

# Homework 5

AE435 - Spring 2018  
Emilio R. Gordon

## Problem 2

```
1 function HW6
2     %Given Parameters
3     MW = 29.948;           % Argon Molecular Weight [amu]
4     ei = 15.76;           % Ionization Energy [eV]
5     G = 2e11;             % Scaling Constant [ $K^{-3/4} \cdot m^{-3/2}$ ]
6     T_low = 5000;         % Lower Temperature Bound [K]
7     T_high = 10000;       % Higher Temperature Bound [K]
8     k = 8.6173303e-5;     % Boltzmann's constant [eV/K]
9     k2 = 1.38064852e-23;  % Boltzmann's Constant [J/K] or [ $m^2 \text{ kg s}^{-2} \text{ K}^{-1}$ ]
10    %Conversions
11    MW = MW*(1.66054e-27); % [amu] to [kg]
12
13    %STP Conditions
14    T_STP = 273.15; % STP Temperature [K]
15    P_STP = 101325; % STP Pressure [Pa] = [kg/ms^2]
16
17    no = P_STP/(k2*T_STP); %Total Density of Heavy Particles
18    %10% from STP
19    no = no*.1
20
21    T = T_low:1:T_high;
22    alpha = zeros(length(T),1);
23    eta = zeros(length(T),1);
24
25    for i=1:length(T)
26        %Small SAHA Equation
27        alpha(i) = G*(no^(-1/2))*(T(i)^(3/4))*exp(-ei/(2*k*T(i)));
28
29        %Collision Cross-Sections
30        Q_en = 10e-20; % Electron-Neutral: Collision Cross Section [m^2]
31        Q_ei = 6.5e-17/((3/2)*k*T(i))^2; % Electron-Ion: Collision Cross Section [m^2]
32
33        %Collision Frequencies
34        np = no*alpha(i);
35        na = no*(1-alpha(i));
36
37        nu_en = na*Q_en;
38        nu_ei = np*Q_ei;
39
40        eta(i) = nu_ei/nu_en;
41    end
42    % Plotting Code Removed
43 end
```

### Problem 3

```
1 function HW6P3
2     %Given Parameters
3     MW = 208.98;           % Bismuth Molecular Weight [amu]
4     ei = 7.29;            % Ionization Energy [eV]
5     u = 1.2566370614e-6;  % N/A^2
6     mdot_3 = 3;           % Mass Flow Rate [g/s]
7     mdot_6 = 6;           % Mass Flow Rate [g/s]
8     J_low = 3;            % Lower Current [kA]
9     J_high = 25;          % Higher Current [kA]
10
11     %Dimensions for the Princeton Benchmark Thruster [cm]
12     PBT = [0.95, 5.1, 9.3, 6.4, 0.95, 10];
13     % [r_c, r_a, r_ao, r_ch, t_a, l_c]
14
15     %Conversions
16     MW = MW*(1.66054e-27); % [amu] to [kg]
17     ei = ei*(1.60218e-19); % [eV] to [Joules]
18     mdot_3 = mdot_3*1e-3; % [g/s] to [kg/s]
19     mdot_6 = mdot_6*1e-3; % [g/s] to [kg/s]
20     J_low = J_low*1000;    % [kA] to [A]
21     J_high = J_high*1000;  % [kA] to [A]
22
23     J = J_low:1:J_high;
24
25     Ct_3 = zeros(length(J),1);
26     Ct_6 = zeros(length(J),1);
27     T_3 = zeros(length(J),1);
28     T_6 = zeros(length(J),1);
29     ue_3 = zeros(length(J),1);
30     ue_6 = zeros(length(J),1);
31     Isp_3 = zeros(length(J),1);
32     Isp_6 = zeros(length(J),1);
33     jet_3 = zeros(length(J),1);
34     jet_6 = zeros(length(J),1);
35
36     ra = PBT(2);
37     rc = PBT(1);
38
39     for i=1:length(J)
40         xi_3 = xi(J(i),u,ra,rc,mdot_3,ei, MW);
41         xi_6 = xi(J(i),u,ra,rc,mdot_6,ei, MW);
42
43         Ct_3(i) = C_T(mdot_3,xi_3,ra,rc);
44         Ct_6(i) = C_T(mdot_6,xi_6,ra,rc);
45
46         T_3(i) = T(Ct_3(i),u,J(i));
47         T_6(i) = T(Ct_6(i),u,J(i));
48
49         ue_3(i) = T_3(i)/mdot_3;
50         ue_6(i) = T_6(i)/mdot_6;
51
52         Isp_3(i) = ue_3(i)/9.81;
```

```

53     Isp_6(i) = ue_6(i)/9.81;
54
55     jet_3(i) = (.5*T_3(i)*ue_3(i))/1000;
56     jet_6(i) = (.5*T_6(i)*ue_6(i))/1000;
57 end
58
59 JJ = linspace(3,25,length(J)); %convenient plotting for current
60
61 % Superficial Plotting Code Removed
62 subplot(2,2,1)
63 plot(JJ,Ct_3,JJ,Ct_6, 'linewidth',2)
64
65 subplot(2,2,2)
66 plot(JJ,T_3,JJ,T_6, 'linewidth',2)
67
68 subplot(2,2,3)
69 plot(JJ,Isp_3,JJ,Isp_6, 'linewidth',2)
70
71 subplot(2,2,4)
72 plot(JJ,jet_3,JJ,jet_6, 'linewidth',2)
73
74 end
75
76 function nu = nu(mdot)
77     mdot_star = 0.066;
78     nu = mdot/mdot_star;
79 end
80
81 function xi = xi(J,u,ra,rc,mdot,ei, MW)
82     xi = (J*sqrt((u/(4*pi))*log(ra/rc)))/(sqrt(mdot)*(((2*ei)/MW)^(1/4)));
83 end
84
85 function C_T = C_T(mdot,xi,ra,rc)
86     C_T = (nu(mdot)/(xi^4)) + log((ra/rc) + (xi^2));
87 end
88
89 function T = T(C_T,u,J)
90     T = C_T*(u/(4*pi))*(J^2);
91 end

```