

# Section 8: Electromagnetic Propulsion

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## 6 Pulsed Plasma Thruster (PPT)

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## 6.1 Pulsed Plasma Thruster

In 1964 Russians decided to avoid problems with tanks, valves, regulators, by using a solid propellant. The first propellant tried was Teflon, and although many others have been tried, Teflon ( $C_2F_4$ ) remains the best choice. It has a density almost twice that of Lexan (2.2 g/cm<sup>3</sup>). It does not have a liquid phase, and evaporates from solid to vapor.

PPT was first flown in 1964 (Zond 2) and in 1968 (LES-6). The classic schematic is:

**Figure 9: PPT Diagram**

Parallel plate design. JxB acceleration.

Stored energy is:

$$\frac{1}{2}CV_o^2 \quad [J] \quad (8.64)$$

For a pulse rate  $f$ , then the average power is

$$P_{\text{avg}} = f E_o \quad [W] \quad (8.65)$$

Typically  $f \sim 1 - 5Hz$ .

Each pulse creates thrust, and evaporates mass,  $m$ .

### Figure 10: Thrust to Propulsion Time

We always use bit when talking about pulse devices. The total is over the entire time, the impulse bit is the amount of impulse for one single pulse. Impulse bit is:

$$\int T dt = I_{\text{bit}} \quad (8.66)$$

Specific impulse is then:

$$I_{SP} = \frac{I_{\text{bit}}}{g_o m} \quad (8.67)$$

Average thrust:

$$\bar{T} \text{ or } T_{\text{avg}} = f I_{\text{bit}} \quad P_{\text{avg}} = f E_o \quad (8.68)$$

The arc heats the Teflon to  $\sim 600 - 700K$ , so it ablates. This ablation continues to occur after the pulsed power is applied. And the ablation that occurs after the initial pulse is referred to as "late time ablation".

This is a major source of inefficiency. As we ablate the mass of teflon and when we heat the surface of the teflon, now we have a hot propellant and some of that propellant mass will ablate which is a neutral and slowly drift out of the thruster at a later time.

### Figure 11: Late Time Ablation Illustration

There is no current being forced through the vapor after the mass has be ablated.

## 6.2 Efficiency

The efficiency is calculated from a simple circuit model:

**Figure 11: Efficiency Circuit Model**

$Z_o$  is the impedance of the drive circuit,  $R_L$  is the load resistance (the joule heating that goes into the heating of the plasma, the switch resistance, the intrinsic PFN resistance),  $Z_L$  is the load impedance

Assume a matched PFN:

$$R_L + Z_L = Z_o \quad (8.69)$$

Thus the voltage drop across the load is equal to that across  $Z_o$ , that is:  $\frac{V_o}{2}$

Thus the current is:

$$J = \frac{\frac{V_o}{2}}{Z_o} = \frac{\frac{V_o}{2}}{R_L + Z_L} \quad (8.70)$$

Then the charge is:

$$Q_o = C_o V_o$$

And the length of the pulse is

$$\frac{Q_o}{J} = \frac{C_o V_o}{(V_o/2)/Z_o} = 2 C_o Z_o = t_p \quad (8.71)$$

Since  $Z_o = \sqrt{\frac{L}{C}}$  where L, C are per section of the PFN, and

$$C_o = N C$$

Where  $N$  is the number of sections in the PFN number. We assume each section has the same capacitance

Then

$$t_p = 2 N \sqrt{L C} \quad (8.72)$$

Recognize that for cases where Electromagnetic energy does work on a mass, thrust in its electromagnetic form is:

$$\text{Thrust} = F_{em} = \frac{1}{2} L' J^2$$

and the resulting power

$$P = \frac{1}{2} F u_e = J^2 Z_o = \frac{1}{2} \left( \frac{1}{2} L' J^2 \right) u_e$$

which means the impedance is

$$Z_o = \frac{1}{4} L' u_e$$

Also, for coaxial electrodes, Maecker (8.22) with (8.53) is:

$$F = \frac{\mu J^2}{4 \pi} \left( \ln \left( \frac{r_a}{r_c} \right) + \frac{3}{4} \right) = \frac{1}{2} L' J^2$$

$$L' = \frac{2 \mu}{4 \pi} \left( \ln \left( \frac{r_a}{r_c} \right) + \frac{3}{4} \right) \quad (8.73)$$

Returning to efficiency,  $R_L$  represents losses in capacitors, switches, electrodes, etc., so let the efficiency:

$$\eta = \frac{Z_L}{R_L + Z_L} = \frac{1}{\frac{R_L}{Z_L} + 1} \quad (8.74)$$

Impedance of the load over entire impedance.  $R_L$  is the resistive lost in the circuit itself,

so

$$Z_L = \frac{1}{4} L' u_e = \frac{1}{2} \frac{\mu}{4 \pi} \left( \ln \left( \frac{r_a}{r_c} \right) + \frac{3}{4} \right) u_e \quad (8.75)$$

Equation 8.75 is the electromagnetic impedance for coaxial electrodes. The electromagnetic impedance for parallel plate electrodes is somewhere in our notes.

**For Example:**

The resistance  $R_L$  is not easy to estimate and is usually done experimentally.

Let  $u_e = 2 \times 10^4 \text{ m/s}$  and our coaxial PPT has the geometry of  $\ln(r_a/r_c) + \frac{3}{4} = 2.5$  then we will calculate the  $Z_L$  for this thruster to be  $Z_L = 2.5 \text{ m}\Omega$ , (Equation 8.54)

If resistive losses amounted to  $R_L = 5 \text{ m}\Omega$  then we would have an efficiency,  $\eta \sim 33\%$ .

Unfortunately those resistive losses tend to be much higher, around 15 mOhms.

The advantage of using parallel plate electrodes: recall that  $Z = 0.5 * k * u_e$  where  $k = 2.57 \times 10^{-7}$  for a coaxial geometry and  $k \sim 5 \times 10^{-7}$  for parallel plate geometry.

Therefore,  $Z_L = 5 \text{ m}\Omega$

We would get a little bit higher load impedance meaning we would get a higher efficiency.

### 6.3 Two Stream Model

Going back to the idea of late time ablation. At the beginning we have the fast plasma current sheet coming out and then the second phase of slow moving particles coming out.

The first complication is that Teflon ( $C_2F_4$ ) is dissociated to  $C + F + F$  and then is partially ionized ( $C^+, F^+$ ). Because of pressure, neutrals are accelerated to lower velocity than ions. This leads to a two-stream model.

The PPT is partially ionized. Neutral and Ions. We have a situation in PPT where the pulse results in "fast" ions and "slow" neutrals coexisting in the same channel. "Two-Fluid" model.

Consider the situation shown here:

**Figure 12: Two Fluid Model**

Total ablated mass is the sum of the fast and slow species.

$$M = M_f + M_s \quad (8.76)$$

Impulse bit is then:

$$I_{\text{bit}} = m_f u_f + m_s u_s \quad (8.77)$$

Thus  $I_{SP}$  and efficiency are:

$$I_{SP} = \frac{I_{\text{bit}}}{m g_o} = \frac{m_f u_f + m_s u_s}{(m_f + m_s) g_o} \quad (8.78)$$

$$\eta = \frac{\frac{1}{2} I_{\text{bit}}^2}{m E_o} \quad (8.79)$$

Experimentally we can measure  $I_{\text{bit}}$ ,  $m$  and  $E_o = \frac{1}{2} C V_o^2$ . But cannot know  $m_f$ ,  $u_f$ ,  $m_s$ ,  $u_s$ . We can also calculate the Electromagnetic portion of  $I_{\text{bit}}$ :

$$I_{\text{bit}} = \int T_{ET} dt + \int \frac{1}{2} L' J^2 dt \quad (8.80)$$

This total impulse bit is due to two main things, 1) the electrothermal acceleration of the particles 2) The electromagnetic flow of the thrust, the acceleration of the charged species. This is the total electromagnetic energy deposited in. We know geometry, we can measure  $J$  so we can evaluate the second integral

This term is equal to the impulse delivered by the fast species, we can solve for the slow species.

$$\int \frac{1}{2} L' J^2 dt = m_f u_f \quad (8.81)$$

Then from measured  $I_{\text{bit}}$ , we also know  $m_s u_s$ ; solve Equation 8.80 for

$$\int T_{ET} dt = m_s u_s$$

Now the problem is, what are  $m_f$  and  $m_s$ , or  $u_f$  and  $u_s$ ?

We don't know how much of our mass is the fast species and how much is the slow species.

We find the answer in Cosmology, more specifically a theory for the origin of the solar system by Hannes Alfvén (Nobel Prize in Physics 1970). Alfvén's paper "On the Origin of the Solar System" assumed the Sun and a magnetized dust cloud.

Alfvén: When  $KE = \text{ionization energy}$ , particle falling in is ionized, losing all energy, so stops. In this way, atoms of H, He, Ne, etc. stop at different radii.

$$\frac{1}{2} m u_c^2 = e \varepsilon_i \quad (8.82)$$

So for the PPT it is assumed that  $u_f$  cannot exceed  $u_s$  by more than  $u_c$ :

If we have this mass of slow species moving along, then the ions cannot leave faster than this critical speed. Whenever the KE = Ionization, the particles transfers all of its KE to Ionization energy and no longer moves, generally caused by collisions.

$$u_f - u_s = u_c \quad (8.83)$$

What is  $u_c$  for Teflon ( $\text{CF}_2$ )?

Critical velocity for Carbon:  $u_c = 13 \text{ [km/s]}$

Critical velocity for Fluorine:  $u_c = 13 \text{ [km/s]}$

Example, assume the net exit velocity is  $10.5 \text{ km/s}$  due to both  $m_s$  and  $m_f$ .

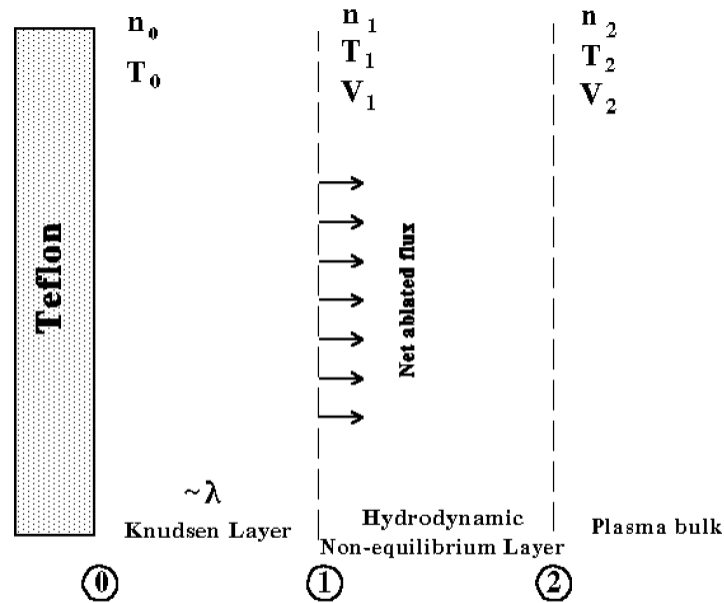
Conclusion: Most efficient PPT has highest degree of ionization, which minimizes  $m_s$  and raises  $I_{sp}$ .



## 6.4 More Advanced PPT Modeling

More advanced PPT models use an ablation model connected with an electrothermal or electromagnetic (i.e., electromechanical) acceleration model connected with a plume model.

### 6.4.1 Ablation Model:

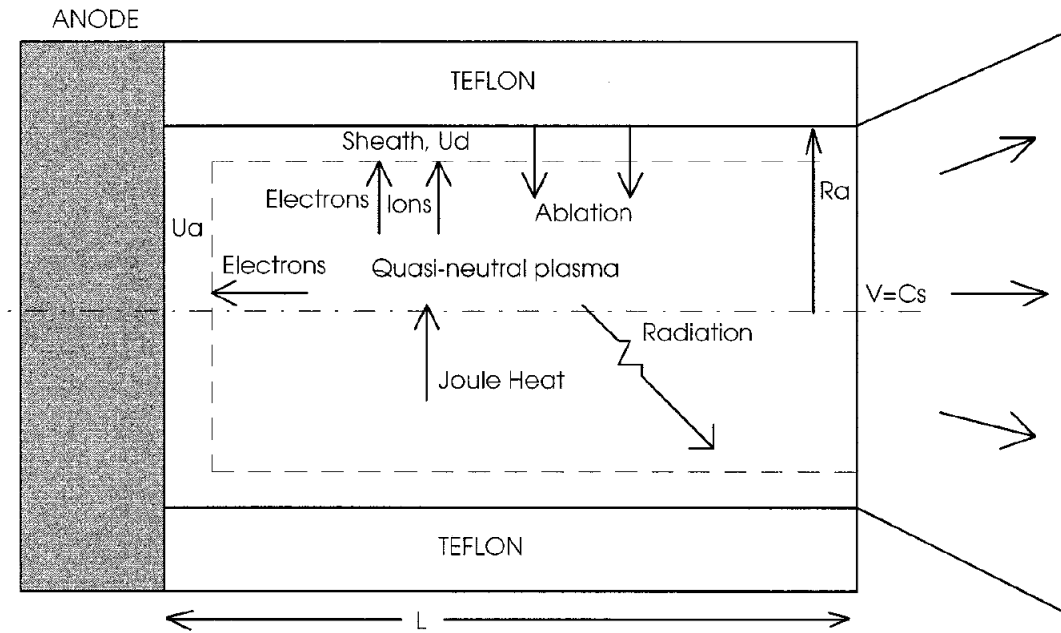


**Figure 1.** Schematic representation of the layer structure near the ablated surface.

Non-equilibrium kinetic layer near the surface of ablating material. Two extremes are the surface and the plasma bulk. In between one can imagine a kinetic non-equilibrium layer close to the surface with a thickness a few mean free paths (Knudsen layer). Then a collision dominated non-equilibrium layer where electron and heavy particle temperatures differ. At the right, all species reach thermal equilibrium.

Michael, K., Iain, D. B., Isak, I. B., "On the model of Teflon ablation in an ablation-controlled discharge," Journal of Physics D: Applied Physics, Vol. 34, No. 11, pp. 1675, 2001.

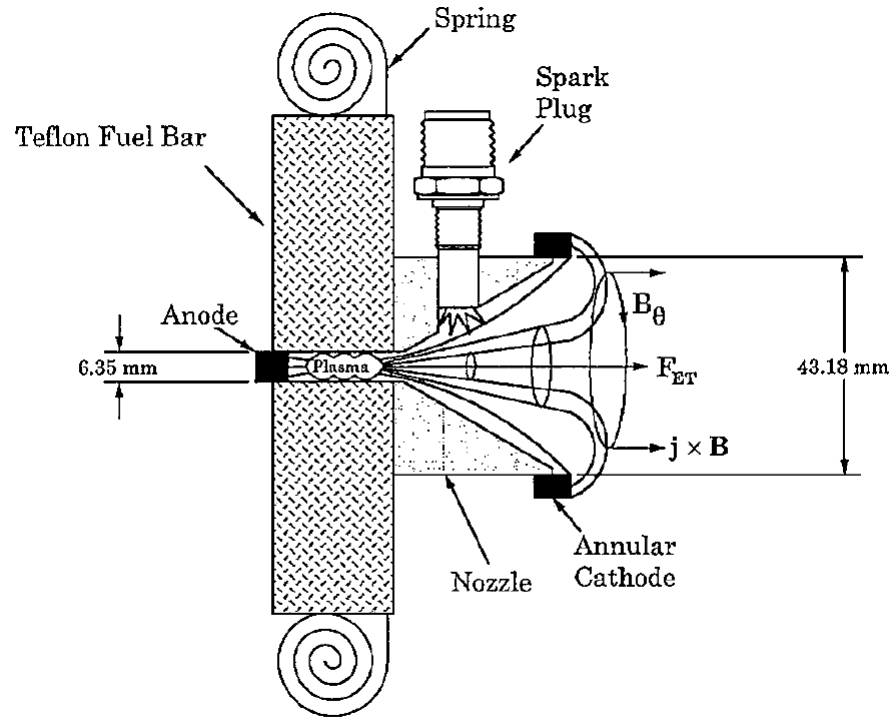
### 6.4.2 Electrothermal Model



Plasma heating with electrothermal acceleration. Energy balance in the quasineutral plasma column is:

Keidar, M., Boyd, I. D., Beilis, I. I., "Electrical discharge in the Teflon cavity of a coaxial pulsed plasma thruster," Plasma Science, IEEE Transactions on, Vol. 28, No. 2, pp. 376-385, 2000

### 6.4.3 Electromagnetic Model with Plume



**Fig. 1 Schematic diagram of the Illinois PPT-4.**

Boyd, I. D., Keidar, M., McKeon, W., "Modeling of a Pulsed Plasma Thruster from Plasma Generation to Plume Far Field," Journal of Spacecraft and Rockets, Vol. 37, No. 3, pp. 399-407, 2000/05/01.