Fault-Recovery of an Underactuated Quadcopter

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1 Manual

1.1 Goal

Your goal over the next three weeks is to implement and test a cascading controller that makes a quadrotor hover with only three operational motors.

1.2 First Week

Your objective this week is to simulate the quadrotor dynamics and implement the inner loop of the cascading controller.

1.2.1 Simulate

Download the lab3_simulate.m MATLAB file. Implement the dynamics as in previous labs, but modify the angular rates equations to match those described by equations 4 - 6 in the attached report. Modify the same function according to equation 7 to account for the time delay of each motor. You will need to modify both the input and the state vector in order to do so.

Implement the inner loop of the controller described in by section 2.5. Hint: start by using the state space given by equation 8 and the equilibrium values given in section 2.5.3 to create a linearized system in the form of $\dot{x} = Ax + Bu$.

1.2.2 Test

Plot the reduced attitude state space variables and their respective equilibrium values over time. Is your controller able to drive these variables to, or close to, the equilibrium?

Plot the quadrotor position over time. What behavior do you see? Why do you see this behavior?

1.3 Second Week

Your objective this week is to implement the outer loop of the cascading controller.

1.3.1 Simulate

Implement the outer loop of the controller described in by section 2.5. Notice that equation 16 provides the input for equation 17. Also, the sum of the forces and the primary axis of rotation can be found from equation 17 by solving with the additional constraint of $\|\boldsymbol{n}_{des}\| = 1$.

1.3.2 Visualize

Download the lab3_visualise.m, quadmodel.mat, and mocapmodel.mat MATLAB files. Create a movie in .mp4 format showing the simulation of your underactuated quadrotor.

1.3.3 Test

Now that your quadrotor has both an inner and outer loop, how does the position response differ from the first week? How does the reduced attitude response differ from the first week? How does the quadrotor respond to a wide variety of different initial conditions? Is the quadrotor's stability more or less sensitive to initial conditions than in previous labs when all four propellers could be used? Try your best to explain why the quadrotor behaves the way it does.

1.4 Third Week

Your objective this week is to implement your cascading controller on a quadrotor and gather experimental data in order to perform analysis.

1.4.1 Implement

Download the AscTec_SDK_v3.0_Lab3.zip archive. Import it into eclipse and implement your controller in the lab3() function inside of the lab.c file. Note: the provided code may not follow the same configuration as defined in class so be careful when setting the motor inputs.

1.4.2 Flight

Since the vehicle safety tether will get in the way of ground takeoff, it will be necessary to suspend and spin the quadrotor before engaging the controller. With the quadrotor disarmed, but the controller switch engaged, suspend the quadrotor near its average equilibrium position and manually spin it the direction of the equilibrium yaw rate. Quickly arm the quadrotor to engage the propellers and controller. Be sure to use the ground station to collect flight data.

1.4.3 Analysis

Plot the experimental position and angular rates and compare them to the respective equilibrium values. How well does the experimental controller match the simulation? Did the inner or outer loop perform better in experiment? What changes could be made to improve the experimental controller? Implement those changes and report the results.

2 Report

2.1 Motivation

As battery and processor technologies become less expensive and less massive, quadrotors become increasingly viable as platforms for enabling services such as package delivery and flying cars. As this technology proliferates, so does the likelihood of collisions and in flight failures. This document provides a control scheme for the case of a single malfunctioning propeller, increasing the robustness and safety of quadrotors.

2.2 Goal

The primary objective of this experiment was to implement a periodic solution for a quadcopter to maintain its height despite the loss of a single propeller. This project was inspired by the work of Dr. Mueller and Dr. Raffaello D'Andrea in their paper "Stability and control of a quadrocopter despite the complete loss of one, two or three propellers." For the loss of a single rotor, the control strategy consist of spinning the quadcopter about a primary axis fixed to the body frame and tilting this axis for translational control. During simulation, the controller performed satisfactorily. It was able to hover about a desired position as well as track a desired trajectory. During experiment, the controller did not preform as well as the one in the cited paper and had a hard time hovering about the desired position. Regardless, the project is considered to be successful given its complexity and the short duration given for completion. To explore the produced controller and flight code, follow this link here. On this page are the resources for flashing the quadcopter, ground-station code, and the simulation software.

2.3 Method of Approach

2.4 Dynamics

The typical quadrotor has five forces acting on it. Four forces from each of its propellers, f_i and the additional force of gravity from its weight, mg which acts in body-fixed direction $\mathbf{z}$. In addition to the forces, there are five torques acting on the quadrotor. Four torques from the propeller τ_i and an additional drag torque, τ_d. The propeller torques oppose the propellers rotation while the drag torque opposes the vehicles angular velocity. The quadrotors angular velocity is expressed as $\omega = (p, q, r)$ and the rotation of the body frame is expressed with respect to the inertial frame via a rotation matrix $\mathbf{R}$. The translational equations of motion are given by the following equation:

$$m\ddot{\mathbf{O}} = \mathbf{R}[0\ 0\ -1]^T \sum_{i=1}^{4} f_i + m[0\ 0\ g]^T \tag{1}$$

The principle moment of inertia for the quadrotor body and propellers are defined as J^B and J^P respectively.

$$J^B = \begin{bmatrix} J^B_{xx} & 0 & 0 \\ 0 & J^B_{yy} & 0 \\ 0 & 0 & J^B_{zz} \end{bmatrix} = \begin{bmatrix} 4093 & 0 & 0 \\ 0 & 3944 & 0 \\ 0 & 0 & 7593 \end{bmatrix} \times 10^{-6}\ kg\, m^2 \tag{2}$$

$$J^P = \begin{bmatrix} J^P_{xx} & 0 & 0 \\ 0 & J^P_{yy} & 0 \\ 0 & 0 & J^P_{zz} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 15 \end{bmatrix} \times 10^{-12}\ kg\, m^2 \tag{3}$$

The equations of motion for the vehicle's angular rates given in the cited paper were adapted to match the standard quadrotor configuration previously defined in class.

$$\begin{aligned} \dot{p} = -\frac{1}{J^B_{xx}} [q\, r \left(J^B_{zz} + 4J^P_{zz}\right) + q\, J^P_{zz}\ (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \\ - l\, k_F\ \left(\sigma_1^2 - \sigma_2^2\right) - q\, r\ \left(J^B_{yy} + 4J^P_{yy}\right)] \end{aligned} \tag{4}$$

$$\begin{aligned} \dot{q} = -\frac{1}{J^B_{yy}} [-p\, r \left(J^B_{zz} + 4J^P_{zz}\right) - p\, J^P_{zz}\ (\sigma_1 + \sigma_2 + \sigma_3 + \sigma_4) \\ - l\, k_F\ \left(\sigma_3^2 - \sigma_4^2\right) + p\, r\ \left(J^B_{xx} + 4J^P_{xx}\right)] \end{aligned} \tag{5}$$

$$\begin{aligned} \dot{r} = -\frac{1}{J^B_{zz}} [-p\, q \left(J^B_{xx} + 4J^P_{xx}\right) + p\, q\ \left(J^B_{yy} + 4J^P_{yy}\right) \\ + \gamma\, r + k_M\ \left(-\sigma_1^2 - \sigma_2^2 + \sigma_3^2 + \sigma_4^2\right)] \end{aligned} \tag{6}$$

To increase the fidelity of the simulation, the motor responses can be modeled as a first order system. The system time constant is given by T_{mot}.

$$\dot{\sigma}_i = -1/T_{mot}(\sigma_i - \sigma_{i_{set}}) \tag{7}$$

2.5 Underactuated Control

With the loss of one propeller, only the the quadrotor's primary axis of rotation can be controlled. The following state vector is used to describe this reduced attitude:

$$s = [p\ q\ n_x\ n_y]^T \tag{8}$$

2.5.1 Reduced Attitude Equilibrium

With one lost propeller, no static equilibrium exists for the quadrotor's position. Instead a periodic equilibrium will result in a constant average position. The following equations provide the necessary number of constraints to solve for a periodic solution:

$$\bar{\boldsymbol{n}} = \epsilon\bar{\boldsymbol{\omega}} \tag{9}$$

$$\|\bar{\boldsymbol{n}}\| = \|\epsilon\bar{\boldsymbol{\omega}}\| \tag{10}$$

$$(\bar{f}_1 + \bar{f}_2 + \bar{f}_3)\bar{n_z} = m\|\boldsymbol{g}\| \tag{11}$$

$$f_1 = f_2 \tag{12}$$

$$\rho = f_3/f_1 \tag{13}$$

$$\dot{\boldsymbol{\omega}} = \boldsymbol{0} \tag{14}$$

$$\sigma_4 = 0 \tag{15}$$

For the solution with one lost propeller, it is assumed that propeller 4 has failed. The ratio ρ acts a tuning factor that can be used to affect the specifics of the periodic solution.

2.5.2 Controller Design

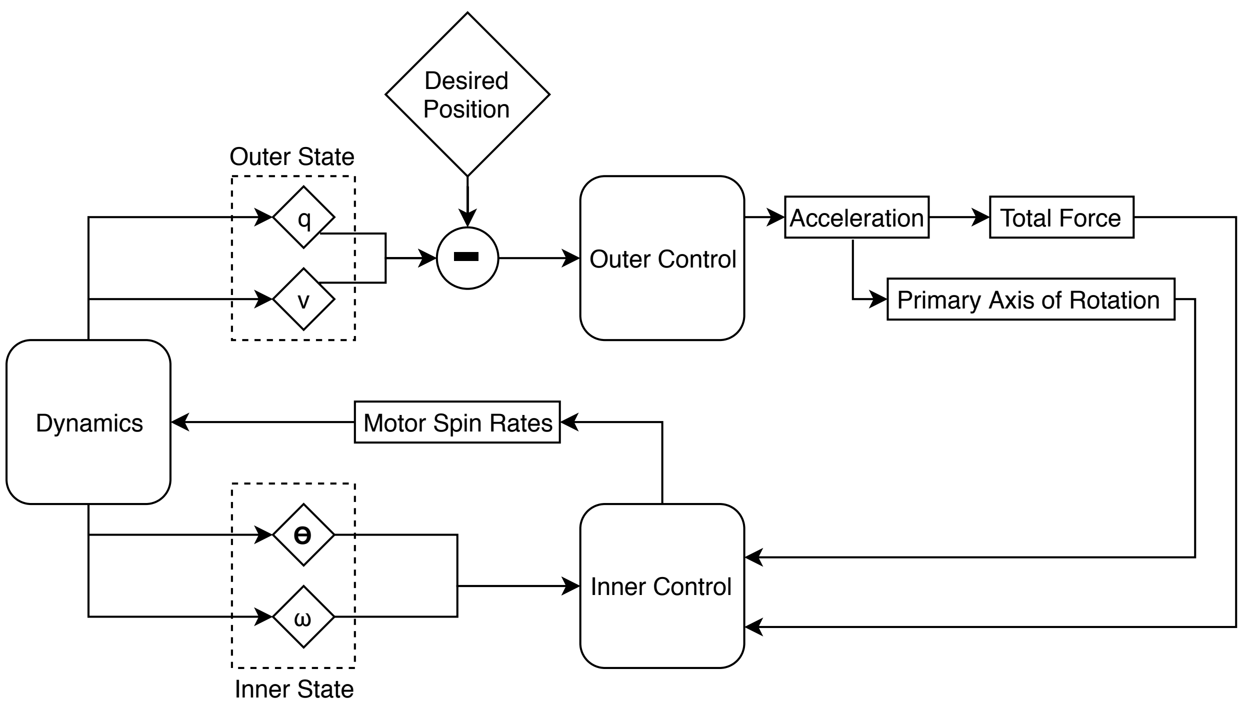


Figure 1: Cascade control scheme for underactuated quadcopter

To control the quadrotor, a cascading control scheme consisting of an inner and an outer loop was implemented. The outer translational controller is modeled as a second order system with damping ratio ζ and natural frequency ω_n. The desired acceleration found by equation 16 is used with equation 19 to set the total force and primary rotational axis that he inner loop must enforce.

$$\ddot{\boldsymbol{O}}_{des} = -\omega_n^2 \boldsymbol{O} - 2\zeta\omega_n \dot{\boldsymbol{O}} \tag{16}$$

$$\boldsymbol{n}_{des}\bar{n}_z(f_1 + f_2 + f_3 + f_4) = m\boldsymbol{R}^{-1}(\ddot{\boldsymbol{O}}_{des} - [0\ 0\ g]) \tag{17}$$

The inner angular controller uses a LQR control scheme with the reduced state given by equation 8. Consistent with the cited paper, the inner loop input vector $u = (u_1, u_2)$ describes the deviation of the actual propeller forces from the total force set by the outer loop. This configuration requires an additional constraint which is given by equation 19.

$$u_1 = (f_1 - \bar{f}_1) - (f_3 - \bar{f}_3), \qquad u_2 = (f_3 - \bar{f}_3) \tag{18}$$

$$f_1 + f_2 + f_3 = \bar{f}_1 + \bar{f}_2 + \bar{f}_3 \tag{19}$$

2.5.3 Implementation

Before attempting to control a quadrotor with only three functioning rotors, a MATLAB simulation of the dynamics and controller was created. During simulation, the inner and outer loop gains were tuned to achieve a desirable system response. The inner and outer loops were executed at 1000 Hz and 50 Hz, respectively. The outer loop was designed as a close to critically damped system. Critically damping the outer loop is important to ensure that the inner loop is able to reach the target values set by the outer loop before the equilibrium target is set to a new value. The outer loop gains were set by $\omega_n = 1.5$ and $\zeta = 0.7$.

The LQR controller for the inner loop placed the following costs on the input and reduced attitude state: $1\, s^2 rad^{-1}$ on the angular rates, 20 on the deviation from the primary axis, and $1\, N^{-2}$ on the force inputs.

Some constants presented in the equations of motion are unknown, but were able to be estimated using values listed in the cited document. These estimations include $\gamma = 2.75$ x 10^{-3}, $T_{mot} = 0.015s$, and the propeller moment of inertia given by equation 3.

When solving for equilibrium, a value of $\rho = 0.5$ was used to achieve equilibrium motor spin rates far away from saturation, ensuring maneuverability. In this case, the attitude equilibrium values are:

$$\bar{f}_1 = \bar{f}_2 = 2.822 \quad \text{[N]} \tag{20}$$

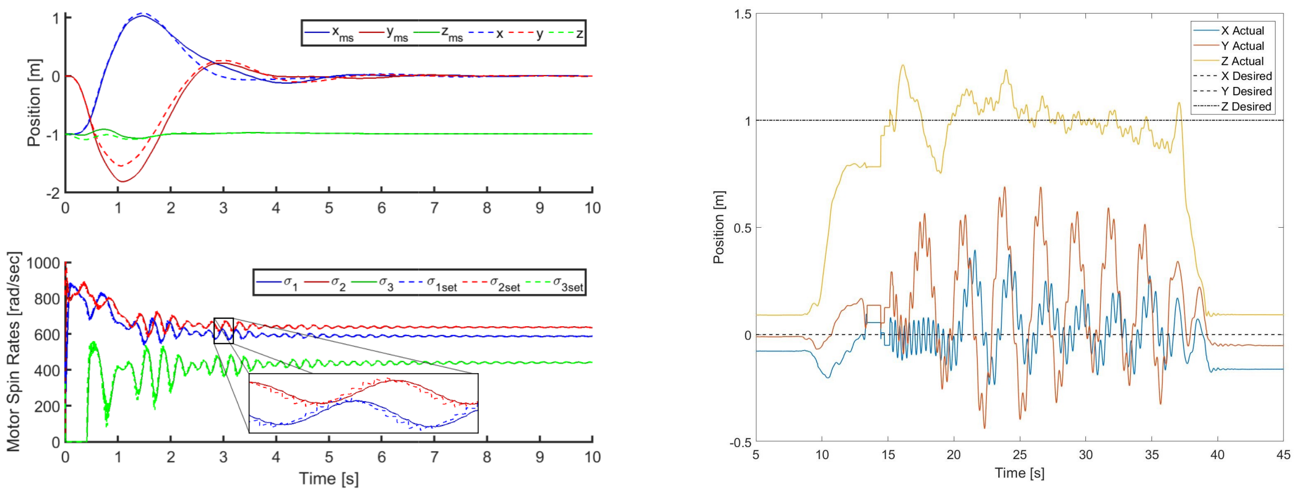
$$\bar{f}_3 = 1.411 \quad \text{[N]} \tag{21}$$

$$\bar{\omega}^B = (-2.70, 0, 25.38) \quad \text{[rad/s]} \tag{22}$$

$$\bar{\mathbf{n}} = (-0.106, 0, 0.994) \tag{23}$$

The resulting controller was implemented in simulation and the system was shown to stabilize the quadrotor around a desired position and follow a desired trajectory. Once the controller was proven, it was implemented in C code. The resulting code was complied and flashed onto a physical quadrotor to collect experimental data.

2.6 Results



((a)) Simulated results of quadrotor position and motor spin rates. The position plot compares the system response with and without simulating the motor state (ms) time delay. The motor spin rate plot compares the set and simulated motor rates.

((b)) Experimental results obtained from cascade control

Figure 2: Simulated and Experimental Results

The simulation results are shown in figure 2.6, sub-figure ((a)). The position plot shows that the system responds similarly with and without the simulated motor states. The main difference lies in the amount of overshoot. The motor spin rates plot shows that, from a macroscopic view, the set and actual inputs do not differ greatly. The zoomed better shows the delay between set an actual input. This plot also highlights that while the input is set at discrete time steps, the actual motors operate in continuous time.

The goal for the x and y position was to have smooth oscillations about the desired position of zero. As shown in figure 2, the oscillations of the x positions from time 15 seconds to 20 seconds was far too sporadic for complete stability. The y positions were the goal period of oscillation, but they had very jagged edges which could be because of one of two reasons. The first reason could be the fact that the MoCap system may not be able to accurately measure the rapidly changing attitude of the quadrotor. The second reason could be that the propellers are just not able to react fast enough for the inputs, which made the x and y positions so jagged and sporadic. The z position was almost ideal, however from time 15 seconds to 20 seconds there was a sizable jump from the ideal position. This was in accordance to the fast oscillations of the x position, which was most likely caused by the sub-ideal propellers.

2.7 Discussion

The quadrotor was able to maintain a stable flight under the underactuated condition. However, hardware limitations, the presence of a disturbing force, limited reliability of sensor data, and the lag in the motor response time contributed to the divergence of the experiment presented here with the experiment presented in Dr. Mueller and Dr. Raffaello D'Andrea's paper.

The requirement of having a tether attached to the quadcopter as it flew created a disturbing for that was unaccounted for. Although the system preformed reasonably well in the presence of this disturbing force, further tests should be done in an fully equipped arena that can facilitate free flight.

The motors had an unknown response time. Within the inner loop, a slight lag time occurs when the inner loop tries to match the targets set by the outer loop. The cited paper suggests extending the controllable state of the quadrotor to include the motors in order to improve the response time. While this was considered in this experiment, a lack of time led to this being treated as a secondary goal and thus was descoped from this experiment. Implementing a controller for the extended motor states would be of high priority in future iterations of this project.

The quadrotor sensors are often less accurate in rapidly changing environments such as the quadrotor's equilibrium. For this reason, it is believed that inaccuracies in the MoCap and IMU sensor data may have lead to a degraded performance during experiment. In the future, this might be overcome by implementing a Kalman filter to estimate the state and filter sensor noise.

This project focused on the failure of a single propeller. The cited paper also presents solutions for the two and three failed propeller cases. Future work on this project could implement these additional failure modes. Additionally, continuous failure monitoring could be developed to detect when and which failure mode occurs and automatically switch to the appropriate controller.