Section 2: Electromagnetics

AE435

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In this section, we will review the basics of charge, electricity, magnetism, and Maxwell equations.

4 Magnetostatics with Magnetic Media

Contents

4.1 Effect of Magnetic Media

In our previous discussion, we only considered magnetostatics involving steady currents in a vacuum. Now we will examine...

- **Question:** What happens if matter is present?
- **Answer:** The magnetic field $\vec{B}$ changes!
- **Reason:** Matter has micro-currents associated with the motion of the electrons around atoms, "atomic currents"
- **Aftermath:** So now we must consider two kinds of currents:
  - Conduction currents, involving free charges
  - Atomic currents, with no charge transport (to the first order)

Each atom has a magnetic dipole moment.

Magnetic Dipole Moment

$$\vec{m}_i = \frac{1}{2} J_i \oint_c \vec{r}_i \times \mathrm{d}\,\vec{l} \tag{75}$$

We can define a macroscopic vector quantity analogous to polarization, known as the **magnetization** or the magnetic dipole moment per unit volume.

Magnetization

$$\vec{M} = \lim_{\Delta V \to 0} \frac{1}{\Delta V} \sum_i \vec{m}_i \tag{76}$$

In the **unmagnetized state**, $\vec{M} = 0$ because $\vec{m}_i$ have random orientations that cancel out. In the presence of an external $\vec{B}$, matter becomes organized and $\vec{M}$ can become nonzero depending on the material properties.

Magnetization Current: How does magnetization give rise to currents?

Figure 20

For a uniform $\overrightarrow{M}$, currents cancel in the interior but not on the exterior. The result is a net surface current as shown in Figure 20.

Similarly, if $\overrightarrow{M}$ is non-uniform, we can have an internal net current.

We can define a **Magnetization Current Density:**

$$\vec{j}_m = \nabla \times \overrightarrow{M} \tag{77}$$

4.2 Total Magnetic Field

To incorporate $\vec{j}_m$ into Ampere's Law (Equation 72) we have to modify the magnetic field equations, just as we modified Gauss' law to include ρ_e.

As before, we still have no monopoles:

$$\nabla \cdot \vec{B} = 0$$

But now, Ampere's law becomes:

$$\nabla \times \vec{B} = \mu_o (\vec{j} + \vec{j}_m) \tag{78}$$

Using Equation 77, we can write this as:

$$\nabla \times \left(\frac{1}{\mu_o} \vec{B} - \vec{M} \right) = \vec{j}$$

Where $(\frac{1}{\mu_o} \vec{B} - \vec{M})$ depends only on conduction current density $\vec{j}$ as its source. As a results, we define a vector field:

Magnetic Intensity or "H"-field

$$\vec{H} = \frac{1}{\mu_o} \vec{B} - \vec{M} \qquad \left[\frac{\mathrm{A}}{m} \right] = [\text{Oersted}] \tag{79}$$

Note: $1 \ \frac{\mathrm{A}}{m} = 0.01257$ Oersted

Finally, **Ampere's Law for Magnetic Media** is:

$$\nabla \times \vec{H} = \vec{j} \tag{80}$$

A comparison of Magnetostatics and Electrostatics:

Electrostatics	**Magnetostatics**
In vacuum (no ρ_p)	In vacuum (no $\vec{j}_m$)
$\nabla \cdot \vec{E} = \frac{q}{\epsilon_o}$ (isolated charges)	$\nabla \cdot \vec{B} = 0$
$\nabla \cdot \vec{E} = \frac{\rho_e(\vec{r})}{\epsilon_o}$ (distributed charges)	
$\nabla \times \vec{E} = 0$	$\nabla \times \vec{B} = \mu_o \vec{j}$
With media effects (finite ρ_p)	With media effects (finite $\vec{j}_m$)
$\nabla \cdot \vec{E} = (\rho_{f} + _{p})/\epsilon_o$	$\nabla \cdot \vec{B} = 0$
$\nabla \cdot \vec{D} = \rho_f$	
$\nabla \times \vec{E} = 0$	$\nabla \times \vec{B} = \mu_o(\vec{j} + \vec{j}_m)$
	$\nabla \times \vec{H} = \vec{j}$

We can also derive the intergral equation for magnetic intensity. From Equation 80, if we integrate over a surface element and apply Stokes theorem on a closed curve surrounding the surface, we get:

$$\int_S \nabla \times \vec{H} \cdot \hat{n}\, \mathrm{d}A = \oint_C \vec{H} \cdot \mathrm{d}\,\vec{l} = \int_S \vec{j} \cdot \hat{n}\, \mathrm{d}A = J \tag{81}$$

Important Note: This only applies for Magnetostatics. It does not work for time-varying fields.

4.3 Constitutive Equations/Relations

Define magnetization as a response to external $\vec{H}$:

$$\vec{M} = \chi_m(\vec{H})\,\vec{H} \tag{82}$$

If material is linear and isotropic, the magnetic susceptibility χ_m is constant.

$$\vec{M} = \chi_m\,\vec{H}$$

This is analogous to the electric susceptibility leading to $\vec{D} = \chi\,\vec{E}$ (Equation 39).

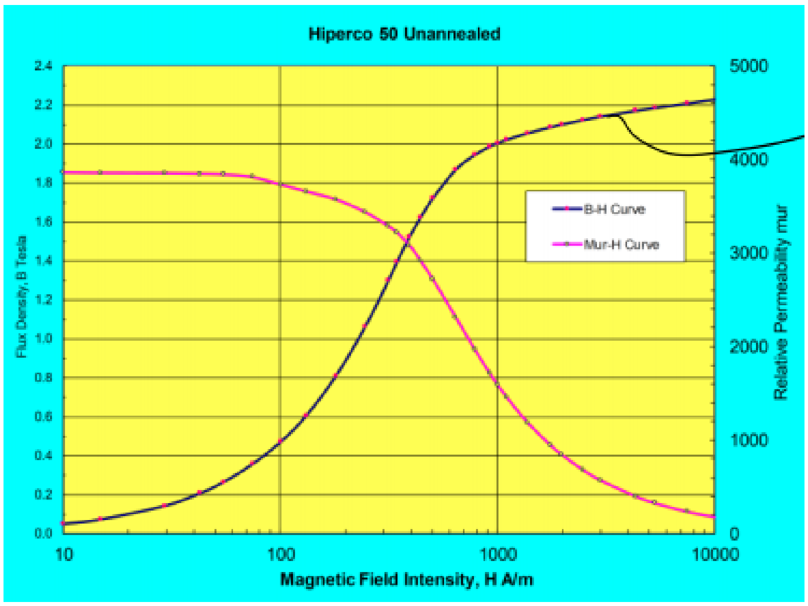
- All dielectrics oppose applied $\vec{E}$ due to dipole orientation with $\vec{E}$
- Magnetic materials can either add to or subtract from the external $\vec{H}$
  - Positive χ_m = **paramagnetic** materials, add to $\vec{H}$. Rare gases, like neon, also titanium, oxygen.
  - Negative χ_m = **diamagnetic** material, subtracts from $\vec{H}$. Bismuth floats over permanent magnet
  - For both paramagnetic and diamagnetic materials, , on the order of $10^{-5} - 10^{-6}$, very small, $|\chi_m| << r1$

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		**Paramagnetic:**	
Bismuth	-1.7×10^{-4}	Oxygen (O_2)	1.7×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.2×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.0×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.7×10^{-4}
Carbon Dioxide	-1.1×10^{-8}	Liquid Oxygen (-200° C)	3.9×10^{-3}
Hydrogen (H_2)	-2.1×10^{-9}	Gadolinium	4.8×10^{-1}

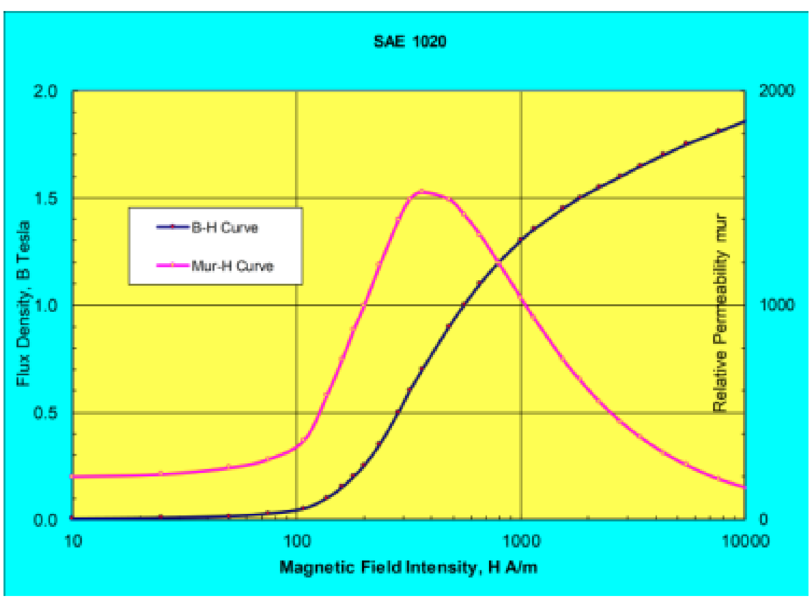
Ferromagnetic materials are different. They're super-paramagnetic (really, really add to the external $\vec{H}$), and are highly nonlinear. Ex: Iron (Fe) and Hiperco (Cobalt-Iron, Co-Fe) where we need the experimental B-H curve to accurately model.

In ferromagnets, each magnetic dipole likes to point in the same direction as its neighbors. But this common alignment occurs over a "domain", a small region (microscopic but containing billions of dipoles). With no H-field, domains are oriented randomly, no net effect, no permanent net magnetization. But when H-field applied, domains line up, resulting in strong magnetization. For permanent magnets (also ferromagnets) the domains remain aligned.

Hiperco 50 (50-50 Co-Fe), also 30, 15 varieties (Co%)



Low Carbon Steel



Curie Temperature refers to the temperature at which material loses its ferromagnetic properties and becomes paramagnetic.

- Fe - 770° C
- Co - 1130° C

Samarium Cobalt (SmCo) is an example of high-temperature permanent magnets. SmCo is commonly found in electric propulsion systems since it operates at 300-500° C and has a Curie Temperature of 700° C.

Relative Permeability

We know from Equation 79 that,

$$\vec{B} = \mu_o\,(\vec{H} + \vec{M})$$

Substituting in the magnetic susceptibility (Equation 82), such that:

$$\vec{B} = \mu_o\,(\vec{H} + \vec{M}) = \mu_o\,(\vec{H} + \chi_m\,\vec{H}) = \mu_o\,(1 + \chi_m)\vec{H} \tag{83}$$

We define **Permeability** (a material property) as

$$\mu = \mu_o\,(1 + \chi_m) \tag{84}$$

And (as we did with permittivity) we can define relative permeability as:

$$K_m = \frac{\mu}{\mu_o} = 1 + \chi_m \tag{85}$$

4.4 Boundary Conditions

Similar to our Electrostatic Boundary Conditions, we can show that for a surface current density, $\vec{j}_s$ (Equation 71)

$$(B_\perp)_2 = (B_\perp)_1 \tag{86}$$

And from Equation 80

$$(H_{||})_2 - (H_{||})_1 = \vec{j}_s \cdot \hat{n} \tag{87}$$

Figure 21

4.5 Magnetic Flux

Magnetic flux is the flux through a given surface.

$$\Phi_m = \int_S \vec{B} \cdot \hat{n} \,\mathrm{d}A \qquad \left[\text{Webers}\right] = \left[T \cdot m^2\right] \tag{88}$$

Note that flux through a closed surface is zero by Gauss's theorem and the magnetic monopole law.

$$\int_S \vec{B} \cdot \hat{n} \,\mathrm{d}A = \int_{\forall} \nabla \cdot \vec{B} \,\mathrm{d}\forall = 0 \tag{89}$$