Section 2: Electromagnetics

AE435

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In this section, we will review the basics of charge, electricity, magnetism, and Maxwell equations.

4 Magnetostatics with Magnetic Media

Contents

4.1 Effect of Magnetic Media

In our previous discussion, we only considered magnetostatics involving steady currents in a vacuum. Now we will examine...

- **Question:** What happens if matter is present?
- **Answer:** The magnetic field $\vec{B}$ changes!
- **Reason:** Matter has micro-currents associated with the motion of the electrons around atoms, "atomic currents"
- **Aftermath:** So now we must consider two kinds of currents:
  - Conduction currents, involving free charges
  - Atomic currents, with no charge transport (to the first order)

Each atom has a magnetic dipole moment.

Magnetic Dipole Moment

$$\vec{m}_i = \frac{1}{2} J_i \oint_c \vec{r}_i \times \mathrm{d}\,\vec{l} \tag{75}$$

We can define a macroscopic vector quantity analogous to polarization, known as the **magnetization** or the magnetic dipole moment per unit volume.

Magnetization

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_i \vec{m}_i \tag{76}$$

In the **unmagnetized state**, $\vec{M} = 0$ because $\vec{m}_i$ have random orientations that cancel out. In the presence of an external $\vec{B}$, matter becomes organized and $\vec{M}$ can become nonzero depending on the material properties.

Magnetization Current: How does magnetization give rise to currents?

Figure 20

For a uniform $\vec{M}$, currents cancel in the interior but not on the exterior. The result is a net surface current as shown in Figure 20.

Similarly, if $\vec{M}$ is non-uniform, we can have an internal net current.

We can define a **Magnetization Current Density:**

$$\vec{j}_m = \nabla \times \vec{M} \tag{77}$$

4.2 Total Magnetic Field

To incorporate $\vec{j}_m$ into Ampere's Law (Equation 72) we have to modify the magnetic field equations, just as we modified Gauss' law to include ρ_e.

As before, we still have no monopoles:

$$\nabla \cdot \vec{B} = 0$$

But now, Ampere's law becomes:

$$\nabla \times \vec{B} = \mu_o \left(\vec{j} + \vec{j}_m \right) \tag{78}$$

Using Equation 77, we can write this as:

$$\nabla \times \left(\frac{1}{\mu_o} \vec{B} - \vec{M} \right) = \vec{j}$$

Where $(\frac{1}{\mu_o} \vec{B} - \vec{M})$ depends only on conduction current density $\vec{j}$ as its source. As a results, we define a vector field:

Magnetic Intensity or "H"-field

$$\vec{H} = \frac{1}{\mu_o} \vec{B} - \vec{M} \qquad \left[\frac{\mathrm{A}}{m} \right] = [\text{Oersted}] \tag{79}$$

Note: $1 \ \frac{\mathrm{A}}{m} = 0.01257$ Oersted

Finally, **Ampere's Law for Magnetic Media** is:

$$\nabla \times \vec{H} = \vec{j} \tag{80}$$

A comparison of Magnetostatics and Electrostatics:

Electrostatics	**Magnetostatics**
In vacuum (no ρ_p)	In vacuum (no $\vec{j}_m$)
$\nabla \cdot \vec{E} = \frac{q}{\epsilon_o}$ (isolated charges)	$\nabla \cdot \vec{B} = 0$
$\nabla \cdot \vec{E} = \frac{\rho_e(\vec{r})}{\epsilon_o}$ (distributed charges)	
$\nabla \times \vec{E} = 0$	$\nabla \times \vec{B} = \mu_o \vec{j}$
With media effects (finite ρ_p)	With media effects (finite $\vec{j}_m$)
$\nabla \cdot \vec{E} = (\rho_f {+}_p)/\epsilon_o$	$\nabla \cdot \vec{B} = 0$
$\nabla \cdot \vec{D} = \rho_f$	
$\nabla \times \vec{E} = 0$	$\nabla \times \vec{B} = \mu_o(\vec{j} + \vec{j}_m)$
	$\nabla \times \vec{H} = \vec{j}$

We can also derive the intergral equation for magnetic intensity. From Equation 80, if we integrate over a surface element and apply Stokes theorem on a closed curve surrounding the surface, we get:

$$\int_S \nabla \times \vec{H} \cdot \hat{n}\,\mathrm{d}A = \oint_C \vec{H} \cdot \mathrm{d}\,\vec{l} = \int_S \vec{j} \cdot \hat{n}\,\mathrm{d}A = J \tag{81}$$

Important Note: This only applies for Magnetostatics. It does not work for time-varying fields.

4.3 Constitutive Equations/Relations

4.4 Boundary Conditions

4.5 Magnetic Flux