CSE 571 – Fall 2015

Homework 2

1) Assume the following KB in DL.

$\exists$ teaches.Course $\sqsubseteq$ ((Student $\sqcap$ $\exists$degree.BS) $\sqcup$ Prof)
Prof $\sqsubseteq$ $\exists$ degree.MS
$\exists$ degree.MS $\sqsubseteq$ $\exists$ degree.BS
MS $\sqcap$ BS $\sqsubseteq$ $\bot$
Teaches(john,ai)
≤ 1 degree(john)
Course(ai)

a) Convert the KB into first-order logic formulas.

σ ={ Teaches/2, Degree/2, Prof/1, Student/1, Course/1, john, MS, BS, course, ai}
$\forall$x[Teaches(x, course) $\Rightarrow$ (Student(x) $\wedge$ Degree(x, BS)) $\vee$ Prof(x)]
$\forall$x[Prof(x) $\Rightarrow$ Degree(x, MS)]
$\forall$x[Degree(x,MS) $\Rightarrow$ Degree(x,BS)]
MS $\wedge$ BS $\Rightarrow$ $\bot$
Teaches(john, ai)
$\exists$x$\forall$y[Degree(john, x) $\wedge$ Degree(john, y) $\Rightarrow$ y=x]
Course(ai)

b) Does the KB entail Prof(john)? Does the KB entail Student(john)? Explain why.

KB does not entail Prof(john) but it entails Student(john).
ai is a course and john teaches ai.
If someone teaches a course, he/she is either a Student and holds BS degree or a Prof.
A Prof holds a MS degree.
If John is a Prof, he must have MS and a BS (as BS is prerequisite for MS)
It is given that BS and MS are disjoint. So they are 2 different courses.
It is also given that john has at most 1 degree.
So it is not possible for john to hold a MS, so he cannot be a Prof.
Hence john is a Student.
KB entails Student(john)

2) Find all answer sets of each of the following formulas. Justify your answer.

a) (p $\wedge$ q) $\vee$ (q $\wedge$ r) $\vee$ (p $\wedge$ r)

$F^x = [(p \wedge q) \vee (q \wedge r) \vee (p \wedge r)]^x$
$= (p \wedge q)^x \vee (q \wedge r)^x \vee (p \wedge r)^x$
$= (p^x \wedge q^x) \vee (q^x \wedge r^x) \vee (p^x \wedge r^x)$
$= (p \wedge q) \vee (q \wedge r) \vee (p \wedge r)$
X = {Φ}

Φ is not a model as atleast {p,q} is required to satisfy the reduct. Hence not an answer set.
Similarly, X = {p}, X = {q}, X = {r} are not models as atleast {p, q} is required to satisfy the reduct. Hence these are not answer sets.
X = {p, q}

{p, q} satisfies the reduct and {p,q} is also a minimal model for the reduct. Hence it is an answer set.

Similarly for X = {q,r} and X = {p,r}; **{q,r} and {p,r}** are also minimal models for the reduct and they are answer sets as well.
X = {p, q, r} satisfies the reduct but it is not a minimal model for the reduct. Hence it is not an answer set.

b) $(p \wedge \neg\neg q) \vee (q \wedge \neg\neg r) \vee (p \wedge \neg\neg r)$

$F^x = [(p \wedge \neg\neg q) \vee (q \wedge \neg\neg r) \vee (p \wedge \neg\neg r)]^x$
$= (p \wedge \neg\neg q)^x \vee (q \wedge \neg\neg r)^x \vee (p \wedge \neg\neg r)^x$
$= (p^x \wedge \neg\neg q^x) \vee (q^x \wedge \neg\neg r^x) \vee (p^x \wedge \neg\neg r^x)$
$= (p \wedge V_x(\neg\neg q)) \vee (q \wedge V_x(\neg\neg r)) \vee (p \wedge V_x(\neg\neg r))$
X = {Φ}
$F^x = (p \wedge \bot) \vee (q \wedge \bot) \vee (p \wedge \bot)$
X doesn't satisfy the reduct. Hence it is not an answer set.
X = {p}
$F^x = (p \wedge \bot) \vee (q \wedge \bot) \vee (p \wedge \bot)$
X doesn't satisfy the reduct. Hence it is not an answer set.
X = {q}
$F^x = (p \wedge \top) \vee (q \wedge \bot) \vee (p \wedge \bot)$
{p} satisfies the reduct. But it is not a minimal model. Hence not an answer set.
X = {r}
$F^x = (p \wedge \bot) \vee (q \wedge \top) \vee (p \wedge \top)$
{p} is enough to satisfy the reduct. But it is not a minimal model. Hence it is not an answer set.

X = {p, q}
$F^x = (p \wedge \top) \vee (q \wedge \bot) \vee (p \wedge \bot)$
{p} is enough to satisfy the reduct. But it is not a minimal model. Hence it is not an answer set.
X = {q, r}
$F^x = (p \wedge \top) \vee (q \wedge \top) \vee (p \wedge \top)$
{p} is enough to satisfy the reduct. But it is not a minimal model. Hence it is not an answer set.
X = {p, r}
$F^x = (p \wedge \bot) \vee (q \wedge \top) \vee (p \wedge \top)$
{p} is enough to satisfy the reduct. But it is not a minimal model. Hence it is not an answer set.
X = {p, q, r}
$F^x = (p \wedge \top) \vee (q \wedge \top) \vee (p \wedge \top)$
{p} is enough to satisfy the reduct. But it is not a minimal model. Hence it is not an answer set.

So this formula has no answer sets.

3) Let Π_n be the n + 1 rule program
p_i <- $\neg p_{i+1}$ (1<=i<=n)
p_{n+1} <- $\neg p_1$
Find all the answer sets of Π_n. Justify your answer.

When n=1,

p_1 <- $\neg p_2$
p_2 <- $\neg p_1$

Answer sets are: 1)p_1 & 2)p_2.

When n=2,

p_1 <- ¬p_2
p_2 <- ¬p_3
p_3 <- ¬p_1

This program has no answer set and hence it is unsatisfiable.

When n=3,

p_1 <- ¬p_2
p_2 <- ¬p_3
p_3 <- ¬p_4
p_4 <- ¬p_1.

Answer sets are: 1) p_1, p_2 and 2) p_2, p_4

This pattern is observed as we increase n.

In general, when n is even, the program is unsatisfiable.

When n is odd, there are 2 answer sets with 1st possessing p_{odd} upto p_n and 2nd possessing p_{even} upto p_{n+1}.

4) How does the auxiliary concept of a "reduct" make a difference? Why not simply define an answer set of a normal program Π as a minimal model of Π? Find a program Π such that the answer sets of Π do not coincide with the minimal models of Π.

For positive programs, the answer sets are a unique minimal set of atoms that satisfy the program. As normal programs have rules with negated atoms, the auxiliary concept of reduct (Π^x) is used to eliminate the negated atoms and convert the normal program to a pseudo positive program. Now, if there is a unique minimal set of atoms that is an answer set of reduct relative to x, we say that the unique minimal set of atoms is an answer set to the normal program. The reduct does not contain negated atoms and its answer sets are already defined. This is the purpose of reduct in normal programs.

Program:

p <- ¬q
q<- ¬r

The answer set for the above program is {q} whereas it has minimal models like {p,q} when X={Φ}, X={p} and so on. Here, it can be seen that the answer set doesn't coincide with the minimal models.

5) Use clingo to verify that Π_n has a unique answer set for n = 10 and n = 100.

Program:

```
index(1..n).
#domain index(I).
p(I) :- not p(I+1).
```

Output:

n = 10

C:\WINDOWS\system32\cmd.exe

```
D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=10 question5
Answer: 1
index(1) index(2) index(3) index(4) index(5) index(6) index(7) index(8) index(9) index(10) p
(10) p(8) p(6) p(4) p(2)
SATISFIABLE

Models      : 1
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000

D:\SEM1\AI\clingo-3.0.5-win64>
```

n = 100

C:\WINDOWS\system32\cmd.exe

```
D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=100 question5
Answer: 1
index(1) index(2) index(3) index(4) index(5) index(6) index(7) index(8) index(9) index(10) inde
x(11) index(12) index(13) index(14) index(15) index(16) index(17) index(18) index(19) index(20)
 index(21) index(22) index(23) index(24) index(25) index(26) index(27) index(28) index(29) inde
x(30) index(31) index(32) index(33) index(34) index(35) index(36) index(37) index(38) index(39)
 index(40) index(41) index(42) index(43) index(44) index(45) index(46) index(47) index(48) inde
x(49) index(50) index(51) index(52) index(53) index(54) index(55) index(56) index(57) index(58)
 index(59) index(60) index(61) index(62) index(63) index(64) index(65) index(66) index(67) inde
x(68) index(69) index(70) index(71) index(72) index(73) index(74) index(75) index(76) index(77)
 index(78) index(79) index(80) index(81) index(82) index(83) index(84) index(85) index(86) inde
x(87) index(88) index(89) index(90) index(91) index(92) index(93) index(94) index(95) index(96)
 index(97) index(98) index(99) index(100) p(100) p(98) p(96) p(94) p(92) p(90) p(88) p(86) p(84
) p(82) p(80) p(78) p(76) p(74) p(72) p(70) p(68) p(66) p(64) p(62) p(60) p(58) p(56) p(54) p(5
2) p(50) p(48) p(46) p(44) p(42) p(40) p(38) p(36) p(34) p(32) p(30) p(28) p(26) p(24) p(22) p(
20) p(18) p(16) p(14) p(12) p(10) p(8) p(6) p(4) p(2)
SATISFIABLE

Models      : 1
Time        : 0.094
  Prepare   : 0.016
  Prepro.   : 0.000
  Solving   : 0.078
```

6) Do Exercise 7. Skip the discussion of how many answer sets this program has, since Question # 3 is about it.

Consider the program obtained from Π_n by adding the rule p(n + 1) ← ¬p(1). How many answer sets does this program have, in your opinion? Check your conjecture for n = 7 and n = 8 using CLINGO.

It has 2 answer sets when n is odd and 0 answer sets when n is even. When n is odd, one of the answer set contains $\{p_{odd}\}$ elements upto n and the other answer set contains elements $\{p_{even}\}$ elements upto n+1.

Program:

index(1..n).
#domain index(I).
p(I) :- not p(I+1).
p(n+1) :- not p(1).

Output:

n=7

```
C:\WINDOWS\system32\cmd.exe

D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=7 question6 0
Answer: 1
index(1) index(2) index(3) index(4) index(5) index(6) index(7) p(7) p(5) p(3) p(1)
Answer: 2
index(1) index(2) index(3) index(4) index(5) index(6) index(7) p(8) p(6) p(4) p(2)
SATISFIABLE

Models      : 2
Time        : 0.016
  Prepare   : 0.000
  Prepro.   : 0.016
  Solving   : 0.000

D:\SEM1\AI\clingo-3.0.5-win64>
```

n=8

```
D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=8 question6 0
UNSATISFIABLE

Models      : 0
Time        : 0.000
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.000

D:\SEM1\AI\clingo-3.0.5-win64>
```

7) Do Exercise 9.

Consider the program

1 {p_{i1}, . . . , p_{in}} ch 1 (1 ≤ i ≤ n), where n is a positive integer.

How many answer sets does this program have, in your opinion? Check your conjecture for n = 3 using CLINGO.

This program has n^3 answer sets. When executed in CLINGO with n=3, it returned 27 answer sets.

Program:

index(1..n).
#domain index(I).
1{p(1,J): index(J)}1.
1{p(2,J): index(J)}1.
1{p(3,J): index(J)}1.

Output:

C:\WINDOWS\system32\cmd.exe

```
index(1) index(2) index(3) p(1,1) p(2,1) p(3,1)
Answer: 2
index(1) index(2) index(3) p(1,1) p(2,1) p(3,2)
Answer: 3
index(1) index(2) index(3) p(1,1) p(2,1) p(3,3)
Answer: 4
index(1) index(2) index(3) p(1,1) p(2,2) p(3,1)
Answer: 5
index(1) index(2) index(3) p(1,1) p(2,2) p(3,2)
Answer: 6
index(1) index(2) index(3) p(1,1) p(2,2) p(3,3)
Answer: 7
index(1) index(2) index(3) p(1,1) p(2,3) p(3,1)
Answer: 8
index(1) index(2) index(3) p(1,1) p(2,3) p(3,2)
Answer: 9
index(1) index(2) index(3) p(1,1) p(2,3) p(3,3)
Answer: 10
index(1) index(2) index(3) p(1,2) p(2,1) p(3,1)
Answer: 11
index(1) index(2) index(3) p(1,2) p(2,1) p(3,2)
Answer: 12
index(1) index(2) index(3) p(1,2) p(2,1) p(3,3)
Answer: 13
index(1) index(2) index(3) p(1,2) p(2,2) p(3,1)
Answer: 14
index(1) index(2) index(3) p(1,2) p(2,2) p(3,2)
Answer: 15
index(1) index(2) index(3) p(1,2) p(2,2) p(3,3)
Answer: 16
index(1) index(2) index(3) p(1,2) p(2,3) p(3,1)
Answer: 17
index(1) index(2) index(3) p(1,2) p(2,3) p(3,2)
Answer: 18
index(1) index(2) index(3) p(1,2) p(2,3) p(3,3)
Answer: 19
index(1) index(2) index(3) p(1,3) p(2,1) p(3,1)
Answer: 20
index(1) index(2) index(3) p(1,3) p(2,1) p(3,2)
Answer: 21
index(1) index(2) index(3) p(1,3) p(2,1) p(3,3)
Answer: 22
index(1) index(2) index(3) p(1,3) p(2,2) p(3,1)
Answer: 23
index(1) index(2) index(3) p(1,3) p(2,2) p(3,2)
Answer: 24
index(1) index(2) index(3) p(1,3) p(2,2) p(3,3)
Answer: 25
index(1) index(2) index(3) p(1,3) p(2,3) p(3,1)
Answer: 26
index(1) index(2) index(3) p(1,3) p(2,3) p(3,2)
Answer: 27
index(1) index(2) index(3) p(1,3) p(2,3) p(3,3)
SATISFIABLE

Models       : 27
Time         : 0.016
  Prepare    : 0.000
```

8) Do Exercise 10.

Use clingo to find all solutions to the 8 queens problem that:

a) have a queen at (1,1):

Program:

```
number(1..8).
#domain number(I).
#domain number(I1).
#domain number(J).
#domain number(J1).
1{q(K,J): number(K)}1.
:- q(I,J), q(I,J1), J<J1.
:- q(I,J), q(I1,J1), J<J1, #abs(I1-I)==J1-J.
q(1,1).
```

Output:

C:\WINDOWS\system32\cmd.exe

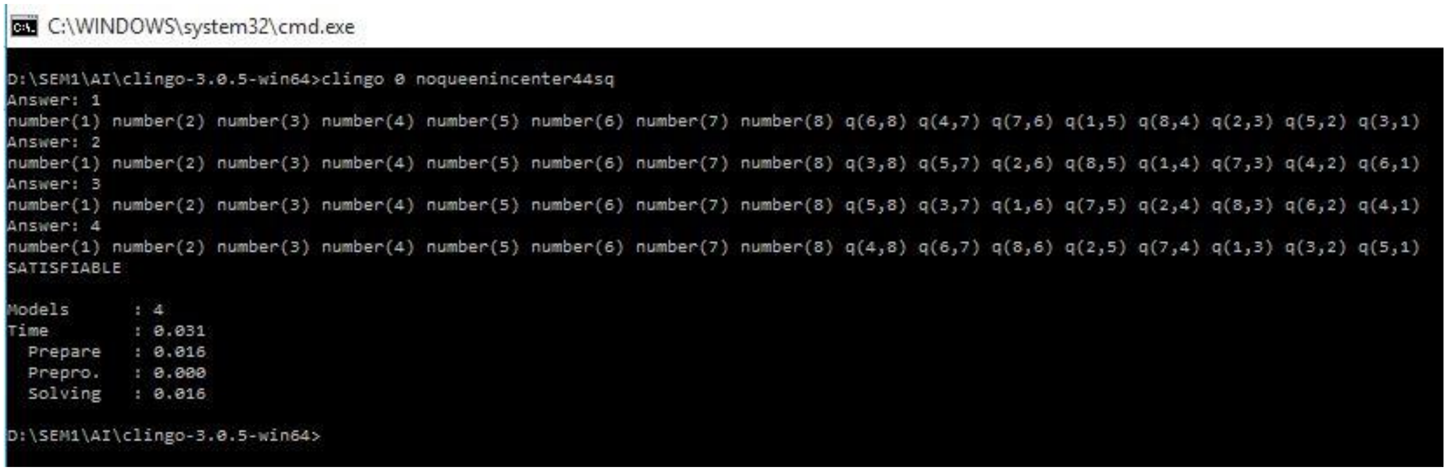
```
D:\SEM1\AI\clingo-3.0.5-win64>clingo 0 queenin11
Answer: 1
q(1,1) number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(5,8) q(2,7) q(4,6) q(7,5) q(3,4) q(8,3) q(6,2)
Answer: 2
q(1,1) number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(4,8) q(2,7) q(7,6) q(3,5) q(6,4) q(8,3) q(5,2)
Answer: 3
q(1,1) number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(3,8) q(5,7) q(2,6) q(8,5) q(6,4) q(4,3) q(7,2)
Answer: 4
q(1,1) number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(3,8) q(6,7) q(4,6) q(2,5) q(8,4) q(5,3) q(7,2)
SATISFIABLE

Models      : 4
Time        : 0.016
  Prepare   : 0.000
  Prepro.   : 0.000
  Solving   : 0.016

D:\SEM1\AI\clingo-3.0.5-win64>
```

b) have no queens in the 4 × 4 square in the middle of the board:

Program:

```
number(1..8).
#domain number(I).
#domain number(I1).
#domain number(J).
#domain number(J1).
1{q(K,J): number(K)}1.
:- q(I,J), q(I,J1), J<J1.
:- q(I,J), q(I1,J1), J<J1, #abs(I1-I)==J1-J.
:- q(I,J), I=3..6, J=3..6.
```

Output:



```
C:\WINDOWS\system32\cmd.exe

D:\SEM1\AI\clingo-3.0.5-win64>clingo 0 noqueenincenter44sq
Answer: 1
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(6,8) q(4,7) q(7,6) q(1,5) q(8,4) q(2,3) q(5,2) q(3,1)
Answer: 2
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(3,8) q(5,7) q(2,6) q(8,5) q(1,4) q(7,3) q(4,2) q(6,1)
Answer: 3
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(5,8) q(3,7) q(1,6) q(7,5) q(2,4) q(8,3) q(6,2) q(4,1)
Answer: 4
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) q(4,8) q(6,7) q(8,6) q(2,5) q(7,4) q(1,3) q(3,2) q(5,1)
SATISFIABLE

Models      : 4
Time        : 0.031
  Prepare   : 0.016
  Prepro.   : 0.000
  Solving   : 0.016

D:\SEM1\AI\clingo-3.0.5-win64>
```

9) Do Exercise 11.

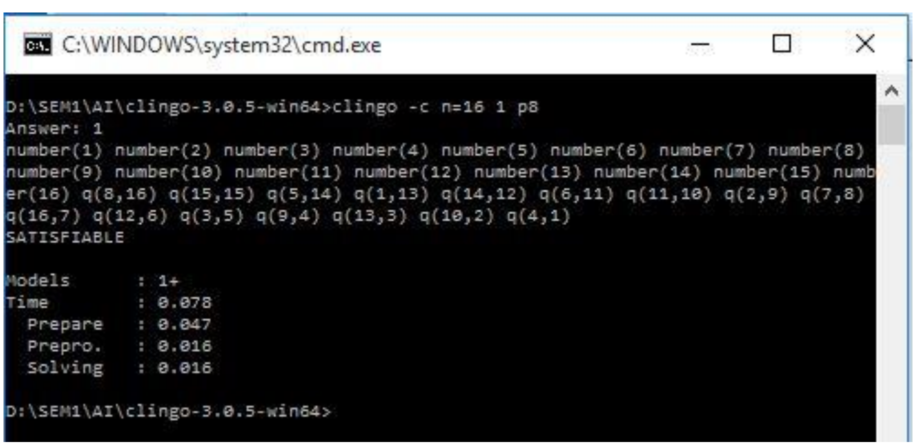
a) Check how long it takes for CLINGO to find one solution to the 16 queens problem using the program above.

It takes about **0.078 seconds** to find one solution to 16 queens problem.

Program:

number(1..n).
#domain number(I).
#domain number(I1).
#domain number(J).
#domain number(J1).
1{q(K,J): number(K)}1.
:- q(I,J), q(I,J1), J<J1.
:- q(I,J), q(I1,J1), J<J1, #abs(I1-I)==J1-J.

Output:



```
C:\WINDOWS\system32\cmd.exe

D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=16 1 p8
Answer: 1
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8)
number(9) number(10) number(11) number(12) number(13) number(14) number(15) numb
er(16) q(8,16) q(15,15) q(5,14) q(1,13) q(14,12) q(6,11) q(11,10) q(2,9) q(7,8)
q(16,7) q(12,6) q(3,5) q(9,4) q(13,3) q(10,2) q(4,1)
SATISFIABLE

Models      : 1+
Time        : 0.078
  Prepare   : 0.047
  Prepro.   : 0.016
  Solving   : 0.016

D:\SEM1\AI\clingo-3.0.5-win64>
```

b) The constraint corresponding to Condition 3 can be alternatively written using a cardinality expression:

<-2{q(i,1),…,q(I,n)} (1<=i<=n).

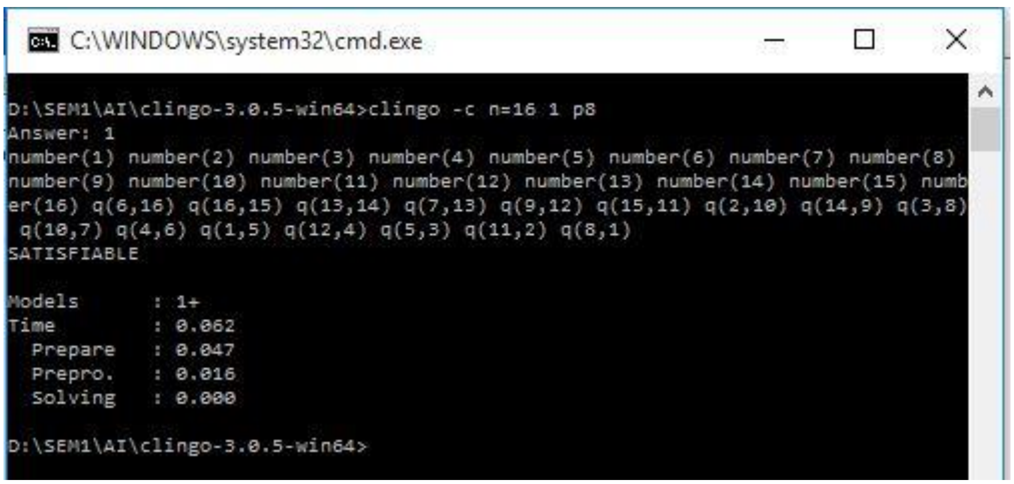
Modify the program accordingly and check how this modification affects the computation time of finding the 1st solution.

After re-writing the program using a cardinality expression, the computation time becomes **0.062 seconds**. (0.016 seconds lesser than the previous approach)

Program:

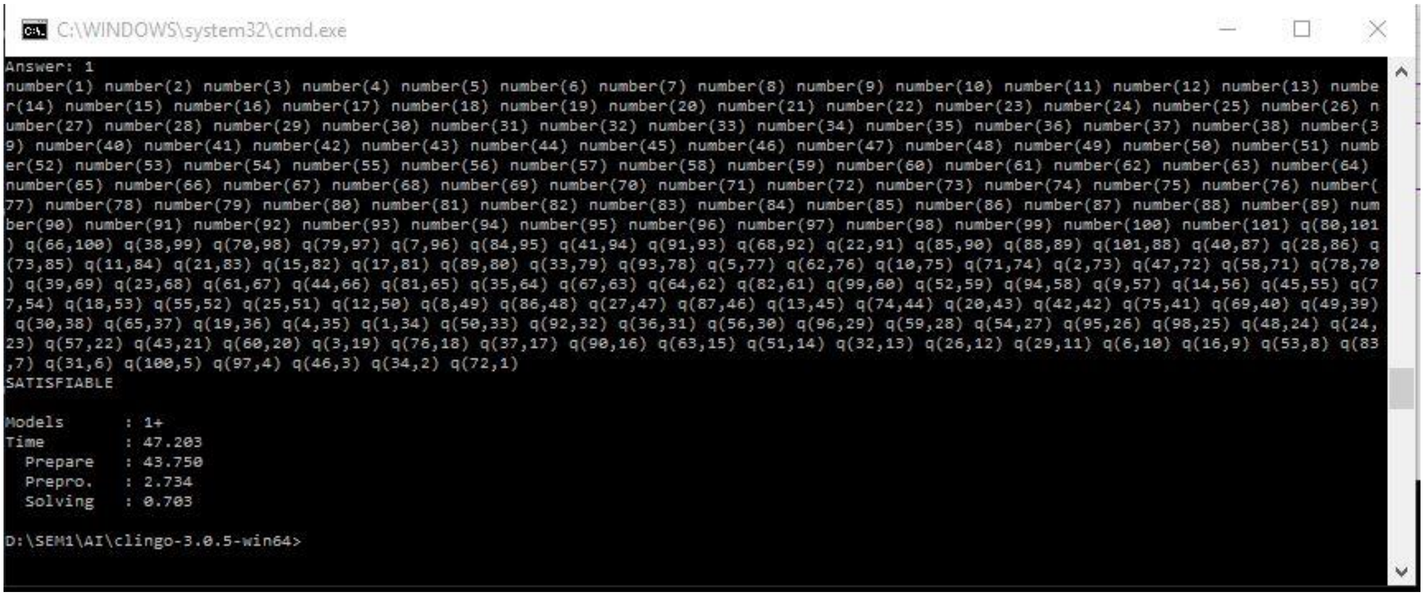
```
number(1..n).
#domain number(I).
#domain number(I1).
#domain number(J).
#domain number(J1).
1{q(K,J): number(K)}1.
:-2{q(I,K) : number(K)}.
:- q(I,J), q(I1,J1), J<J1, #abs(I1-I)==J1-J.
```

Output:



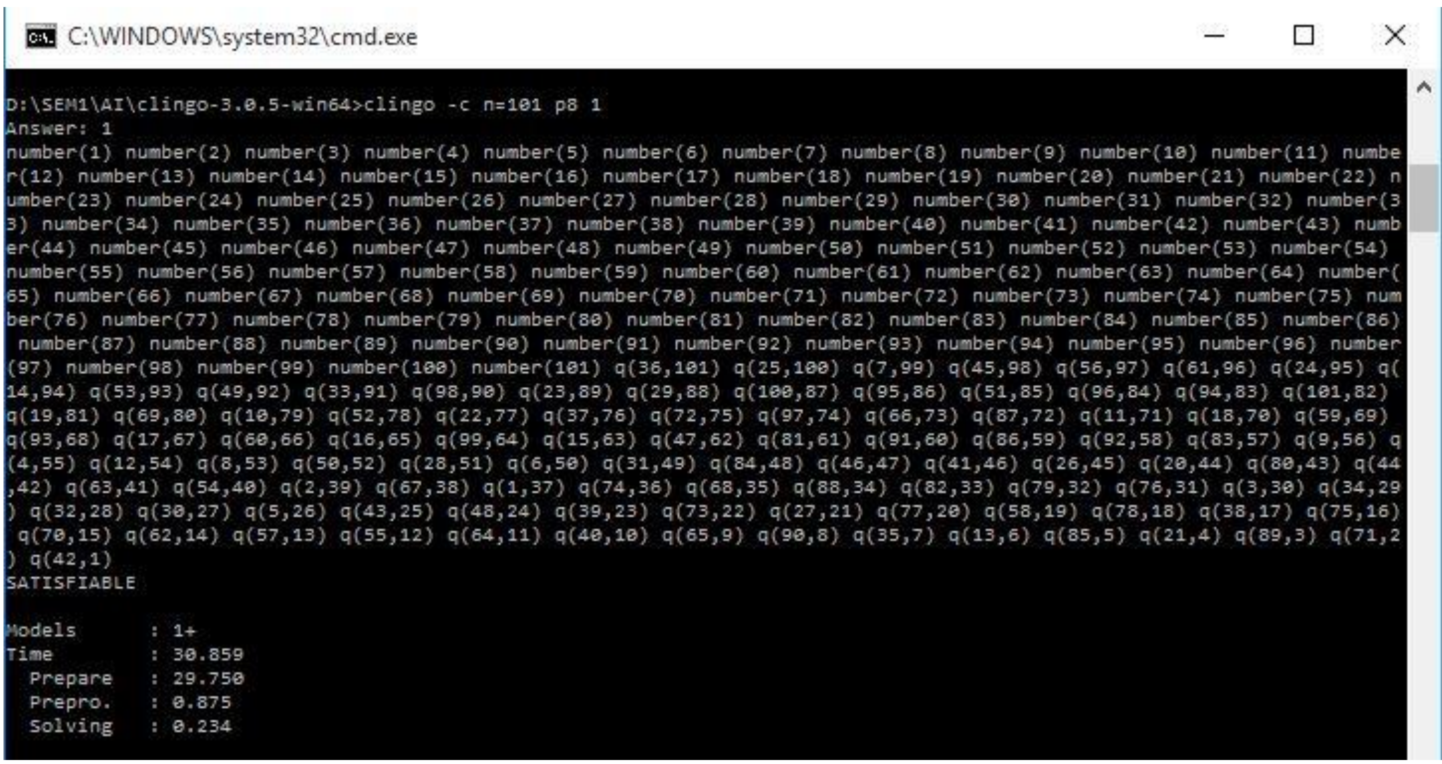
Comparison for 101 queens:

Before optimization: **47.203 seconds**



```
C:\WINDOWS\system32\cmd.exe
Answer: 1
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) number(9) number(10) number(11) number(12) number(13) numbe
r(14) number(15) number(16) number(17) number(18) number(19) number(20) number(21) number(22) number(23) number(24) number(25) number(26) n
umber(27) number(28) number(29) number(30) number(31) number(32) number(33) number(34) number(35) number(36) number(37) number(38) number(3
9) number(40) number(41) number(42) number(43) number(44) number(45) number(46) number(47) number(48) number(49) number(50) number(51) numb
er(52) number(53) number(54) number(55) number(56) number(57) number(58) number(59) number(60) number(61) number(62) number(63) number(64)
number(65) number(66) number(67) number(68) number(69) number(70) number(71) number(72) number(73) number(74) number(75) number(76) number(
77) number(78) number(79) number(80) number(81) number(82) number(83) number(84) number(85) number(86) number(87) number(88) number(89) num
ber(90) number(91) number(92) number(93) number(94) number(95) number(96) number(97) number(98) number(99) number(100) number(101) q(80,101
) q(66,100) q(38,99) q(70,98) q(79,97) q(7,96) q(84,95) q(41,94) q(91,93) q(68,92) q(22,91) q(85,90) q(88,89) q(101,88) q(40,87) q(28,86) q
(73,85) q(11,84) q(21,83) q(15,82) q(17,81) q(89,80) q(33,79) q(93,78) q(5,77) q(62,76) q(10,75) q(71,74) q(2,73) q(47,72) q(58,71) q(78,70
) q(39,69) q(23,68) q(61,67) q(44,66) q(81,65) q(35,64) q(67,63) q(64,62) q(82,61) q(99,60) q(52,59) q(94,58) q(9,57) q(14,56) q(45,55) q(7
7,54) q(18,53) q(55,52) q(25,51) q(12,50) q(8,49) q(86,48) q(27,47) q(87,46) q(13,45) q(74,44) q(20,43) q(42,42) q(75,41) q(69,40) q(49,39)
 q(30,38) q(65,37) q(19,36) q(4,35) q(1,34) q(50,33) q(92,32) q(36,31) q(56,30) q(96,29) q(59,28) q(54,27) q(95,26) q(98,25) q(48,24) q(24,
23) q(57,22) q(43,21) q(60,20) q(3,19) q(76,18) q(37,17) q(90,16) q(63,15) q(51,14) q(32,13) q(26,12) q(29,11) q(6,10) q(16,9) q(53,8) q(83
,7) q(31,6) q(100,5) q(97,4) q(46,3) q(34,2) q(72,1)
SATISFIABLE

Models      : 1+
Time        : 47.203
  Prepare   : 43.750
  Prepro.   : 2.734
  Solving   : 0.703

D:\SEM1\AI\clingo-3.0.5-win64>
```

After optimization: **30.859 seconds**



```
C:\WINDOWS\system32\cmd.exe
D:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=101 p8 1
Answer: 1
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) number(9) number(10) number(11) numbe
r(12) number(13) number(14) number(15) number(16) number(17) number(18) number(19) number(20) number(21) number(22) n
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3) number(34) number(35) number(36) number(37) number(38) number(39) number(40) number(41) number(42) number(43) numb
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14,94) q(53,93) q(49,92) q(33,91) q(98,90) q(23,89) q(29,88) q(100,87) q(95,86) q(51,85) q(96,84) q(94,83) q(101,82)
q(19,81) q(69,80) q(10,79) q(52,78) q(22,77) q(37,76) q(72,75) q(97,74) q(66,73) q(87,72) q(11,71) q(18,70) q(59,69)
q(93,68) q(17,67) q(60,66) q(16,65) q(99,64) q(15,63) q(47,62) q(81,61) q(91,60) q(86,59) q(92,58) q(83,57) q(9,56) q
(4,55) q(12,54) q(8,53) q(50,52) q(28,51) q(6,50) q(31,49) q(84,48) q(46,47) q(41,46) q(26,45) q(20,44) q(80,43) q(44
,42) q(63,41) q(54,40) q(2,39) q(67,38) q(1,37) q(74,36) q(68,35) q(88,34) q(82,33) q(79,32) q(76,31) q(3,30) q(34,29
) q(32,28) q(30,27) q(5,26) q(43,25) q(48,24) q(39,23) q(73,22) q(27,21) q(77,20) q(58,19) q(78,18) q(38,17) q(75,16)
 q(70,15) q(62,14) q(57,13) q(55,12) q(64,11) q(40,10) q(65,9) q(90,8) q(35,7) q(13,6) q(85,5) q(21,4) q(89,3) q(71,2
) q(42,1)
SATISFIABLE

Models      : 1+
Time        : 30.859
  Prepare   : 29.750
  Prepro.   : 0.875
  Solving   : 0.234
```

10) Do Exercise 13.

Program:

```
number(1..n).
#domain number(X).
#domain number(Y).
subset(1..k).
1{a(I,X) : subset(I)}1.
:- a(I,X), a(I,Y), a(I,X+Y), subset(I), X+Y<=n.
:- a(I,X), I>X. %(constraint to remove symmetry)
```

Investigate the effect of this optimization on the computation time of CLINGO for

(a) k = 4, n = 44. SATISFIABLE.

Before optimization: 6552 models and 320.547 seconds

```
Models       : 6552
Time         : 320.547
  Prepare    : 0.016
  Prepro.    : 0.016
  Solving    : 320.516

d:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=44 -c k=4 0 schur
```

After optimization: 426 models and 33.859 seconds

```
Models       : 426
Time         : 33.859
  Prepare    : 0.016
  Prepro.    : 0.000
  Solving    : 33.844

D:\SEM1\AI\clingo-3.0.5-win64>clingo -c k=4 -c n=44 0 schur
```

(b) k = 4, n = 45. UNSATISFIABLE

Before optimization: 14.891 seconds

```
d:\SEM1\AI\clingo-3.0.5-win64>clingo -c n=45 -c k=4 0 schur
UNSATISFIABLE

Models       : 0
Time         : 14.891
  Prepare    : 0.016
  Prepro.    : 0.016
  Solving    : 14.859
```

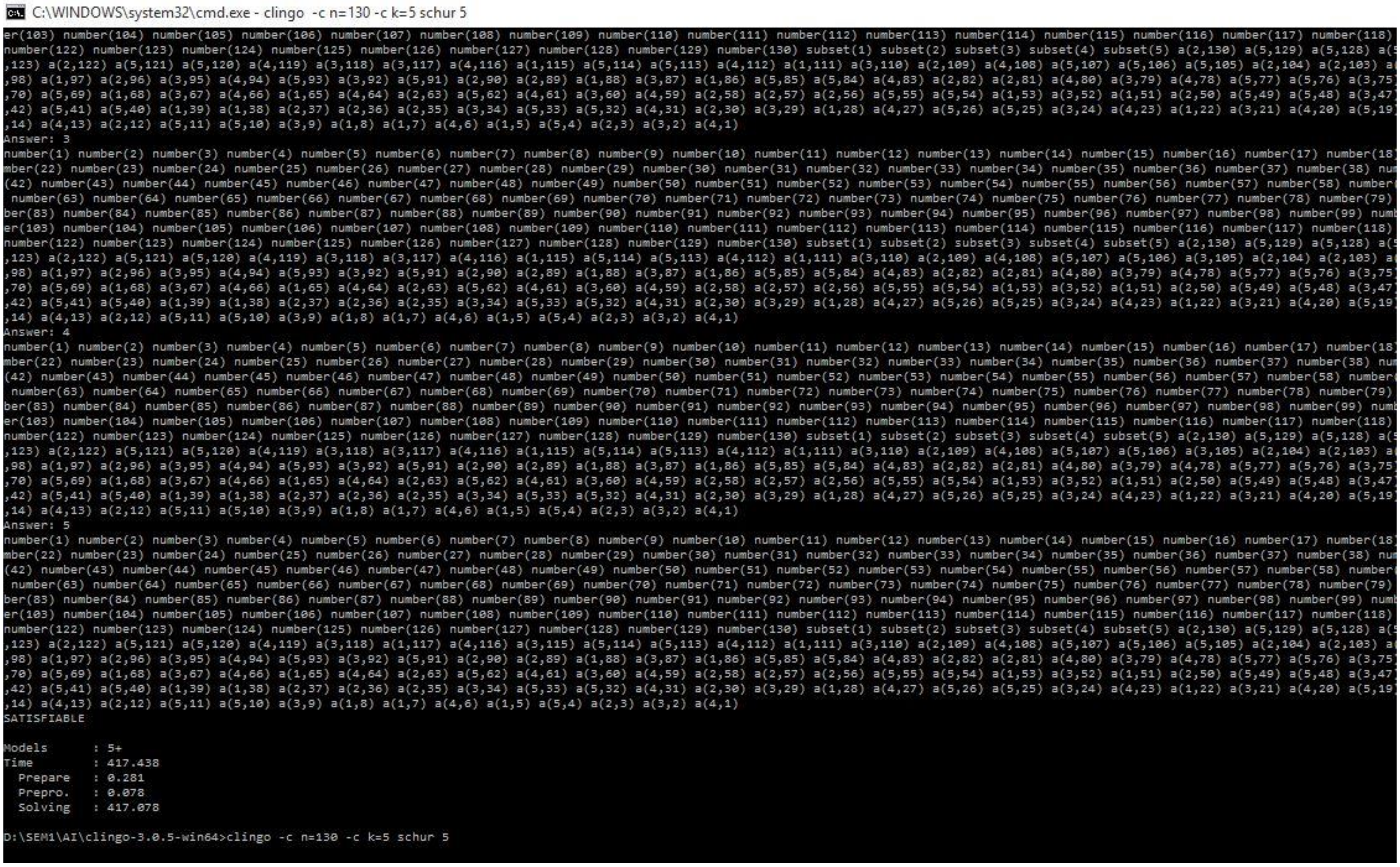
After optimization: 11.141 seconds

```
D:\SEM1\AI\clingo-3.0.5-win64>clingo -c k=4 -c n=45 0 schur
UNSATISFIABLE

Models       : 0
Time         : 11.141
  Prepare    : 0.031
  Prepro.    : 0.000
  Solving    : 11.109
```

(c) k = 5, n = 130.

Before optimization (for 5 models): 417.438 seconds



After optimization (for 5 models): 566.172 seconds

C:\WINDOWS\system32\cmd.exe

```
(42) number(43) number(44) number(45) number(46) number(47) number(48) number(49) number(50) number(51) number(52) number(53) number(54) number(55) number(56) number(57) number(58) number(59) number(60) number(61) n
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number(122) number(123) number(124) number(125) number(126) number(127) number(128) number(129) number(130) subset(1) subset(2) subset(3) subset(4) subset(5) a(5,130) a(2,129) a(4,128) a(3,127) a(4,126) a(3,125) a(4
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Answer: 3
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) number(9) number(10) number(11) number(12) number(13) number(14) number(15) number(16) number(17) number(18) number(19) number(20) numb
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,42) a(5,41) a(4,40) a(2,39) a(4,38) a(1,37) a(4,36) a(5,35) a(4,34) a(5,33) a(2,32) a(5,31) a(1,30) a(3,29) a(1,28) a(3,27) a(5,26) a(2,25) a(5,24) a(4,23) a(5,22) a(4,21) a(5,20) a(2,19) a(5,18) a(1,17) a(5,16) a
,14) a(3,13) a(2,12) a(2,11) a(1,10) a(4,9) a(3,8) a(4,7) a(3,6) a(4,5) a(1,4) a(2,3) a(2,2) a(1,1)
Answer: 4
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) number(9) number(10) number(11) number(12) number(13) number(14) number(15) number(16) number(17) number(18) number(19) number(20) numb
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Answer: 5
number(1) number(2) number(3) number(4) number(5) number(6) number(7) number(8) number(9) number(10) number(11) number(12) number(13) number(14) number(15) number(16) number(17) number(18) number(19) number(20) numb
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,14) a(3,13) a(2,12) a(2,11) a(1,10) a(4,9) a(3,8) a(4,7) a(3,6) a(4,5) a(1,4) a(2,3) a(2,2) a(1,1)
SATISFIABLE

Models      : 5+
Time        : 566.172
  Prepare   : 0.297
  Prepro.   : 0.078
  Solving   : 565.797

D:\SEM1\AI\clingo-3.0.5-win64>
```