Section 2: Electromagnetics

AE435

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In this section, we will review the basics of charge, electricity, magnetism, and Maxwell equations.

3 Magnetostatics

"Charge in motion creates a magnetic field"

Contents

3.1 Electric Current

Current

$$J \equiv \frac{\partial Q}{\partial t} \qquad \left[\frac{c}{s}\right] = \left[A\right] = \text{Amps} \tag{50}$$

We'll use J, although sometimes we see I used for current. Currents can flow in a range of media: metals, semiconductors, fluids, gases and plasmas.

METALS
In metals, there are fixed ionic cores with bound inner ions:

Figure 10

Outer valence electrons get freely traded from ion to ion in response to electric fields. In other words, say we put 10 electrons in one end of a wire and we get 10 electrons out the other end. Those won't be the same 10 electrons we put in though.

GASES AND PLASMAS
In gases and plasmas, both electrons AND ions move:

Figure 11

- Most of the conduction is by electrons, because they're much lighter.
- In thermal motion, both ions and electrons are as likely to cross plane in one direction as another, so no net current.
- Under electric field, drift velocity of species (ions toward cathode, electrons toward anode) gives rise to current.

3.2 Current Density

Consider flux of particles through surface element dS:

Figure 12

Since particle with charge q crossing dS carries incremental current $q\,\vec{v}\cdot\hat{n}$. For N particles per unit volume, current crossing dS is then:

$$\mathrm{d}J = N\,q\,\vec{v}\cdot\hat{n}\,\mathrm{d}S$$

For multiple species, sum over all:

$$\mathrm{d}J = \sum_i N_i\,q_i\,\vec{v}_i\cdot\hat{n}\,\mathrm{d}S$$

Define current density as a vector current per unit area:

Current Density

$$\vec{j} = \sum_i N_i\,q_i\,\vec{v}_i \qquad \left[\frac{A}{m^2}\right] \tag{51}$$

So, the total current across a surface S is:

Total Current Across a Surface

$$J = \int_S \vec{j}\cdot\hat{n}\,\mathrm{d}S \tag{52}$$

3.3 Continuity

Consider volume V enclosed by surface S, within a current density field

Current crossing in or out of V through S is:

Note positive current is into the volume.

We also know from the definition of current (2.50), and from definition of volume charge density, and for a steady control volume:

Combining Eqns. (2.54) (2.53)

This must be true for an arbitrary volume, so the integrand must vanish at every point, giving:

3.4 Ohm's Law

Experiments show that

Where the conductivity has units of Siemens/m. In common conductors (metals, electrolytes, unmagnetized plasmas) a constant. These are called linear media or ohmic media. If you take the reciprocal of the conductivity, you get the resistivity,

which has units of Ohm-m, and is tabulated in lots of easy-to-find places (the CRC Handbook, multiple places on the web).

For steady currents, . By continuity then:

or substituting in conductivity:

For homogeneous media (constant over space): which for electrostatic fields is just Laplace's Eqn.

Given boundary conditions of or at surfaces of a medium, can solve for in the medium.

3.5 Magnetic Field

Recall our description of force on a charge, Coulomb's Law, from section II.A.1.

Again, look at a TWO-PARTICLE model.

The force on q due to q1 is Eqn. (2.1).

But what if the charges are moving? Let q move at a velocity and q1 move at a velocity . In this case, there's now another force on q due to q1 . This force is the magnetic force.

where is the permeability of free space. In SI units, the constant of proportionality is defined as:

By grouping terms relating to the field charge q1 , we can define a vector field that represents the force on the test charge q moving at velocity . So that:

where

This vector field is the magnetic induction a.k.a. magnetic field intensity, and has SI units Tesla.

Total force on test charge q is then Electric AND Magnetic forces, which is called the Lorentz force.

Work done by a stationary magnetic field?

A stationary magnetic field cannot do work/accelerate a fluid/particle/charge.

3.6 Forces on Conductors

Consider a conducting circuit immersed in a B-field. Charges with a number density F move at a velocity along a wire section with a cross-sectional area A.

The total charge in this section is:

And so the total magnetic force on this section is:

Since the current runs along the wire, , then we can swap locations in the vector equation:

Recall that the current density magnitude (2.51) , and that total current (2.52), then:

Integrating over the entire circuit, the total magnetic force on the circuit is:

3.7 Biot-Savart Law

4 Basic Templates

Note 1. This is how you make numbered notes

Exercise 1. This is how you make numbered exercises

Definition 1. This is how you make numbered definitions

Rule 1. This is how you make numbered rules

Equation Name

$$y = mx + b$$

Where:

$$\begin{aligned} variable1 &= \text{Description} \\ variable2 &= \text{Description} \end{aligned}$$