Homework 4

AE370 - Spring 2018

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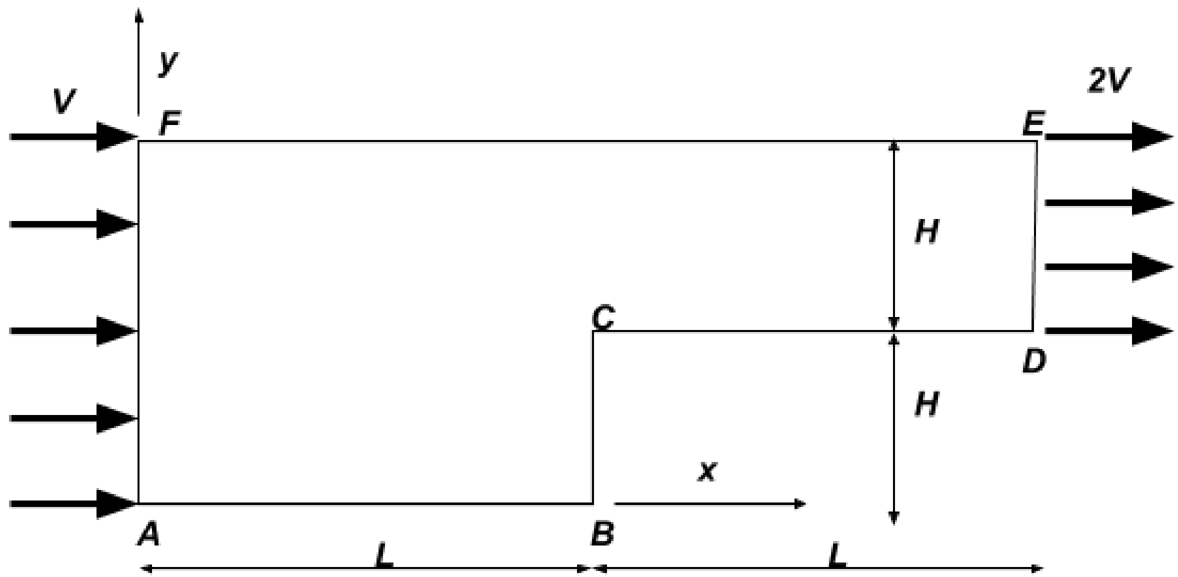
Problem: Steady-state fluid problem

Use the finite difference method to solve the following incompressible/inviscid fluid flow problem:



The flow problem can be described by the following GDE:

$$\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} = 0$$

where the streamline function Φ is related to the x and y components of the velocity vector through:

$$V_x = \frac{\partial \Phi}{\partial y} \qquad V_y = -\frac{\partial \Phi}{\partial x}$$

The boundary conditions are:

$\Phi = V_y$	along AF
$\Phi = 2VH$	along FE
$\Phi = 2V(y - H)$	along DE
$\Phi = 0$	along ABCD

Use a second-order central diffrence scheme to solve the problem using N, grid spacings to discretize L (i.e. $\Delta x = L/N_x$) and N_y grid spacings to discretize H (i.e., $\Delta y = H/N_y$).

(a) Derive the discretized form of the PDE

(b) Choose a numbering of the grid points (Describe it thoroughly in your report!) and put together the matrix equation.

(c) Implement the boundary conditions.

(d) Write a Matlab code that solves this problem. As output you should create the following three plots:

   (a) A contour plot for the streamline function (using the command `CONTOUR`)

   (b) A vector plot for the velocity field (using the central difference approximation for the first derivatives of the streamline function and the command `QUIVER`)

   (c) A x-y plot of the pressure distribution along the edge boundary condition, where the pressure is obtained by $P = \rho(V_x^2 + V_y^2)$ where ρ is the fluid density.

(e) Solve the problem for L=1 [m], H = 0.2 [m], V = 1 [m/s] and $\rho = 1[kg/m^3]$. Perform a convergence study to determine the appropriate values of N_x and N_y. Comment on your solution and especially on the computed solution in the vicinity of the corner.

Solution:

The Jacobi Method converges within a tolerance of 1e-14 after the 74 iteration with a final solution: