

Section 9: Electrostatic Propulsion

AE435
Spring 2018

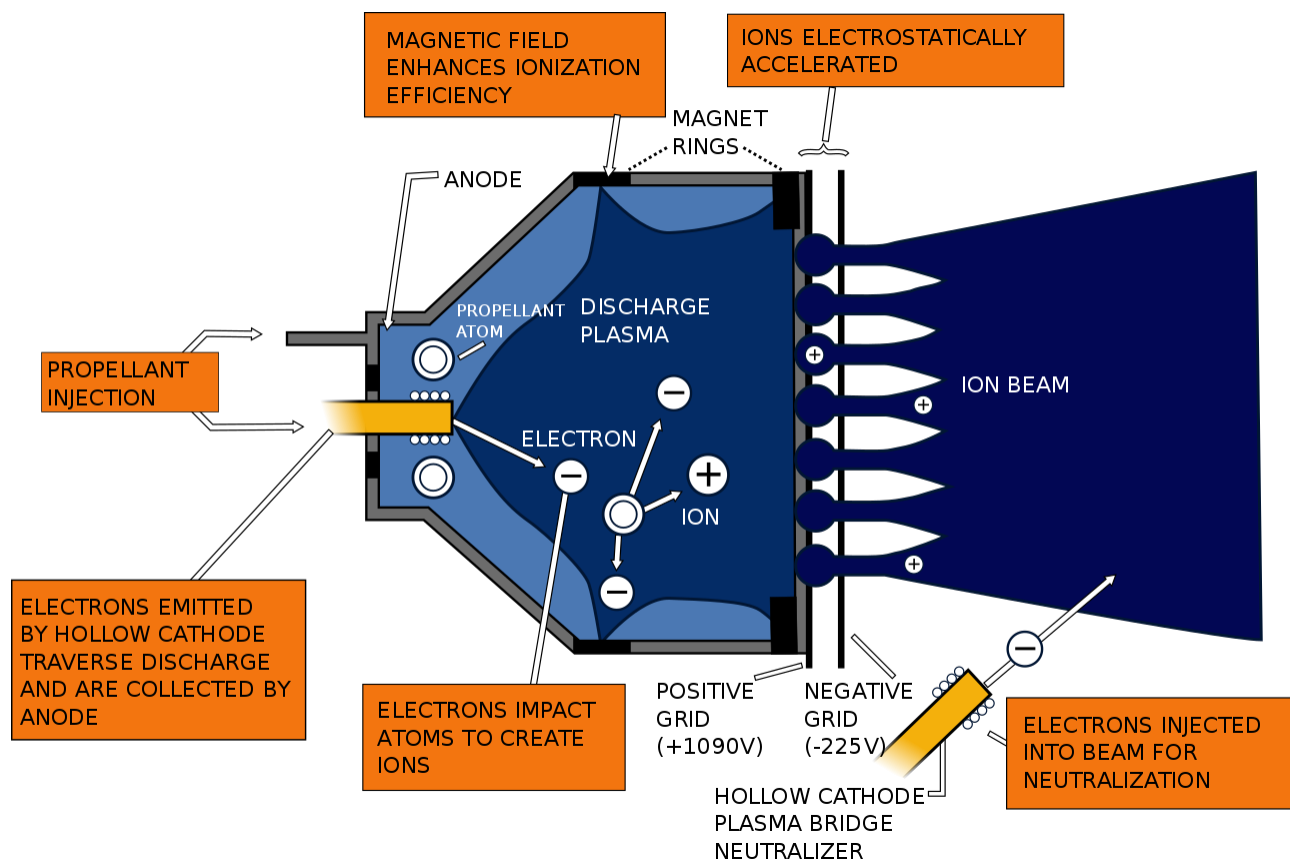
1 Gridded Ion Engines

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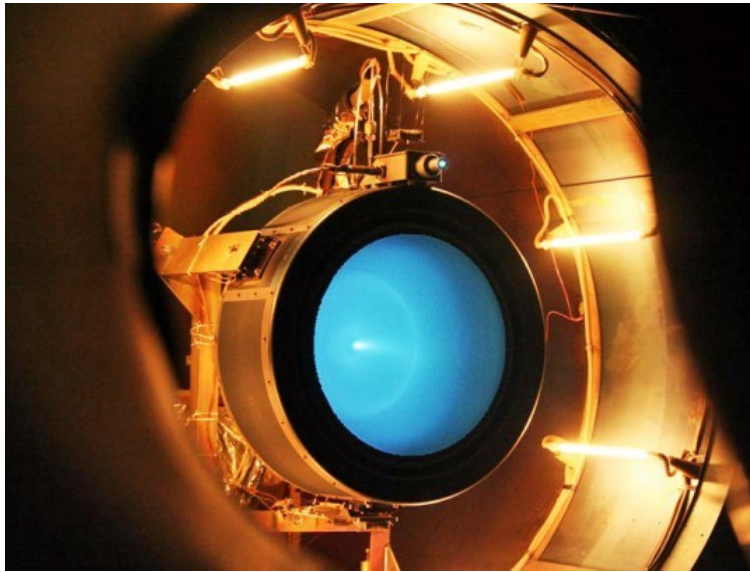
Gridded ion engines, a.k.a. ion thrusters, are characterized by the electrostatic acceleration of ions extracted from a plasma generator. Ion thruster geometries are best described in terms of three basic components:

- Ion accelerator
 - Plasma generator
 - Electron neutralizer
-
- Ion accelerator typically uses electrically biased multi-aperture grids to produce the ion beam.
 - Neutralizer cathode is positioned outside the thruster body to provide electrons to neutralize the ion beam and maintain the potential of the thruster and spacecraft relative to the space plasma potential.
 - Plasma generator can be direct current (DC) electron discharges, radio frequency (RF) discharges, or microwave discharges to produce the plasma.



Section 9: Electrostatic Propulsion - Gridded Ion Engines

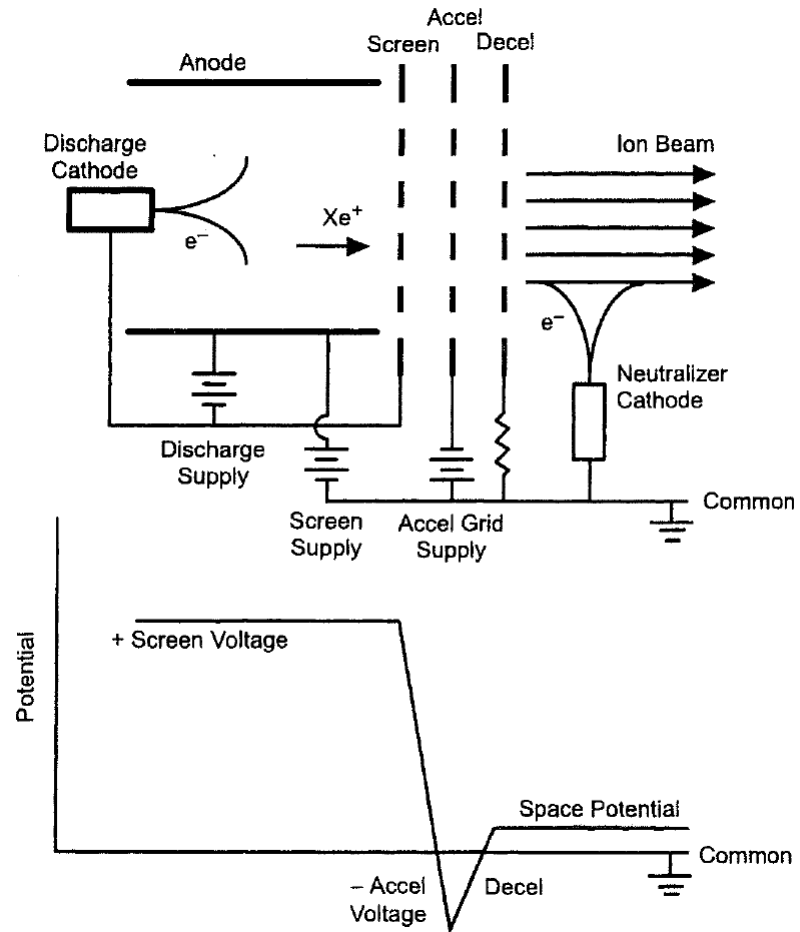
NEXIS 57-cm diameter ion engine (circa 2005), 20kW, 7000 sec, 100khr, JIMO, never flown in space



NEXT, 40-cm diameter ion engine, 7kW, 4200sec, never flown in space

NSTAR, 30-cm diameter ion engine, 2.3kW, 3000sec, flow on Deep-Space One in 1998 (a single unit) and DAWN in 2007 (3 units)

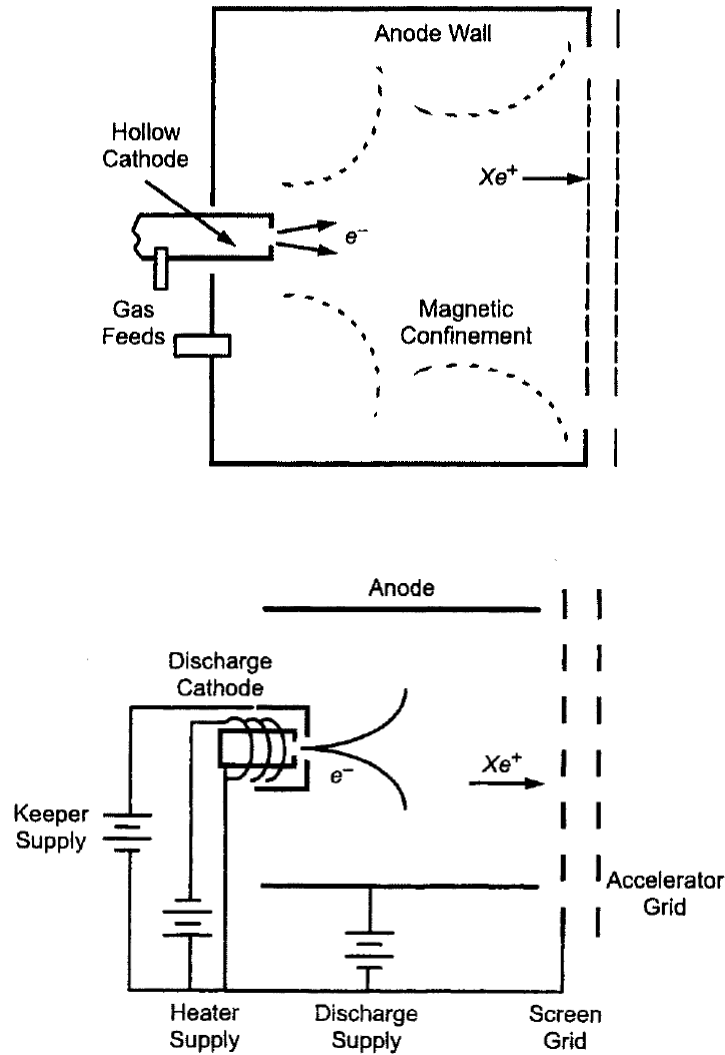
1.0.1 Electrical Schematic:



Decel grid to protect accel grid from CEX ions that cause pit-ngroove erosion of accel grid.

1.1 Plasma Generator - Discharge Chamber

The plasma generator we will focus on here is the DC electron discharge plasma generator, typically just called the discharge chamber. The purpose of the discharge chamber is to create a large volume of uniform plasma from which ions can be extracted and accelerated by the ion accelerator.



The goal is to inject neutral gas atoms, ionize them, and then supply them to the ion accelerator. Therefore there are two main important performance metrics:

1. How efficiently ions are created and supplied to the accelerator. This is called the ion production cost (or discharge loss), which has units of eV/ion (or W/A).

$$\eta_d = \frac{P_{in}}{I_b} = \frac{\text{Power In}}{\text{Beam Current}} \quad (9.1)$$

- How efficiently the injected neutral atoms are converted into ions. Specifically the fraction of injected neutrals that become supplied to the accelerator. This is called the mass utilization efficiency.

Mass Utilization Efficiency

$$\eta_{md} = \frac{I_b}{\frac{\dot{m}_o}{M} e} \quad (9.2)$$

Where

\dot{m}_o = Neutral Particle Input Flow Rate

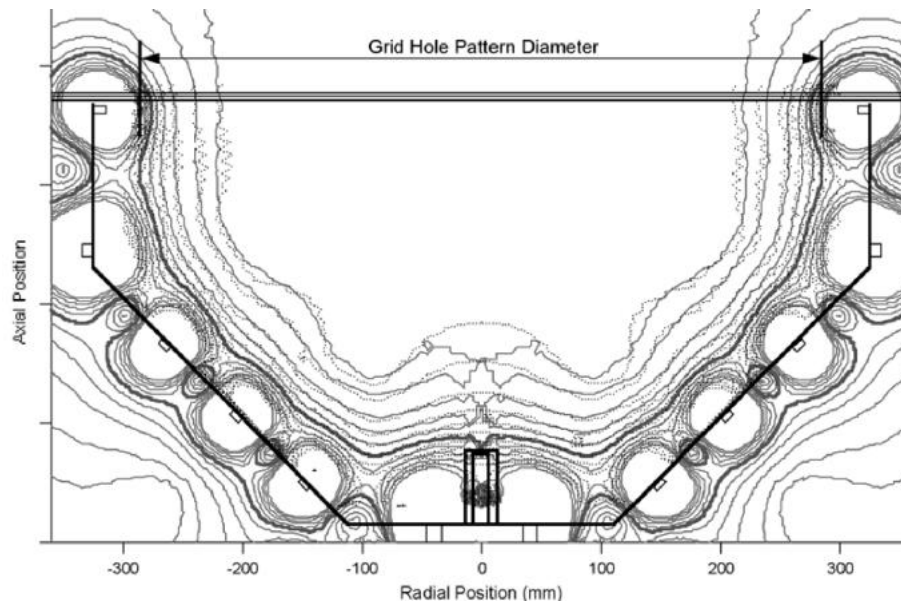
e = Electron Charge

$= 1.60217662 \times 10^{-19} C$

M = Ion Mass

Clearly we want low ion production cost (will never be less than ionization energy! 12.13 eV/xenon-ion), and high mass utilization efficiency.

An example of modern state-of-the-art discharge chamber is shown here, the NEXIS ion engine.



Electrons created and emitted by the cathode (center cylinder) are called primary electrons, and are accelerated toward the anode body (surrounding metallic shell). These primary electrons have ionizing collisions with neutral propellant (typically xenon, neutral pressure of a few mTorr) injected into the chamber.

To reduce ion propulsion cost and increase mass utilization efficiency, the pathlength of the electrons (distance they must travel to get to the anode), and thereby their probability of having an ionizing collision, is increased with a constant magnetic field. A cusped field configuration has been shown

to provide best performance in terms of providing a flat beam profile with low ion production cost and high utilization efficiency.

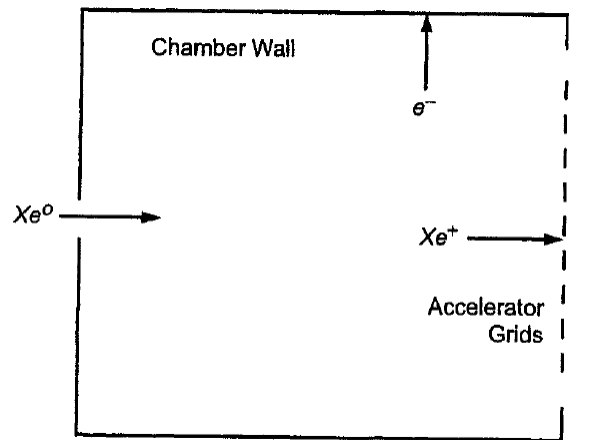
Cusps are created using alternating polarity of permanent (typically samarium cobalt) magnets along the shell of the discharge chamber. A good rule of thumb for high efficiency is to keep the 50-60 G magnetic field contour from intersecting the anode wall.

Models have been developed to predict the effect of cusp magnetic field on electron collection at the anode, and these effects have been incorporated into discharge chamber models. But here, we develop a simple idealized model that neglects the magnetic field entirely. While it is simple, it is still a good approximation to current experimental data.

1.1.1 Idealized Model for Predicting Discharge Chamber Performance

Power is injected by arbitrary means into a volume filled with neutral gas to produce ionization and neutral gas excitation.

- All the ions go to the grids
- Equal number of plasma electrons go to the wall to conserve charge.
- Discharge chamber has a volume, V .



Ions only flow to grids, perfect confinement everywhere else. Current of ions to the grids is the Bohm current (due to the plasma sheath that forms along the surface of the accelerator grids). The Bohm current is:

Bohm Current

$$I_{\text{Bohm}} = I_i = \frac{1}{2} n_i e v_a A \quad (9.3)$$

Where

$$v_a = v_{\text{Bohm}} = \sqrt{\frac{k T_e}{M}}$$

= Ion Acoustic/Bohm Velocity
 A = Area of Grid

The fraction of ions that arrive at the grids and get accelerated into the beam is T_g , the "effective grid transparency", so the beam current is:

$$I_b = \frac{1}{2} n_i e v_a A T_g \quad (9.4)$$

Ions in the discharge chamber are produced at a rate given by the ionization rate coefficient:

$$I_p = n_o n_e e \langle \sigma_i v_e \rangle V \quad (9.5)$$

Where

$$n_o = \text{Neutral Density}$$

$$\langle \sigma_i v_e \rangle = \text{Ionization Rate Coefficient; Average over Velocity Distribution Function}$$

Power is conserved in the system, so the power put into the plasma is equal to the power that comes out in the form of charged particles and radiation. To first order, the power put in goes into ionization and excitation of neutral gas, heating of the electrons, and power that is carried to the walls and the grids by the ions and electrons. Then:

Plasma Power Input

$$P_{in} = I_p U^+ + I^* U^* + I_i \varepsilon_i + \frac{n_e V}{\tau} \varepsilon_e \quad (9.6)$$

where

$$U^+ = \text{Ionization potential}$$

$$U^* = \text{Excitation potential}$$

$$\varepsilon_i = \text{Energy carried by ions to grids}$$

$$\varepsilon_e = \text{Energy carried by electrons to anode/leaving plasma}$$

$$I^* = \text{Production rate of excited species}$$

$$\tau = \text{Mean confinement time of electrons}$$

The production rate of excited species is given by (similar to (9.5)):

$$I^* = \sum_j n_o n_e e \langle \sigma^* v_e \rangle_j V \quad (9.7)$$

Where σ^* is the cross-section for excitation to level j .

And we sum over all possible excitation levels.

Then, with (9.5), (9.7), Eqn. (9.6) becomes:

$$P_{in} = n_o n_e \langle \sigma_i v_e \rangle V \left[U^+ + \frac{\langle \sigma^* v_e \rangle_j}{\langle \sigma_i v_e \rangle} U^* \right] + I_i \varepsilon_i + \frac{n_e V}{\tau} \varepsilon_e \quad (9.8)$$

where we assume there is one "average" excited state, j , with corresponding energy U^* . ($U^* \sim 10$ eV for Xe)

Assuming the plasma is quasi-neutral, $n_e \approx n_i$, and that ions and electrons leave at the same rate, such that the confinement time of ions and electrons is the same, then:

$$I_i = \frac{1}{2} n_i e v_a A = \frac{n_i e V}{\tau} \quad (9.9)$$

such that:

$$\tau = \frac{2V}{v_a A} \quad (9.10)$$

We see that larger volume to surface increases confinement time. Therefore we expect discharge chambers with larger characteristic size (e.g., radius) will have higher confinement time of charged particles (good so electrons can have ionization collisions). Larger discharge chambers tend to have better performance (but there's a mass and size penalty for the spacecraft!).

Now to determine how much energy an ion and electron takes with it when it leaves the discharge chamber, ε_i and ε_e . We must consider the potential profile at the anode wall. It is an electron repelling sheath, since the plasma potential is higher than the anode potential.

Figure: Potential Profile at the Anode Wall

Assuming Maxwellian electrons in the bulk plasma, these electrons are decelerated and repelled by the sheath potential. The electron current density reaching the wall is given by moments (integral) of the distribution function (see 4.26):

$$\begin{aligned}
 j_e &= e n \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{\sqrt{\frac{2e\phi}{m}}}^{\infty} v_z \left(\frac{m}{2\pi k T_e} \right)^{\frac{3}{2}} \exp \left(\frac{-mv^2}{2k T_e} \right) dv_z \\
 &= \frac{1}{4} e n \sqrt{\frac{8k T_e}{\pi m}} \exp \left(\frac{-e\phi}{k T_e} \right)
 \end{aligned} \tag{9.11}$$

The flux of kinetic energy (or kinetic power) reaching the wall is:

$$\begin{aligned}
 P_e &= n \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_{\sqrt{\frac{2e\phi}{m}}}^{\infty} v_z \left(\frac{1}{2} m v^2 \right) \left(\frac{m}{2\pi k T_e} \right)^{\frac{3}{2}} \exp \left(\frac{-mv^2}{2k T_e} \right) dv_z \\
 &= \frac{1}{4} n e \sqrt{\frac{8k T_e}{\pi m}} \left(2 \frac{k T_e}{e} + \phi \right) \exp \left(\frac{-e\phi}{k T_e} \right)
 \end{aligned} \tag{9.12}$$

Therefore the average energy transported by an electron from the plasma is the ratio of the power per electron to the flux of electrons:

$$\varepsilon_e = \frac{P_e}{j_e} = 2 \frac{k T_e}{e} + \phi = 2 T_{e[\text{eV}]} + \phi \tag{9.13}$$

Energy removed by ions when they leave the plasma. Ions first fall through the pre-sheath potential, which is approximately $T_e/2$ [eV] to produce the Bohm velocity, and then through the sheath potential, ϕ . Each ion then removes from the plasma a total energy per ion of:

$$\varepsilon_i = \frac{1}{2} \frac{k T_e}{e} + \phi = \frac{1}{2} T_{e[\text{eV}]} + \phi \tag{9.14}$$

The plasma potential, ϕ , is found from electron current leaving the plasma, which for a stable sheath is:

$$I_{\text{anode}} = \frac{1}{4} \sqrt{\frac{8k T_e}{\pi m}} e n_e A_a \exp \left(\frac{-e\phi}{k T_e} \right) \tag{9.15}$$

Since we have assumed ambipolar ion and electron flow to the wall (ion and electron loss rates are same), we can equate (9.15) and (9.3):

Floating Potential

$$\phi = \frac{k T_e}{e} \ln \left[\frac{A_a}{A} \sqrt{\frac{2M}{m\pi}} \right] \tag{9.16}$$

Where

A = Grid Area

A_a = Anode Area

This is normally called the "floating potential" since it's the potential such that the ion and electron current are equal (we set (9.15) equal to (9.3)). But in this model there are no applied potentials to draw a net current.

This potential, ϕ , is the potential setup to balance the electron and ion loss rate. Since electrons are less massive, they leave the plasma first, leaving behind a slight positive charge (or potential, ϕ). This potential retards the loss of electrons to keep the plasma quasi-neutral.

Need Electron Temperature for this model. Find electron temperature by equating ion production and loss rates, (9.3) equal to (9.5):

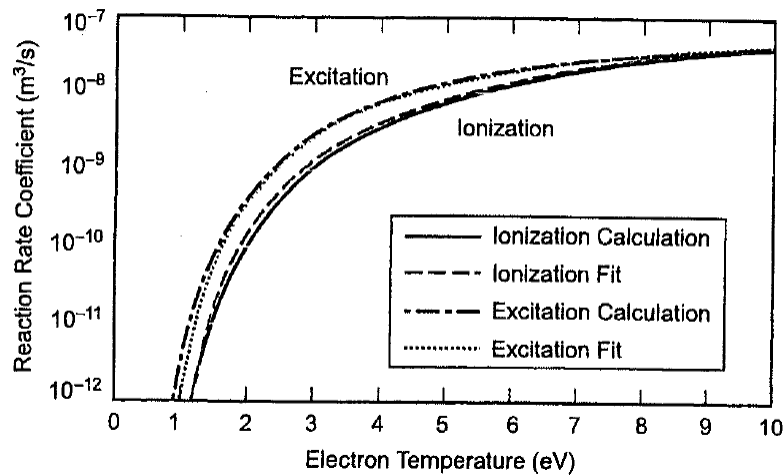
$$\frac{\sqrt{\frac{kT_e}{M}}}{\langle \sigma_i v_e \rangle} = \frac{2n_o V}{A} \quad (9.17)$$

Where

A = Ion Loss/ Grid Area

n_o = Neutral Densities

This equation can be solved for T_e , but not analytically. The reaction rate coefficient has been calculated using empirically determined collision cross-sections:



Electron Energy (eV)	Ionization (m ³ /s)	Excitation (m ³ /s)
0.5	4.51×10^{-25}	1.99×10^{-22}
0.6	3.02×10^{-23}	4.01×10^{-21}
0.7	6.20×10^{-22}	3.61×10^{-20}
0.8	6.04×10^{-21}	1.95×10^{-19}
0.9	3.58×10^{-20}	7.44×10^{-19}
1.0	1.50×10^{-19}	2.21×10^{-18}
1.5	1.16×10^{-17}	6.64×10^{-17}
2.0	1.08×10^{-16}	4.02×10^{-16}
2.5	4.24×10^{-16}	1.23×10^{-15}
3.0	1.08×10^{-15}	2.66×10^{-15}
3.5	2.13×10^{-15}	4.66×10^{-15}
4.0	3.59×10^{-15}	7.12×10^{-15}
4.5	5.43×10^{-15}	9.93×10^{-15}
5.0	7.61×10^{-15}	1.30×10^{-14}
5.5	1.01×10^{-14}	1.61×10^{-14}
6.0	1.28×10^{-14}	1.94×10^{-14}
6.5	1.57×10^{-14}	2.26×10^{-14}
7.0	1.88×10^{-14}	2.57×10^{-14}
7.5	2.20×10^{-14}	2.87×10^{-14}
8.0	2.53×10^{-14}	3.14×10^{-14}
8.5	2.86×10^{-14}	3.34×10^{-14}
9.0	3.20×10^{-14}	3.41×10^{-14}
9.5	3.55×10^{-14}	3.21×10^{-14}
10.0	3.90×10^{-14}	2.48×10^{-14}

Now to calculate the discharge loss (a.k.a. Ion Production Cost):

Discharge Loss

$$\eta_d = \frac{P_{in}}{I_b} = \frac{2 n_o \langle \sigma_i v_e \rangle V}{v_a A T_g} \left[U^+ + \frac{\langle \sigma^* v_e \rangle}{\langle \sigma_i v_e \rangle} U^* \right] + \frac{1}{T_g e} \left[2.5 k T_e + 2 k T_e \ln \left(\frac{A_a}{A} \sqrt{\frac{2 M}{m \pi}} \right) \right] \quad (9.18)$$

Where

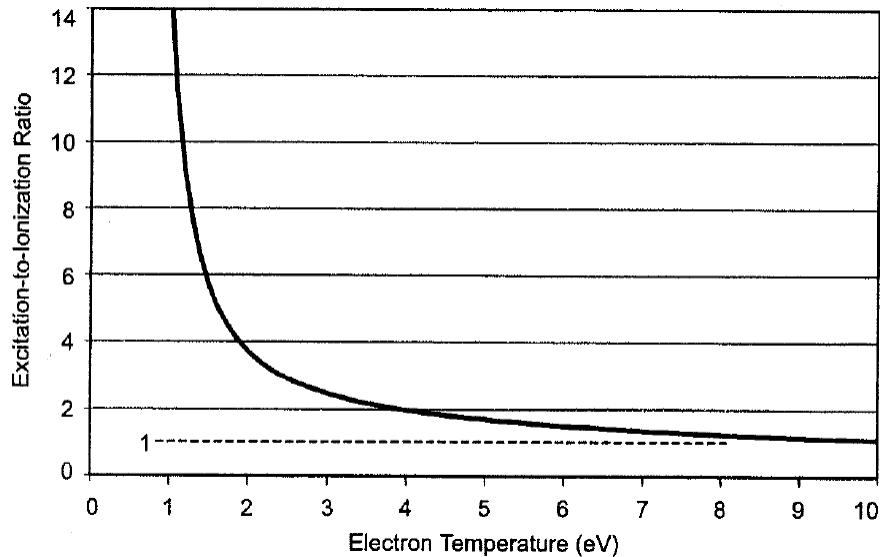
$$v_a = v_{Bohm} = \sqrt{\frac{k T_e}{M}}$$

M = Ion Mass

- Grid transparency, T_g , directly affects the discharge loss

- Input power is distributed between:
 1. 1st term: producing ions and excited neutrals
 2. 2nd term: heating the electrons that are lost to the walls

Need the ratio of the excitation to ionization reaction rates:



At temperatures below about 8eV, the excitation rate exceeds the ionization rate. This higher excitation rate results in more of the power being radiated to the walls than producing ions. This explains at least part of the inefficiency inherent in XENON plasma generators, but similar results are found for other inert gas propellants.

Also need T_g (80excitation potential (10 eV), diameter of grids (A), electron loss area (Aa chamber wall and screen grid).

The other performance parameter is the mass utilization efficiency. To calculate it we recognize that neutrals entering the discharge chamber can leave the discharge chamber as neutrals through the grid, or become ionization and leave as ions through the grids.

The mass flow rate of neutrals out of the discharge chamber can be written as: Neutral velocity
Neutral density Grid area Grid transparency to neutrals Clausing factor

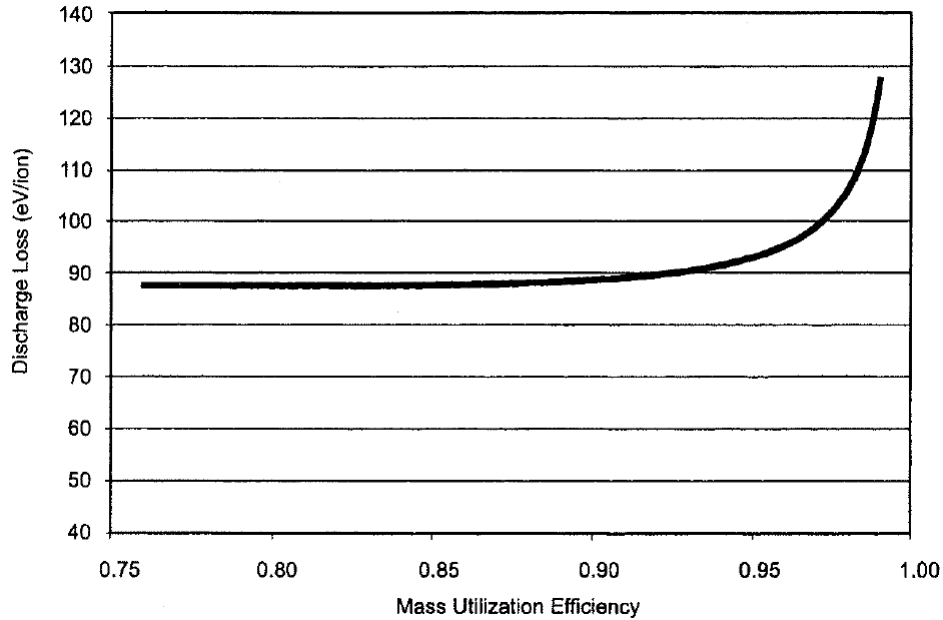
The Clausing factor accounts for the fact that the grids are finite thickness (not infinitely thin), related to gas flow restriction in short tubes. Typical grids have small thickness-to-length ratios, so Clausing factor must be calculated with numerical kinetic theory/Monte Carlo techniques, but a value of 0.5 is typical for ion engines.

The mass utilization efficiency was given as (9.2), which will us (9.4) for the beam current.

Using (9.19) and (9.20), we can solve for the neutral density in the discharge chamber:

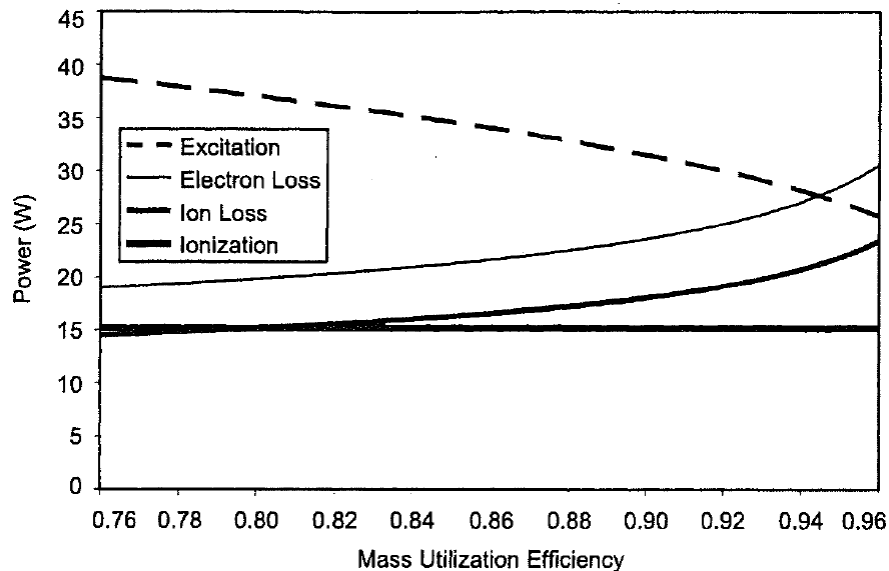
Typical efficiencies for a discharge chamber:

20-cm diameter, 30-cm long.



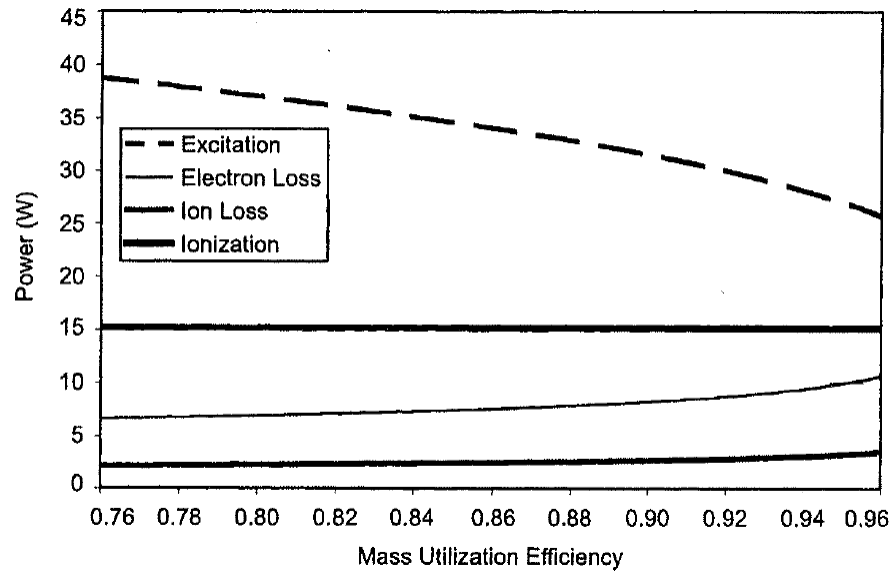
Amount of power to produce 1A of beam current (with 80% transparency T_g) is about 90W. While it only takes 12.13eV to ionize a xenon atom, even in an idealized thruster it takes 7.5 times this energy to produce and deliver an ion into the beam due to other losses.

What are the losses??



This plot assumes the ionization power is constant. Major power loss is excitation at low mass utilization where electron temp is low. Electron and ion convection losses to the wall increase with mass utilization because there are less neutrals, so have higher electron temp, higher plasma potential, and thereby increased energy lost per electron and ion.

State-of-the-art discharge chambers use magnetic confinement to reduce the effective anode area (A_a), the Bfield reduces the area of the anode the electrons can actually get to. This effect is illustrated below:



Effective anode area reduced to 1cm^2