

Section 7: Electrothermal Propulsion

AE435
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1 Fundamentals

Contents

1	Fundamentals	2
1.1	Simple Analysis	3
1.2	Deviation from Ideal Behavior	5
1.3	Efficiency Terms	7
1.4	Enthalpy of High Temperature Gas	8
1.4.1	Effect of Temperature on Internal Degrees of Freedom	12
1.5	Equilibrium Composition	16
1.5.1	Hydrazine Thruster	20
1.6	Nozzle Flow	23

1.1 Simple Analysis

Propellant flexibility leads us to expect high I_{SP} . Consider 1D adiabatic nozzle flow from the heating chamber to the nozzle exit. Thus...

Figure 1:

Stagnation Enthalpy Balance:

$$h_{oc} = h_{oe} \quad (7.1)$$

$$\frac{1}{2} u_e^2 + h_e = \frac{1}{2} u_c^2 + h_c \quad (7.2)$$

Where

u_e	= The Exit Velocity	[m/s]
u_c	= The Chamber Velocity	[m/s]
h_e	= The Exit Enthalpy	[J/kg]
h_c	= The Chamber Enthalpy	[J/kg]

If we assume constant specific heats, and negligible inlet speed ($u_c \approx 0$), the exit velocity is:

Exit Velocity

$$u_e = \sqrt{2 c_p (T_c - T_e)}$$

Where

$$h_e = c_p T_e \quad (7.3)$$

$$h_c = c_p T_c \quad (7.4)$$

Further, assuming complete expansion ($T_e \ll T_c$):

$$u_e \cong \sqrt{2 c_p T_c} \quad (7.5)$$

And using

Where

$$c_p = \frac{\gamma \Re}{(\gamma - 1) MW}$$

$$\Re = 8314 \frac{J}{K mol - K}$$

MW = Molecular Weight

(7.6)

The exit velocity is then:

$$u_e \cong \sqrt{\frac{\gamma \Re 2}{(\gamma - 1) MW} T_c} \quad (7.7)$$

The specific heat c_p is critical: it defines the achievable stagnation enthalpy. **For H2,**

$$T_{oc} = 3000 [K] \text{ The limit for refractory metals.}$$

$$C_p = 2 \times 10^4 [J/kg \cdot K] \text{ at 1atm and 3000K}$$

$$u_e = 11 [km/s]$$

$$I_{SP} = 1100 [sec]$$

Assuming an efficiency $\eta = 60\%$ the thrust-to-power ratio of this combination is:

$$\frac{T}{P} = \frac{2 \eta}{g_o I_{SP}} = 0.11 \left[\frac{N}{kW_e} \right]$$

So, Electrothermal Propulsion can readily provide

- 3-4x The I_{SP} of bipropellant system
- 5x The I_{SP} of monopropellant system

Typical commsats have 4-5 kW available for propulsion, so this could provide ~ 0.5 N of thrust.

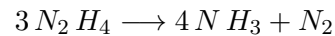
1.2 Deviation from Ideal Behavior

The biggest loss factor in Electrothermal Propulsion is **Frozen Flow Losses**. Recall our stagnation enthalpy balance:

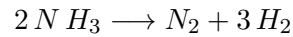
$$\frac{1}{2} u_e^2 = h_{oc} - h_e \quad (7.8)$$

Frozen flow losses results from non-negligible amounts of unrecoverable internal energy, h_e .

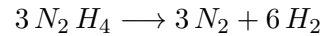
As an example, consider a hydrazine arcjet. We start with the exothermic decomposition of hydrazine:



There's an associated endothermic dissociation of ammonia:



In the discharge chamber, there's enough heat and time to drive both to the end products:



Two Extremes: Equilibrium and Endothermic Reactions.

If we exhaust this through a nozzle (so T and p drop with axial distance, x), we can get two extremes. If the flow stays in equilibrium, the ammonia composition rises with axial distance in the nozzle as nitrogen and hydrogen recombine.

Figure 2: Equilibrium Flow

But, if the expansion is quick enough that the endothermic reaction doesn't have time to reverse (i.e., release heat into the flow), we have frozen flow. In this case, the energy tied up in dissociation is unavailable for acceleration of the gas.

Figure 3: No Recombination, No Ammonia, Frozen Flow

Note that dissociation is good in Nuclear Thermal Rockets (NTRs):

- Lower mean molecular weight of exhaust products
- Increases maximum I_{SP}

But this is bad in Electrothermal Propulsion thrusters.

Question: Why?

Answer: Energy cost

NTRs have lots of cheap power, but making up the lost investment means more solar array area for Electrothermal Propulsion (need more electrical power). And thus more dry mass.

1.3 Efficiency Terms

Frozen flow is one of several losses that can be described by efficiency terms:

Frozen Flow Efficiency:

$$\eta_f = \frac{h_{oc} - h_e}{h_{oc}} \quad (7.9)$$

Where

h_{oc} = Stagnation Enthalpy in the Chamber

h_e = Enthalpy at the Exit Plane

Physically you can think of this as

$$\eta_f = \frac{\text{Power in Fluid "Available" for Thrust}}{\text{Power in Fluid}} \quad (7.10)$$

Some power is also lost by heat-transfer inefficiencies (remember we assumed adiabatic in (7.1)), this is radiated or conducted heat loss from the system, and can be described by:

Heating Efficiency

$$\eta_{th} = \frac{\text{Power into Fluid}}{\text{Electrical Power}} \quad (7.11)$$

Non-ideal nozzle flow is another loss (viscosity, heat transfer in the nozzle, profile losses) costs power too, described by the:

Nozzle Efficiency

$$\eta_n = \frac{\text{Thrust Power}}{\text{Available Power in Fluid}} \quad (7.12)$$

The overall thruster efficiency is the product of these component efficiencies:

Overall Thruster Efficiency

$$\eta = \eta_f \eta_{th} \eta_n = \frac{\text{Thrust Power}}{\text{Electrical Power}} \quad (7.13)$$

If we can't keep the overall efficiency high, Electrothermal Propulsion systems lose any competitive advantage over chemical rockets. This is the major challenge in Electrothermal Propulsion thruster development.

1.4 Enthalpy of High Temperature Gas

In 1-D energy equation for nozzle flow:

$$\frac{1}{2} u_e^2 = \frac{1}{2} u_c^2 + (h_c - h_e)$$

The driving term is the specific enthalpy:

$$h \equiv e + \frac{P}{\rho} \quad (7.14)$$

The partial differential of the enthalpy with respect to temperature at constant pressure is the specific heat:

$$c_p = \left(\frac{\partial h}{\partial T} \right)_P \quad (7.15)$$

The enthalpy of a gas mixture depends on its constituents, and on what internal energy modes they have available. Consider one kg of a diatomic gas. The original number of molecules is:

$$N_o = \frac{1}{M_A} \quad (7.16)$$

where M_A is the molecular mass. (for example, one kg of nitrogen has

$$(N_o)_{N_2} = \frac{1}{2.325867 \times 10^{-26} [kg]} = 2.150 \times 10^{25} [kg^{-1}]$$

If you heat this to a temperature where dissociation and ionization are important, you get:

- $\alpha_2 N_o$ Neutral molecules
- $\alpha_1 N_o$ Neutral atoms
- $\alpha_2^+ N_o$ Molecular single ions
- $\alpha_1^+ N_o$ Atomic single ions
- $\alpha_e N_o$ Free electrons

Where α 's are species fractions. Note, we ignore multiple ionization levels.

By conservation of atomic particles, we get:

$$\alpha_2 + \alpha_2^+ + \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_1^+ = 1 \quad (7.17)$$

While conservation of electric charge gives:

$$\alpha_e = \alpha_1^+ + \alpha_2^+ \quad (7.18)$$

We want an equation for the Enthalpy of (7.14), so we will start with the P/ρ term, then do the internal energy , e, term.

Now that we've defined the species fractions, we can use this in the perfect gas equation of state:

$$\frac{P}{\rho} = (\alpha_2 + \alpha_2^+ + \alpha_1 + \alpha_1^+ + \alpha_e) N_o k T \quad (7.19)$$

Rewriting conservation of atomic particles (7.17) as

$$\alpha_2 = 1 - \alpha_2^+ - \frac{1}{2} \alpha_1 - \frac{1}{2} \alpha_1^+$$

(7.19) then becomes:

$$\frac{P}{\rho} = (1 + \frac{1}{2} \alpha_1 + \frac{1}{2} \alpha_1^+ + \alpha_e) N_o k T$$

If we substitute in conservation of charge (7.18), we get:

$$\frac{P}{\rho} = (1 + \frac{1}{2} \alpha_1 + \alpha_2^+ + \frac{3}{2} \alpha_1^+) N_o k T \equiv \alpha_P N_o k T \quad (7.20)$$

which we can write in terms of an **Indicated Factor:** α_P .

Now we move on to the internal energy, e, term in (7.14).

We can consider the specific internal energy, e , for the individual species:

1. Neutral molecules

$$e_2 = \alpha_2 N_o \left(\frac{3}{2} k T + \beta_r k T + \beta_v k T + \sum_j \beta_j \varepsilon_j \right) \quad (7.21)$$

Where

$\frac{3}{2} k T$ = Translational Energy for all Molecules

$\beta_r k T$ = Rotational Energy for the Rotationally-Excited Fraction, β_r

$\beta_v k T$ = Vibrational Energy for the Vibrationally-Excited Fraction, β_v

β_j = Electronic Energy Fraction in the j^{th} Excited State

ε_j = The Energy of that Electronic State above Ground State

2. Neutral atoms

$$e_1 = \alpha_1 N_o \left(\frac{3}{2} k T + \sum_k \beta_k \varepsilon_k \right) \quad (7.22)$$

3. Molecular ions

$$e_2^+ = \alpha_2^+ N_o \left(\frac{3}{2} k T + \beta_r^+ k T + \beta_v^+ k T + \sum_l \beta_l \varepsilon_l \right) \quad (7.23)$$

4. Atomic ions

$$e_1^+ = \alpha_1^+ N_o \left(\frac{3}{2} k T + \sum_m \beta_m \varepsilon_m \right) \quad (7.24)$$

5. Free electrons

$$e_e = \alpha_e N_o \left(\frac{3}{2} k T \right) \quad (7.25)$$

Also need to include the specific dissociation energy (the energy tied up in dissociation):

$$e_d = N_o \frac{\alpha_1 + \alpha_1^+}{2} \varepsilon_d \quad (7.26)$$

Where

ε_d = Dissociation energy for a single molecule

The $1/2$ term exists because ε_d is split between a pair of atom.

Also need to include the specific ionization energy (the energy tied up in dissociation):

$$e_i = N_o (\alpha_2^+ \varepsilon_i + \alpha_1^+ \varepsilon'_i) \quad (7.27)$$

Where

ε_i = Molecular Ionization Potential

ε'_i = Atomic Ionization Potential

Combine all these terms:(7.20), (7.21), (7.22), (7.23), (7.24), (7.25), (7.26), (7.27), to get :

Total Specific Enthalpy of the Mixture

$$\begin{aligned}
 h = e + \frac{P}{\rho} = & \alpha_2 N_o \left[(5/2 + \beta_r + \beta_v) k T + \sum_j \beta_j \varepsilon_j \right] \\
 & + \alpha_1 N_o \left[\frac{5}{2} k T + \sum_k \beta_k \varepsilon_k + \frac{\varepsilon_d}{2} \right] \\
 & + \alpha_2^+ N_o \left[(5/2 + \beta_r^+ + \beta_v^+) k T + \sum_l \beta_l \varepsilon_l + \varepsilon_i \right] \\
 & + \alpha_1^+ N_o \left[\frac{5}{2} k T + \sum_m \beta_m \varepsilon_m + \frac{\varepsilon_d}{2} + \varepsilon_i' \right] \\
 & + \alpha_e N_o \left[\frac{5}{2} k T \right]
 \end{aligned} \tag{7.28}$$

Let's now look at how the internal degrees of freedom is affected by temperature.

Internal Degrees of Freedom

$$\mathcal{N} = \frac{2 c_v}{N_o k} \tag{7.29}$$

The available energy modes for storing internal energy.

1.4.1 Effect of Temperature on Internal Degrees of Freedom

- Monatomic species:

Figure 4:

Monatomic species have just translation giving it just 3 degrees of freedom (3 dimensions to move around). Monatomic species do not have rotation or vibration and so $\mathcal{N} = 3$ up until 1000K, then electronic energy modes become important and so the internal degrees of freedom increases. Monatomic species have fewer degrees of freedom, so their specific heats change less over a range of temperatures up to electronic excitation around 10^4 K.

- Diatomic gas:

Figure 5:

At low temperatures, the internal degrees of freedom for diatomic molecules is just translational. But, as the particle heats up, they start to gain rotational energy and there for \mathcal{N} increases to 5 (because particles can spin in two directions). Continue to heat and vibration is the next energy mode at around 1000-10000K and \mathcal{N} increases to 7. Increasing temperature again and we start to see electronic energy modes. In a diatomic gas, rotation becomes fully-excited at very low temperature (cryogenic temps, a few K), and vibrational excitation in the high 100s of K.

The adiabatic index,

$$\gamma = \frac{c_p}{c_v} = \frac{N_o k + c_v}{c_p} = \frac{\mathcal{N} + 2}{\mathcal{N}} \quad (7.30)$$

is then:

Figure 6:

Finally, note that since $h \sim N_o$ and $N_o \sim \frac{1}{m}$ the specific enthalpy is higher for lower molecular weight gases.

The α and β fractions adjust to changes in the flow field at rates that are often slower than translational changes (which are fast):

1. Rotation β_r, β_r^+
 - Adjust very rapidly
 - Fully-excited even at cryo temperatures
2. Vibration β_v, β_v^+
 - Adjusts at a rate that depends on the mode and molecule
 - Some modes several order of magnitude slower than translation or rotation
 - May be only partially excited at EP temperatures
 - Can get significant vibrational nonequilibrium
3. Electronic $\beta_j, \beta_k, \beta_l, \beta_m$
 - Adjusts at rates depending on density and temperature
 - Optical thinness results in loss of radiative equilibrium
 - Higher levels may be sparsely populated and thus negligible
4. Dissociation α_1, α_2
 - Need collisions with $\varepsilon \approx \varepsilon_d$ so slow to adjust
 - Rates highly dependent on density, $\frac{\varepsilon_d}{kT}$
5. Ionization α_1^+, α_2^+
 - Needs $\varepsilon \geq \varepsilon_i$ so slow to adjust

Example: Assume we have H_2 at equilibrium with a pressure of 0.01 atm and temperature of 3000K. The gas is at a dissociation of 60%. What is the enthalpy of this hydrogen gas?

$$\begin{aligned}\alpha_2 &= (1 - 0.6) = 0.4 && \text{Fraction of Neutral Molecules} \\ \alpha_1 &= 2(0.6) = 1.2 && \text{Fraction of Atoms} \\ \alpha_p &= (0.4 + 1.2) = 1.6 && \text{Fraction of Particles}\end{aligned}$$

At this temperature, there is lots of vibrational excitation, little electronic excitation, so the combustion chamber enthalpy is (7.28):

For atoms, we only have translation, there are no ions present at 3000K. If we were to do SAHA equation, we would see that the ionization fraction would in fact be very small. We would have to double the temperature in order to see ionization.

$$h_c = N_o \left[\alpha_2 \left(\frac{9}{2} kT \right) + \alpha_1 \left(\frac{5}{2} kT + \frac{1}{2} \varepsilon_d \right) \right] \quad (7.31)$$

Assume that all degrees of freedom except dissociation reach equilibrium after complete expansion, so

$$h_e = \frac{1}{2} \xi \alpha_1 N_o \varepsilon_d \quad (7.32)$$

where ξ is the fraction of the original dissociation remaining at the exit:

$$\begin{aligned}\xi &= 0 && \text{Equilibrium Flow} \\ \xi &= 1 && \text{Frozen Flow}\end{aligned}$$

Solving energy equation for exhaust speed gives:

$$\begin{aligned}\frac{1}{2} u_e^2 &= h_c - h_e \\ u_e &= \begin{cases} 1.95 \times 10^4 & , \quad \xi = 0 & , \quad \text{Equilibrium Flow} \\ 1.10 \times 10^4 & , \quad \xi = 1 & , \quad \text{Frozen Flow} \end{cases}\end{aligned}$$

We should expect higher energies with equilibrium flow compared to frozen flow. Hydrogen atoms have recombined and released some of that disassociation energy as translational energy.

Dissociation increases c_p (and thus enthalpy) but the increased enthalpy may not be recoverable (frozen flow loss). The gas may expand out so fast that there is no time for the energy to change modes and be released into the exchanged flow.

Jahn suggests three ways to avoid frozen-flow losses:

1. Increase nozzle length (which runs into practical scaling limits, increased weight, and viscous losses). If we make the nozzle longer, then the flow remains in the nozzle longer and then the particles have more time to change energy modes.
2. Operate at higher pressure (to increase the rate of recombination). Jahn's figure 6-3 shows how the complete frozen flow efficiency varies with specific impulse for hydrogen (the worst-case frozen flow efficiency, with no recombination at all). Note that higher pressure does help to control frozen-flow losses.

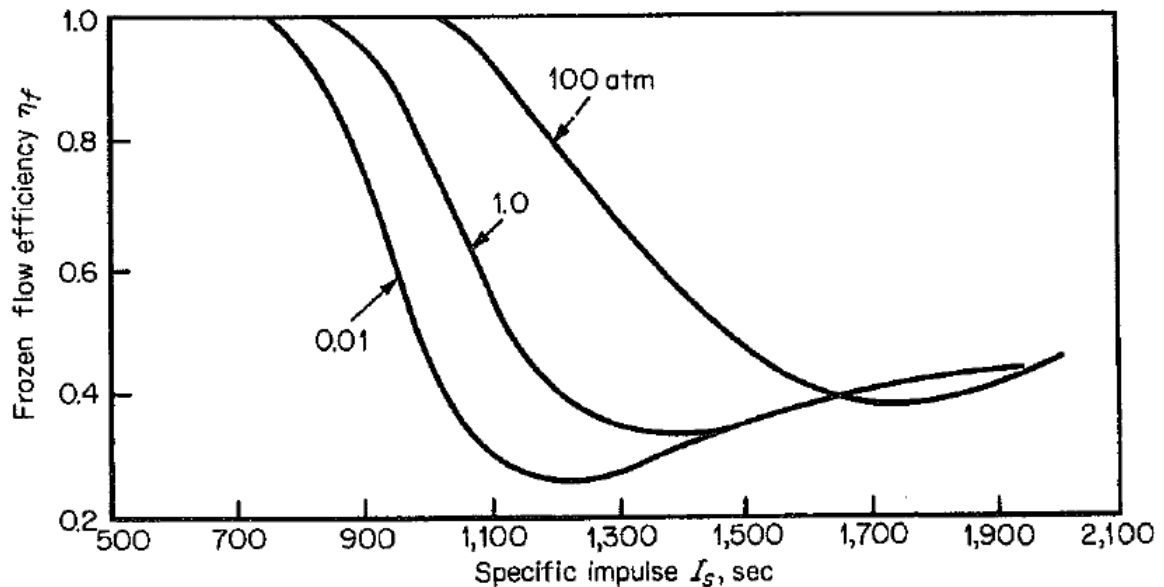
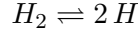


Fig. 6-3 Theoretical variation of frozen flow efficiency with specific impulse for hydrogen at various pressures. (From J. R. Jack, *Theoretical Performance of Propellants Suitable for Electrothermal Jet Engines*, ARS J., vol. 31, p. 1685, 1961.)

3. Choose a better propellant. Jahn gives a range of candidates at the end of 6-2. We can choose a propellant that doesn't disassociate as readily, or even a switching to a monopropellant.

1.5 Equilibrium Composition

We can define equilibrium composition as the point where forward and backward reactions balance. For example, the dissociation of hydrogen:



Showing us that the rate at which H_2 is associating is equal to the rate that it is disassociating. Where it is breaking up in the forward reaction and recombining in the backward.

The dissociation of hydrogen in an arcjet is at equilibrium when:

$$\frac{d}{dt}(H_2) = \frac{d}{dt}(H) = 0$$

Where the parenthesis represent the mean concentration [mol/m³]. In this equation, there is no change in the species concentrations while at equilibrium.

For the generic reaction



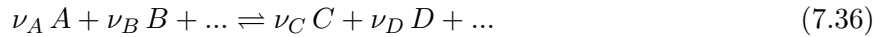
the rate of change of the concentration of species C is given by:

$$\frac{d}{dt}(C) = k_{\text{forward}}(A)(B) - k_{\text{backward}}(C)(D) \quad (7.34)$$

At equilibrium, $\frac{d}{dt} = 0$:

$$\frac{(A)(B)}{(C)(D)} = \frac{k_{\text{backward}}}{k_{\text{forward}}} \quad (7.35)$$

Very generally then, the reaction



Where ν_i is the number of moles per species. The reaction has an equilibrium constant

$$\kappa_P = \frac{P_C^{\nu_C} P_D^{\nu_D}}{P_A^{\nu_A} P_B^{\nu_B}} \quad (7.37)$$

where P_i is the partial pressure of species i . For a mixture of perfect gases,

$$P_i = \chi_i p \quad (7.38)$$

where

$$\begin{aligned} \chi_i &= \text{Mole Fraction of Species } i \\ p &= \text{Total Pressure} \end{aligned}$$

Thus, the equilibrium constant can be expressed as

$$\kappa_P = \frac{\chi_C^{\nu_C} \chi_D^{\nu_D} \cdots}{\chi_A^{\nu_A} \chi_B^{\nu_B} \cdots} P^{(\nu_C + \nu_D + \cdots) - (\nu_A + \nu_B + \cdots)} \quad (7.39)$$

But this has units! We have units of pressure in this form.

We can define a **Dimensionless Equilibrium Constant** for this generic reaction as:

$$\kappa_P = \frac{\chi_C^{\nu_C} \chi_D^{\nu_D} \cdots}{\chi_A^{\nu_A} \chi_B^{\nu_B} \cdots} \left(\frac{P}{P_o} \right)^{(\nu_C + \nu_D + \cdots) - (\nu_A + \nu_B + \cdots)} \quad (7.40)$$

by defining a reference pressure p_o (typically 1 atm).

Note:

- κ is dimensionless
- κ_p is NOT dimensionless

For ideal gases, K is a function of Temp only.

Values for κ can be found tabulated in numerous combustion texts, e.g., appendix of "Fundamentals of Classical Thermodynamics" by vanWylen and Sonntag. The next page has this table. Note that the values of κ in this table are the natural log values.

Table A.12
Logarithms to the Base e of the Equilibrium Constant K

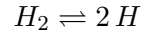
For the reaction $\nu_A A + \nu_B B \rightleftharpoons \nu_C C + \nu_D D$ the equilibrium constant K is defined as

$$K = \frac{a_C^{\nu_C} a_D^{\nu_D}}{a_A^{\nu_A} a_B^{\nu_B}}$$

Base on thermodynamic data given in the JANAF Thermochemical Tables, Thermal Research Laboratory, The Dow Chemical Company, Midland, Michigan.

Temp. K	$\text{H}_2 \rightleftharpoons 2\text{H}$	$\text{O}_2 \rightleftharpoons 2\text{O}$	$\text{N}_2 \rightleftharpoons 2\text{N}$	$\text{H}_2\text{O} \rightleftharpoons \text{H}_2 + \frac{1}{2}\text{O}_2$	$\text{H}_2\text{O} \rightleftharpoons \frac{1}{2}\text{H}_2 + \text{OH}$	$\text{CO}_2 \rightleftharpoons \text{CO} + \frac{1}{2}\text{O}_2$	$\frac{1}{2}\text{N}_2 + \frac{1}{2}\text{O}_2 \rightleftharpoons \text{NO}$
298	-164.005	-186.975	-367.480	-92.208	-106.208	-103.762	-35.052
500	-92.827	-105.630	-213.372	-52.691	-60.281	-57.616	-20.295
1000	-39.803	-45.150	-99.127	-23.163	-26.034	-23.529	-9.388
1200	-30.874	-35.005	-80.011	-18.182	-20.283	-17.871	-7.569
1400	-24.463	-27.742	-66.329	-14.609	-16.099	-13.842	-6.270
1600	-19.637	-22.285	-56.055	-11.921	-13.066	-10.830	-5.294
1800	-15.866	-18.030	-48.051	-9.826	-10.657	-8.497	-4.536
2000	-12.840	-14.622	-41.645	-8.145	-8.728	-6.635	-3.931
2200	-10.353	-11.827	-36.391	-6.768	-7.148	-5.120	-3.433
2400	-8.276	-9.497	-32.011	-5.619	-5.832	-3.860	-3.019
2600	-6.517	-7.521	-28.304	-4.648	-4.719	-2.801	-2.671
2800	-5.002	-5.826	-25.117	-3.812	-3.763	-1.894	-2.372
3000	-3.685	-4.357	-22.359	-3.086	-2.937	-1.111	-2.114
3200	-2.534	-3.072	-19.937	-2.451	-2.212	-0.429	-1.888
3400	-1.516	-1.935	-17.800	-1.891	-1.576	0.169	-1.690
3600	-0.609	-0.926	-15.898	-1.392	-1.088	0.701	-1.513
3800	0.202	-0.019	-14.199	-0.945	-0.501	1.176	-1.356
4000	0.934	0.796	-12.660	-0.542	-0.044	1.599	-1.216
4500	2.486	2.513	-9.414	0.312	0.920	2.490	-0.921
5000	3.725	3.895	-6.807	0.996	1.689	3.197	-0.686
5500	4.743	5.023	-4.666	1.560	2.318	3.771	-0.497
6000	5.590	5.963	-2.865	2.032	2.843	4.245	-0.341

Example: Say we have a Hydrogen gas at a temperature of 3000K and pressure of 20 atm. The dissociation reaction



can be described by the dimensionless equilibrium constant

$$\kappa = \frac{\chi_H^2}{\chi_{H_2}} \left(\frac{P}{P_o} \right)^{2-1} = \frac{\chi_H^2}{\chi_{H_2}} 20$$

Now, look up tabulated value of $K = K(T)$ at 3000 K (from the table above):

$$\kappa(3000K) = e^{-3.685} = 2.51 \times 10^{-2}$$

so, the mole fraction ratio is:

$$\frac{\chi_H^2}{\chi_{H_2}} = \frac{2.51 \times 10^{-2}}{20} = 1.255 \times 10^{-3}$$

We still have two unknowns but only one equation. However, from the definition of mole fraction,

$$\chi_H + \chi_{H_2} = 1$$

And so

$$1.255 \times 10^{-3} (1 - \chi_H) = \chi_H^2$$

which is a quadratic equation. The solution has two roots, but only one of them is positive, so:

$$X_h = 3.482 \times 10^{-2}$$

and so we have that...

At 3000 K and 20 atm pressure, hydrogen is about 3.5% dissociated.

1.5.1 Hydrazine Thruster

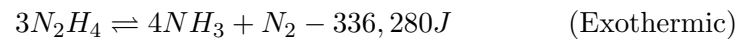
A more relevant example is the hydrazine thruster

Figure 7:

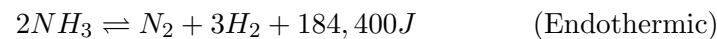
These have a long history

- Early N_2H_4 thrusters (Ranger and Mariner midcourse maneuvering engines) used a hypergolic slug to warm a non-spontaneous catalyst bed to temperatures at which N_2H_4 decomposed.
- The Shell 405 catalyst (alumina pellets coated with iridium) was developed in 1962, permitting spontaneous decomposition of hydrazine.

As we saw before, we have two competing reactions. The first is decomposition of hydrazine:



If this exothermic reaction was one-way, there would be enough energy to raise the products to an equilibrium temperature of 1650 K. But at this temperature ammonia dissociates:



This reaction is endothermic, which drives the equilibrium temperature down.

Equilibrium temperature assumes that both reactions have had time to complete! But the

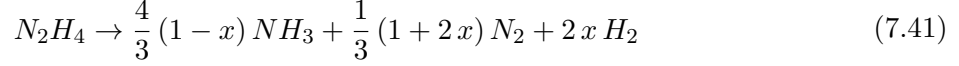
- Exothermic reaction is fast, $\sim < 1$ [ms].
- Endothermic reaction is slow, $\sim 10 - 100$ [ms].

This means that using hydrazine, we are definitely not going to get equilibrium flow. We are going to get energy loss in disassociation from ammonia.

For a catalyst bed/reaction chamber volume V_{ch} , we can reduce the residence time to control the dissociation fraction, x .

$$\tau_{res} = \frac{\rho V_{ch}}{\dot{m}}$$

The final reaction then becomes:



and the enthalpy balance for an adiabatic reaction is:

$$h_{N_2H_4, l \text{ at } 298K} = \frac{4}{3}(1-x)h_{NH_3}(T) + \frac{1}{3}(1+2x)h_{N_2}(T) + 2xh_{H_2}(T) \quad (7.42)$$

which can be solved for T , given enthalpies. (subscript l denotes liquid phase).

Need the enthalpies, curve-fits to enthalpy data, using

$$\begin{aligned} h_{NH_3}(\theta) &= (-16.83 + 12.35\theta + 0.983\theta^2) \left[\frac{\text{kcal}}{\text{mole}} \right] \\ h_{N_2}(\theta) &= (-2.83 + 7.75\theta + 0.183\theta^2) \left[\frac{\text{kcal}}{\text{mole}} \right] \\ h_{H_2}(\theta) &= (-1.967 + 6.60\theta + 0.367\theta^2) \left[\frac{\text{kcal}}{\text{mole}} \right] \\ h_{N_2H_4}(l, 298 K) &= 12 \left[\frac{\text{kcal}}{\text{mole}} \right] \end{aligned} \quad (7.43)$$

We can solve for T at various levels of dissociation:

Disassociation Level x	Equilibrium Temperature T_c [K]
0.0	1659
0.2	1502
0.4	1343
0.6	1182
0.8	1023
1.0	863

For comparison, the equilibrium composition at ~ 1000 K is

- 0.23% NH_3 at 10 atm
- 0.68% NH_3 at 20 atm

So, if allowed to equilibrate, ammonia would nearly completely dissociate.

Dissociation:

- Decreases chamber temperature T_c , but also
- Decreases the mean product mass (molecular weight)

To see how these competing effects influence I_{SP} , we look at values for $\varepsilon = 50$, $\frac{P_e}{P_c} = 8 \times 10^{-4}$ and Area Ratio of ~ 50

x	T_c [K]	Adiabatic Index γ	\bar{m} [g/mole]	I_{SP} [s]
0.0	1659	1.330	19.2	229.4
0.2	1502	1.336	17.6	226.9
0.4	1343	1.342	15.8	225.4
0.6	1182	1.348	14.0	223.6
0.8	1023	1.354	12.3	220.9
1.0	863	1.362	10.67	216.7

Clearly, best performance for a purely-chemical hydrazine thruster results at minimal dissociation $x \rightarrow 0$. Design is typically a compromise between:

- High performance at high T_c
- Long lifetime at low T_c

Typical choice is to have a disassociation fraction of $x \sim 0.4$, 40%. So we compromise between long life and good performance.

We'll explore more when we get into resistojets shortly.

1.6 Nozzle Flow

Electrothermal thrusters use a converging-diverging nozzle to convert high-enthalpy chamber gases to high-speed supersonic exhaust:

Figure 8:

Here, we see that temperature is high in the chamber and drops as we move to the exit. The velocity experiences the opposite and accelerates as it moves across the chamber.

If we describe two planes, x, and y, that are perpendicular to the mean flow direction, we can use simple 1-D compressible flow equations to arrive at area ratio equation:

$$\frac{A_y}{A_x} = \frac{M_x}{M_y} \left(\frac{1 + \frac{\gamma-1}{2} M_y^2}{1 + \frac{\gamma-1}{2} M_x^2} \right)^{\frac{\gamma+1}{2(\gamma-1)}} \quad (7.44)$$

where we use Mach number:

$$M = \frac{u}{\sqrt{\gamma R T}} \quad (7.45)$$

Where

$$R = \text{Specific Gas Constant} = \frac{\Re}{MW}$$

to nondimensionalize the axial velocity component, u. Note that the species gas constant:

$$R = k N_o \quad (7.46)$$

The most useful way to use this equation is to calculate properties:

- A_e, M_e at the exit
- A_t, M_t at the throat

If the throat is choked, (ie, flow becomes sonic, $M_t = 1$).

And so:

$$\frac{A_e}{A_t} = \frac{1}{M_e} \left(\frac{2 + (\gamma - 1) M_e^2}{\gamma + 1} \right)^{\frac{\gamma + 1}{2(\gamma - 1)}} \quad (7.47)$$

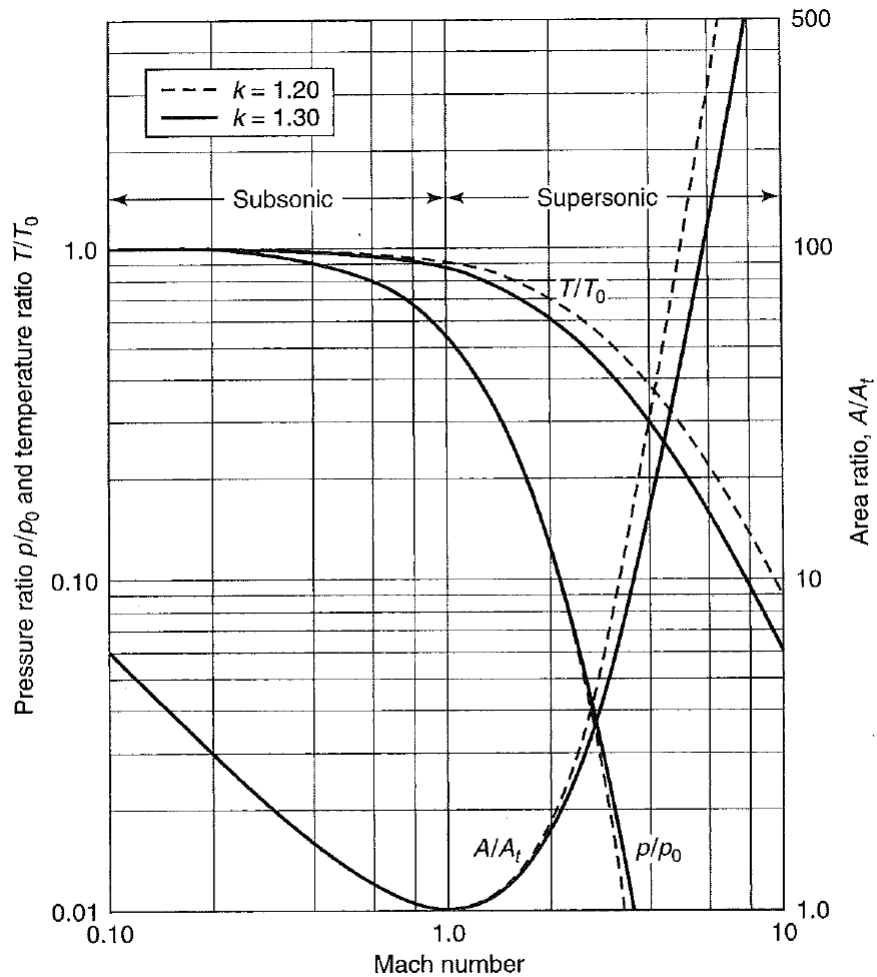


FIGURE 3–1. Relationship of area ratio, pressure ratio, and temperature ratio as functions of Mach number in a De Laval nozzle for the subsonic and supersonic nozzle regions.

Note that as γ decreases, we need to increase the area ratio for a fixed exit Mach number.

We can use this equation (7.47) and the isentropic relations to relate exit conditions to fluid conditions in the combustion chamber:

$$u_e = \sqrt{\frac{2\gamma R T_c}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} + u_c^2 \quad (7.48)$$

We can say that the chamber velocity is negligible, $u_c \approx 0$

By inspection, if we want to maximize u_e , we need

- High chamber temperature
- Low molecular-weight gas
- Large ratio of chamber pressure to exit pressure

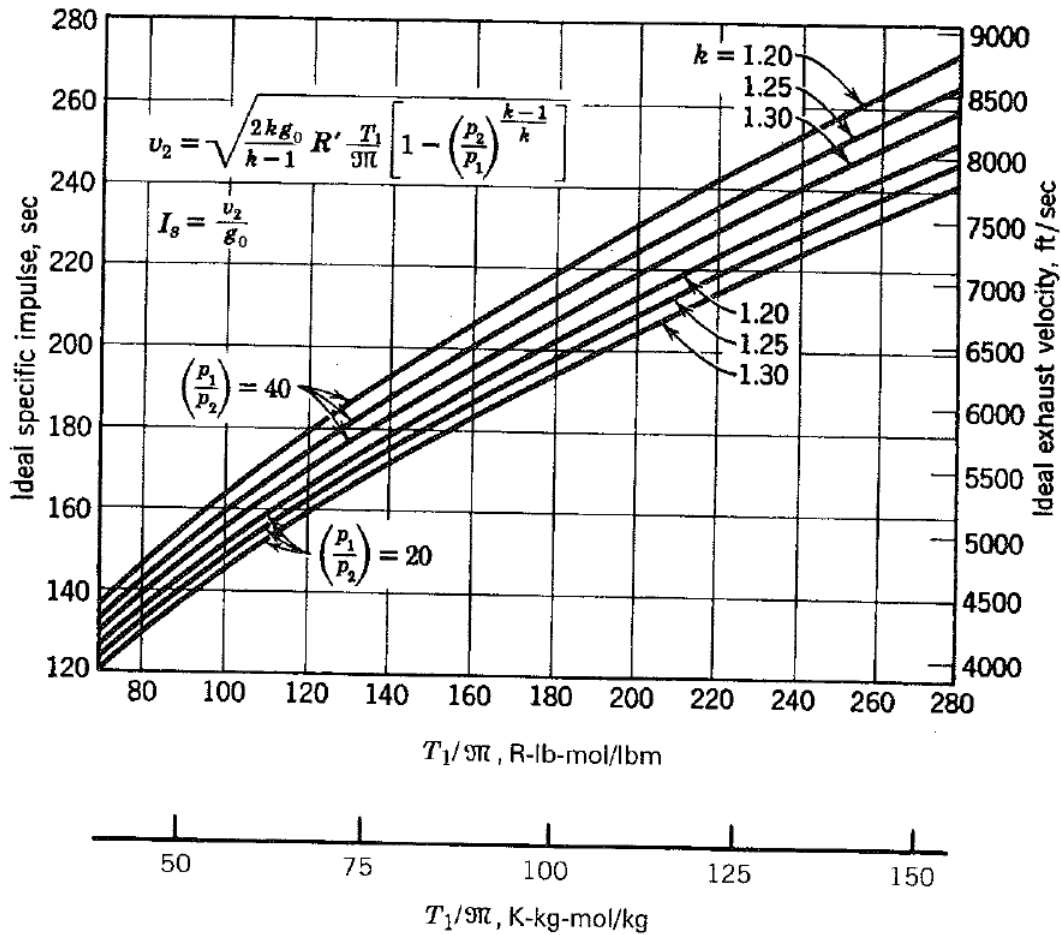


FIGURE 3–2. Specific impulse and exhaust velocity of an ideal rocket at optimum nozzle expansion as functions of the absolute chamber temperature T_1 and the molecular mass M for several values of k and p_1/p_2 .

Note that as γ decreases, ideal I_{SP} increases for fixed pressure ratio and chamber temperature.

For a perfectly expanded nozzle ($P_e = P_\infty$), the thrust is given by:

$$T = \dot{m} u_e \quad (7.49)$$

And the specific impulse is given by:

$$I_{SP} = \frac{u_e}{g_o} \quad (7.50)$$

For an imperfectly expanded nozzle, we can define an effective exhaust velocity:

$$C_e = u_e + \frac{A_e}{\dot{m}} (P_e - P_\infty) \quad (7.51)$$

And the specific impulse becomes:

$$I_{SP} = \frac{C_e}{g_o} \quad (7.52)$$

For launch vehicles, Isp always increase with altitude. For ET, $P_\infty \sim 0$.

CHOKED FLOW

If the pressure ratio across a nozzle is low enough, we will get sonic flow at the throat (min area location). It's most convenient to calculate the mass flow rate \dot{m} based on the throat conditions or to reference the area and velocity to the throat.

Critical Pressure Ratio

$$\frac{P_t}{P_c} = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \quad (7.53)$$

Where

P_t = Throat Pressure P_c = Heating Chamber Pressure

The specific volume $v = \frac{1}{\rho}$ at the throat is given by:

$$\frac{v_t}{v_c} = \left(\frac{\gamma + 1}{2} \right)^{\frac{1}{\gamma - 1}} = \frac{\rho_c}{\rho_t} \quad (7.54)$$

while the throat temperature is related to the bulk temperature in the heating zone by:

$$\frac{T_t}{T_c} = \frac{2}{\gamma + 1} \quad (7.55)$$

Finally, $M_t = 1$, so the velocity in the throat is the speed of sound:

$$u_t = a_t = \sqrt{\gamma R T_t} \quad (7.56)$$

and the mass flow rate:

$$\dot{m} = \rho_t u_t A_t \quad (7.57)$$

becomes

$$\dot{m} = P_c A_t \sqrt{\frac{\gamma}{R T_c} \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma+1}{\gamma-1}}} \quad (7.58)$$

The area ratio for choked flow at a location, x (subsonic or supersonic), is given by:

$$\frac{A_t}{A_x} = \left(\frac{\gamma + 1}{2} \right)^{\frac{1}{\gamma-1}} \left(\frac{P_x}{P_c} \right)^{\frac{1}{\gamma}} \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{P_x}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} = \frac{1}{\varepsilon} \quad (7.59)$$

where ε is the area ratio, A_x/A_t

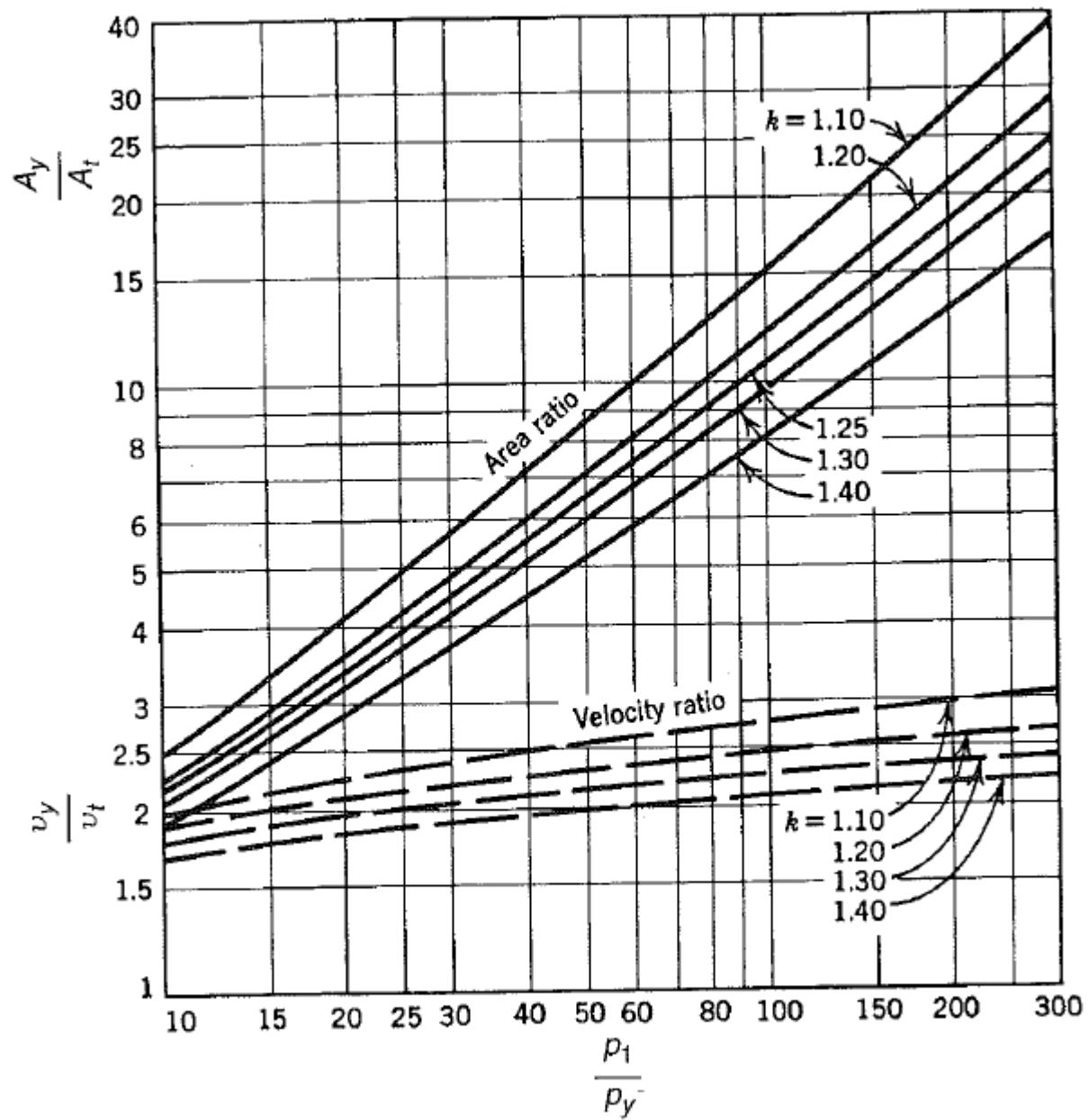
The general velocity ratio is then:

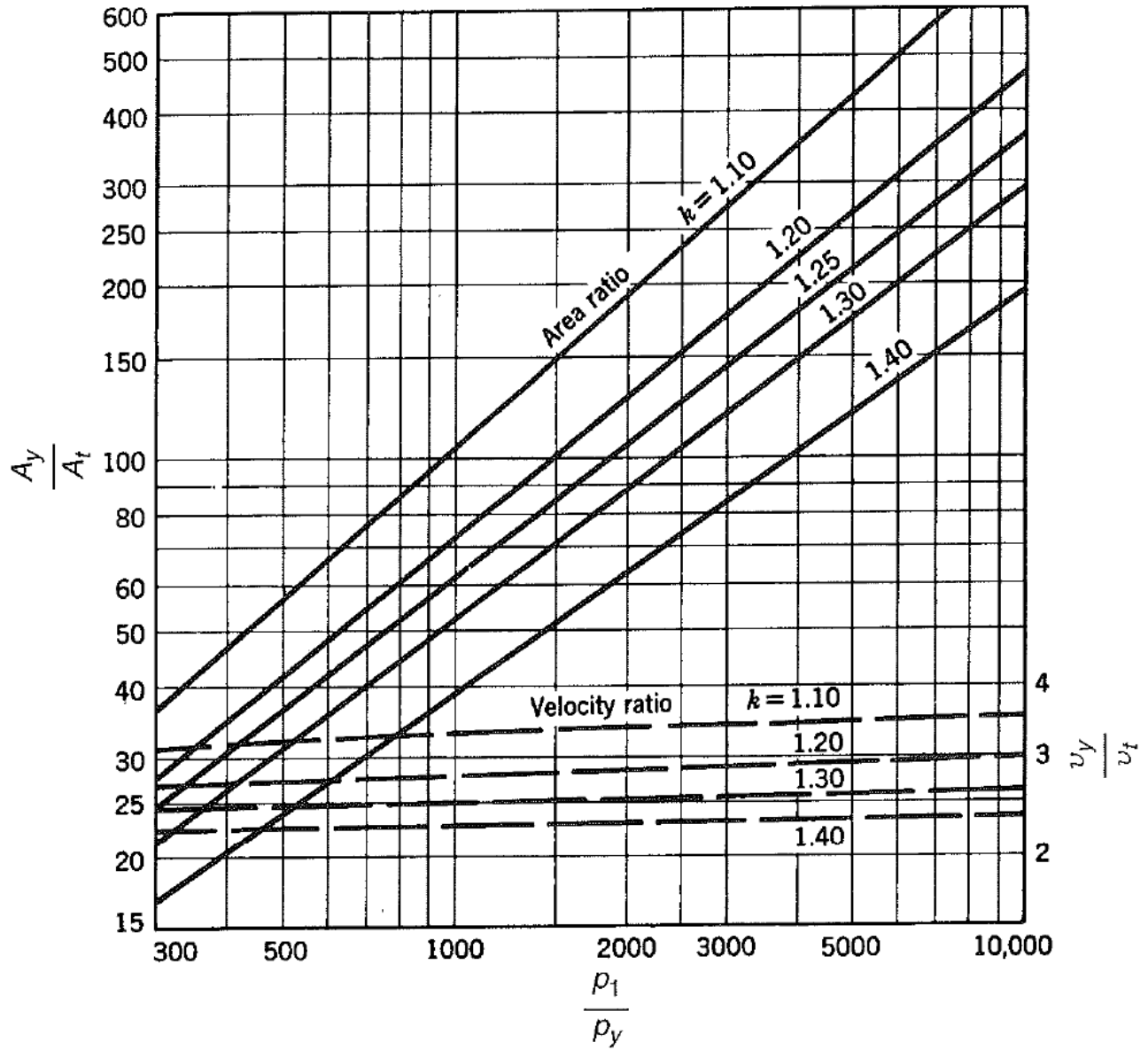
$$\frac{u_x}{u_t} = \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{P_x}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (7.60)$$

Therefore, the exit velocity ratio is:

$$\frac{u_e}{u_t} = \sqrt{\frac{\gamma + 1}{\gamma - 1} \left[1 - \left(\frac{P_e}{P_c} \right)^{\frac{\gamma-1}{\gamma}} \right]} \quad (7.61)$$

Both relations are shown below (7.59) and (7.60)





Finally, the thrust

$$T = \dot{m} u_e + A_e (P_e - P_\infty) \quad (7.62)$$

can be written in terms of chamber and exit conditions as:

$$T = P_c A_t \sqrt{\frac{2\gamma^2}{\gamma-1} \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}} \left[1 - \left(\frac{P_e}{P_c}\right)^{\frac{\gamma-1}{\gamma}}\right]} + A_e (P_e - P_\infty) \quad (7.63)$$

This is commonly nondimensionalized as the thrust coefficient:

$$C_t = \frac{T}{P_c A_t} \quad (7.64)$$

which is shown below for ideal expansion conditions.

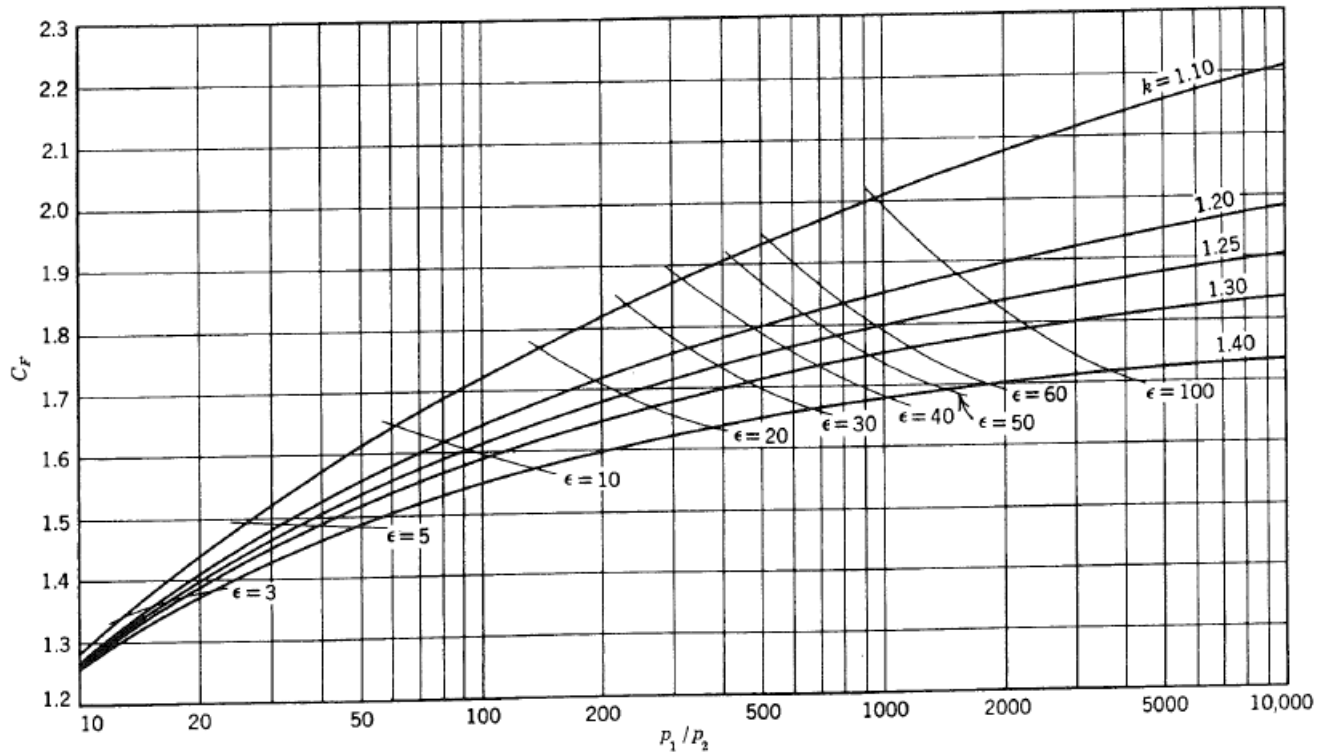


FIGURE 3-6. Thrust coefficient C_F as a function of pressure ratio, nozzle area ratio, and specific heat ratio for optimum expansion conditions ($p_2 = p_3$).

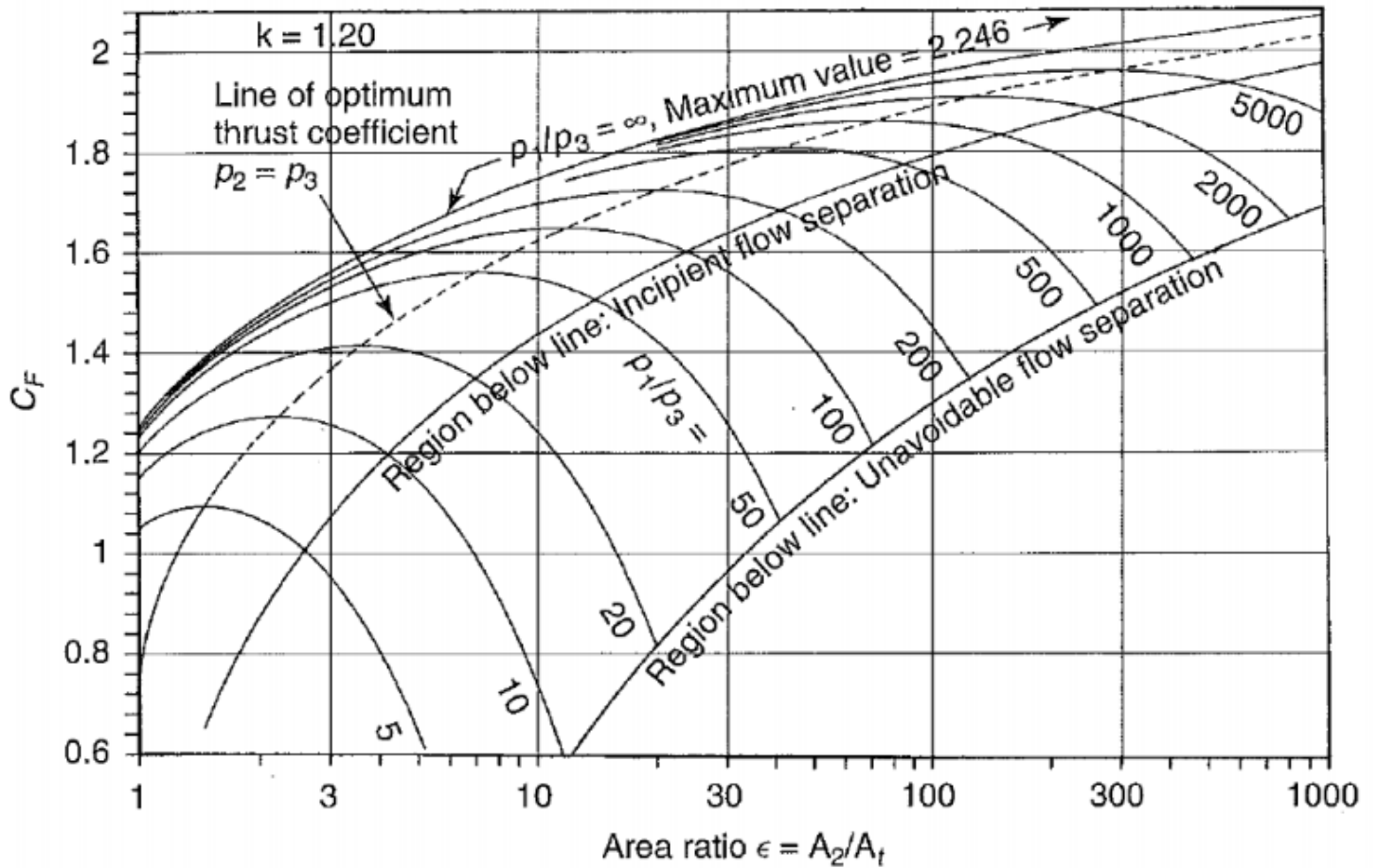


FIGURE 3-7. Thrust coefficient C_F versus nozzle area ratio for $k = 1.20$.

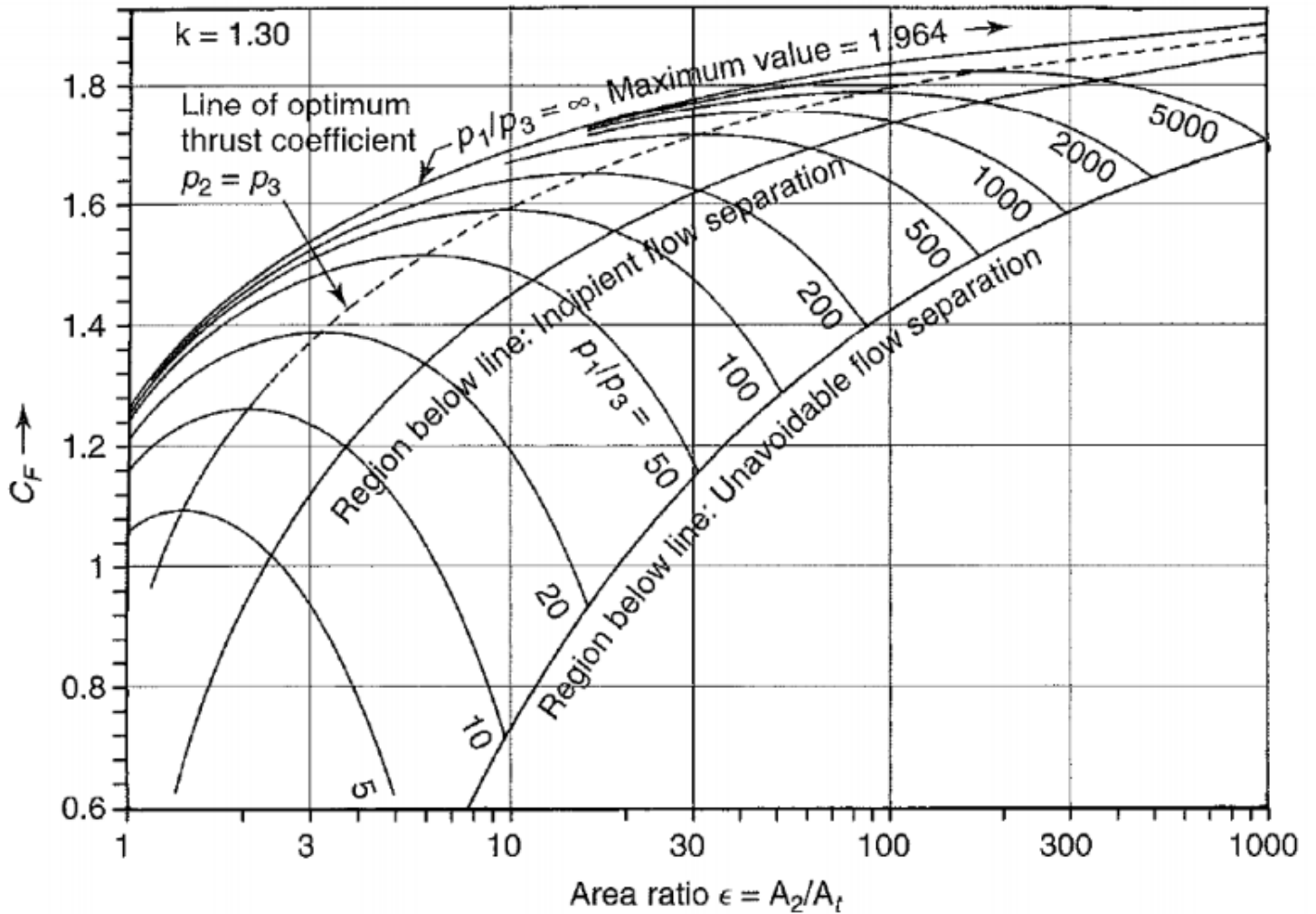


FIGURE 3-8. Thrust coefficient C_F versus nozzle area ratio for $k = 1.30$.