

# Adimensionnement

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Novembre 2025

## 1 Introduction

$$\eta = \frac{\mu}{2\beta}, \quad \beta = \sqrt{\frac{\mu}{\rho}}$$
$$\eta v = -k(l^* + \delta - v_p t) - f_0 \sigma - a\sigma \ln\left(\frac{v(t)}{v_p}\right) - b\sigma \ln\left(\frac{v_p \theta(t)}{d_c}\right) \quad (1)$$

On dérive par rapport au temps :

$$\eta \frac{dv}{dt} = -k(v - v_p) - a\sigma \frac{1}{v} \frac{dv}{dt} - b\sigma \frac{1}{\theta} \frac{d\theta}{dt} \quad (2)$$

On pose les variables adimensionnées suivantes et on obtient:

$$\bar{t} = \frac{tv_p}{d_c}, \quad \bar{v} = \frac{v}{v_p}, \quad \bar{\theta} = \frac{\theta v_p}{d_c}$$
$$\eta \frac{dv}{dt} = -kv_p(\bar{v} - 1) - \frac{v_p a \sigma}{d_c} \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{v_p b \sigma}{d_c} \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (3)$$

On multiplie par  $\frac{v_p b \sigma}{d_c}$  :

$$\frac{\eta v_p}{b\sigma} \frac{d_c}{v_p^2} \frac{dv}{dt} = -\kappa(\bar{v} - 1) - \alpha \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (4)$$

$$\begin{cases} \kappa = \frac{kd_c}{b\sigma} \\ \alpha = \frac{a}{b} \end{cases}$$

Comme  $\frac{d\bar{v}}{d\bar{t}} = \frac{d_c}{v_p^2} \frac{dv}{dt}$  et en posant  $\bar{\eta} = \frac{\eta v_p}{b\sigma}$ , on obtient:

$$\bar{\eta} \frac{d\bar{v}}{d\bar{t}} = -\kappa(\bar{v} - 1) - \alpha \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (5)$$

Or  $\frac{d\bar{\theta}}{d\bar{t}} = \frac{d\theta}{dt} = 1 - \frac{v\theta}{d_c} = 1 - \bar{v}\bar{\theta}$  on a finalement :

$$\frac{d\bar{v}}{d\bar{t}} \left( \bar{\eta} + \frac{\alpha}{\bar{v}} \right) = -\kappa(\bar{v} - 1) + \bar{v} - \frac{1}{\bar{\theta}} \quad (6)$$

On en déduit la formule utilisée dans le programme Python :

$$\boxed{-\kappa(\bar{v} - 1) + \bar{v} - \frac{1}{\bar{\theta} \bar{\eta} + \frac{\alpha}{\bar{v}}}} \quad (7)$$