Section 5: Collisions

AE435

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1 Cross Sections

The collision "cross-section" is like the target size a particle "sees" as it moves through a collection of particles.

Some cross-sections tell us how (to what angle) the incident particle will be scattered due to a particle type of collision, these are "differential cross-sections".

If we integrate the differential cross-section over all possible scatter angles you get the "total" or "effective" cross-section for that particular type of collision.

Finally, if you sum the effective (or total) cross-section for all possible collision types, you get the "total" collision cross-section.

Contents

1.1 Effective Cross Sections

Consider

- A particle of type j encountering
- A particle of type k
- In collision of type β

The probability of this over a path length $\mathrm{d}x$ is:

Probability of Encounter

$$\mathrm{d}P_{jk}^{(\beta)} = n_k\, Q_{jk}^{(\beta)}\, \mathrm{d}x \tag{2}$$

Where

$$\begin{aligned} Q_{jk}^{(\beta)} &= \text{Corresponding Collision Cross-Section} \\ &= [m^{-3}\, m^2\, m^1] = [1] \quad \text{Unitless} \end{aligned}$$

For a flux (Γ) of test particles, j, entering a cloud of field particles, k, the flux leaving the cloud is:

$$\Gamma + \mathrm{d}\Gamma = \Gamma(1 - \mathrm{d}P_{jk}^{(\beta)}) = \Gamma(1 - n_k\, Q_{jk}^{(\beta)}\, \mathrm{d}x) \tag{3}$$

Figure 3

Cancelling like terms yields:

$$d\Gamma = \Gamma\, n_k\, Q_{jk}^{(\beta)}\, dx) \tag{4}$$

Which integrates to:

$$\Gamma = \Gamma_o \,\exp\left(-\, n_k\, Q_{jk}^{(\beta)}\, x\right) = \Gamma_o \,\exp\left(-\,\frac{x}{\lambda_{\mathrm{mfp}}}\right) \tag{5}$$

where Γ_o is the initial flux. The mean free path now becomes

$$\lambda_{\mathrm{mfp}} = \frac{1}{n_k\, Q_{jk}^{(\beta)}} \tag{6}$$

A good first estimate for $Q_{jk}^{(\beta)}$ is the... atomic cross section

Atomic Cross Section

$$a_o = \frac{\epsilon_o\, h^2}{\pi\, m_e\, q_e^2} \tag{7}$$

Where the Bohr radius πa_o^2 describes the orbit for an electron with the lowest possible non-zero momentum.

Returning to the collisional cross-section, note that

$$Q_{jk}^{(\beta)} = Q_{kj}^{(\beta)} \tag{8}$$

We can define a...

Total ("Effective") Cross-Section

$$Q_{jk} = \sum_{\beta} Q_{jk}^{(\beta)} \tag{9}$$

which is the sum of all the different types of collisions (elastic, excitation, ionization, etc.).

1.2 Differential Cross-Sections

In addition to Effective Cross Sections, we also have differential cross-sections $q(\theta)$ defined by the scattering angle,

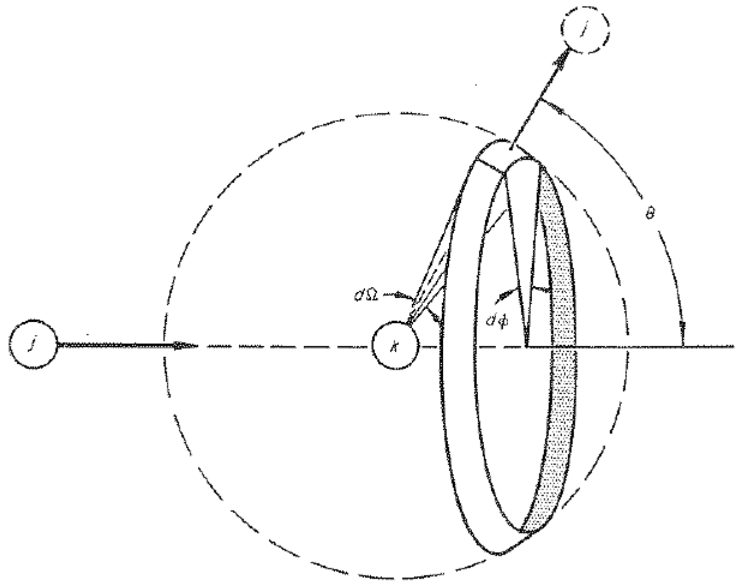

Fig. 4-2 Nomenclature for definition of differential cross section.

The probability of scattered particle emerging into solid angle:

$$\mathrm{d}\Omega = \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{10}$$

is

$$\mathrm{d}P(\theta) = q(\theta) \, \mathrm{d}\Omega = q(\theta) \, \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{11}$$

Integrating over the full 4π of the solid angle around the scattering center gives us the:

Total Collisional Cross-Section

$$Q = \int_0^{2\pi} \int_0^{\pi} q(\theta) \, \sin\theta \, \mathrm{d}\theta \, \mathrm{d}\phi \tag{12}$$

For a type $-\beta$ collision, the mean free path is

$$\lambda_{\mathrm{mfp}}^{(\beta)} = \frac{1}{n_k \, Q_{jk}^{(\beta)}} \tag{13}$$

as previously defined in Equation 6 and Equation 4.15

We can now define...

Collision Frequency

$$\nu_{jk}^{(\beta)} = n_k \, Q_{jk}^{(\beta)} \, \overline{v_{jk}} \tag{14}$$

Where

$$\overline{v_{jk}} = \text{Collision Speed} \quad = \text{Relative Speed Between j and k}$$

The collision speed $\overline{v_{jk}}$ is well-defined in beam experiments, but less so for thermal plasmas.

General Equation for Collision Speed in Thermal Plasmas:

$$\overline{v_{jk}} \cong v_{th} = \left(\frac{8\,k\,T}{\pi\,m}\right)^{\frac{1}{2}} \qquad \text{Equation 4.32} \tag{15}$$

This relation applies for the faster particle species in mixed thermal plasmas.

Example: Consider a xenon plasma with equal electron and ion temperature ($T_e = T_i = 3\text{eV}$).

As a result...

$$\begin{cases} V_e = 750 \, \frac{km}{s} \\ V_i = 1.5 \, \frac{km}{s} \end{cases}$$

Meaning that...

$$\begin{aligned} \nu &= n \; < Q_{jk} \, V_{jk} > \\ &= n \, \overline{Q_{jk} \, V_{jk}} \end{aligned}$$

Such that

$$\overline{Q\,V} = \int_0^\infty Q \, v \, f(v) \, \mathrm{d}v$$

Finally, can define a:

Mean Collision Rate for a Swarm of Particles

$$\nu_{jk}^{(\beta)} = n_k \, Q_{jk}^{(\beta)} \, \overline{v_{jk}} \tag{16}$$

Where

- We can approximate and as Maxwellians,
- shows the dependence of Q on relative velocity.