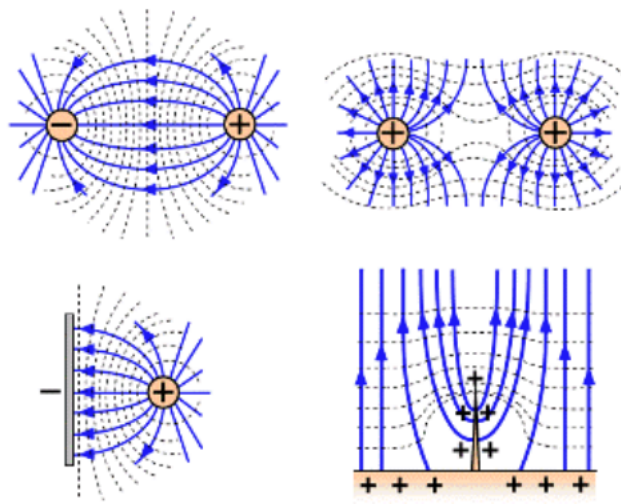
Electric Field Equation

$$\vec{E} = \frac{q_i}{4\pi\epsilon_0} \left[\sum_{i=1}^N q \frac{\vec{r} - \vec{r}_i}{|\vec{r} - \vec{r}_i|^3} + \int_V \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \rho_e(\vec{r}') dV + \int_S \frac{\vec{r} - \vec{r}'}{|\vec{r} - \vec{r}'|^3} \sigma_e(\vec{r}') dS \right] \quad (7)$$

Where:

$$\vec{E} = \vec{E}(\vec{r}) \quad \text{Electric Field is a function of position } \vec{r}$$

We can use integral approach to solve problems, but this can get complex. We can also visualize the Electric Field via field lines, curves that are everywhere tangent to the field.



1.3 Conductors and Insulators

Definition 2. Conductor: Free charges, respond to external Electric field with charge motion.

Definition 3. Insulator: Bound charges, no motion. Also often called a "dielectric"

1.4 Gauss's Law

Definition 4. Gauss's Law: Relates the electric field at a surface to the charge enclosed within that surface. The total flux that passes through any closed surface is proportional to the electric charge enclosed by a surface. This surface is the Gaussian Surface.

Case 1: Single Charge

The electric field for a single charge is

$$\vec{E} = \frac{q}{4\pi\epsilon_0} \frac{\vec{r}}{r^3} \quad (8)$$

If we take the surface integral around an arbitrary volume surrounding the charge we get:

Figure 2

Gauss' Law for Electric Fields

$$\oint_S \vec{E} \cdot \hat{n} \, dA = \frac{q}{4\pi\epsilon_0} \oint_S \frac{\vec{r} \cdot \hat{n}}{r^3} \, dA \quad (9)$$

Where:

$$\left(\frac{\vec{r}}{r} \right) \cdot \hat{n} \, dA = \text{The project of } dA \text{ on a plane perpendicular to } \vec{r}$$

If we divide the projected area, $\left(\frac{\vec{r}}{r}\right) \cdot \hat{n} dA$, by r^2 , we arrive at the solid angle $d\Omega$.

Figure 3

The total solid angle subtended by a surface totally enclosing the charge is 4π , so

$$\oint_S \frac{\vec{r} \cdot \hat{n}}{r^3} dA = 4\pi$$

As a result, Equation 9 becomes:

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{q}{\epsilon_0} \quad (10)$$

Case 2: Many Point Charges

For the case of many point charges, we take the sum:

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \sum_{i=1}^N qi$$

Case 3: Distributed Charge

For a distributed charge, ρ_e , we take the integral over the enclosed volume:

Integral Form of Gauss's Law

$$\oint_S \vec{E} \cdot \hat{n} dA = \frac{1}{\epsilon_0} \int_V \rho_e dV \quad (11)$$

Also recall Divergence Theorem (also known as Gauss's Theorem):

Divergence Theroem

$$\oint_S \vec{F} \cdot \hat{n} \, dA = \int_V \nabla \cdot \vec{F} \, dV \quad (12)$$

which applies to any vector field \vec{F} , so we arrive that...

$$\oint_S \vec{E} \cdot \hat{n} \, dA = \int_V \nabla \cdot \vec{E} \, dV = \frac{1}{\epsilon_0} \int_V \rho_e \, dV$$

Differential Form of Gauss's Law

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} \rho_e \quad (13)$$

1.5 Electrostatic Potential and Energy

One can show that the curl of the electric field is zero for an electrostatic field. In general $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = 0$ as we will see later.

$$\nabla \times \vec{E} = 0 \quad (14)$$

We also use the vector identity:

$$\nabla \times \nabla \phi = 0 \quad (15)$$

From Equation 14 and Equation 15, we can see the relation $\vec{E} = \nabla \phi$, such that the vector field \vec{E} is related to the gradient in some scalar field.

We call ϕ the **electric potential**. And actually use

$$\vec{E} = -\nabla \phi(\vec{r}) \quad (16)$$

For a single point charge q_i at \vec{r}_i , by integrating Equation 16, we arrive at:

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \frac{q_1}{|\vec{r} - \vec{r}_1|} + \text{constant}$$

On any curve linking point P_1 to point P_2 ,

$$\phi_{12}(\vec{r}) = \int_{P_1}^{P_2} \nabla \phi(\vec{r}) \cdot d\vec{l} = - \int_{P_1}^{P_2} \vec{E}(\vec{r}) \cdot d\vec{l}$$

Which tells us the work per unit charge to move from P_1 to P_2 .

More generally,

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \left[\sum_{i=1}^N \frac{q_i}{|\vec{r} - \vec{r}_i|} + \int_V \frac{\rho_e(\vec{r})}{|\vec{r} - \vec{r}'|} dV + \int_S \frac{\sigma_e(\vec{r})}{|\vec{r} - \vec{r}'|} dA' \right] + \text{constant}$$

Taking line integral from some reference location to \vec{r} ,

$$\int_{\text{ref}}^{\vec{r}} \vec{E}(\vec{r}) \cdot d\vec{r} = - \int_{\text{ref}}^{\vec{r}} \nabla \phi(\vec{r}) \cdot d\vec{r} = - \int_{\text{ref}}^{\vec{r}} d\phi = -\phi(\vec{r}) - \phi_{\text{ref}}$$

Now if we set $\phi_{\text{ref}} = 0$ at $r \rightarrow \infty$. Then

$$\phi(\vec{r}) = - \int_{\text{ref}}^{\vec{r}} \vec{E}(\vec{r}) \cdot d\vec{r} \quad (17)$$

The Potential Energy, U , associated with a force \vec{F} is

$$U(\vec{r}) = - \int_{\text{ref}}^{\vec{r}} \vec{F}(\vec{r}) \cdot d\vec{r}$$

Since $\vec{F} = q\vec{E}$ for electrostatic force as seen in Equation 16, the electrostatic potential is

Electrostatic Potential

$$\phi(\vec{r}) = \frac{U(\vec{r})}{q} \tag{18}$$

The electrostatic potential defines the potential energy per unit charge.

If we define $\phi(\infty) = 0$ then $U(\vec{r})$ is the energy required to bring a test charge from $\vec{r} = \infty$ to \vec{r} .

1.6 Poisson and Laplace Equations

For a charge distribution $q(\vec{r})$:

$$\phi(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\partial q}{|\vec{r} - \vec{r}'|} \quad (19)$$

Such that

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{(\vec{r} - \vec{r}') dq}{|\vec{r} - \vec{r}'|^3} \quad (20)$$

Since $\vec{E} = -\nabla\phi$ from Equation 16. We can solve for $\phi(\vec{r})$ and $\vec{E}(\vec{r})$ if we know $q(\vec{r})$ charge distribution. Alternatively, we can start with the differential form of Gauss's law (Equation 13) and Equation 16,

$$\nabla \cdot \vec{E} = \frac{\rho_e}{\epsilon_o} \quad \text{and} \quad \vec{E} = -\nabla\phi$$

$$\nabla \cdot (-\nabla\phi) = \frac{\rho_e}{\epsilon_o}$$

Use laplacian: $\nabla^2 = \nabla \cdot \nabla \rightarrow$

Poisson Equation

$$\nabla^2\phi = -\frac{\rho_e}{\epsilon_o} \quad (21)$$

Where:

$$\begin{aligned} \rho_e &= \rho_e(\vec{r}) && \text{charge density distribution} \\ \phi &= \phi(\vec{r}) && \text{electric potential distribution} \end{aligned}$$

The Poisson Equation relates charge density distribution $\rho_e(\vec{r})$ to electric potential $\phi(\vec{r})$ distribution.

Alternatively,

Laplace Equation

$$\nabla^2\phi = 0 \quad (22)$$

For region with no free (space) charge $\rho_e = 0$

2 Electrostatics with Dielectric Media

2.1 Polarization

Many molecules have a positive end and a negative end.

Water, H_2O , for example is a "polar molecule".

Figure 4

Oxygen gets slightly more than its share of the electron cloud while hydrogen gets less.

Water is an example of a "polar molecule". It is always polarized, even without any electric field present. But even non-polar atoms/molecules can become polarized in an electric field. Polarized, meaning a redistribution of the electron cloud creating an asymmetric charge distribution that aligns with the electric field.

We model this by thinking of each polarized atom/molecule as a dipole.

Figure 5

Polarization (how skewed the electron cloud is) depends on \vec{E} . However, part of \vec{E} is produced by polarization. In addition, the redistribution of charge in a dielectric (insulator) can affect the external charge distribution, which changes \vec{E} . In short, we have a non-linear system!

A dipole is two equal and opposite charges, $\pm q$, separated by small distance, \vec{l} .

Figure 6

We start by defining the dipole potential field.

Dipole Potential Field

$$\phi(\vec{r}) = \frac{q}{4\pi\epsilon_0} \frac{(\vec{r} - \vec{r}') \cdot \vec{l}}{|\vec{r} - \vec{r}'|^3} \quad (23)$$

Note: We ignore the higher order terms in \vec{l} .

This equation is exact for a point dipole, where $|\vec{l}| \rightarrow 0$ as $q \rightarrow 0$. All nonlinear terms vanish for a point dipole, which has no net charge or spatial extent, and has a constant dipole moment.

$$\vec{p} = \lim_{l \rightarrow 0} q \vec{l} = \text{constant} \quad (24)$$

For dielectric media, we can generalize this to

$$\Delta \vec{p} = \int_V \vec{r} dq$$

which is the dipole moment of charge distribution in a small volume.

Usually, it is more convenient to work with the dipole moment per unit volume

Polarization of the Dielectric

$$\vec{P} = \lim_{\Delta V \rightarrow 0} \frac{\Delta \vec{p}}{\Delta V} \quad (25)$$

This is a point property, $\vec{P}(\vec{r})$ unit [C/m³], called the Polarization of the dielectric. It's a vector with direction defined by charge separation.

Figure 7

Another way to think of this is through physical molecules, rather than just point dipoles:

$$\begin{aligned} \vec{p}_m &= \text{Polarization of a Molecule} \\ \vec{P} &= N \vec{p}_m \quad \left[\frac{\#}{m^3} \right] \end{aligned} \quad (26)$$

where N is the number density of molecules in a dielectric.

2.2 Surface and Volume Charge Density

Charge displacement due to induced dipoles results in a net surface charge density

Net Surface Charge Density

$$\sigma_p = \vec{P} \cdot \hat{n} \quad \left[\frac{c}{m^2} \right] \quad (27)$$

and a net volume charge density.

Net Volume Charge Density

$$\rho_p = -\nabla \cdot \vec{P} \quad \left[\frac{c}{m^3} \right] \quad (28)$$

The rest of this section will be spend proving Equation 27 and Equation 28.

Consider the boundary of a dielectric shown below. Assume all positive charges in the slab move a displacement vector \vec{S} in response to an applied electric field, \vec{E} , and that negative charges are stationary.

Figure 8

We find that the differential volume defined by displacement \vec{s} and projected area $d\vec{A} = \hat{n} dA$ is:

$$dV = \vec{s} \cdot d\vec{A}$$

If N is the number of positive charges per unit volume in the medium, the charge crossing $d\vec{A}$ is:

$$dq = Nq \vec{s} \cdot d\vec{A}$$

Recall polarization per molecule (Equation 24) $\vec{p}_m = q \vec{s}$

And polarization per unit volume (Equation 26) $\vec{P} = N \vec{p}_m$

So charge crossing $d\vec{A}$,

$$dq = \vec{P} \cdot d\vec{A}$$

Then the **Net Surface Charge** is the charge per unit area:

$$\frac{dq}{dA} = \sigma_p = \vec{P} \cdot \hat{n} \quad (29)$$

So whenever \vec{P} and \hat{n} are in the same direction, we get a net positive (+) surface charge. Whenever \vec{P} and \hat{n} are in the opposite direction, we get a net negative (-) surface charge.

Similar for Volume Charge Density

Again, the charge flow out of any surface dA with normal \hat{n} is:

$$dq = \vec{P} \cdot d\vec{A} = \vec{P} \cdot \hat{n} dA$$

So the net flow out of a differential volume equals the total flux integrated over the surface,

$$\int_S \vec{P} \cdot d\vec{A} = \int_S \frac{dq}{dA} dA$$

No charge is created or destroyed, so the differential loss of charge within dV is:

$$-\int_S dq = \int_V \rho_p dV = -\int_S \vec{P} \cdot d\vec{A} = -\int_V \nabla \cdot \vec{P} dV$$

Thus, similar to Gauss' Law, you get a differential form valid for any arbitrary volume.

Then the **Net Volume Charge** is the charge per unit volume:

$$\rho_p = -\nabla \cdot \vec{P} \tag{30}$$

So, a negative gradient in polarization creates a positive volumetric charge. Meanwhile, a positive gradient in polarization creates a negative volumetric charge.

2.3 Gauss' Law for Dielectrics

Starting with the free-space Gauss's law, we add dielectric charges

Gauss' Law with Dielectric Charges

$$\oint_S \vec{E} \cdot d\vec{A} = \int_V \nabla \cdot \vec{E} dV = \frac{\sum q}{\epsilon_0} = \frac{1}{\epsilon_0} \int_V (\rho_f + \rho_p) dV \quad (31)$$

Where:

ρ_f = volume charge density of free charges

ρ_p = volume charge density of charge due to dielectric polarization

The differential form is then (compare with 2.13):

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} (\rho_f + \rho_p) \quad (32)$$

Total electric field is due to free charges AND polarization "charges" .

E is due to ALL charges, but usually don't care about, ρ_p .

What we'd like is an equation in terms of free charge as a source term. Recall (2.30), so that

$$\rho_p = -\nabla \cdot \vec{P}$$

$$\nabla \cdot \vec{E} = \frac{1}{\epsilon_0} (\rho_f - \nabla \cdot \vec{P})$$

Then:

$$\nabla \cdot (\epsilon_0 \vec{E} + \vec{P}) = \rho_f \quad (33)$$

Define Electric Displacement

$$\vec{D} = \epsilon_0 \vec{E} + \vec{P} \quad (34)$$

The above equation (2.33) becomes:

$$\nabla \cdot \vec{D} = \rho_f \quad (35)$$

Which is the most common form of Poisson's equation seen in Maxwell equations.

Free charge is the source of displacement . In integral form,

$$\oint_S \vec{D} \cdot d\vec{A} = \int_V \rho_f dV \quad (36)$$

In contrast, total charge is the source of the Electric Field.

$$(\rho_f + \rho_p)$$

2.4 Susceptibility, Permittivity, & Dielectric Constant

Material properties are characterized by constitutive relations (a relationship between two physical quantities that is specific to a material). For isotropic materials,

Equation Name

$$\vec{P} = \chi(E) \vec{E} \quad (37)$$

Where:

χ = Electrical Susceptibility of the Material

Can define permittivity as:

$$\epsilon(E) = \epsilon_0 + \chi(E) \quad (38)$$

So that

$$\vec{D} = \epsilon(E) \vec{E} \quad (39)$$

More generally, for time-varying fields, permittivity is a function of both the wave number and frequency.

$$\vec{D}(\vec{K}, w) = \epsilon(\vec{K}, w) \vec{E}(\vec{K}, w)$$

Often χ and ϵ are independent of \vec{E} , these materials are called linear dielectrics.

$$\vec{P} = \chi \vec{E} \quad (40)$$

$$\vec{D} = \epsilon \vec{E}$$

For these, can also specify material's electrical behavior by dielectric constant.

$$\epsilon = K \epsilon_0 \quad (41)$$

So

$$\epsilon_r = K = \frac{\epsilon}{\epsilon_0} = 1 + \frac{\chi}{\epsilon_0} \quad (42)$$

To analyze problems involving dielectrics, you need:

1. Boundary Conditions
2. Governing Equations

2.5 Boundary Conditions at Interfaces

Consider a generalized surface with external surface charge density, σ_e . This does NOT include polarization charge, σ_p .

Figure 9

Where

\vec{D}_1 = The Electric Displacement just below the surface

\vec{D}_2 = The Electric Displacement just above the surface

Note that \vec{D}_2 changes only because of the effect of the external electric field. The change is related to the charge on the surface.

From the figure, it can be shown that...

The difference in the vertical components of the Electric Displacement above and below the generalized surface is equal to the external surface charge density.

$$(D_{\perp})_1 - (D_{\perp})_2 = \sigma_e \quad (43)$$

The perpendicular component of Electric Displacement changes proportional to σ

If we apply Gauss' law to a small volume slightly above and slightly below the surface and then look at the field around the surface, we can derive Equation 44.

The horizontal components of Electric Displacement above and below the generalized surface are equal. Since we did not include Polarization, \vec{P} , our equation for electric displacement, $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$, becomes $\vec{D} = \epsilon_0 \vec{E}$ and so...

$$(E_{\parallel})_1 = (E_{\parallel})_2 \quad (44)$$

The parallel components of the Electric Field, \vec{E} , do not change above and below the surface.

2.6 Poisson & Laplace's Eqns. for Dielectrics

When we include dielectric media, the divergence equation is: (Derived earlier in Equation 35)

$$\nabla \cdot \vec{D} = \rho_f \quad (45)$$

where ρ_f is the volume density of free charges, and the displacement is

$$\vec{D} = \epsilon \vec{E} \quad (46)$$

So, in dielectric media, the divergence of the electric field is given by:

$$\nabla \cdot \vec{E} = \frac{\rho_f}{\epsilon} \quad (47)$$

Which is just what we had for Gauss' Law except now it includes dielectrics. But we still have electrostatic fields, so can put in terms of scalar potential as:

$$\nabla^2 \phi = -\frac{\rho_f}{\epsilon} \quad (48)$$

which is Poisson's Equation for dielectrics.

In most cases, there is no free charge distributed through the dielectric, it concentrates either on the surface or (more rarely) concentrates in clumps within the dielectric. As a result, you can usually use Laplace's Equation for Dielectrics

$$\nabla^2 \phi = 0 \quad (49)$$

in the body of a dielectric.

3 Magnetostatics

”Charge in motion creates a magnetic field”

3.1 Electric Current

Current

$$J \equiv \frac{\partial Q}{\partial t} \quad \left[\frac{C}{s} \right] = [A] = \text{Amps} \quad (50)$$

We'll use J, although sometimes we see I used for current. Currents can flow in a range of media: metals, semiconductors, fluids, gases and plasmas.

METALS

In metals, there are fixed ionic cores with bound inner ions:

Figure 10

Outer valence electrons get freely traded from ion to ion in response to electric fields. In other words, say we put 10 electrons in one end of a wire and we get 10 electrons out the other end. Those won't be the same 10 electrons we put in though.

GASES AND PLASMAS

In gases and plasmas, both electrons AND ions move:

Figure 11

- Most of the conduction is by electrons, because they're much lighter.
- In thermal motion, both ions and electrons are as likely to cross plane in one direction as another, so no net current.
- Under electric field, drift velocity of species (ions toward cathode, electrons toward anode) gives rise to current.

3.2 Current Density

Consider flux of particles through surface element dS :

Figure 12

Since particle with charge q crossing dS carries incremental current $q \vec{v} \cdot \hat{n}$. For N particles per unit volume, current crossing dS is then:

$$dJ = N q \vec{v} \cdot \hat{n} dS$$

For multiple species, sum over all:

$$dJ = \sum_i N_i q_i \vec{v}_i \cdot \hat{n} dS$$

Define current density as a vector current per unit area:

Current Density

$$\vec{j} = \sum_i N_i q_i \vec{v}_i \quad \left[\frac{A}{m^2} \right] \quad (51)$$

So, the total current across a surface S is:

Total Current Across a Surface

$$J = \int_S \vec{j} \cdot \hat{n} dS \quad (52)$$

3.3 Continuity

3.4 Ohm's Law

3.5 Magnetic Field

3.6 Forces on Conductors

3.7 Biot-Savart Law

4 Electromagnetic Induction

5 Ampere's Law

6 Maxwell's Equations

7 Basic Templates

Note 1. This is how you make numbered notes

Exercise 1. This is how you make numbered exercises

Definition 5. This is how you make numbered definitions

Rule 1. This is how you make numbered rules

Equation Name

$$y = mx + b$$

Where:

variable1 = Description

variable2 = Description

