Section 4: Kinetic Theory

AE435

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2 Mean Free Path

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2.1 Mean Free Path

Assume a particle with diameter, d.

Collisions occur when the center of a another particle falls within a volume of diameter, $2d$, swept out by the initial particle.

Figure 2

For an average speed:

$$\bar{c} = \frac{\sum c_i}{N} \tag{13}$$

The volume swept out per unit time is: $\pi\, d^2\, \bar{c}$

Given a number density, n, $\frac{\#}{m^3}$

The number of collisions will be:

$$\theta = n\, \pi\, d^2\, \bar{c} \tag{14}$$

If only one particle is moving, we can derive...

Mean Free Path

$$\lambda_1 = \frac{\bar{c}}{\theta} = \frac{1}{n\, \pi\, d^2} \tag{15}$$

The average distance between collisions for a particle

If all the particles are moving at the same speed, the relative velocity becomes $\frac{\bar{\mathbf{c}}}{\sqrt{\mathbf{2}}}$ such that the mean free path becomes:

$$\lambda = \frac{\bar{c}}{\sqrt{2}\, \theta} = \frac{1}{\sqrt{2}\, n\, \pi\, d^2} \tag{16}$$

Example

Consider air at STP with number density $n_o = 2.69 \times 10^{25} \quad m^{-3}$ which is the number of particles per cubic meter.

The average space between particles:

$$\begin{aligned} \delta = n_o^{-\frac{1}{3}} &= 3.34 \times 10^{-9} \quad [m] \\ &= 3.34 \quad [nm] \end{aligned}$$

While the molecular diameter is:

$$\begin{aligned} d &\approx 0.37 \quad [nm] \\ &= 3.7 \quad [\dot{A}] \end{aligned}$$

As a result, the Mean Free Path is:

$$\lambda = \frac{1}{\sqrt{2}\, n\, \pi\, d^2} = 61.1 \quad [nm]$$

Giving us the general relation that

$$d << \delta << \lambda$$

Molecular Diameter << Average Space Between Particles << Mean Free Path

Question: When would we have to use these variables?

Answer: We typically would know when to use them via the Knudsen Number.

Knudsen Number

$$K_n = \frac{\lambda}{L}$$

Where L is the characteristic size of the system.

$$K_n >> 1$$

- If the Knudsen Number is large, the distance between the collision of particles is much larger than the size of the particles. As a result kinetic theory should be used and our system is likely to have a non-equilibrium gas. As a result, the governing equations would be Boltzmann equation.

$$K_n << 1$$

- If the Knudsen Number is small, the distance is smaller and so there are lots of collisions happening. By the time a particle goes from one side to the next it experiences lots of collisions while covering that distance so macroscopic fluid equations useful. As a result, the gas is in equilibrium and so the governing equations would be Navier Stokes or Euler's Equations.