Section 8: Electromagnetic Propulsion

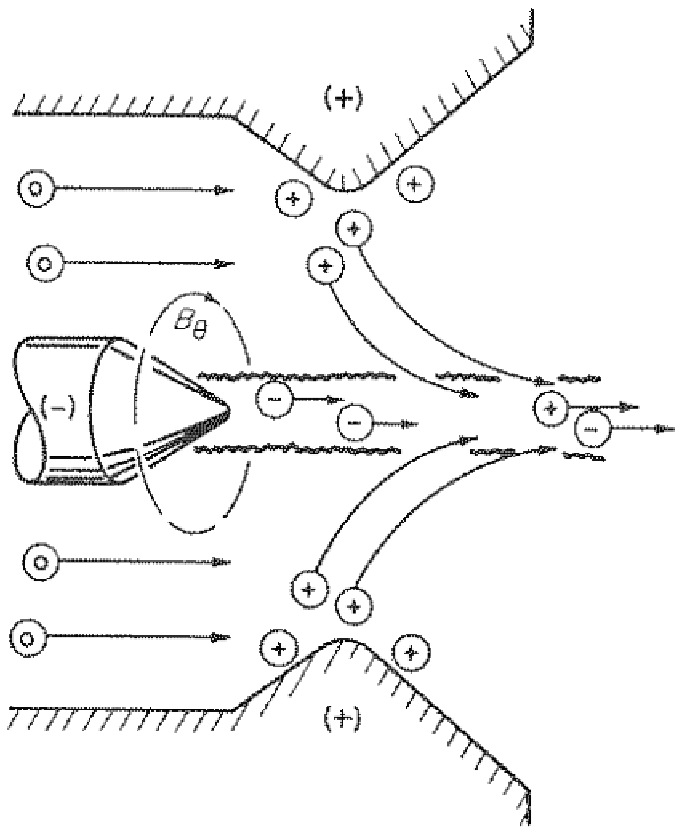
AE435

Spring 2018

2 Particle Description

Contents

In the particle description the MPD acts like an ion thruster but with a neutral plasma. Ions are driven from the anode and electrons from the cathode. Particles are collisionless.



Note as ions approach the cathode, 3 things happen:

1. Ions are accelerated by E_r inward gaining velocity.
2. $B_\theta \sim \frac{1}{r}$ so r_L gets smaller as B_θ increases. As they move inward, B_θ has more of an influence on them causing them to bend (v x B), do gyro motion.
3. Ions are turned axially

r_L is Larmor radius

$$r_L = \frac{m\, v_\perp}{|q|\, B}$$

Current and mass flow related by:

$$J = \frac{\dot{m}\, e}{M} \tag{8.34}$$

For singly-ionized particles with M being the mass of ions.

Using Equation 8.18, $B = \frac{\mu_o J}{2 \pi r}$, and Equation 3.11, $r_L = \frac{M v_+}{e B}$ (mass of ion, velocity of ion): it can be said that the r_L must be smaller than radius, in other words,

$$r \geq r_l = \frac{M v_+}{e B} = \frac{2 \pi r M v_+}{e \mu_o J} \tag{8.35}$$

Looking at this from a particle point of view, we are going to find a limit on the current or how the current scales.

If we replace v_+ with u_e (fluid exit velocity, speed of ions as they leave the MPD), we can write a requirement on J, which will be large enough to turn ions if:

$$J > \frac{2 \pi M u_e}{e \mu_o} \tag{8.36}$$

Where

$$M = A \, m_{\text{proton}}$$

$$A = \text{Atomic Weight}$$

$$\begin{aligned} m_{\text{proton}} &= 1.6726219 \times 10^{-27} \, [kg] \\ &= 0.992776097 \, [amu] \end{aligned}$$

such that

$$J > \frac{2 \pi m_{\text{proton}}}{e \mu_o} A \, u_e = 0.0522 \, A \, u_e \sim \frac{A \, u_e}{20}$$

If we want higher specific impulse, we want smaller mass ions and larger current.

Example: Consider an Argon particle with atomic mass, A = 40 [amu] with an exit velocity $u_e = 40,000\,[m/s]$. We want the resulting current to be greater than 80 kA, $J > 80\,[kA]$.

We are looking at the scaling of a current from a scaling point of view.

In practice this current level is higher than is actually used. But the relation does illustrate two points:

1. Current and ion mass, M, are coupled
2. Since J is limited for practical reasons (higher current means higher mass power electronics, high current switching, etc.), lighter ions have higher I_{SP}

Current and mass flow rate are coupled (Equation. 8.34), so

$$J \sim \frac{A\,u_e}{20} = \frac{e\,\dot{m}}{A\,m_{\text{proton}}} \quad \rightarrow \quad \dot{m} \cong \frac{1}{20}\,\frac{m_{\text{proton}}}{e}\,A^2\,u_e$$

Thus

$$F = \dot{m}\,u_e = \frac{m_{\text{proton}}}{20\,e}\,A^2\,u_e^2$$

$$Z = \frac{1}{2}\,k\,u_e \quad \rightarrow \quad V = Z\,J = \frac{1}{2}\,k\,u_e\,J = \frac{1}{2}\,k\,u_e\,\frac{A\,u_e}{20}$$

$$V = \frac{k\,A\,u_e^2}{40}$$

2.1 The mass injection problem

It was found that performance was improved by mass injection at large radius. Performance depends on where the ions are produced. Larger radius is better cause get more energy from E_r. For the ions to build up as much energy, you will have to have it start at a large radius away from the cathode that way it can accelerate towards it for a longer period of time. In general, you want the mass injection to be along the outer radius such that the ions can be created close to the anode surface.

Figure 1