Theory and Implementation of Peridynamic Theory on Elastictiy

# Peridynamic Theory and Formulation

For a given undeformed state of a body, each material point is referred with its coordinates,

Each material point associated with incremental volume and mass density in which body is decomposed of.

Similarly, position for a material in deformed state is,

Under a deformation, each material point exhibits a displacement,

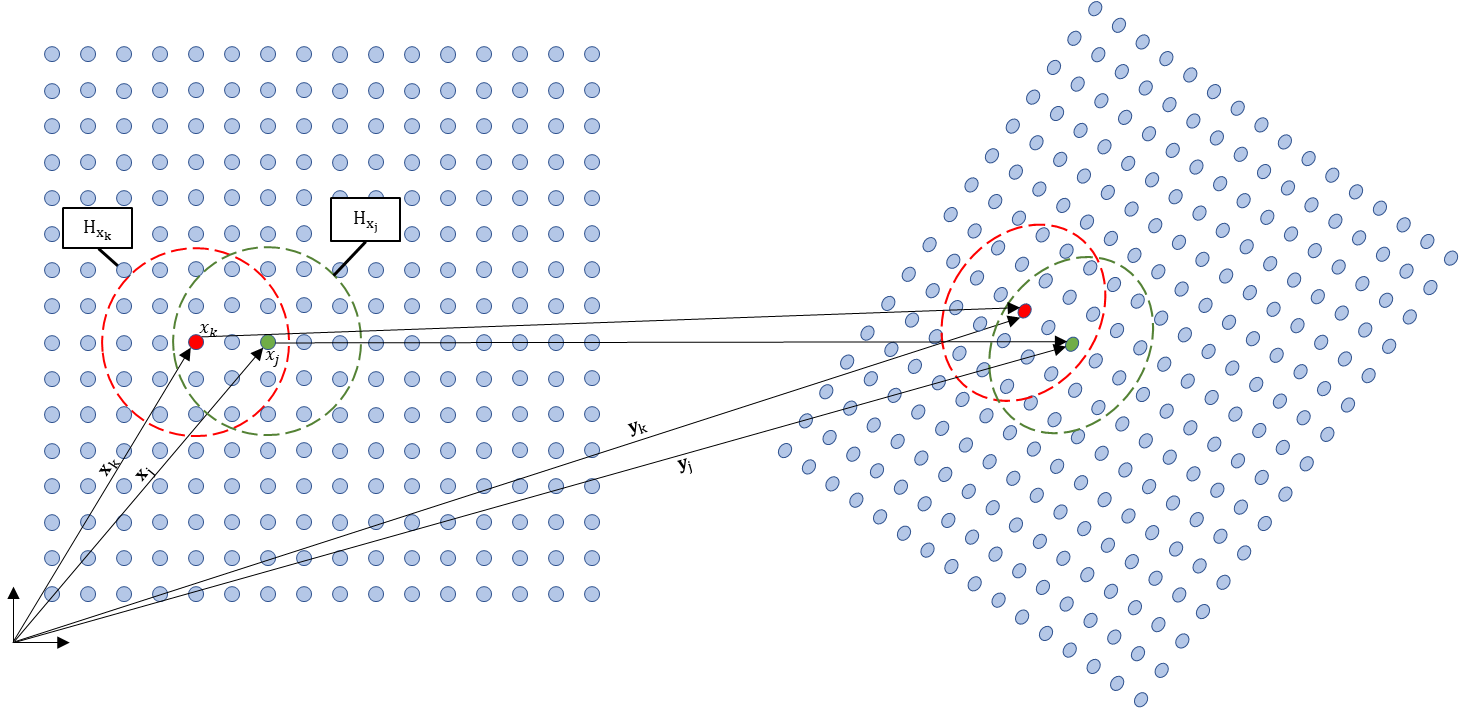


Figure 1 - Reference and Deformed Configurations within a Body

Interaction of material points in a body is defined with a subdomain. Each material point is assigned with a subdomain where it exhibits influence of other material points within this subdomain referred as neighborhood of xk or .

Definition of a neighborhood can be made with a constant range of from the its position.

Stretch between two material points due to a deformation, is the ratio of relative positions after and before deformation such that,

Due to the collective deformation of material points, force interactions between two material points are defined with force density vector functions. ()

Definition of these force densities are not only dependent on pairwise motion of its owners and but also collective deformation of points inside their neighborhoods and .

Diagram

Description automatically generated

Figure 2 - Force Density Vectors due to a collective deformation

Energy density of these interactions are defined with a scalar value micropotential function and . Strain energy density of a material point is then the average of summation of these micropotentials;

where;

# Equation of Motion

Total kinetic and potential energies in the body that consists of N material points are defined as,

With principle of virtual work at material point ,

which is satisfied by Lagrange’s equation,

Using micropotentials, total potential energy becomes,

Lagrangian is then becomes,

Rewriting with terms associated with material point ,

Rearranging associated terms with kth material point,

Then,

Substituting into Lagrange’s equation,

Since micropotential functions are functions of relative displacement vectors in deformed state,

Similarly,

By definition of relative position vectors are,

And similarly,

Thus,

which results in,

Then the Lagrange’s equation become,

or,

With the force density definition, equation of motion for material point becomes;

where,