Homework 12
COMP543 Fall 2020 - Modern Cryptography

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1. Quesitons

Q1: What are the properties of a random oracle?

A1: The oracle is a black-box, we do not know the internal works. We only give it a binary string as input, and it returns a binary string as output. Everyone (honest and adversarial) can interact with the box, that is: they can query the oracle on x. Such queries are said to be private, and no one else learns what x is during the query. In fact, they do not know that oracle was queried at all. (This is because such queries are done locally in practice.) Call this oracle H for now:

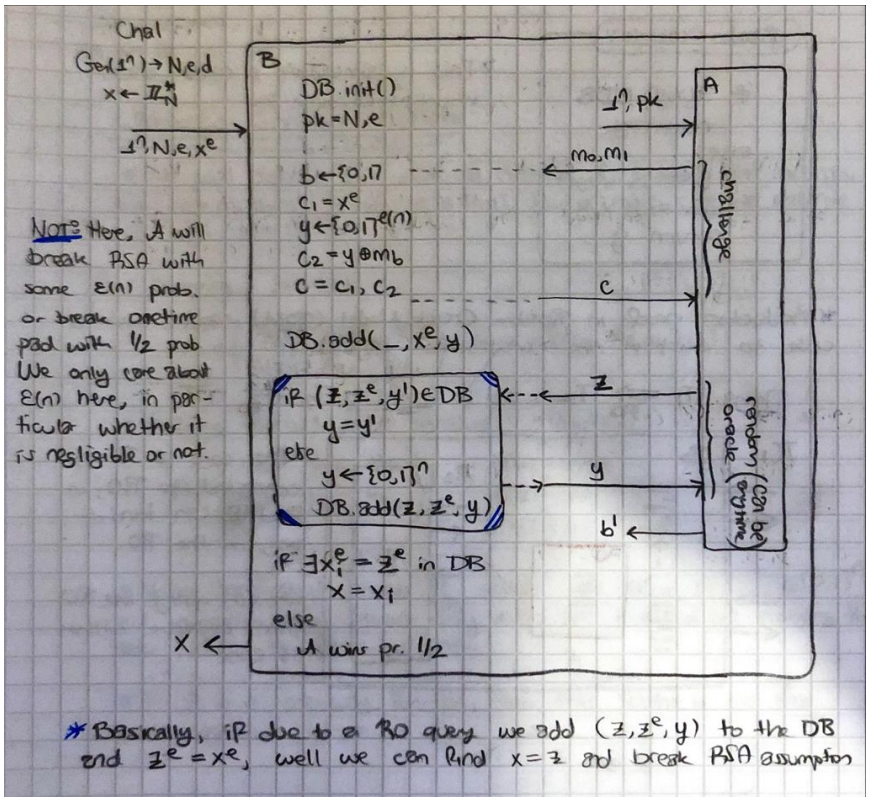
- **Consistency**: H is *consistent*. If returned y to some query $H(x)$, it will always do that for everyone when they query $H(x)$.
- **Uniformity**: If x has not been queried to H, then the value of $H(x)$ is *uniform*.
- **Extractability**: If $\mathcal{A}$ queries $H(x)$, the reduction (i.e. algorithm B that we construct from $\mathcal{A}$) can see the query and learn (*extract*) the query x.
- **Programmability**: The reduction (i.e. algorithm B that we construct from $\mathcal{A}$) can set (*program*) the value of $H(x)$, which is the response to some query x on H, to a value of it's own choice. However, this chosen value must be uniformly distributed.

Q2: Let $GenRSA$ be a PPT algorithm that, on input 1^n , outputs a modulus N that is the product of two n-bit primes, along with integers e, d satisfying $ed = 1 \bmod \phi(N)$. Let H be a function with domain $\{0,1\}^*$ and range $\mathbb{Z}_N^*$ for any N. Construct a signature scheme as follows:

- Gen: on input 1^n, run $GenRSA(1^n)$ to compute (N, e, d) and set the range of H to be $\mathbb{Z}_N^*$. The public key is (N, e) and the private key is (N, d).
- $Sign$: on input a private key (N, d) and a message $m \in \{0,1\}^*$, compute $\sigma := [H(m)^d \bmod N]$.
- $Vrfy$: on input a public key (N, e), a message m, and a signature σ, output 1 if and only if $\sigma^e = H(m) \bmod N$.

Formally prove that if the RSA problem is hard relative to $GenRSA$ and H is modeled as a random oracle, then the construction above is existentially unforgeable under an adaptive chosen-message attack.

A2: Basically we want to show that RSA assumption $\implies {}_R O\Pi$ is CPA-secure. Suppose there is an algorithm $\mathcal{A}$ that breaks Π. Then we will show it would be possible to construct an algorithm B that breaks the RSA assumption.



Q3: Consider some of the schemes you have seen, try to perform the following replacements to see where the reduction proof fails.

(1) Replace a random oracle with a PRF
(2) Replace a random oracle with a collision-resistant hash function
(3) Replace a collision-resistant hash function with a second-preimage-resistant hash function
(4) Replace a collision-resistant hash function with a preimage-resistant hash function

A3: First note two things:

- If the range of RO is larger than domain, then it is indistinguishable from a PRG.
- $F_k(x) = RO(k||x)$ is a PRF using random oracle RO.
- The success probability of any PPT $\mathcal{A}$ in the following experiment is negligible:
  (1) A random function H is chosen.
  (2) $\mathcal{A}$ wins if it outputs distinct x, x' with $H(x) = H(x')$.
  In fact, $\mathcal{A}$ succeeds with negligible probability $\mathcal{O}(q^2/2^{l_{out}})$, which is known from the Birthday Problem.

to do...