Adimensionnement

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1 Introduction

$$\eta = \frac{\mu}{2\beta}, \qquad \beta = \sqrt{\frac{\mu}{\rho}}$$

$$\eta v = -k\left(l^* + \delta - v_p t\right) - f_0\sigma - a\sigma \ln\left(\frac{v(t)}{v_p}\right) - b\sigma \ln\left(\frac{v_p\,\theta(t)}{d_c}\right) \tag{1}$$

On dérive par rapport au temps :

$$\eta\frac{dv}{dt} = -k(v - v_p) - a\sigma\frac{1}{v}\frac{dv}{dt} - b\sigma\frac{1}{\theta}\frac{d\theta}{dt} \tag{2}$$

On pose les variables adimensionnées suivantes et on obtient:

$$\bar{t} = \frac{tv_p}{d_c}, \qquad \bar{v} = \frac{v}{v_p}, \qquad \bar{\theta} = \frac{\theta v_p}{d_c}$$

$$\eta\frac{dv}{dt} = -kv_p(\bar{v} - 1) - \frac{v_p a\sigma}{d_c}\frac{1}{\bar{v}}\frac{d\bar{v}}{d\bar{t}} - \frac{v_p b\sigma}{d_c}\frac{1}{\bar{\theta}}\frac{d\bar{\theta}}{d\bar{t}} \tag{3}$$

On multiplie par $\frac{d_c}{v_p b\sigma}$:

$$\frac{\eta v_p}{b\sigma}\frac{d_c}{{v_p}^2}\frac{dv}{dt} = -\kappa(\bar{v} - 1) - \alpha\frac{1}{\bar{v}}\frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}}\frac{d\bar{\theta}}{d\bar{t}} \tag{4}$$

$$\begin{cases} \kappa = \frac{kd_c}{b\sigma} \\ \alpha = \frac{a}{b} \end{cases}$$

Comme $\frac{d\bar{v}}{d\bar{t}} = \frac{d_c}{{v_p}^2}\frac{dv}{dt}$ et en posant $\bar{\eta} = \frac{\eta v_p}{b\sigma}$, on obtient:

$$\bar{\eta}\frac{d\bar{v}}{d\bar{t}} = -\kappa(\bar{v} - 1) - \alpha\frac{1}{\bar{v}}\frac{d\bar{v}}{d\bar{t}} - \frac{1}{\bar{\theta}}\frac{d\bar{\theta}}{d\bar{t}} \tag{5}$$

Or $\frac{d\bar{\theta}}{d\bar{t}} = \frac{d\theta}{dt} = 1 - \frac{v\theta}{d_c} = 1 - \bar{v}\bar{\theta}$ on a finalement :

$$\frac{d\bar{v}}{d\bar{t}}(\bar{\eta} + \frac{\alpha}{\bar{v}}) = -\kappa(\bar{v} - 1) + \bar{v} - \frac{1}{\bar{\theta}} \tag{6}$$

On en déduit la formule utilisée dans le programme Python :

$$\boxed{\frac{-\kappa(\bar{v}-1)+\bar{v}-\frac{1}{\theta}}{\alpha+\bar{\eta}\bar{v}}d\bar{t}=\frac{d\bar{v}}{\bar{v}}} \tag{7}$$