

# Adimensionnement

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## 1 Introduction

$$\eta = \frac{\mu}{2\beta}, \quad \beta = \sqrt{\frac{\mu}{\rho}}$$

$$\eta v = -k(l^* + \delta - v_p t) - f_0 \sigma - a \sigma \ln\left(\frac{v(t)}{v_p}\right) - b \sigma \ln\left(\frac{v_p \theta(t)}{d_c}\right) \quad (1)$$

On dérive par rapport au temps :

$$\eta \frac{dv}{dt} = -k(v - v_p) - a \sigma \frac{1}{v} \frac{dv}{dt} - b \sigma \frac{1}{\theta} \frac{d\theta}{dt} \quad (2)$$

On pose les variables adimensionnées suivantes et on obtient:

$$\bar{t} = \frac{tv_p}{d_c}, \quad \bar{v} = \frac{v}{v_p}, \quad \bar{\theta} = \frac{\theta v_p}{d_c}$$

$$\eta \frac{d\bar{v}}{d\bar{t}} = -kv_p(\bar{v} - 1) - \frac{v_p a \sigma}{d_c} \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{v_p b \sigma}{d_c} \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (3)$$

On multiplie par  $\frac{v_p b \sigma}{d_c}$  :

$$\frac{\eta v_p}{b \sigma} \frac{d_c}{v_p^2} \frac{d\bar{v}}{d\bar{t}} = -\kappa(\bar{v} - 1) - \alpha \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (4)$$

$$\begin{cases} \kappa = \frac{k d_c}{b \sigma} \\ \alpha = \frac{a}{b} \end{cases}$$

Comme  $\frac{d\bar{v}}{d\bar{t}} = \frac{d_c}{v_p^2}$  et en posant  $\bar{\eta} = \frac{\eta v_p}{b \sigma}$ , on obtient:

$$\bar{\eta} \frac{d\bar{v}}{d\bar{t}} = -\kappa(\bar{v} - 1) - \alpha \frac{1}{\bar{v}} \frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}} \frac{d\bar{\theta}}{d\bar{t}} \quad (5)$$

Or  $\frac{d\bar{\theta}}{d\bar{t}} = \frac{d\theta}{dt} = 1 - \frac{v\theta}{d_c} = 1 - \bar{v}\bar{\theta}$  on a finalement :

$$\frac{d\bar{v}}{d\bar{t}} (\bar{\eta} + \frac{\alpha}{\bar{v}}) = -\kappa(\bar{v} - 1) + \bar{v} - \frac{1}{\bar{\theta}} \quad (6)$$

On en déduit la formule utilisée dans le programme Python :

$$\boxed{\frac{d\bar{v}}{\bar{v}} = \frac{-\kappa(\bar{v} - 1) + \bar{v} - \frac{1}{\bar{\theta}}}{\bar{\eta} + \frac{\alpha}{\bar{v}}} d\bar{t}} \quad (7)$$

Par ailleurs, on obtient immédiatement la seconde équation nécessaire :

$$\boxed{\frac{d\bar{\theta}}{\bar{\theta}} = (\frac{1}{\bar{\theta}} - \bar{v}) d\bar{t}} \quad (8)$$