Many Problems For Fun and Profit

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1. **(Wager)** "I will bet you one pound," said Fred, "that if you give me two pounds, I will give you three pounds in return." "Done!" said Jack. How did this bet turn out?

2. **(Another Wager)** This time Fred offers Jack the "three pennies" bet: Jack flips three coins, winning a dime if all three land on the same face and losing a nickel otherwise. Fred says that the bet is in Jack's favor because it is guaranteed that at least two coins will land on the same face, and then the third coin has a 50/50 chance of agreeing with the first two. Should Jack take the bet?

3. **(Beehive)** One fifth of a hive flew to a rose bush, one third to a patch of tulips, three times the difference went to a nearby arbor, and one bee remained. How large was the hive?

4. **(Apple Orchard)** I went through seven gates to an apple orchard, where I picked some apples. On the way back I had to bribe the guard at each gate, and the size of each bribe was half of my remaining apples, plus one. If I made it past the seventh gate with a single apple left for myself, how many apples did I have to begin with?

5. **(The Bridges of Konigsberg)** In days of old the town of Konigsberg (Figure 1) was joined by seven bridges. A riddle was posed for the inhabitants of the town: how can one tour the town by crossing each bridge exactly once?

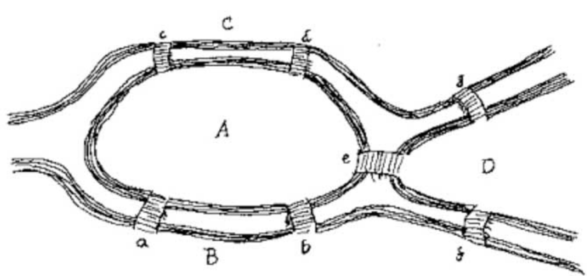

Figure 1: How to cross the bridges? (Problem 5)

6. **(Dominoes)** Can you tile a normal 8×8 chessboard with dominoes if you remove two adjacent corners first? What if you remove two opposite corners?

7. **(Triominoes)** Remove a single corner from a $n \times n$ chessboard. We attempt to tile the board by triominoes. (A triomino is like a domino except it consists of three squares in a row; each cell can cover one cell on a chessboard) For what values of n can you find a tiling?

8. **(Band-Aids)** Consider three pairwise adjacent faces of an $n \times n \times n$ cube. For what values of n is it possible to tile the three faces with 3×1 band-aids? A band-aid may wrap around an edge, but cannot otherwise be bent.

9. **(Missing Sums)** The sums of four numbers, omitting each of the numbers in turn, are 22, 24, 27, and 20, respectively. What are the four numbers?

10. **(The Problem of the Calissons)** *Calissons* are a French sweet, often in the shape of an almond (or, for the purposes of this puzzle, a rhombus). You want to pack your calissons in a hexagonal box. As the example in Figure 2 shows, the sweets may be pointing in any one of three different directions. Your puzzle is this: to explain why, no matter how you pack the box, there must be an equal number

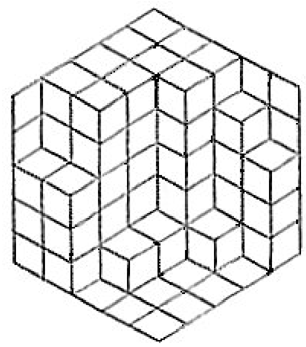

Figure 2: Packing calissons in a hexagonal box (Problem 10)

of calissons pointing in each direction.

11. **(Sock Drawer)** In a drawer you have ten red socks and ten blue socks, identical except for the color. Fumbling around in the dark, you begin drawing socks from the drawer at random. How many must you draw in order to guarantee that you have a matching pair?

12. **(Knights and Knaves)** In your travels you find yourself stranded on the island of Knights and Knaves—a kingdom where the Knights always tell the truth and Knaves always lie. You come across five people and, asking them who among them is trustworthy, receive the following responses:

    A: Exactly one of us is a knave.

    B: Exactly two of us are knaves.

    C: Exactly three of us are knaves.

    D: Exactly four of us are knaves.

    E: All of us are knaves!

    Can you tell who is a knight and who is a knave?

13. **(More Knights and Knaves)** You carry on and come across another five people. Asking again for helpful information, you get the following replies:

    A: C and D are knaves!

    B: A and E are knaves!

    C: B and D are knaves!

    D: C and E are knaves!

    E: A and B are knaves!

    Can you tell who the knaves are this time?

14. **(Knights and Knaves III)** You make your way to the nearby island of Knights and Knaves and Normals. Knights still tell the truth and knaves always lie, but Normals might either tell the truth or lie. Fortunately for you, the islanders have a habit of traveling in groups of three—one person of each type. You come across such a trio, and they offer you the following information:

    A: C is a knight.

    B: A is a knight.

    Which person is which?

15. **(Fifty Points)** A problem from Sam Loyd's 1914 *Cyclopedia of Puzzles*: in the carnival game shown in Figure 3, you win a fabulous prize if you knock down a set of targets whose numbers add up to 50. Can you find a way to win the game?

Figure 3: How to score fifty points? (Problem 15)

16. **(Archery Puzzle)** Another by Loyd: How many arrows does it take to score exactly 100 on the target shown in Figure 4? What if instead the target has seven regions worth 11, 13, 31, 33, 42, 44, and 46 points?

17. **(Cumulative Dice)** You roll a die and keep adding the numbers you roll until the total exceeds 12. What is the most likely final total?

18. **(Digital Puzzle)** What is the final digit of 7^{7^7}? What about the final three digits?

19. **(Painted Cubes)** You have a large number of wooden cubes and wish to paint each face of the cubes solid red or blue. How many different types of painted cubes could you end up with? Two cubes count as the same if one can be rotated to look just like the other. How many cubes might there be if you also had green paint available?

20. **(Cheese)** A mouse wishes to eat a $3 \times 3 \times 3$ block of cheese one unit cube at a time, so that each cube it eats is horizontally or vertically adjacent to the previous one and so that it finishes with the cube in the center. Can it accomplish this feat?

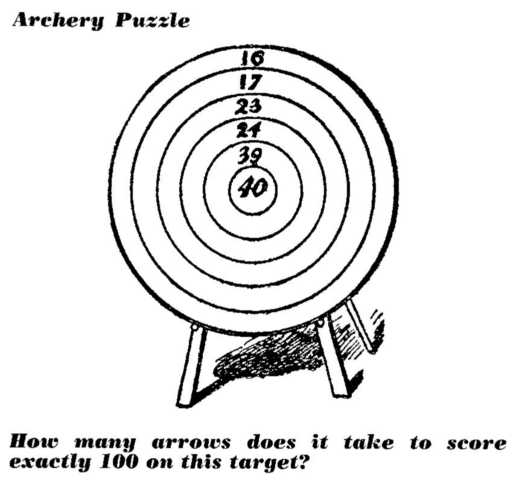

Figure 4: How to score 100 points? (Problem 16)

21. **(Cookie)** Alice and Bob are sharing a cookie. Alice takes half and passes the rest to Bob. Bob takes half of the remainder and passes the rest to Alice. If they continue in this manner, what fraction will Alice get?

22. **(Pizza Slicing)** What is the greatest number of pieces of pizza I can cut using just six straight cuts with a knife? My friends aren't picky, so the pieces can be any size or shape and don't need to include the crust.

23. **(Cheese Slicing)** What if I am instead dividing a large cylindrical block of cheese—how many pieces can I cut using just six straight plane cuts with the knife?

Figure 5: Into how many pieces may the cheese be cut? (Problem 23)

24. **(Houses)** Mr. Gardner lives on a very long block where the houses are numbered $1, 2, 3$, and so on in succession. One day he noticed that the sum of the the house numbers less than his own is equal to the sum of house numbers greater than his. If Mr. Gardner's house number is somewhere in the thirties, what is it?

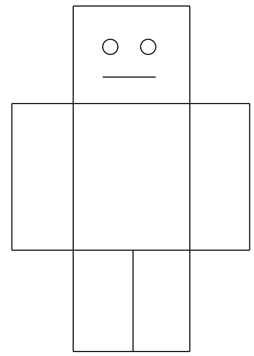

Figure 6: Adorable robot (Problem 25)

25. **(Draw the Robot)** Aww, what an adorable robot (Figure 6)! Ignoring the eyes and mouth, can you draw the little fella by tracing each line only once and without picking your pencil off the page?

26. **(Catching the Hogs)** In the illustration (Figure 7) Hendrik and Katrün are attempting to capture a couple of hogs. Why did they fail?

    Represent the couple's challenge by the following game: on the grid shown in the figure, the Dutchman and his wife move one square each (horizontally or vertically). Then the pigs move one square each. This continues until Hendrick catches one hog and Katrün catches theother.

    This you will find would be absurdly easy if the hogs moved first, but this is just what Dutch pigs will not do.

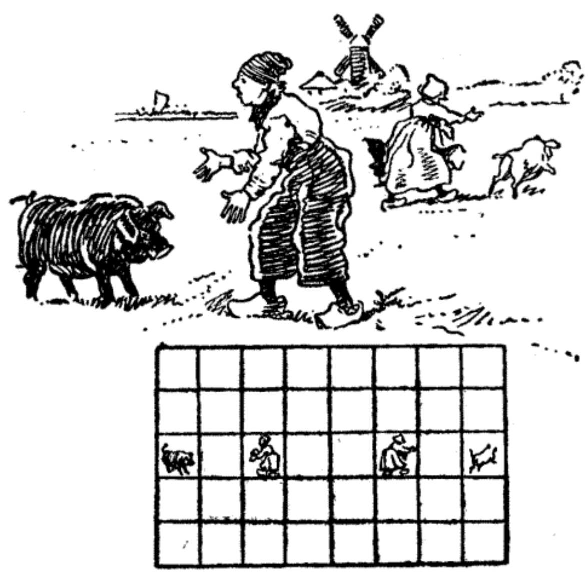

Figure 7: Can the farmers catch the hogs? (Problem 26)

27. **(Jigsaw Puzzle)** If fitting together two pieces (or a piece to a block of pieces) counts as a "move", how many moves will it take to solve a 27×37 (999-piece) jigsaw puzzle, and what is the optimal strategy?

28. **(Elimination Tournament)** If a single-round elimination tournament has 2, 4, 8, or 16 people, it is easy to see how the tournament will be structured and how many matches will be played. What if there are 37 people in a tournament—how many matches must there be in order to produce an eventual champion?

29. **(The Game of Gale)** In this two player game (Figure 8), Red and Blue take turns connecting dots of their own color. Blue wins by creating a bridge that spans the board vertically, and Red wins by creating a bridge that spans the board horizontally. If both players play perfectly is it possible for the first player to force a victory? What about the second player? Can the game ever end in a draw?

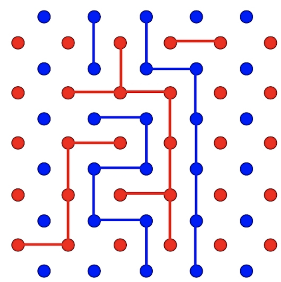

Figure 8: The Game of Gale (Problem 29)

30. **(The Game of Hex)** A two-player game similar to Gale, but this time played on a board with a hexagonal grid (Figure 9). What can you conclude about this game?

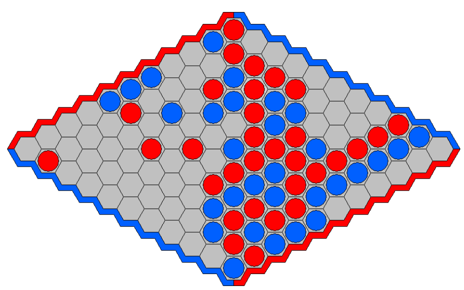

Figure 9: The Game of Hex (Problem 30)

31. **(The Missing Dollar)** Three friends dine out and pay 10 dollars each for the 30 dollar bill. Later the manager realizes that there was a mistake and that the bill should have been only 25 dollars. He gives the waiter 5 singles to return to the diners, but since he has no way to divide the money evenly the waiter returns 1 dollar to each of them and pockets 2 for himself.

    In all, the diners paid 27 dollars for the meal (9 dollars each) and the waiter kept 2 dollars, but $27 + 2 = 29$ and the friends originally paid 30 dollars! What happened to the extra dollar?

32. **(Infection)** Some of the 64 cells of a chessboard are initially infected. The infection spreads according to the following rule: if two neighbors of a cell (horizontal and/or vertical) are infected then the cell gets infected. No cell is ever cured. What is the minimum number of cells that need to be initially infected to guarantee that the infection spreads all over the chessboard? It is easy to see that 8 are sufficient in many ways. Can you show that 7 are not enough? (This is an AH-HA problem. The main idea of a clear and convincing solution can be summarized in a single 9-letter word.)

33. **(Towns)** Seven towns lie on a straight line, each town an integer distance from each of the others. If all of the distances are distinct, how small can the distance between the first and last towns be?

34. **(Product-Sum)** It is somewhat interesting to note that $2 + 2 = 2 \times 2$. How many other pairs of numbers can you find whose product is equal to their sum?

35. **(Double Swap)** Find a six-digit number so that if the two leftmost digits are moved over to the right end, the resulting number is twice the original.

36. **(Quadruple Swap)** A certain number whose last digit is 4, becomes 4 times larger when the 4 is removed from the end and placed at the front. What is the number?

37. **(Six Knights)** Exchange the positions of the three white knights and the three black knights (Figure 10) in as few moves as possible.

Figure 10: Exchange the knights (Problem 37)

38. **(Knight's Tour)** How can a knight make a tour of an 8×8 chessboard, visiting each square exactly once and ending where it began? What is the smallest board for which it can do this?

39. **(Dance Party)** At a small party each man danced with exactly three women, and each woman with exactly three men. Furthermore, each pair of men had exactly two dance partners in common. How many were at the party? [Wel92]

40. **(Triangular Numbers)** The numbers $1, 3, 6, 10, \ldots$ are known as *triangular numbers* and Figure 11 demonstrates the reason why. Can you find any numbers that are both triangular and square? How many? Can you find any triangular number that can be written as the sum of two other triangular numbers?

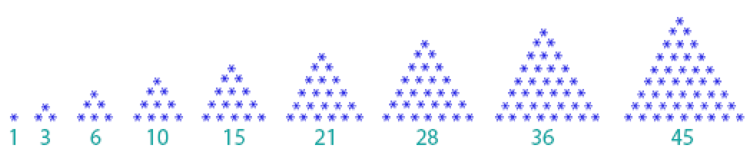

Figure 11: Triangular numbers (Problem 40)

41. **(More Triangular Numbers)** As it turns out, every triangular number past the first can be written as the sum of a square number and two triangular numbers. Can you find a way to write the first seven triangular numbers in this manner? Can you find an explanation for the pattern?

42. **(Chelsea Pensioners)** A puzzle by Lewis Carroll: if 70 percent in a group of soldiers have lost an eye, 75 percent an ear, 80 percent an arm, and 85 percent a leg, how many *at least* must have lost all four?

43. **(Stamp Problem)** If I have a collection of 5-cent and 17-cent stamps, what is the largest number that cannot be made from them?

44. **(Orange Packing)** For the holidays I decided to pack boxes of oranges to send out as care packages. When I packed them 10 to a box, the final box I tried to pack had only nine oranges. When I tried packing them them 9 to a box instead, the last box had only 8 oranges. If this pattern continued to the final box having only a single orange when I packed two to a box, what is the smallest number of oranges I might have had to begin with?

45. **(Cube Sum)** How many three-digit numbers are equal to the sum of its digits' cubes?

46. **(White to Play)** The board in Figure 12 looks like the position after White has made a rather unusual first move, but in fact this position resulted after Black just played! What is the smallest number of

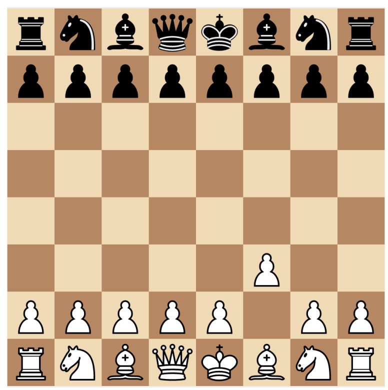

Figure 12: White to play. How? (Problem 46)

moves that could have been played in order to reach this bizarre scenario with White to play?

47. **(Infinite Tic-Tac-Toe)** In a regular game of Tic-Tac-Toe, players alternate putting Xs and Os on a 3×3 board with the goal of getting 3 in a row in any direction. If the board were infinite, it would be simple for the first player (X) to win. What if X needs to get 4 in a row to win? 5? 6?

48. **(The Garage Door)** My garage door has a lock with a 3-digit code, which I seem to have forgotten. Since there are 1000 possible codes, I could find the correct one by brute force in a maximum of $3 \times 1000 = 3000$ key presses. Fortunately, my lock is a little more forgiving than that: if I enter a string of numbers such as 34758, then if the correct code is any of 347, 475, or 758 the door will open. Is there a string of 1002 keys I can press that contains all possible codes?

49. **(The Ferry Problem)** An old classic: you wish to take a chicken, a wolf, and a bag of grain across a river, but your boat is so small that you can only take one thing at a time. Furthermore, if left unattended the wolf will eat the chicken, and similarly the chicken will eat the grain. How can you cross the river without anything being eaten?

50. **(Wine Pouring)** How can you take exactly 4 pints from a large cask if you only have two unmarked jars that hold 3 and 5 pints?

51. **(Share the Wealth)** You and a friend have 12 pints of wine in a cask as well as jars that hold 5 and 7 pints. How can you divide the wine evenly between the two of you?

52. **(Dueling Dice)** Archibald, Bertrand, and Chauncey have agreed to a competition with dice. The rules are simple: each person rolls a die and the higher number wins. Each person brings in a fair six-sided die, but each with their own custom set of numbers on the faces. After many rounds they find themselves unable to agree who the winner should be: Archibald rolled a higher number than Bertrand over half the time and Bertrand beat Chauncey over half the time, but Chauncey also outperformed Archibald over half the time! What numbers could possibly be on their dice that would explain this odd result?

53. **(Fool's Duel)** Archibald, Bertrand, and Chauncey agree to face off with pistols to resolve their dispute. As it happens they only have one pistol between them, so they plan to take turns shooting at each other until only one person remains—on each person's turn that person will be allowed to fire a single bullet at the target of his choice. If Archibald can hit his mark half the time, Bertrand hits 2/3 of the time, and Chauncey is a sure shot, which of the three has the best chance of surviving?

54. **(Around the World)** In Figure 13, each letter represents a city and each line a path between two cities. Can you find a way to tour the world, visiting each city exactly once and ending up where you began?

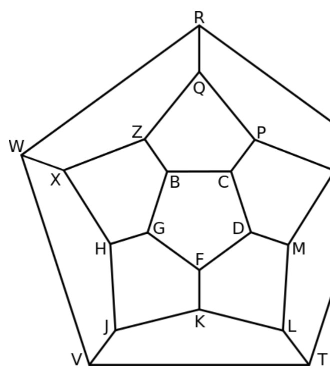

Figure 13: Can you visit each city exactly once? (Problem 54)

55. **(Rule of Sixes)** Which is more likely: to throw at least one six with six dice, at least two sixes with 12 dice, or at least three sixes with 18 dice?

56. **(Pan Balance)** With a pan balance, sort five distinct weights from heaviest to lightest in only seven weighings.

57. **(Odd Marble Out)** You have nine marbles, one of which is slightly heavier than the others. Using only a pan balance, what is the smallest number of weighings you need to locate the heavy marble?

58. **(Game of Elevens)** How many orderings of the digits 123456789 give a number divisible by 11?

59. **(Photo Finish)** A photographer wishes to arrange 10 people (of all different heights) in two rows of five so the heights increase from left to right and so that each person is taller than the one directly in front. In how many ways might this be done?

60. **(Prime Friends)** When are the numbers p and $p^2 + 2$ both prime?

61. **(Prime Triplets)** A pair of numbers p and $p + 2$ are known as *twin primes* if both are prime. For example, 17 and 19 are twin primes, as are 71 and 73. How many sets of prime *triplets* can you find?

62. **(Relative Primes)** What are the chances that two random positive integers are relatively prime?

63. **(Primes and Squares)** The sum and difference of two squares may be primes (Say, $3^2 + 2^2 = 13$ and $3^2 - 2^2 = 5$). Can the sum and difference of two primes ever be squares?

64. **(The Quarrelsome Neighbors)** Three neighbors who shared a small park had a falling out. The owner of the large house, complaining that his neighbors' chickens annoyed him, build an enclosed path from his house to the gate at the bottom of the picture (Figure 14). Then the owner on the right built a path to the gate on the left and the owner on the left built a path to the gate on the right, but none of the three paths crossed! How could this be?

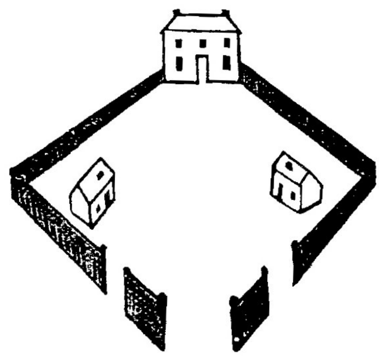

Figure 14: How did the paths avoid crossing? (Problem 64)

65. **(The Three Utilities Puzzle)** A similar but more classic problem: can you connect the three houses to the three utilities (Figure 15) so that no two lines cross?

Figure 15: The Three Utilities Puzzle (Problem 65)

66. **(Return of the Sock Drawer)** In an even worse organizational system than before, I now have three drawers: the first contains three red socks and three blue, the second has three blue and three green, and the third has three red and three green. Drawing at random, how many socks must I draw in order to know for sure which drawer is which?

67. **(Family Planning)** In a kingdom ravaged by war, much of the kingdom's male population had died in battle. The king issued a decree that each family should have children until they have one girl, then stop. His reasoning was that some families may have just one girl or a boy and a girl, but other families may have a long string of boys before having a single girl and so help balance out the population again. How did his plan work out?

68. **(Grand Prix)** Sophie plans to tour 20 cities in her new car, ending up in her starting city. She has planned the route carefully, and at each city she has prepared a certain amount of gasoline. The amount differs from city to city, but Sophie knows that the total number of miles the gasoline will let her travel is precisely equal to the length of her circuit.

    With a poor choice of starting city she might find herself in a pickle: for example, she could start in City A with 10 miles' worth of gas, leaving her unable to cross the 100-mile desert to get to City B. The question is this: is there guaranteed to be a city such that Sophie will be able to complete the circuit by choosing that city as her starting point?

69. **(The Strange Library)** A certain library has more books on its shelves than any single one of its books contains words. If no two of its books contain the same number of words, what can you say about the number of words in one of the books?

70. **(Palindrome)** You can move through the grid (Figure 16) to spell out the palindrome "Was it a cat I saw?" If you are allowed to travel in any horizontal or vertical direction, in how many different ways can you do this?

Figure 16: Was it a cat I saw? (Problem 70)

71. **(Birdhouse)** What is the largest number of pigeonholes that may be occupied by 100 pigeons if each hole is occupied but no 2 have the same number of pigeons?

72. **(Lights Out)** You have 1000 light swiches in a row, corresponding to 1000 lights that are currently off. First you flip every switch. Then you flip every other switch (those that were on now get flipped off and vice versa), then every third switch, and so on until you flip every thousandth switch (i.e. just the last one). How many lights are on at the end?

73. **(Fowl Play)** 100 birds are sold for 100 dollars: the ducks for 5 apiece, the hens for 3 apiece, and the quail for $\frac{1}{3}$ apiece. How many of each were sold?

74. **(The States Game)** Here is a game to play: pick a U.S. state (say, Idaho), then name a state whose first letter is the same as the last letter of the previous state (e.g. **O**regon). If you are not permitted to name the same state twice, what is the longest this game could possibly last?

75. **(Counting Squares)** How many squares can you find on an 8×8 chessboard?

76. **(How Many Friends)** At any party (meaning any gathering of at least two people), at least two people will have the same number of friends present —true or false?

77. **(Hotel Rooms)** Seven friends wish to stay at a hotel for the night, but the manager only has six rooms available. The manager resolves the issue in the following manner: he puts the first person in room one, and has another wait with her in the room temporarily. Then he puts the third person in room two, the fourth in room three, the fifth in room four, and the sixth in room five. Then he calls the seventh person from where she was waiting in room 1 and puts her in the sixth and final room. Should the manager receive a promotion for accomplishing such a feat?

78. **(The Tower of Hanoi)** It is said that in a certain monastery the monks labor to move a great tower made of 64 stone disks from one peg to another. They have three pegs available (Figure 17) and at each step may move the disk on top of one pile to the top of any other pile, just as long as a larger disk never gets placed on a smaller one. When they complete their task, the legend goes, the world will come to an end.

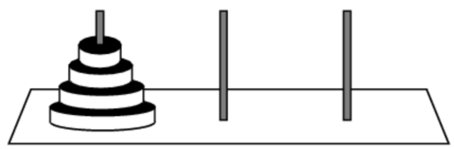

Figure 17: How long will it take to move the tower? (Problem 78)

Moving the stones at a rate of one per second, how long will the monks take to move the whole tower?

79. **(The Apple Sellers)** Two farmers each have thirty apples that they wish to sell at market. One of them normally sells the apples at 2 for 1 dollar and the other at 3 for 2 dollars, but in the spirit of cooperation they agree to combine their stock and sell it all at 5 for 3 dollars. Normally they would have made $15 + 20 = 35$ dollars, but after selling the combined stock they found that they made 36 dollars. . . an extra dollar in profit!

    Two other farmers heard their story, and decided to combine their wares as well. They also had 30 apples apiece, one batch selling at 2 for 1 and the other at 3 for 1, but when they combined to sell at 5 for 2 dollars they found that they made one less dollar than they would have individually! How could this be?

80. **(Penny Game)** Ten pennies lie in a circle. Each of two players takes turns removing either 1 penny or 2 adjacent pennies. The person to remove the final penny wins. Who has a winning strategy? What if the pennies are in a line rather than a circle?

81. **(The N-Queens Problem)** Can you put 4 queens on a 4×4 board so that none attacks any of the others? What about 5 queens on a 5×5 board? 6 queens on a 6×6?

82. **(Wonderful Sagacity)** Nikonorov the bookkeeper asked each of four children to think of a four-digit number. "Now please transfer the first digit to the end and add the new number to the old one. For example, $1,234 + 2,341 = 3,575$. Tell me your results."

    Kolya: 8,612.

    Polya: 4,322.

    Tolya: 9,867.

    Olya: 13,859.

    "Everyone except Tolya is wrong," Nikonorov responded. How did he know?

83. **(The Largest Product)** What is the largest number that can be written as the product of natural numbers that add up to 100?

84. **(A Crime Story)** An elementary school teacher in New York had her purse stolen. The thief had to be Lillian, Judy, David, Theo, or Margaret. When questioned, each child made three statements.

    Lillian:

    i) I didn't take the purse.
    ii) I have never in my life stolen anything.
    iii) Theo did it.

    Judy:

    iv) I didn't take the purse.
    v) My daddy is rich enough, and I have a purse of my own.
    vi) Margaret knows who did it.

    David:

    vii) I didn't take the purse.
    viii) I didn't know Margaret before I enrolled in this school.
    ix) Theo did it.

    Theo:

    x) I am not guilty.
    xi) Margaret did it.
    xii) Lillian is lying when she says I stole the purse.

    Margaret:

    xiii) I didn't take the teacher's purse.
    xiv) Judy is guilty.
    xv) David can vouch for me because he knows me since I was born.

    Later, each child admitted that two of their statements were true and one was false. Who stole the purse?

85. **(Hair)** Why must there be two people in the world with the same number of hairs on their head?

86. **(Squares)** Find three numbers such that their sum is a square and the sum of any pair is a square.

87. **(Hat Problem)** Three students, all wearing white or black hats, are sitting in a column of three chairs so that each one can see the hats of the students in front of her but not her own hat. They are informed that at least one of them is wearing a white hat, and asked one by one if they know the color of their own hat.

    The student in back says "No, I don't know."

    The middle student says "Hmm. . . no, I don't know either."

    The third student thinks for a bit, then says "Okay, I know what color my hat is!"

    What color was the student's hat, and how did she know?

88. **(Second Hat Problem)** This time the students are sitting in a circle, so each one can see the others. Each one is given a white or black hat, and they are told to raise their hand if they see someone else wearing a white hat. All three students raise their hand. Asked to determine the color of their own hat, the students sit in silence for a while before one ventures "I think my hat is white."

    How did the student determine this?

89. **(The Wrong Envelopes)** A correspondent writes ten letters and addresses ten envelopes. In how many ways can all of the letters be put in the wrong envelopes?

90. **(A Pair of Lists)** Find two infinite lists of non-negative numbers—call them A and B—so that every non-negative number can be written *uniquely* as the sum of one number from A and one from B.

91. **(The Four Weights)** You have a pan balance and four weights that allow you to measure any integer weight from one to forty pounds. What are the values of the weights?

92. **(Proofreaders)** Two proofreaders are checking a manuscript. The first finds thirty errors and the second finds twenty-four. On comparing their notes, it turns out that only twenty errors were found by both of them. How many errors do you suspect remain, not detected by either?

93. **(Count the Triangles)** How many triangles can you find in Figure 18? If you vary the number of interior lines, can you find a pattern to the number of triangles that appear?

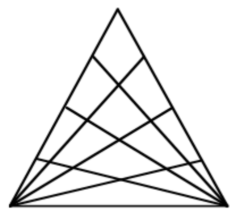

Figure 18: How many triangles? (Problem 93)

94. **(Odd Marble Out II)** You have twelve marbles, one of which is either slightly heavier or slightly lighter than the rest (you don't know which). Using only a pan balance, what is the smallest number of weighings you need to determine which marble is the odd one out?

95. **(Kirkman's Schoolgirl Problem)** Fifteen girls walk to school seven days a week in five rows of three. How may the girls arrange themselves so that each pair of girls shares a row exactly once per week?

96. **(Byzantine Basketball)** In Byzantine basketball the only two ways to score are free throws and goals—goals are worth more, but I do not recall how many points each is worth. If there are 35 scores that are impossible to achieve and 58 is one of them, how much is each worth?

97. **(Tower Escape)** A mad king has lock the queen along with their daughter and son in a tall tower, as mad kings are liable to do. Outside the tower window is a pulley and long rope. The queen finds a 75-pound weight in the tower, and hopes to use it as a counterbalance so that the family can escape.

    The queen weight 195 pounds, the daughter 165, and the son 90. When two loads are attached to the rope the heavier one will descent and the lighter one rise, but if the imbalance is any more than 15 pounds the descending people will fall too fast and get injured. How can the queen and family escape?

98. **(Parentheses)** You wish to evaluate the sum $a + b + c + d + e + f + g + h$. In how many different ways can you add parentheses to the expression to change the order of summation? For example,

    $$(a + (b + (c + d))) + (e + ((f + g) + h))$$

    and

    $$(((a + (b + c)) + (d + e)) + f) + (g + h)$$

    will result in two different orders.

99. **(Triangulation)** You can divide a regular octagon into six triangles by connecting pairs of edges. How many different ways can you do so?

100. **(Deadly Logic)**

   i) The number of the first true statement here added to the number of the second false statement gives the number of a statement which is true.

   ii) There are more true statements than false.

   iii) The number of the second true statement added to the number of the first false statement gives the number of a statement which is true.

   iv) There are no two consecutive true statements.

   v) There are at most three false statements.

   vi) If this puzzle consisted of statements 1 to 5 only, then the answer to the following question would still be the same.

   Which statements are true?

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