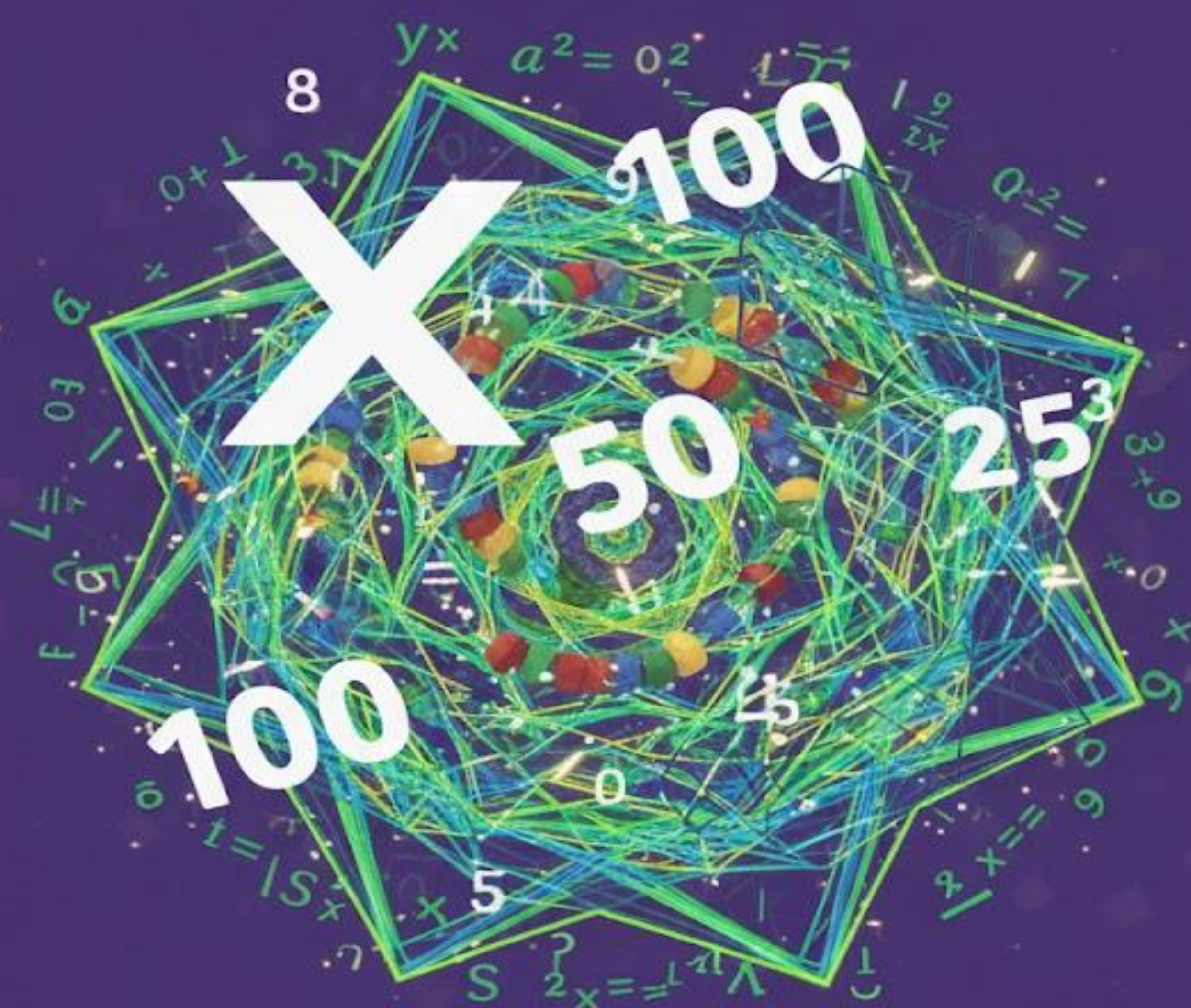
MATHEMATICS

FOR BASIC 6



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Strand 1: Number

Sub-strand 1: Counting, Representation, Cardinality & Ordinality

We will now work with numbers up to 1 billion, understanding their quantities and place values.

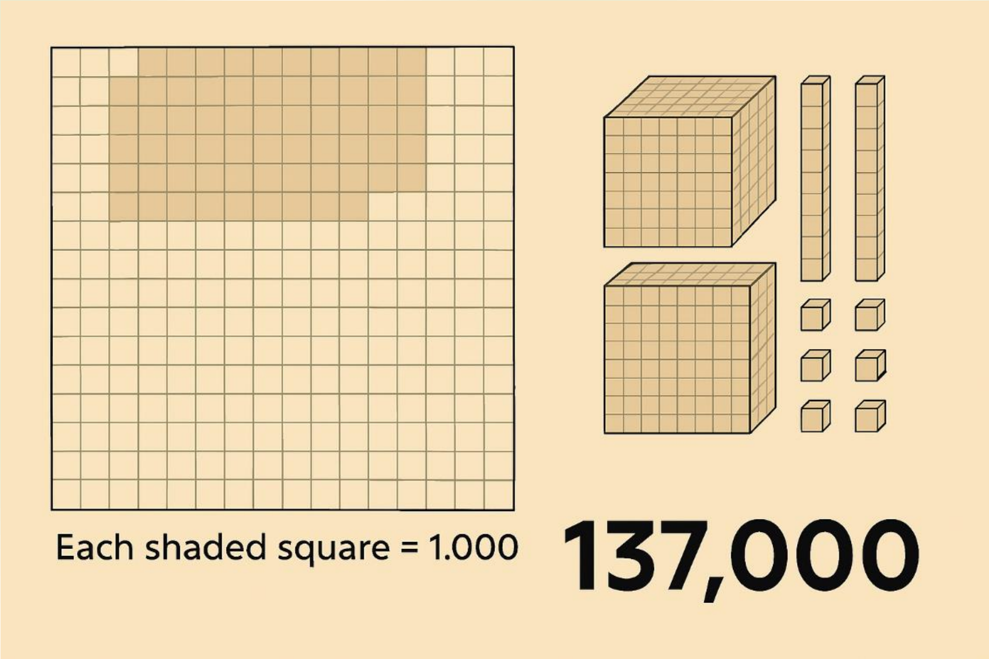
Topic 1: Modeling Large Number Quantities

We can use different materials to represent large numbers and understand their size.

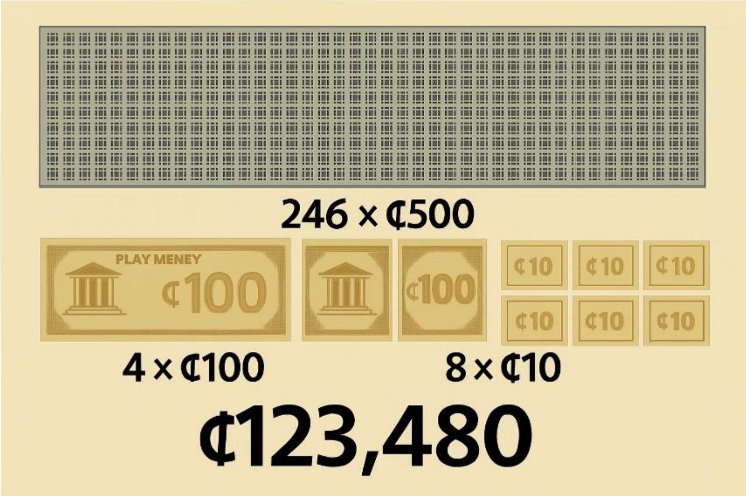
- **Multi-base blocks:** For numbers up to 1,000,000, we can think of:
  - A unit cube = 1
  - A rod = 10 units
  - A flat = 100 units
  - A small cube (often shown as a larger single cube in some sets) = 1,000 units
  - A rod of thousands = 10,000 units
  - A flat of thousands = 100,000 units
  - A large cube = 1,000,000 units

  To model 436,000, you would use 4 flats of thousands (400,000), 3 rods of thousands (30,000), and 6 small cubes (6,000).

- **Graph sheets:** We can represent numbers by shading squares. If one small square represents 1,000, then to model 137,000, you would shade 137 of these 'thousand' squares.



- **Token Currency Notes:** Using play money, like ¢10, ¢100, and ¢500 notes, we can model amounts. To model ¢123,480, you would need:
    - 246 × ¢500 notes (for ¢123,000)
    - 4 × ¢100 notes (for ¢400)
    - 8 × ¢10 notes (for ¢80)



Question 1.1: How would you model 250,000 using flats of thousands in multi-base blocks?

Question 1.2: If each shaded square on a graph sheet represents 10,000, how many squares would you need to shade to represent 320,000?

Topic 2: Reading and Writing Large Numbers

We use place value to read and write large numbers. The place values up to billions are:

Billions, Hundred Millions, Ten Millions, Millions, Hundred Thousands, Ten Thousands, Thousands, Hundreds, Tens, Ones.

Example 1: Reading Numbers

The number 500,000 is read as "five hundred thousand". The number 1,362,524,513 is read as "one billion, three hundred sixty-two million, five hundred twenty-four thousand, five hundred thirteen".

Example 2: Writing Numbers

"Seven hundred twenty-five million, forty thousand, six" is written as 725,040,006.

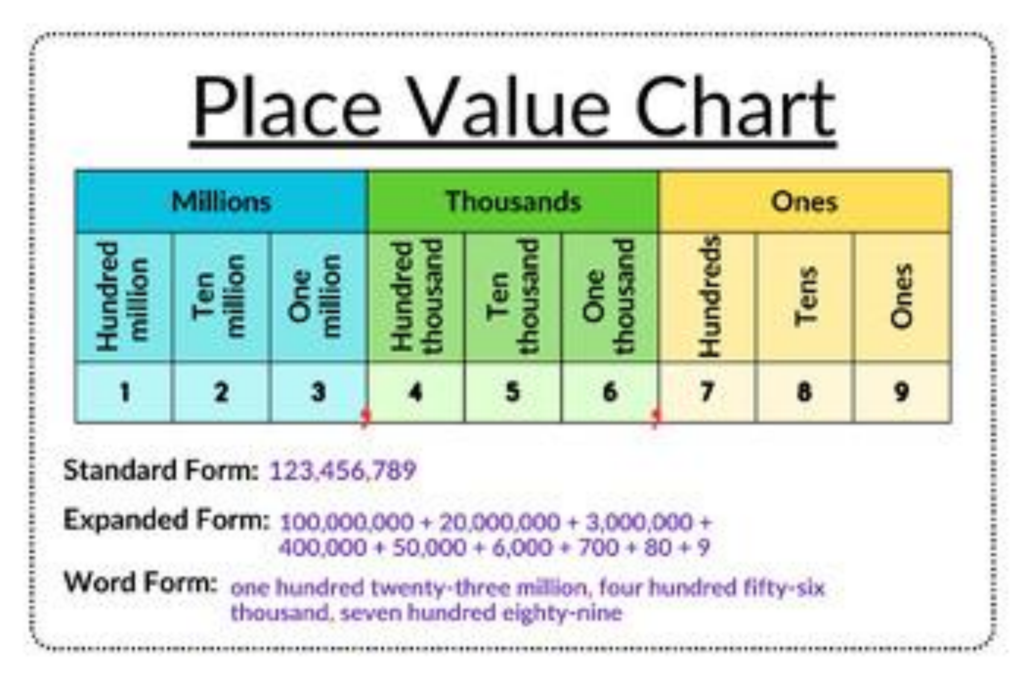
Expanded Form: We can also write numbers in expanded form to show the value of each digit.

1,362,524,513

$= (1 \times 1{,}000{,}000{,}000) + (3 \times 100{,}000{,}000) + (6 \times 10{,}000{,}000) + (2 \times 1{,}000{,}000)$

$+ (5 \times 100{,}000) + (2 \times 10{,}000) + (4 \times 1{,}000) + (5 \times 100) + (1 \times 10) + (3 \times 1)$

$= 1{,}000{,}000{,}000 + 300{,}000{,}000 + 60{,}000{,}000 + 2{,}000{,}000$



+ 500,000 + 20,000 + 4,000 + 500 + 10 + 3 **Question 2.1:** Write the number 456,789 in words.

Question 2.2: Write "two hundred million, thirty thousand, one hundred fifty" in figures.

Topic 3: Numbers on a Number Chart

A number chart can help us see the relationship between numbers. By looking at the position of a number, we can easily find numbers that are greater or smaller than it by a certain amount.

Example: Consider the number chart provided (multiples of 1,500):

1500	3000	4500	6000	7500
9000	10500	12000	13500	15000
16500	18000	19500	21000	22500
24000	25500	27000	28500	30000
31500	33000	34500	36000	37500
39000	40500	42000	43500	45000
46500	48000	49500	51000	52500
54000	55500	57000	58500	60000

If we pick the number 33,000:

- The number to its right (34,500) is 1,500 more.
- The number to its left (31,500) is 1,500 less.
- The number directly below it (40,500) is 7,500 more.
- The number directly above it (25,500) is 7,500 less.

1500	3000	4500	6000	7500
9000	10500	12000	13500	15000
16500	18000	19500	21000	22500
24000	25500	27000	28500	30000
31500	33000	34500	36000	37500
39000	40500	42000	43500	45000
46500	48000	49500	51000	52500
54000	55500	57000	58500	60000

Question 3.1: Using the number chart, what number is to the right of 41,500? How much more is it?

Question 3.2: Using the number chart, what number is directly above 53,000? How much less is it?

Topic 4: Comparing and Ordering Numbers

To compare numbers, we look at their **place values from left to right**.

• Greater than (>):

274,679 > 264,679

(because the hundred-thousands digit 2 < 2, but 7 > 6)

• Less than (<):

123,400 < 133,400

(because the ten-thousands digit 2 < 3)

• Equal to (=):

100,200 = 100,200

We can also order a set of numbers from smallest to largest (ascending order) or from largest to smallest (descending order).

Example: Order the following numbers in ascending order:

140, 230. 17, 025. 75. 267,389. 287, 368.

Ascending order: 75 < 17,025 < 140,230,< 267,389, < 287,368.

Question 4.1: Compare the numbers 56,789 and 56,978 using >, <, or =.

Question 4.2: Order the following numbers in descending order: 92,000, 87,500, 95,100, 87,050.

Topic 5: Rounding Whole Numbers

Rounding helps us estimate and work with numbers that are close to a certain place value.

- **Rounding off:** We look at the digit to the right of the place we are rounding to. If it is 5 or more, we round up. If it is less than 5, we round down.
  - 129,500 rounded to the nearest ten thousand is 130,000 (because the thousands digit is 9).
  - 19,100 rounded to the nearest thousand is 19,000 (because the hundreds digit is 1).
- **Rounding up:** We always round to the next higher multiple of the place value.
  - 214,765 rounded up to the nearest ten is 214,770.
  - 214,765 rounded up to the nearest hundred is 214,800.
  - 214,765 rounded up to the nearest thousand is 215,000.
- **Rounding down:** We always round to the next lower multiple of the place value.
  - 214,765 rounded down to the nearest ten is 214,760.
  - 214,765 rounded down to the nearest hundred is 214,700.
  - 214,765 rounded down to the nearest thousand is 214,000.

Question 5.1: Round off 78,345 to the nearest thousand.

Question 5.2: Round up 45,672 to the nearest hundred.

Question 5.3: Round down 91,238 to the nearest ten thousand.

Topic 6: Skip Counting with Large Numbers

Skip counting involves adding or subtracting a fixed amount repeatedly. We can do this with larger numbers as well.

Example 1: Skip counting forwards in 5000s

Starting at 287,940: 287,940, 292,940, 297,940, 302,940, 307,940, 312,940, 317,940, ...

Example 2: Skip counting backwards in 10,000s

Starting at 827,685: 827,685, 817,685, 807,685, 797,685, 787,685, 777,685, ...

Activity:

1. Start at 150,000 and skip count forwards in 10,000s up to 200,000.

2. Start at 900,000 and skip count backwards in 50,000s down to 700,000.

Error Spotting:

Here's a skip counting sequence in 20,000s with a mistake: 340,000, 360,000, 380,000, 410,000, 420,000. Can you identify the error? (410,000 should be 400,000).

Question 6.1: Skip count forwards in 25,000s starting from 500,000 for three steps.

Question 6.2: Skip count backwards in 100,000s starting from 1,000,000 for two steps.

Topic 7: Recognizing Roman Numerals up to C (100)

Roman numerals use letters to represent numbers. The main symbols we need to know up to 100 are:

- I = 1
- V = 5
- X = 10
- L = 50
- C = 100

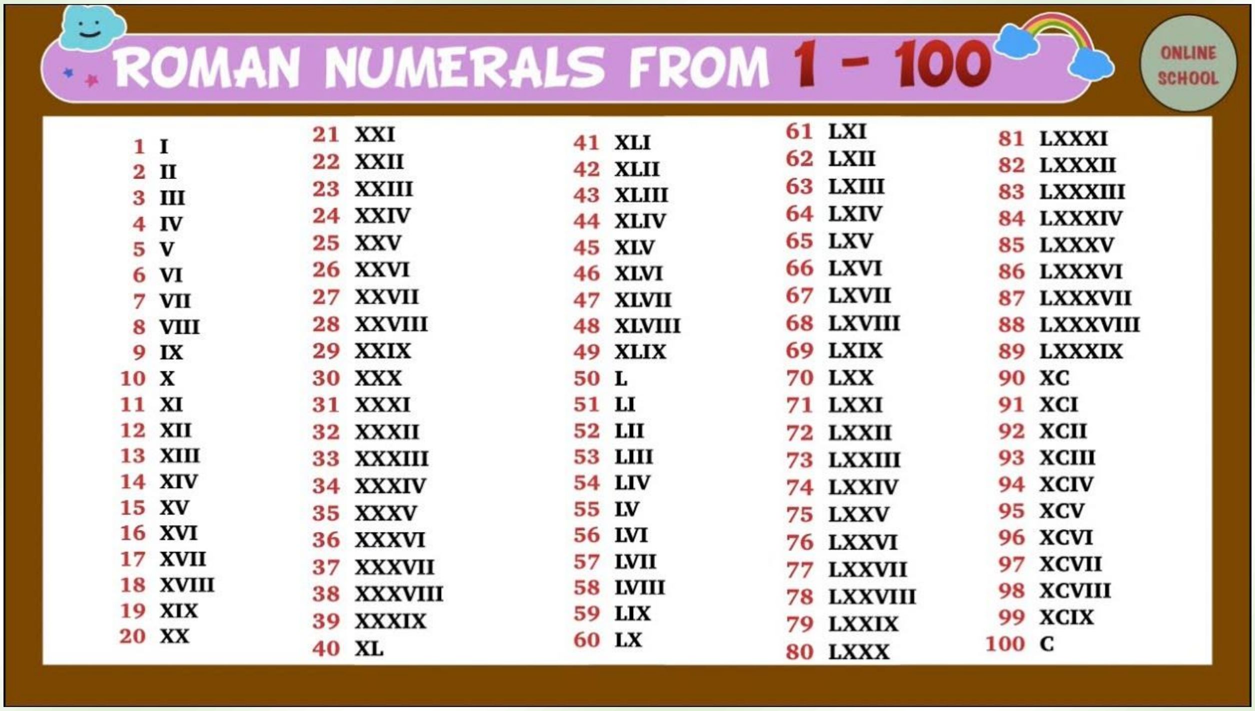
We combine these symbols to make other numbers.

- Repeating a symbol means adding its value (e.g., III = 1 + 1 + 1 = 3).
- Placing a smaller value symbol before a larger one means subtracting (e.g., IV = 5 - 1 = 4, IX = 10 - 1 = 9).
- Placing a smaller value symbol after a larger one means adding (e.g., VI = 5 + 1 = 6, XI = 10 + 1 = 11).

Here are some examples:

- I = 1
- V = 5
- X = 10
- XX = 20
- XXX = 30
- XL = 40 (50 - 10)
- L = 50
- LX = 60 (50 + 10)

- XC = 90 (100 - 10)
- C = 100



1	I	21	XXI	41	XLI	61	LXI	81	LXXXI
2	II	22	XXII	42	XLII	62	LXII	82	LXXXII
3	III	23	XXIII	43	XLIII	63	LXIII	83	LXXXIII
4	IV	24	XXIV	44	XLIV	64	LXIV	84	LXXXIV
5	V	25	XXV	45	XLV	65	LXV	85	LXXXV
6	VI	26	XXVI	46	XLVI	66	LXVI	86	LXXXVI
7	VII	27	XXVII	47	XLVII	67	LXVII	87	LXXXVII
8	VIII	28	XXVIII	48	XLVIII	68	LXVIII	88	LXXXVIII
9	IX	29	XXIX	49	XLIX	69	LXIX	89	LXXXIX
10	X	30	XXX	50	L	70	LXX	90	XC
11	XI	31	XXXI	51	LI	71	LXXI	91	XCI
12	XII	32	XXXII	52	LII	72	LXXII	92	XCII
13	XIII	33	XXXIII	53	LIII	73	LXXIII	93	XCIII
14	XIV	34	XXXIV	54	LIV	74	LXXIV	94	XCIV
15	XV	35	XXXV	55	LV	75	LXXV	95	XCV
16	XVI	36	XXXVI	56	LVI	76	LXXVI	96	XCVI
17	XVII	37	XXXVII	57	LVII	77	LXXVII	97	XCVII
18	XVIII	38	XXXVIII	58	LVIII	78	LXXVIII	98	XCVIII
19	XIX	39	XXXIX	59	LIX	79	LXXIX	99	XCIX
20	XX	40	XL	60	LX	80	LXXX	100	C

Question 7.1: What Hindu-Arabic numeral does the Roman numeral X represent?

Question 7.2: What Roman numeral represents the Hindu-Arabic numeral 50?

Topic 8: Counting and Converting Between Number Systems

Now let's practice converting between our usual numbers (Hindu-Arabic) and Roman numerals.

Converting Hindu-Arabic to Roman:

To convert a number like 37 to Roman numerals:

- 30 = XXX
- 7 = VII
- So, 37 = XXXVII

To convert 94:

- 90 = XC
- 4 = IV
- So, 94 = XCIV

Converting Roman to Hindu-Arabic:

- XXIV: X = 10, IV = 4.

  So, XXIV = 10 + 4 = 14.

- LX: L = 50, X = 10.

  So, LX = 50 + 10 = 60.

Activity:

1. Convert the following Hindu-Arabic numbers to Roman numerals: 19, 45, 88.
2. Convert the following Roman numerals to Hindu-Arabic numbers: XI, LIX, XCII.

Question 8.1: Convert the Hindu-Arabic number 63 to a Roman numeral.

Question 8.2: Convert the Roman numeral LXXXVII to a Hindu-Arabic number.

Topic 9: Prime Factorization

Before finding the HCF and LCM using prime factors, let's quickly review how to find the prime factors of a number. The **prime factors** of a number are the prime numbers that multiply together to give that number. We can use a **factor tree** to find them.

Example: Find the prime factors of 24.

Start with 24, and break it down into any two factors:

```
     24
    /  \
   2   12
      /  \
     2    6
         / \
        2   3
```

The prime factors at the end of the branches are 2, 2, 2, and 3. So, the prime factorization of 24 is 2×2×2×3, or 23×3.

Example: Find the prime factors of 36 and 48.

For 36:

```
    36
   /  \
  2    18
      /  \
     2     9
          / \
         3   3
```

Prime factors of 36 are 2, 2, 3, 3, so 36=22×32.

For 48:

```
    48
   /  \
  2    24
      /  \
     2    12
         /  \
        2     6
             / \
            2   3
```

Prime factors of 48 are 2, 2, 2, 2, 3, so 48=24×3.

Question 9.1: Find the prime factors of 30 using a factor tree.

Question 9.2: What are the prime factors of 16?

Topic 10: Finding HCF using Prime Factors

The **Highest Common Factor (HCF)** is the largest number that divides exactly into two or more numbers. Using prime factorization:

1. Find the prime factorization of each number.
2. Identify the common prime factors.
3. For each common prime factor, take the lowest power that appears in any of the factorizations.
4. Multiply these lowest powers together to get the HCF.

Example: Find the HCF of 36 and 48.

- 36=22×32
- 48=24×31

The common prime factors are 2 and 3. The lowest power of 2 is 22. The lowest power of 3 is 31.

HCF = 22×31=4×3=12.

Question 10.1: Find the HCF of 18 (2×32) and 24 (23×3).

Question 10.2: Find the HCF of 12 (22×3), 20 (22×5), and 30 (2×3×5).

Topic 11: Finding LCM using Prime Factors

The **Lowest Common Multiple (LCM)** is the smallest number that is a multiple of two or more numbers. Using prime factorization:

1. Find the prime factorization of each number.
2. List all the prime factors that appear in any of the factorizations.
3. For each prime factor, take the highest power that appears in any of the factorizations.
4. Multiply these highest powers together to get the LCM.

Example: Find the LCM of 36 and 48.

- 36=22×32
- 48=24×31

The prime factors involved are 2 and 3. The highest power of 2 is 24. The highest power of 3 is 32.

LCM = 24×32=16×9=144.

Question 11.1: Find the LCM of 18 (2×32) and 24 (23×3).

Question 11.2: Find the LCM of 12 (22×3), 20 (22×5), and 30 (2×3×5).

Topic 12: Using a Table for HCF and LCM

We can also use a table to find the HCF and LCM of three numbers by dividing by prime factors.

Example: Find the HCF and LCM of 18, 24, and 30.

To find the HCF, we look for prime factors that divide *all* the numbers at each step. Here, 2 and 3 did this.

÷	18	24	30
2	9	12	15
3	3	4	5

HCF = 2×3 =6

To find the LCM, we continue until all numbers become 1, and then multiply all the prime factors on the left and the final row.

÷	18	24	30
2	9	12	15
3	3	4	5
3	1	4	5
2	1	2	5
2	1	1	5
5	1	1	1

LCM = 2×3×3×2×2×5 = 360.

Question 12.1: Use the table method to find the HCF and LCM of 12, 15, and 18.

Sub-strand 2: Number Operations

We will learn and apply mental mathematics strategies to solve multiplication and division problems.

Topic 13: Mental Math Strategies for Multiplication and Division (Part 1)

Mental math involves solving problems in your head without relying on paper or a calculator. Here are some strategies:

1. Skip Counting from a Known Fact

• If you know $5 \times 7 = 35$, then:

$6 \times 7 = 35 + 7 = 42$

$7 \times 7 = 42 + 7 = 49$

• If you know $8 \times 8 = 64$, then:

$7 \times 8 = 64 - 8 = 56$

$6 \times 8 = 56 - 8 = 48$

2. Doubling

• To find 8×3:

$4 \times 3 = 12$

$8 \times 3 = 12 + 12 = 24$

3. Patterns when Multiplying by 9

• 9×6:

$10 \times 6 = 60$, then $60 - 6 = 54$

• 7×9:

$7 \times 10 = 70$, then $70 - 7 = 63$

4. Repeated Doubling

• If $2 \times 6 = 12$, then:

$4 \times 6 = 12 + 12 = 24$

$8 \times 6 = 24 + 24 = 48$

5. Repeated Halving (for Division)

• To find $60 \div 4$:

Half of 60 = 30

Half of 30 = 15

So, $60 \div 4 = 15$

6. Relating Division to Multiplication

• To find 64 ÷ 8:

"What number multiplied by 8 equals 64?"

8 × 8 = 64, so 64 ÷ 8 = 8

Question 13.1: Use skip counting from 3×9=27 to find 5×9.

Question 13.2: Use doubling to find 6×4 (think of 3×4).

Question 13.3: Use the pattern for multiplying by 9 to find 9×8.

Topic 14: More Mental Math Strategies for Multiplication

1. Annexing Zeros (for multiples of 10, 100, 1000)

• 3 × 200:

3 × 2 = 6

Add two zeros → 600

2. Halving and Doubling

• 32 × 5:

(32 ÷ 2) × (5 × 2) = 16 × 10 = 160

3. Using the Distributive Property

• 6 × 18:

18 = 10 + 8

(6 × 10) + (6 × 8) = 60 + 48 = 108

• 29 × 7:

29 = 30 − 1

(30 × 7) − (1 × 7) = 210 − 7 = 203

4. Multiplying by One

• Any number × 1 = the number itself

Example: 15 × 1 = 15

5. Multiplying by Zero

• Any number × 0 = 0

Example: 23 × 0 = 0

Question 14.1: Use annexing zeros to find 7×300.

Question 14.2: Use halving and doubling to find 25×8.

Question 14.3: Use the distributive property to find 8×12.

Topic 15: Multi-Digit Multiplication

There are several ways to multiply multi-digit numbers:

- **Expand and Box Method (Partial Decomposition):** For example, 448×2:

×	400	40	8
2	800	80	16

$$448 \times 2 =$$
$$(400\times2)+(40\times2)+()$$
$$= 800+80+16$$
$$= 896$$
$$= 800+80+16=896$$

- **Column or Vertical Method:** For example, 25×32:

$$\begin{array}{r} 25 \\ \times\ \ 32 \\ \hline 50 \\ 750 \\ \hline 800 \end{array}$$

1. Distributive Property

• Example: 25 × 32

25 × (30 + 2)

= (25 × 30) + (25 × 2)

= 750 + 50

= 800

2. Lattice Method

• Example: 345 × 27

1. Draw a 2 × 3 grid

2. Write 345 across the top and 27 down the side

3. Multiply each digit; place tens above the diagonal and ones below

4. Add along the diagonals to get the final product

Question 15.1: Use the expand and box method to calculate 312×3.

Question 15.2: Use the column method to calculate 45×23.

Topic 16: Basic Division Facts up to 81

Basic division facts are related to basic multiplication facts. For example, since 7×9=63, we know that 63÷9=7 and 63÷7=9.

Divisibility Tests: Understanding divisibility rules can help with division:

- A number is divisible by 6 if it is divisible by both 2 and 3.
- A number is divisible by 8 if the last three digits are divisible by 8.
- A number is divisible by 9 if the sum of its digits is divisible by 9.
- A number is divisible by 11 if the alternating sum of its digits is divisible by 11.

Activity: 3-in-a-Line Division Game

Imagine a 6 by 6 multiplication chart (products up to 36). Roll a die (results 1-6). Find a number on the chart that is divisible by the result on the die and mark it. The first player to get three marked numbers in a line (horizontally, vertically, or diagonally) wins. The marked number represents the dividend, the die roll is the divisor, and the marked spot's original factors relate to the quotient.

Factor	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

Question 16.1: What is 56÷8? How do you know?

Question 16.2: Is the number 72 divisible by 9? How do you know?

Topic 17: Dividing 3-Digit Numbers by 1-Digit Numbers

We can use the long division method, which can be thought of as repeated subtraction, to divide larger numbers.

Example: Divide 468 by 3.

```
      156
    ------
 3 | 468
     -3
     ---
      16
     -15
     ----
        18
       -18
       ----
         0
```

Explanation:

1. How many times does 3 go into 4? Once (1 x 3 = 3). Subtract 3 from 4, leaving 1.
2. Bring down the next digit (6) to make 16. How many times does 3 go into 16? Five times (5 x 3 = 15). Subtract 15 from 16, leaving 1.
3. Bring down the last digit (8) to make 18. How many times does 3 go into 18? Six times (6 x 3 = 18). Subtract 18 from 18, leaving 0.

So, 468÷3=156.

Question 17.1: Divide 575 by 5 using long division.

Question 17.2: Divide 864 by 4 using long division.

Topic 18: Solving Multi-Step Word Problems

To solve multi-step word problems, we need to read carefully, identify the operations needed, and solve them in the correct order.

Example 1: Ama bought 3 bags of oranges with 12 oranges in each bag. She also bought 5 apples. How many fruits did she buy in total?

- Step 1: Find the total number of oranges: $3\times12=36$ oranges.
- Step 2: Add the number of apples: $36+5=41$ fruits. Mathematical sentence: $(3\times12)+5=41$.

Example 2: Kofi had 50 marbles. He gave 12 to his friend and then shared the rest equally among 4 other friends. How many marbles did each of these 4 friends receive?

- Step 1: Find the number of marbles left after giving some away: $50-12=38$ marbles.
- Step 2: Divide the remaining marbles among 4 friends: $38\div4=9$ with a remainder of 2. (Each friend received 9 marbles, and 2 were left over). Mathematical steps: $(50-12)\div4=38\div4=9R2$.

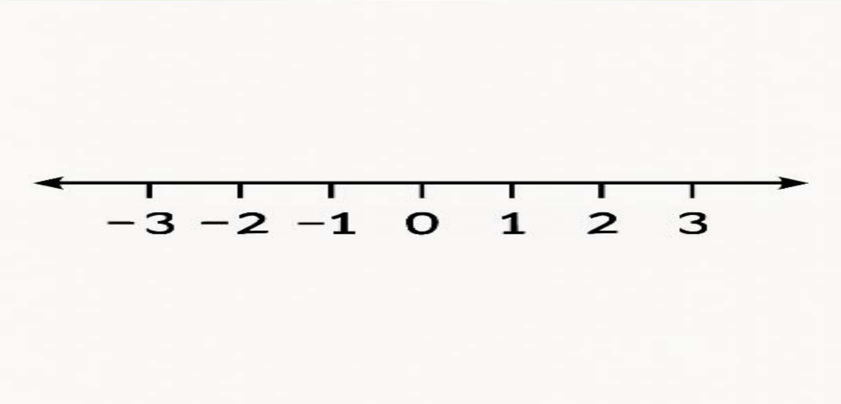
Question 18.1: A baker made 2 trays of cupcakes with 24 cupcakes on each tray. He sold 35 cupcakes. How many cupcakes are left?

Question 18.2: Sarah had ₵100. She bought 2 notebooks at ₵15 each and a pen for ₵22. How much money does she have left?

Topic 19: Understanding Integers

Integers are whole numbers and their opposites (negatives). They include: ..., -3, -2, -1, 0, 1, 2, 3, ...

We can use a **number line** to visualize integers:



- Numbers to the right are greater than numbers to the left.
- Positive integers are greater than zero.
- Negative integers are less than zero.

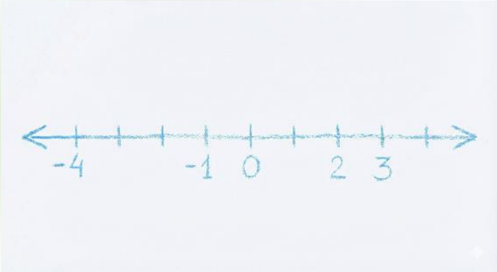
Comparing Integers:

- $3 > 1$ (3 is greater than 1)
- $-1 > -3$ (-1 is greater than -3, it's to the right on the number line)
- $-2 < 0$ (-2 is less than 0)

Ordering Integers:

To order a set of integers, we place them in increasing or decreasing order based on their position on the number line.

Example: Order -4, 2, -1, 0, 3 in increasing order: -4, -1, 0, 2, 3.

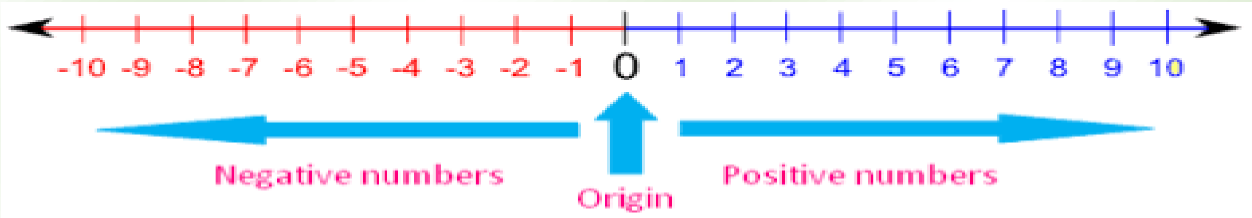


Question 19.1: Which integer is larger: -5 or -2?

Question 19.2: Order the integers 1, -3, 0, -2, 4 in decreasing order.

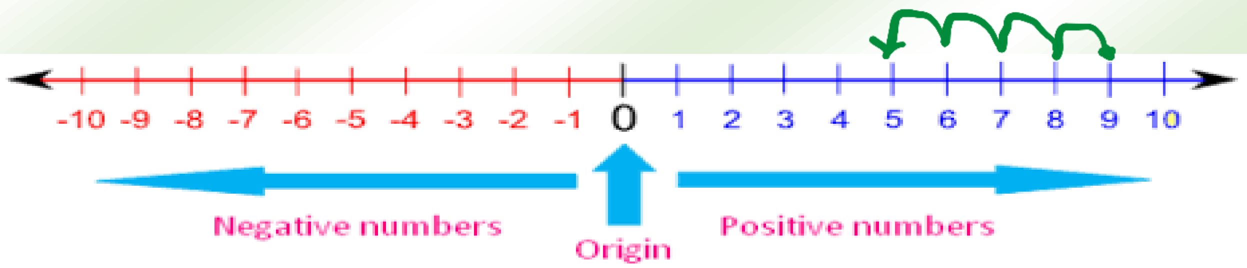
Topic 20: Simple Addition and Subtraction of Integers

We can use the number line to help with adding and subtracting integers.



Addition:

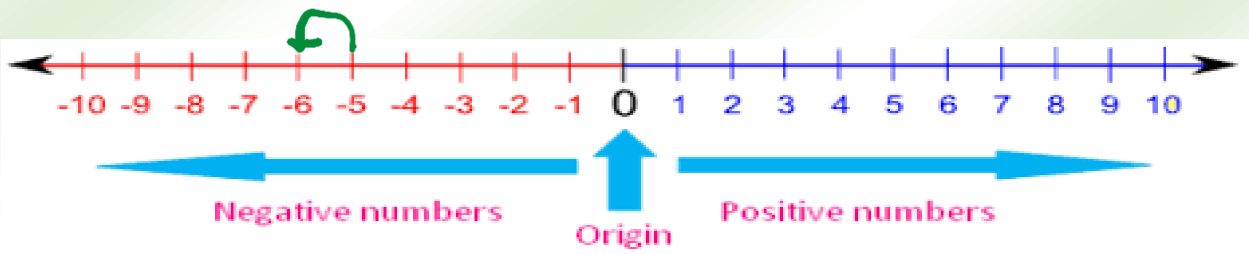
- 9+ (−4): Start at 9, move 4 steps to the left = 5.



- −8 + 4: Start at -8, move 4 steps to the right = -4.
- −3+(−5): Start at -3, move 5 steps to the left = -8.

Subtraction (not subtracting negatives):

- −5−1: Start at -5, move 1 step to the left = -6.



- 2−6: Start at 2, move 6 steps to the left = -4.

Question 20.1: Calculate $-7+3$.

Question 20.2: Calculate $1-5$.

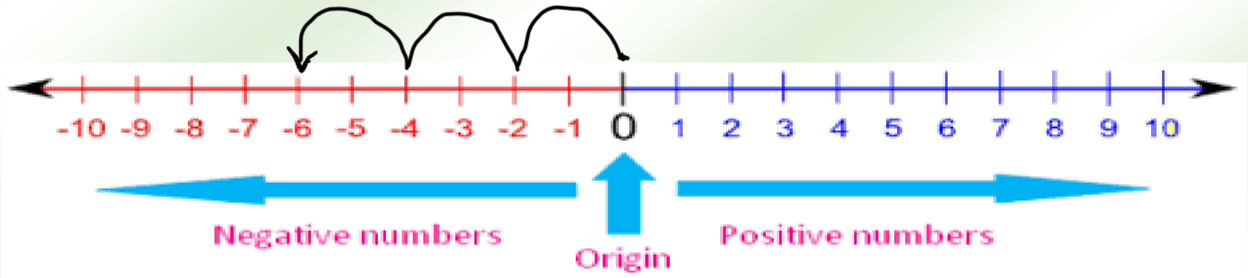
Topic 21: Simple Multiplication with Integers

Here are the rules for multiplying integers:

- Positive × Positive = Positive (e.g., $2\times3=6$)
- Negative × Positive = Negative (e.g., $-2\times3=-6$)
- Positive × Negative = Negative (e.g., $2\times-3=-6$)

We can visualize multiplication using a number line as repeated addition (or subtraction for negatives).

Example: $3\times(-2)$: Start at 0, and move 3 times in the negative direction by 2 steps each time. $0\rightarrow -2\rightarrow -4\rightarrow -6$. So, $3\times(-2)=-6$.



Question 21.1: Calculate -4×2.

Question 21.2: Calculate 5×-1.

Sub-strand 3: Fractions

We will learn strategies for comparing, adding, subtracting, multiplying, and dividing common, decimal, and percent fractions.

Topic 22: Comparing and Ordering Mixed Fractions

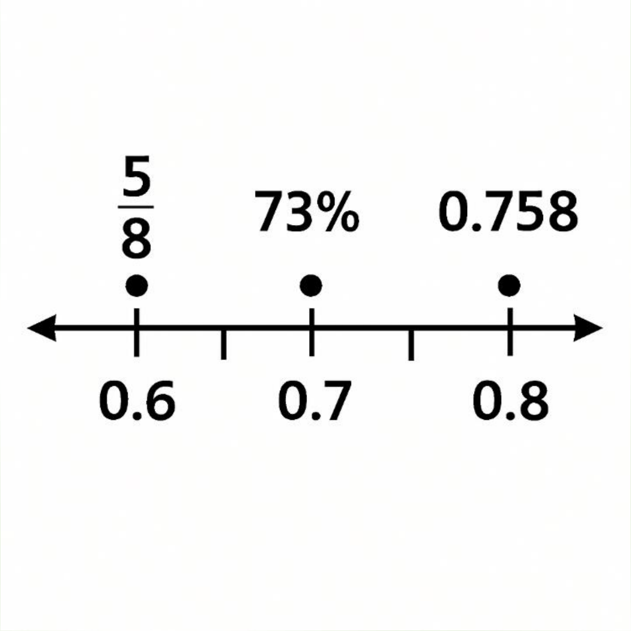
To compare and order different types of fractions, it's easiest to convert them all into the same form (either common, decimal, or percent).

Example: Order 0.758, 85, and 73%.

1. **Convert to decimals:**
   - 85 = 5÷8 = 0.625
   - 73% = $\frac{73}{100}$ =0.73
   - 0.758 is already a decimal.
2. **Compare the decimals:** 0.625 < 0.73 < 0.758
3. **Order the original fractions:** 85, 73%, 0.758

Alternatively, we could convert them all to percentages or common fractions and then compare.

Question 22.1: Order the following from least to greatest: 0.5, 43, 40%.



Question 22.2: Which is larger: 0.8 or 85%?

Topic 23: Adding and Subtracting Unlike and Mixed Fractions

To add or subtract fractions with different denominators (unlike fractions), we first need to find a **common denominator**. The easiest common denominator to use is often the **Lowest Common Denominator (LCD)**.

Example 1: Adding mixed fractions with the same denominator:

$2\frac{1}{3} + 3\,\frac{2}{3} = (2+3)+(\frac{1}{3}+\frac{2}{3})$

$= 5+\frac{1}{3}+2$

$=5+\frac{3}{3}$

=5+1=6.

Example 2: Subtracting mixed fractions with the same denominator: $3\frac{2}{3} - 2\frac{1}{3}$

- Convert to improper fractions:
    - $3\frac{2}{3} = \frac{11}{3}$
    - $2\frac{1}{3} = \frac{7}{3}$
- Subtract:
    - $\frac{11}{3} - \frac{7}{3} = \frac{4}{3}$
- Convert back to a mixed fraction:
    - $\frac{4}{3} = 1\frac{1}{3}$

Example 3: Adding unlike mixed fractions:

To add $2\frac{1}{3}+3\frac{2}{5}$

1. Find the least common denominator (LCD) of 3 and 5, which is 15.

2. Convert the fractions:
   - $2\frac{1}{3}=2\frac{5}{15}$
   - $3\frac{2}{5}=3\frac{6}{15}$
3. Add whole numbers: 2+3 = 5
4. Add fractions: $\frac{5}{15}+\frac{6}{15}=\frac{11}{15}$
5. Combine the results: $5\frac{11}{15}$

Question 23.1: Calculate $1\frac{2}{7}+2\frac{7}{3}$.

Question 23.2: Calculate 341−121.

Topic 24: Multiplying Fractions

Multiplying a whole number by a mixed fraction: $3\times 2\frac{2}{3}$ can be thought of as

$2\frac{2}{3}+2\frac{2}{3}+2\frac{2}{3}$.

$= (2+2+2)+(\frac{2}{3}+\frac{2}{3}+\frac{2}{3})$

$= 6+\frac{6}{3}$

= 6+2=8.

Multiplying a fraction by a whole number: $\frac{2}{3}\times 5$ means $\frac{2}{3}$ of 5. This is like having 5 items, and we want two-thirds of each.

$\frac{2}{3}\times 5$

$=\frac{2\times 5}{3}\times 5=\frac{10}{3}=3\frac{1}{3}$.

Multiplying a fraction by a fraction: Multiply the numerators together and the denominators together. $\frac{a}{b}\times\frac{c}{d}=\frac{a\times c}{b\times d}=\frac{ac}{bd}$

Example: $\frac{1}{2}\times\frac{3}{4}=\frac{1\times3}{2\times4}=\frac{3}{8}$

Question 24.1: Calculate $4\times1\frac{1}{2}$.

Question 24.2: Calculate $\frac{3}{5}\times2$.

Question 24.3: Calculate $\frac{2}{3}\times\frac{1}{4}$.

Sub-strand 4: Ratios and Proportion

We will explore the concept of ratios, their relationship to fractions, and how to work with them.

Topic 25: Understanding Ratios

A **ratio** compares two quantities. It tells us how many times one quantity is contained in another. We can write a ratio using a colon (:).

Example 1: Imagine shape A has an area of 1 unit and shape B has an area of 4 units. The ratio of the area of A to the area of B is 1:4. This means the area of A is $\frac{1}{4}$ of the area of B.

If shape C has an area of 3 units and shape A has an area of 1 unit, the ratio of the area of C to the area of A is 3:1. This means the area of C is three times the area of A.

Simplest Form of a Ratio: Just like fractions, ratios can be simplified by dividing both parts by their highest common factor (HCF).

Example 2: Shape C has 6 squares, and shape A has 2 squares. The ratio of the area of C to A is 6:2. The HCF of 6 and 2 is 2. Dividing both parts by 2, we get the simplest form: 6÷2 : 2÷2 = 3:1.

If shape C has 6 squares and shape B has 8 squares, the ratio of C to B is 6:8. The HCF of 6 and 8 is 2. Simplifying, we get 6÷2 : 8÷2 = 3:4.

Question 25.1: Simplify the ratio 10:15.

Question 25.2: Write the ratio of 3 apples to 7 oranges.

Question 25.3: In a class of 30 students, 12 are boys. Find the ratio of girls to boys in its simplest form.

Topic 26: Equivalent Ratios, Comparing, and Ordering Ratios

Equivalent Ratios:

Just like equivalent fractions, equivalent ratios represent the same comparison. We can find equivalent ratios by multiplying or dividing both parts of the ratio by the same number.

Example: The ratio 2:3 can be written as the fraction $\frac{2}{3}$.

- Multiplying both parts by 2 gives 4:6, which is equivalent to the fraction $\frac{4}{6}$. Notice that $\frac{2}{3} = \frac{2\times2}{3\times2} = \frac{4}{6}$.
- Multiplying both parts by 3 gives 6:9, which is equivalent to the fraction $\frac{6}{9}$. Notice that $\frac{2}{3} = \frac{2\times3}{3\times3} = \frac{6}{9}$.
- Dividing both parts by 2 gives 1:1.5, which is equivalent to the fraction $\frac{1}{5.1}$. Notice that $\frac{2}{3} = \frac{2\div2}{3\div2}$=1.51.

Comparing Ratios:

One way to compare ratios is to express them as fractions and then compare the fractions (e.g., by finding a common denominator).

Example:

- Afia mixes squash (S) and water (W) in the ratio 3:14. This can be expressed as the fraction $\frac{3}{14}$ (representing the proportion of squash to water).

- Bedu mixes squash and water in the ratio 2:7. This can be expressed as the fraction $\frac{2}{7}$. To compare it with Afia's ratio, we can find an equivalent fraction with a denominator of 14: $\frac{2}{7} = \frac{2\times2}{7\times2} = \frac{4}{14}$.
- Caro mixes squash and water in the ratio 1:4. This can be expressed as the fraction $\frac{1}{4}$. To compare it, we can find an equivalent fraction with a denominator of 14: $\frac{1}{4} = \frac{1\times3.5}{4\times3.5} = \frac{3.5}{4}$

To see whose drink tastes strongest of squash, we compare the fractions of squash in the total mixture (if we consider the second number in the ratio as a fixed amount of water, or simply compare the ratio of squash to water).

- Afia: $\frac{3}{14}$
- Bedu: $\frac{4}{14}$
- Caro: $\frac{3.5}{4}$

Solving Problems with Ratios:

Example: Given that 10:q = 2:3, find q.

We can write this as $\frac{10}{q} = \frac{2}{3}$.

Cross-multiplying, we get 10×3 = 2×q,

so 30 = 2q

divide both side by 2 , $\frac{30}{\cancel{2}} = \frac{\cancel{2}q}{\cancel{2}}$

15 = q

q =15

Example: The ratio of boys to girls in a classroom is 7 to 11. If there are a total of 49 boys, how many girls are there? Ratio of boys to girls = 7:11. If 7 parts represent 49 boys, then 1 part represents 49 ÷ 7 = 7 students. The number of girls is 11 parts, so there are 11×7 = 77 girls. The total number of students is 49+77=126.

Question 26.1: Write two ratios equivalent to 3:5.

Question 26.2: Which ratio is greater: 2:3 or 5:8? (Hint: Convert to fractions).

Question 26.3: If the ratio of red balls to blue balls is 2:5, and there are 10 red balls, how many blue balls are there?

Topic 27: Understanding Proportion

A **proportion** is a statement that two ratios are equal. It's like saying two fractions are equivalent.

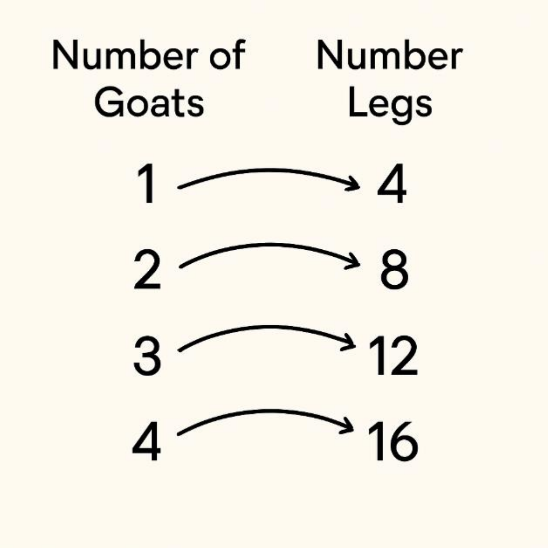
Example 1: Consider goats and their legs. The ratio of the number of goats to the number of legs is always 1:4 (since each goat has 4 legs).

- 3 goats have 12 legs. The ratio of goats to legs is 3:12.
- 4 goats have 16 legs. The ratio of goats to legs is 4:16.

Since the ratio of goats to legs remains constant, we can say that the number of goats is proportional to the number of legs. This can be written as equal ratios or fractions:

3:12 = 4:16 or $\frac{3}{12} = \frac{4}{16}$

We can see that both $\frac{3}{12}$ and $\frac{4}{16}$ simplify to $\frac{1}{4}$.



Example 2: If 200 bottles of equal capacity hold 350 litres of water, we can use proportion to find how much water each bottle holds.

Let x be the amount of water one bottle holds. The ratio of bottles to litres is 200:350. The ratio of 1 bottle to the amount it holds is 1:x.

These ratios are proportional: 200:350 =1:x or $\frac{200}{350} = \frac{1}{x}$

To solve for x, we can cross-multiply: 200× x = 350×1

200x=350

x= $\frac{35\cancel{0}}{20\cancel{0}}$

=$\frac{35}{20}$

= $\frac{7}{4}$ =1.75 litres.

So, each bottle holds 1.75 litres of water.

Question 27.1: Are the ratios 2:5 and 6:15 proportional? Explain.

Question 27.2: If 3 identical books cost ¢12, how much would 5 of the same books cost? Use proportion to solve.

Topic 28: Rates and Scales

Rate: A **rate** is a ratio that compares two different quantities with different units. A **unit rate** compares a quantity to one unit of another quantity.

Example 1: If the cost of 2 kg of meat is ¢8, the rate of cost to weight is ¢8:2 kg. The unit rate (cost per 1 kg) is ¢4:1 kg, or ¢4/kg.

(Indication for Image: A picture showing meat being sold with the price and weight indicated.)

Example 2: If a litre of sachet water costs ¢0.40:

- Cost of $\frac{1}{2}$ litre: $\frac{1}{2}$ × ¢0.40 = ¢0.20.
- Cost of 7 litres: 7× ¢0.40= ¢2.80.
- Cost of 9 litres: 9× ¢0.40= ¢3.60.

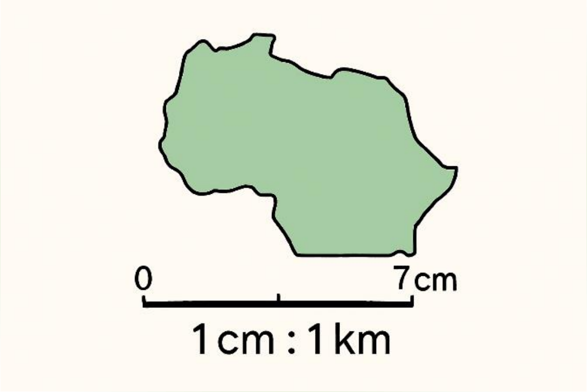
Scale: A **scale** is a ratio that compares the size of a model or a drawing to the actual size of the object. It often appears on maps and plans.

Example 3: A plan of a bedroom has a scale of 1:100. This means 1 cm on the drawing represents 100 cm (or 1 metre) in the actual room.

- If the length of the room on the plan is 421cm, the actual length is 4.5×100 = 450 cm, or 4.5 metres.
- If the length of the bed on the plan is 1.8 cm, the actual length is 1.8×100 =180 cm, or 1.8 metres.

Example 4: On a map, 10 cm stands for 10 km. The scale is 10 cm : 10 km, which simplifies to 1 cm : 1 km.

- A distance of 1 km would be represented by 1 cm on the map.
- A distance of 7 km would be represented by 7 cm on the map.
- A distance of 41 km would be represented by 41 cm on the map.



Question 28.1: If a car travels 150 km in 3 hours, what is its speed in km per hour (km/h)?

Question 28.2: A map has a scale of 1 cm : 5 km. What actual distance does 8 cm on the map represent?

Strand 2: Algebra

Sub-strand 1: Pattern and Relationships

We will learn how to identify, represent, and describe patterns, and write rules to predict subsequent elements.

Topic 29: Visual Verification of Patterns

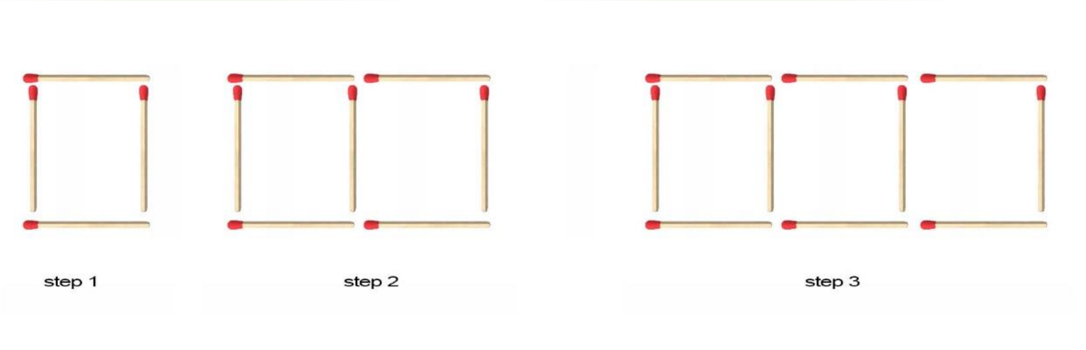
We can use visual representations like diagrams or tables to understand and predict patterns.

Example 1: Matchstick Pattern

Imagine a pattern made with matchsticks where each step adds a square:

- Step 1: (4 matchsticks)
- Step 2: (7 matchsticks)
- Step 3: (10 matchsticks)

To verify a prediction, say for Step 4, we can draw it: (13 matchsticks). We can see that each step adds 3 matchsticks.



Example 2: Patterns of Squares

Consider a pattern of squares made with matchsticks:

Pattern Number	Matchsticks in the perimeter	Matchsticks enclosed
1	4	0
2	8	1
3	12	4

We can visualize these patterns to understand the relationship between the pattern number and the number of matchsticks. For example, for the perimeter, it increases by 4 each time. For the enclosed matchsticks, it increases by consecutive odd numbers (1, 3, ...).

(Indication for Image: Drawings of the first three square patterns with matchsticks, showing the perimeter and enclosed matchsticks.)

Activity: Use matchsticks to make the first few patterns of triangles (where each step adds one triangle along one side) and complete the table:

Number of triangles	1	2	3	4	5	6	7	8	9
Math sticks	3	5	?	?	?	?	?	?	?

Question 29.1: For the square pattern, how many matchsticks would you predict for the perimeter of the 4th pattern? Verify by drawing.

Question 29.2: Complete the table for the triangle matchstick pattern up to 5 triangles. What do you notice about how the number of matchsticks increases?

Topic 30: Making Predictions from Patterns

By identifying the rule of a pattern, we can predict future elements.

Example: Look at the number pattern: 2, 5, 8, 11, ... The rule is to add 3 to the previous number. The next number would be $11+3=14$.

Example from the content:

Number of triangles	1	2	3	4	5	6	7	8	9
Math sticks	3	5	?	?	?	?	?	?	?

From our activity, we found the number of matchsticks increases by 2 each time. So the completed table might look like:

Number of triangles	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Math sticks	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**

To answer the question, "how many match sticks will be used for the 9th pattern of triangles?", the answer is 19.

For the square perimeter pattern (4, 8, 12, ...), the rule is to multiply the pattern number by 4. So, for the 8th pattern, there would be $8 \times 4 = 32$ matchsticks in the perimeter.

Question 30.1: What is the rule for the number pattern: 10, 8, 6, 4, ...? What is the next number?

Question 30.2: Using the triangle matchstick pattern rule, how many matchsticks would be in the 15th pattern?

Topic 31: Writing Rules in Words and Algebra

We can describe the rule of a pattern using words and also using algebraic expressions. Let 'n' represent the term number (or input).

Term/Input (n)	**1**	**2**	**3**	**4**	**5**	**Rule in words**	**Rule in Algebra**
Result/Output (A)	9	18	27	36	45	9 times n	9n
B	0	4	8	12	16	4 times (n minus 1)	
C	4	7	10	13	16	1 more than 3 times n	
D	20	18	16	14	12	20 minus 2 times (n minus 1)	
E	15	19	23	27	31	4 more than 4 times n	
F	12	17	22	27	32	5 more than 5 times n	
Result/Output (A)	100	85	70	55	40	115 minus 15 times n	

Let's analyze Result/Output E: The sequence is 15, 19, 23, ... Each term increases by 4. Rule in words: Start at 11, then add 4 times (n).

(For n=1, 11+4×1=15; for n=2, 11+4×2=19, etc.)

Rule in algebra: 4n+11.

Let's analyze Result/Output F: The sequence is 12, 17, 22, ... Each term increases by 5. Rule in words: Start at 7, then add 5 times (n).

(For n=1, 7+5×1=12; for n=2, 7+5×2=17, etc.)

Rule in algebra: 5n+7.

Let's analyze the last sequence: 100, 85, 70, ... Each term decreases by 15. Rule in words: Start at 115, then subtract 15 times (n).

(For n=1, 115−15×1=100; for n=2, 115−15×2=85, etc.)

Rule in algebra: 115−15n.

Term/Input (n)	**1**	**2**	**3**	**4**	**5**	**Rule in words**	**Rule in Algebra**
Result/Output (A)	9	18	27	36	45	9 times n	9n
B	0	4	8	12	16	4 times (n minus 1)	4(n−1)
C	4	7	10	13	16	1 more than 3 times n	1+3n
D	20	18	16	14	12	20 minus 2 times (n minus 1)	20−2(n−1)
E	15	19	23	27	31	4 more than 4 times n	4n+11
F	12	17	22	27	32	5 more than 5 times n	5n+7
Result/Output (A)	100	85	70	55	40	115 minus 15 times n	115−15n

Question 31.1: Consider the pattern 3, 6, 9, 12, ... Write the rule in words and in algebra.

Question 31.2: For the pattern with rule 2n−1, what are the first three terms? Write the rule in words.

Topic 32: Describing Relationships with Expressions

A mathematical expression can clearly show how one quantity depends on another in a pattern presented in a table or chart.

Example 1: Cost of Boxed Lunches

Consider the table showing the cost of boxed lunches for students:

Number of students	1	2	3	4	5	?
Cost of lunch (¢)	3	6	9	12	15	90

(i) **Explain the pattern:** We can see that for each student, the cost of lunch is ¢3. As the number of students increases by 1, the cost increases by ¢3.

(ii) **Use the pattern to determine the number of students for a cost of ¢90:** Let n be the number of students and C be the cost of lunch. The relationship between the number of students and the cost can be expressed as: C = 3×n or C = 3n

If the cost C is ¢90, we can find the number of students n: 90 = 3n To find n, we divide both sides by 3: n = 390=30

So, if the cost of lunches is ¢90, then 30 students went on the trip.

Number of students	1	2	3	4	5	30
Cost of lunch (¢)	3	6	9	12	15	90

Example 2: Imagine a table showing the number of hours worked and the amount earned:

Hours worked (h)	**10**	**20**	**30**	**40**
Amount earned (¢)	150	300	450	600

What is the mathematical expression that describes the relationship between the hours worked (h) and the amount earned (E)?

We can see that for every 10 hours, ¢150 is earned. So, for 1 hour, 10150=¢15 is earned. The relationship is E=15×h or E=15h.

Activity: Create a simple table showing a pattern (e.g., the number of squares and the number of sides) and ask a friend to describe the relationship using a mathematical expression.

Question 32.1: Look at the boxed lunch table again. Write a question based on this pattern for a friend to solve.

Question 32.2: The following table shows the number of trees planted and the total amount of carbon dioxide absorbed. Find the mathematical expression that describes this relationship.

Number of trees	**1**	**2**	**3**	**4**
CO2 absorbed (kg)	25	50	75	100

Topic 33: Understanding Algebraic Expressions

An **algebraic expression** is a combination of numbers, variables (letters that represent numbers), and mathematical operations.

Example 1: Writing expressions

1. Sum of 8 and s:

$8 + s$

2. Take away 4 from m:

$m - 4$

3. 9 times the sum of 8 and q:

$9 \times (8 + q)$ or $9(8 + q)$

4. Subtract 4 from 7 times g:

$7g - 4$

5. One-sixth of n is added to the product of 9 and y:

$(1/6 \times n) + (9 \times y)$ or $(1/6)n + 9y$

6. Three-fourths of the sum of c and 2:

$(3/4 \times (c + 2))$ or $3/4(c + 2)$

7. 8 divided by r:

$8 \div r$ or $8/r$

8. 8 times the sum of c and 7:

$8 \times (c + 7)$ or $8(c + 7)$

Example 2: Perimeter of shapes

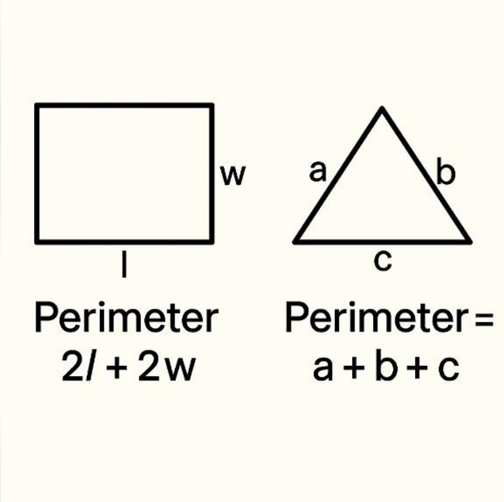
If a rectangle has length l and width w, its perimeter is:

$l + w + l + w$

$= 2l + 2w$

$= 2(l + w)$

If a triangle has sides a, b, and c, its perimeter is a+b+c.



Example 3: Area of shapes

If a rectangle has length l and width w, its area is l×w or lw.

If a triangle has base b and height h, its area is 21×b×h or 21bh.

Question 33.1: Write an algebraic expression for "5 more than twice a number x".

Question 33.2: Write an algebraic expression for the perimeter of a square with side length s.

Topic 34: Simplifying Algebraic Expressions

To simplify an algebraic expression, we **combine like terms**.
Like terms have **the same variable** raised to **the same power**.

1. $5 + 4z - 1 + 2z$

   $(5 - 1) + (4z + 2z)$

   $= 4 + 6z$

2. $10s - 1 + 3 - 5s$

   $(10s - 5s) + (-1 + 3)$

   $= 5s + 2$

3. $-6t - 7 - 2 - 3t$

   $(-6t - 3t) + (-7 - 2)$

   $= -9t - 9$

4. $2 - z - 1 + 4z$

   $(2 - 1) + (-z + 4z)$

   $= 1 + 3z$

5. $-9c + 7c + 8 - 3c$

   $(-9c + 7c - 3c) + 8$

   $= (-2c - 3c) + 8$

   $= -5c + 8$

6. $-5p + 3px - 7 - 5px + 10x$

   $-5p + (3px - 5px) - 7 + 10x$

   $= -5p - 2px - 7 + 10x$

Question 34.1: Simplify the expression $3a+2b-a+5b$.

Question 34.2: Simplify the expression $-4y+6-2y-1$.

Topic 35: Substituting Values into Expressions

To evaluate an algebraic expression, we **replace the variables** with their given values and then **perform the arithmetic operations**.

1. $z + 7d$ use $z = 7$ and $d = 4$:

$7 + (7 \times 4)$

$= 7 + 28$

$= 35$

2. $8k + d$ use $k = 2$ and $d = 3$:

$(8 \times 2) + 3$

$= 16 + 3$

$= 19$

3. $7(5f - 3n) - 8$ use $n = 3$ and $f = 7$:

$7((5 \times 7) - (3 \times 3)) - 8$

$= 7(35 - 9) - 8$

$= 7(26) - 8$

$= 182 - 8$

$= 174$

4. $7d - 2f + 9$ use $d = 2$ and $f = 5$:

$(7 \times 2) - (2 \times 5) + 9$

$= 14 - 10 + 9$

$= 4 + 9$

$= 13$

5. $-5f + 8b + 4 - 9$ use $f = 9$ and $b = 3$:

$(-5 \times 9) + (8 \times 3) + 4 - 9$

$= -45 + 24 + 4 - 9$

$= -21 + 4 - 9$

$= -17 - 9$

$= -26$

6. $-6(2x - 7h)$ use $h = 2$ and $x = 4$:

$-6((2 \times 4) - (7 \times 2))$

$= -6(8 - 14)$

$= -6(-6)$

$= 36$

Example with matchsticks:

If Sena used 13 matchsticks in making the next pattern of squares, let's assume the pattern starts with 4 matchsticks for 1 square and adds 3 for each additional square. The expression for the number of matchsticks could be $1+3s$, where s is the number of squares. If 13 matchsticks were used: $13=1+3s \Longrightarrow 12=3s \Longrightarrow s=4$ squares.

(i) A mathematical sentence for the matchsticks for s squares: $M=1+3s$. (ii) For 10 squares: $M=1+3(10)=1+30=31$ matchsticks.

(iii) If 64 matchsticks were used: $64=1+3s \Longrightarrow 63=3s \Longrightarrow s=21$ squares.

Question 35.1: Evaluate the expression $2a+3b$ if $a=5$ and $b=2$.

Question 35.2: Evaluate the expression $5(x-y)+4$ if $x=10$ and $y=3$.

Sub-strand 3: Variables and Equations

We will learn to identify unknowns, represent problems with equations, and solve simple one-step equations.

Topic 36: Identifying Unknowns and Representing Problems with Equations

In many problems, there is a value we don't know. This is the **unknown**, and we often use a variable (like x, y, n) to represent it. We can write an **equation** to show the relationship between the known and unknown values.

Example 1: "Four added to a number is equal to eleven."

- The unknown is "the number". Let's represent it with x.
- The problem can be written as the equation: $4+x=11$.

To solve for x, we need to find the value that makes the equation true. We can do this by using the inverse operation. To get x by itself, we subtract 4 from both sides of the equation:

$4+x-4$

$=11-4$

$x=7$

So, the unknown number is 7.

Example 2: "x minus three is equal to ten."

- Unknown: x.
- Equation: $x-3=10$.

To solve for x, we add 3 to both sides:

$x-3+3$

$=10+3$

$x=13$

Example 3: "Four times x is equal to twelve."

- Unknown: x.

- Equation: 4x=12.

To solve for x, we divide both sides by 4:

$\frac{4x}{4}=\frac{12}{4}$ x =3

Example 4

Problem: "Twenty-four is equal to three times x."
Unknown: x
Equation:

$$\mathbf{24 = 3x}$$

To solve for x, divide both sides by 3:

$$\mathbf{\frac{24}{3} = \frac{3x}{3}}$$
$$\mathbf{8 = x}$$

Final Answer:

$$\mathbf{x = 8}$$

Example 5: x divided by two is equal to five."

- **Unknown: x**
- **Equation:**

$$\mathbf{\frac{x}{2} = 5}$$

To solve for x, multiply both sides by 2:

$$\mathbf{\frac{x}{2} \times 2 = 5 \times 2}$$
$$\mathbf{x = 10}$$

Question 36.1: Write an equation for "A number multiplied by 5 equals 20" and solve for the number.

Question 36.2: Write an equation for "A number plus 7 equals 15" and solve for the number.

Topic 37: Creating Problems for Given Equations

We can also go the other way around: given an equation, we can create a word problem that it represents.

Example 1: Equation $4-x=9$

Here are two possible stories:

1. "I had 4 apples, and I ate some (x). Now I owe my friend 9 apples because I needed more. How many apples did I eat (including the ones I now owe)?" (This interpretation might lead to negative numbers, which might be beyond the current scope but good for discussion).

   Solving this as initially stated: $4-x=9 \Longrightarrow -x=9-4 \Longrightarrow -x=5 \Longrightarrow x=-5$. This suggests an initial misunderstanding of the scenario if we expect a positive number of eaten apples.

   Let's rephrase the story to fit the math more directly: "I had 4 items, and after some change (x), the result is 9 less than what I started with. What was the change?" (Still a bit awkward for this equation resulting in a negative if we strictly interpret 'take away').

   A better fit for $4-x=9$ might be: "The temperature started at 4 degrees Celsius and changed by some amount (x). The final temperature is 9 degrees lower than some reference. What was the change?" (Again, not directly leading to a simple positive scenario).

   Let's try to create a problem where the solution for x would be more straightforward if we were solving for a different variable.

   Consider a slight modification: "I started with 9 items and gave away 4. How many are left (x)?". This would be $9-4=x$, so $x=5$. This isn't the given equation.

   Let's stick to the given equation and think abstractly: "The difference between 4 and a certain number is 9. What is the number?" Here, $4-x=9$ or $x-4=9$. If $4-x=9$, then $x=-5$. If $x-4=9$, then $x=13$. The exemplar seems to be looking for story creation, even if the direct solution might involve concepts not yet fully covered (like negatives).

   Story 1: "Sarah had 4 cookies. She gave some (x) to her friend. Now she has 9 fewer cookies than her brother initially had. If her brother had 15 cookies, how many did Sarah give away?" (This doesn't directly translate to $4-x=9$).

   Revised Story 1 (closer fit): "The temperature was 4 degrees. It changed by some amount (x) and is now -5 degrees. What was the change?" Here, $4+x=-5 \Longrightarrow x=-9$. Not $4-x=9$.

Let's use the exemplar's intent: describe stories where the equation *could* represent a step.

Story 1: "John had 4 marbles. He lost some (x). He now has 9 marbles less than Peter. The equation $4-x=(\text{number of Peter's marbles}-9)$ could be part of solving the problem."

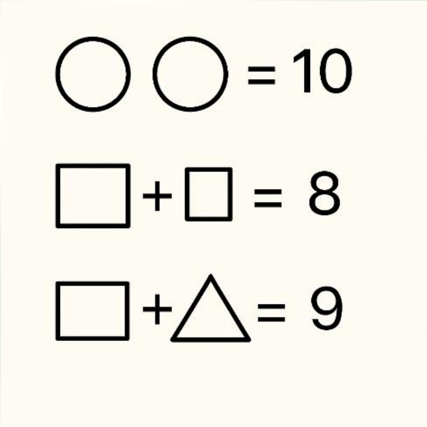
Story 2: "A length of 4 meters was reduced by x meters. The remaining length is 9 meters shorter than a certain target length. The equation $4-x=(\text{target length}-9)$ could be used."

Example 2: Solving a puzzle with equations (from the figure)

(Since I don't have the figure, I'll create a similar puzzle concept).

Puzzle: Find the value of each shape.

- Circle + Circle = 10 $\Longrightarrow 2\times\text{Circle}=10 \Longrightarrow \text{Circle}=5$
- Circle + Square = 8 $\Longrightarrow 5+\text{Square}=8 \Longrightarrow \text{Square}=3$
- Square + Triangle = 9 $\Longrightarrow 3+\text{Triangle}=9 \Longrightarrow \text{Triangle}=6$



Question 37.1: Create a word problem that could be represented by the equation $y+6=15$.

Question 37.2: Create a word problem that could be represented by the equation $3z=21$.

Strand 3: Geometry and Measurement

Sub-strand 1: Shapes and Space

We will learn about prisms, identify examples, and construct them from their nets.

Topic 38: Identifying Rectangular and Triangular Prisms

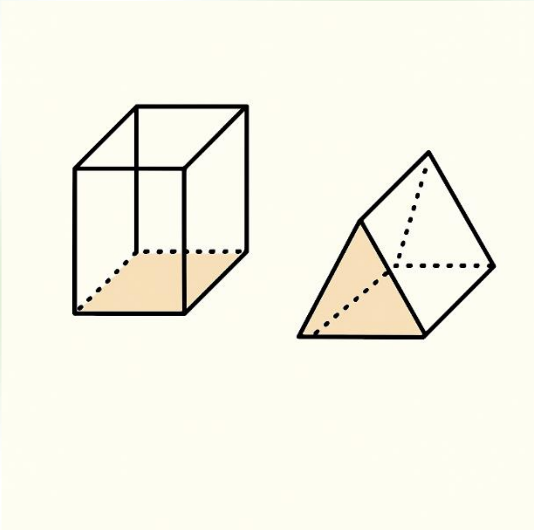
A **prism** is a 3D shape with two identical ends (bases) that are parallel polygons, and its sides (lateral faces) are parallelograms. Prisms are named after the shape of their bases.

- A **rectangular prism** has rectangular bases. Think of a cereal box, a book, or a brick. Its cross-section parallel to the bases is always a rectangle.
- A **triangular prism** has triangular bases. Think of some tents or the shape of a Toblerone chocolate box. Its cross-section parallel to the bases is always a triangle.

Activity 1: Look around the classroom and your community. Can you find any objects that are rectangular prisms? How about triangular prisms?

Example from the content: Examining cardboard 3D shapes and their cross-sections.

If we cut a rectangular prism parallel to its rectangular bases, the cross-section will also be a rectangle. If we cut a triangular prism parallel to its triangular bases, the cross-section will also be a triangle.



Question 38.1: Name three objects in your classroom that are rectangular prisms.

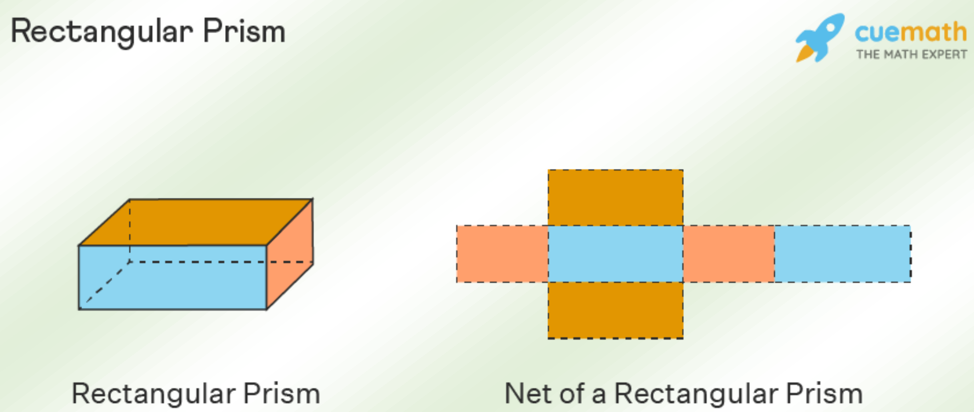
Question 38.2: Can you think of an object in your community that is a triangular prism?

Topic 39: Constructing Prisms from Nets

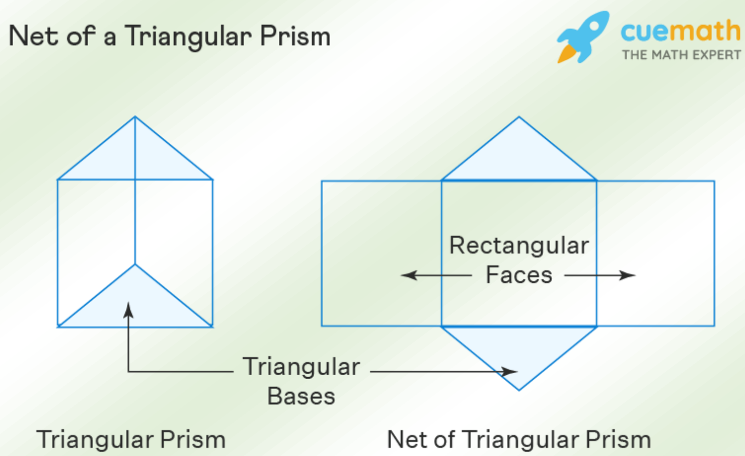
A **net** is a 2D shape that can be folded to form a 3D shape.

Rectangular Prism Net: A net of a rectangular prism typically consists of six rectangles. For a square prism (where the bases are squares), the net has two squares and four identical rectangles.

Triangular Prism Net: A net of a triangular prism consists of two triangles (the bases) and



three rectangles (the sides).



Activity 2:

1. Take a rectangular or triangular prism made of cardboard and carefully open it up to see its net. Draw the net.
2. Draw a net of a square prism with square bases of 10cm by 10cm and a height of 10cm. Include tabs for gluing.

3. Draw a net of a rectangular prism with square bases of 10cm by 10cm and a height of 15cm. Include tabs for gluing.
4. Draw a net of a triangular prism with equilateral triangle bases of side 10cm and a height of 10cm. Include tabs for gluing.

(Indication for Image: Examples of the nets asked to be drawn in Activity 2.)

Question 39.1: How many rectangular faces does a triangular prism have?

Question 39.2: Describe the shapes that make up the net of a square prism.

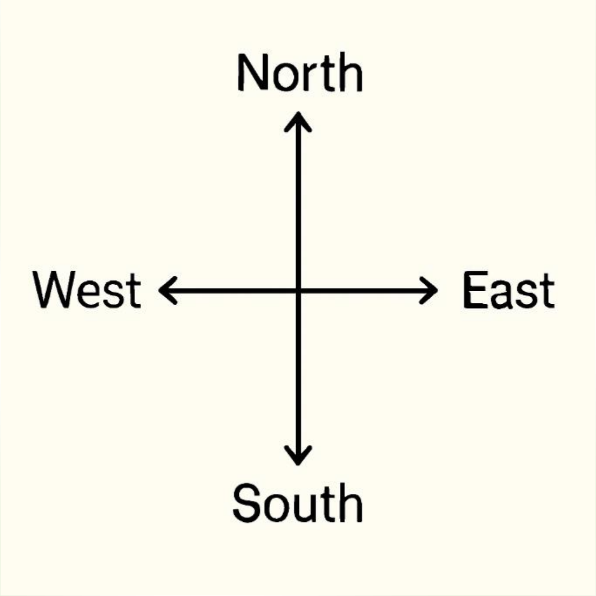
Sub-strand 3: Geometric Reasoning

We will learn to describe the position of objects using cardinal points and perform single transformations on 2D shapes.

Topic 40: Describing Position Using Cardinal Points

We use cardinal points (North, South, East, West) and their combinations (Northeast, Northwest, Southeast, Southwest) to describe the position and movement of objects.

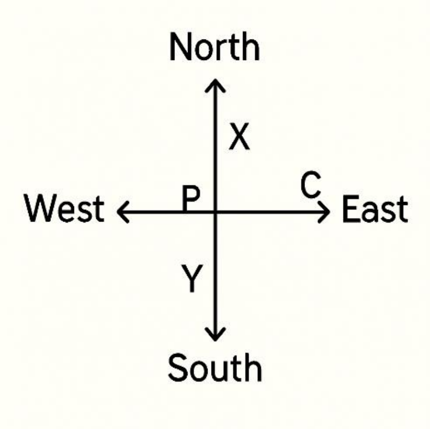
- **North-east:** Up and to the right.



- **North-west:** Up and to the left.
- **South-east:** Down and to the right.
- **South-west:** Down and to the left.

Example 1: If point X is directly north of point P, and point C is to the east of point P, then point P is south of X and south-west of C.

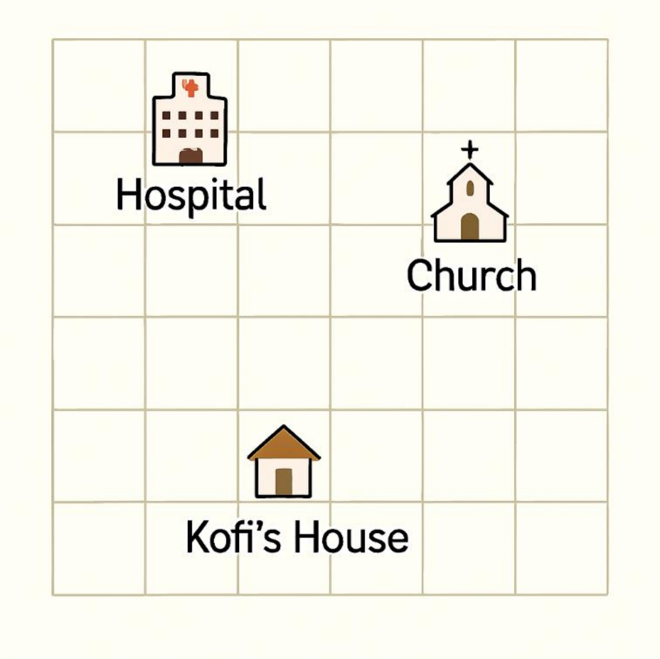
If point Y is directly south of point T, and point C is to the north-east of Y, then Y is north of T and south-west of C.



Example 2: Consider Kofi's house on a grid. If the church is two squares to the east and two squares to the north of Kofi's house, we can say the church is northeast of Kofi's house. If the hospital is three squares to the west and one square to the north, it's northwest of Kofi's house.

To get from Kofi's house to the church, you would move 2 squares to the east and 2 squares to the north.

Question 40.1: If a school is directly east of a market, what is the market's position relative to



the school?

Question 40.2: Describe the location of a point that is southeast of your current position.

Topic 41: Performing and Identifying Reflections

A **reflection** is a transformation that flips a shape over a line, called the **line of reflection** or the **mirror line**. The reflected image is a mirror image of the original shape.

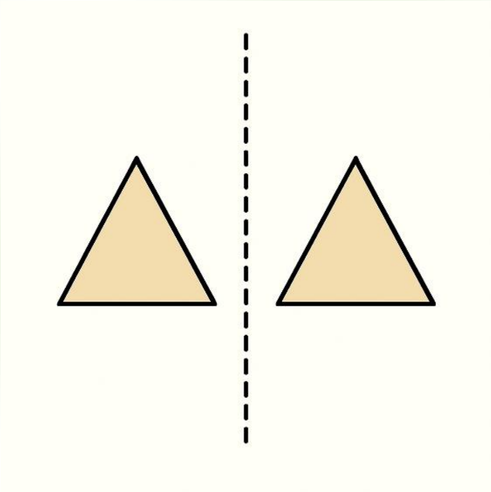
Properties of Reflection:

- The size and shape of the object and its image are the same.
- The distance from any point on the object to the mirror line is the same as the distance from the corresponding point on the image to the mirror line.
- The line joining a point on the object to its corresponding point on the image is perpendicular to the mirror line.

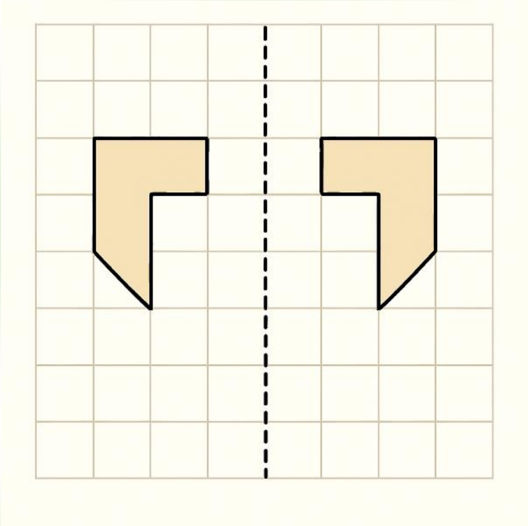
Activity 1: Imagine a shape drawn on paper. If you fold the paper along a line (the mirror line), the image of the shape will overlap perfectly with its reflection.

Example 1: Consider a triangle reflected over a vertical line. Each point of the triangle will have a corresponding point on the other side of the line, at the same distance.

Activity 2: On a grid, draw a simple 2D shape and a line of reflection. Draw the reflected image



of the shape. For each point on the original shape, find its corresponding point on the reflected image such that the mirror line is the perpendicular bisector of the segment joining the two points.



Question 41.1: If you reflect the letter 'b' over a vertical line, what letter does its image look like?

Question 41.2: Draw a square on a grid and reflect it over a horizontal line that does not pass through the square. What does the image look like?

Topic 42: Understanding Translation

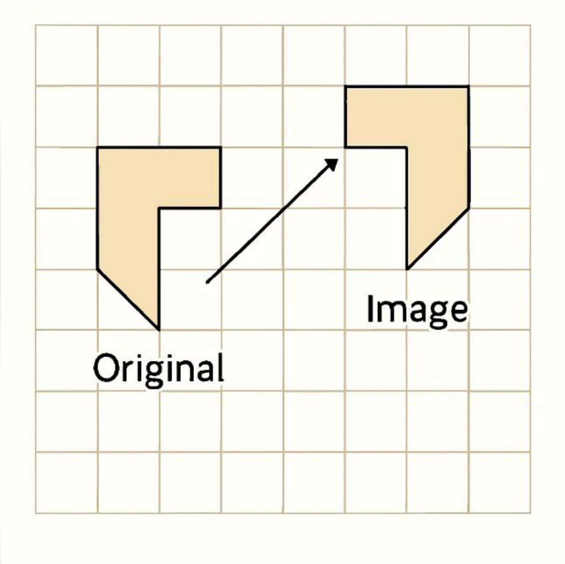
(Part of Indicator B6.3.3.5.1, focusing on translation)

A **translation** is a transformation that moves every point of a shape the same distance in the same direction. We often describe a translation using a **translation vector**, which tells us how far to move horizontally and vertically.

A translation vector is written as (ab), where a is the horizontal movement (positive for right, negative for left) and b is the vertical movement (positive for up, negative for down).

Example 3: A translation by the vector (4−1) means moving every point of the shape 4 units to the right and 1 unit down.

If point P has coordinates (1, 2) and it's translated by (4−1), its image Q will have coordinates (1+4,2+(−1))=(5,1).



Properties of Translation:

- The size and shape of the object and its image are the same.
- Every point of the object moves the same distance in the same direction.
- Lines in the object are parallel to their corresponding lines in the image.

Activity 3: On a grid, draw a 2D shape and a translation vector. Draw the image of the shape after the translation.

Question 42.1: If a triangle with vertices (0, 0), (1, 0), and (0, 1) is translated by the vector (23), what are the coordinates of the vertices of the image?

Question 42.2: Describe a translation that moves a shape 5 units to the left and 2 units up. Write the translation vector.

Strand 4: Data

Sub-strand 1: Data Collection, Organization, Presentation, Interpretation and Analysis

We will learn how to create, label, and interpret line graphs to draw conclusions from data.

Topic 45: Drawing Line Graphs

A **line graph** is useful for showing how something changes over time or in relation to something else. To draw a line graph, we need:

1. **Title:** A descriptive title that tells us what the graph is about.
2. **Axes:**
    - A horizontal axis (x-axis) which usually represents time or the independent variable.
    - A vertical axis (y-axis) which usually represents the quantity being measured or the dependent variable.
3. **Labels:** Clear labels for both axes, including the units of measurement.
4. **Intervals:** Appropriate and consistent intervals marked along each axis to represent the scale of the data.
5. **Points:** Plot the data points corresponding to the values in the table.
6. **Lines:** Connect the plotted points with straight lines.

Example: Temperature of water increasing with time.

Time (minutes)	0	1	2	3	4	5	6	7	8	9	10	11
Temperature (°C)	15	25	35	45	53	60	70	75	80	85	90	95

To draw this, we would:

- Title: "Temperature of Water Over Time"
- Horizontal axis: Time (minutes), with intervals of 1 minute.
- Vertical axis: Temperature (°C), with suitable intervals (e.g., 10°C).
- Plot each point (0, 15), (1, 25), (2, 35), and so on.
- Connect the points with lines.

(Indication for Image: A line graph drawn using the data above, with all attributes labeled.)

Questions based on the graph:

- After what time will the water temperature reach 70°C? (Answer: 6 minutes)
- What is the water temperature after 5 minutes? (Answer: 60°C)

Activity: Given the following data, determine the attributes and draw a line graph:

Day	1	2	3	4	5
Height(cm)	2	4	6	8	10

Topic 46: Discrete vs. Continuous Data for Line Graphs

- **Continuous data** can take on any value within a range (e.g., height, temperature, time). Line graphs are suitable for continuous data because the lines connecting the points suggest the values between the measured points.
- **Discrete data** can only take on specific, separate values (e.g., number of students, number of items). While you can plot discrete data on a graph, connecting the points with a line might not always be meaningful, as there are no values between the plotted points. Sometimes, a series of points or a bar graph is more appropriate for discrete data.

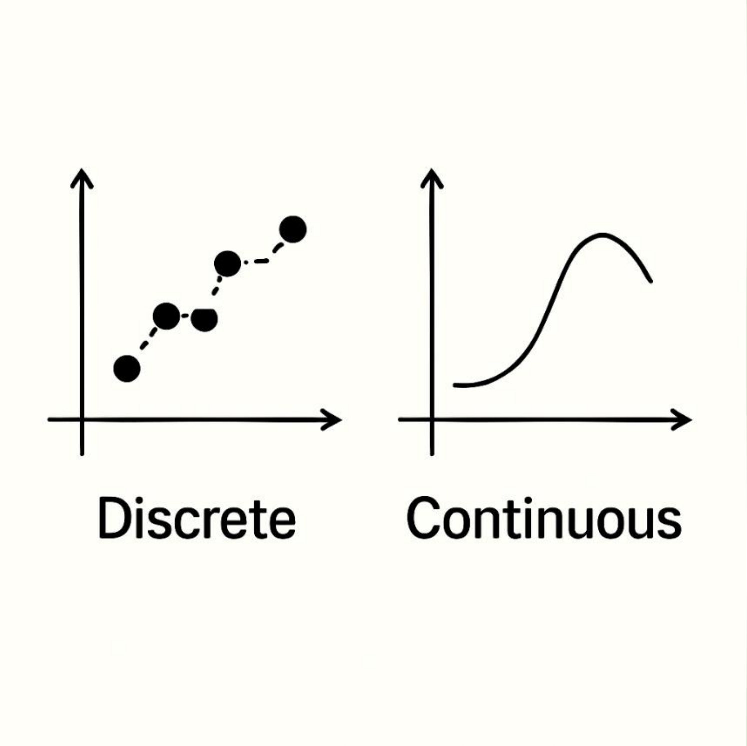
Example 1:

(i) Number of matchsticks vs. number of squares: This is **discrete** data because you can only have a whole number of squares and a corresponding number of matchsticks. A line graph *can* be drawn by connecting the points, but the intermediate points on the line would not represent actual scenarios.

(ii) Plant's growth over a week (height in cm): This is **continuous** data because the plant's height can take any value within the range over time. A line graph is suitable here.

(iii) Number of people in a family vs. number of students: This is **discrete** data. A bar graph would likely be more appropriate than a line graph.

(iv) Abu's distance from home over time: This is **continuous** data (distance and time). A line graph is suitable.



Activity: Categorize the following data sets as discrete or continuous and explain why:

1. Number of cars passing a point each hour.
2. The weight of a growing puppy recorded daily.

Topic 47: Interpreting Line Graphs

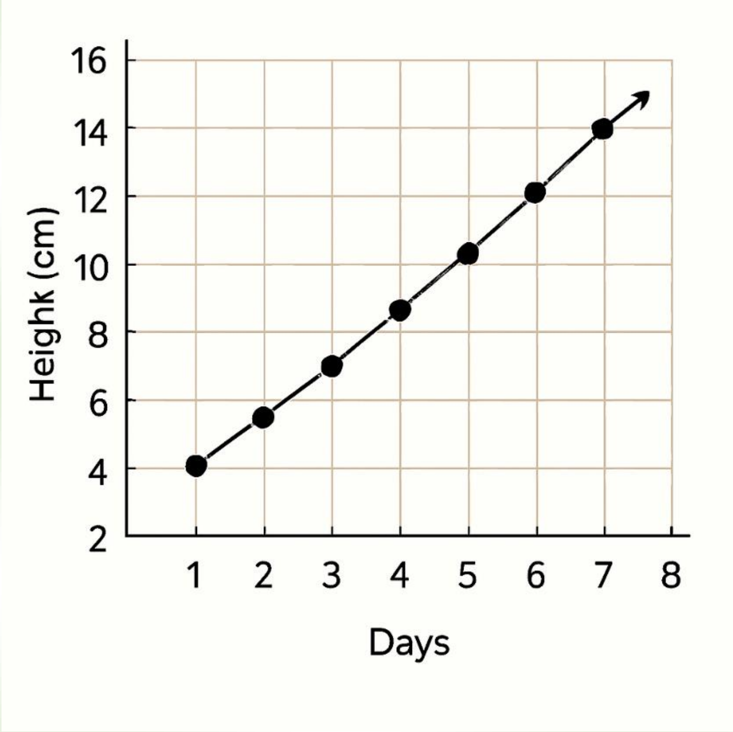
Once a line graph is drawn, we can interpret it to understand trends, make predictions, and draw conclusions.

Example: Using the plant's growth data:

Days (d)	1	2	3	4	5	6	7	8
Height (h) (cm)	5	6.5	8	9.5	11	12.5	14	15.5

We can draw a line graph and then answer questions like:

- What was the height of the plant on day 4? (Answer: 9.5 cm)
- Between which days did the plant's growth rate appear to be the fastest? (We'd look for the steepest part of the line).
- Predict the approximate height on day 9 (by extending the trend).



Activity: Draw line graphs for the other data sets provided in the content (cost of lunches, Abu's travel, rainfall) and create questions for a friend to interpret them.

Question 47.1: Look at the water temperature graph again. Between which minutes did the temperature increase the most?

Question 47.2: For Abu's bicycle travel graph, at what time was he furthest from home?

Topic 48: Selecting and Justifying Data Collection Methods

There are several ways to collect data, and the best method depends on the question we want to answer. Some common methods include:

- **Questionnaires:** A set of written questions given to people to answer. Useful for collecting opinions, preferences, and factual information from a large group.

- **Interviews:** Asking questions directly to individuals, either face-to-face or remotely. Good for in-depth understanding and exploring perspectives.
- **Observation:** Watching and recording what happens in a situation. Useful for studying behavior or events.
- **Experiments:** Conducting controlled procedures to test hypotheses and gather data on cause and effect.
- **Databases:** Using existing collections of organized data. Efficient for accessing large amounts of information.
- **Electronic media/Internet:** Collecting data through online surveys, social media, or information available on the internet. Useful for reaching a wide audience or gathering digital information.

Example 1: Situations and Potential Data Collection Methods

(a) The type of drinks to buy for a class party:

* Method: **Questionnaire** (ask students their favorite drinks) or **Interview** (ask a smaller sample).

* Justification: To find out the preferences of the students who will be attending.

(b) The make of sport shoes to buy for all P6 students:

* Method: **Questionnaire** (ask about preferred brands, comfort) or **Interview** (discuss needs and preferences).

* Justification: To understand what factors (brand, comfort, price) are important to the students.

(c) The make of school bag to buy for all P6 students:

* Method: **Questionnaire** (ask about desired features, durability) or **Observation** (of current school bags).

* Justification: To identify important features and preferences for school bags.

(d) The number of desks in each classroom:

* Method: **Observation** (go and count).

* Justification: This is a direct count of a physical object.

(e) How much money P6 students spend on bus fare to school every month?

* Method: **Questionnaire** or **Interview**.

* Justification: To gather specific financial information from the students.

(f) To buy drinks for people in the immediate family of all P6 students at a party.

* Method: **Questionnaire** (ask each student about their family's drink preferences).

* Justification: To cater to the tastes of all attendees.

(g) Buy a mobile phone from an online shop:

* Method: **Electronic media/Internet** (read reviews, compare specifications).

* Justification: To gather information available online before making a purchase.

Activity: For each of the situations above, discuss in a small group and decide on the best method of data collection and why.

Question 48.1: You want to find out the favorite colors of people in your community. Which data collection method would you choose and why?

Question 48.2: You want to know if a new type of fertilizer helps plants grow faster. Which data collection method would you use and why?

Topic 49: Designing and Administering a Questionnaire

A **questionnaire** is a tool for collecting data by asking a set of questions. When designing a questionnaire, it's important to:

- Have a clear purpose.
- Use clear and simple language.
- Ask questions that are easy to understand and answer.
- Consider the types of questions (e.g., multiple choice, open-ended).
- Organize the questions logically.

Example 1: Class Survey Question Form

1. Hello, What is your name? __________________
2. How old are you? __________________
3. What is your favourite school subject? __________________
4. What is your worst subject? __________________
5. What is the most important school subject? __________________
6. The size of your shoe? __________________
7. What is your favourite drink? __________________
8. How much do you spend on bus fare to school every day? __________________

After administering the questionnaire (giving it to people to answer), the results need to be recorded and organized.

Name	Age	Favourite Subject	Worst Subject	Important Subject	Shoe Size	Favourite Drink	Daily Bus Fare (¢)
Kojo	12	English	Maths	English	6	Coke	2
Aku	11	Science	P.E.	English	6	Coke	1
Ami	11	Maths	Art	Maths	6	Fanta	3
Abu	13	Maths	Art	Maths	7	Sprite	1
Ama	12	Science	Art	Maths	7	Fanta	2
Paapa	11	Maths	P.E.	Science	6	Fanta	1

Activity: In small groups, design a short questionnaire (3-4 questions) to find out the favorite snacks of students in your class. Administer it to your classmates and record the results in a table.

Question 49.1: What is an advantage of using a questionnaire to collect data?

Question 49.2: What are some things to keep in mind when designing a questionnaire?

Topic 50: Analyzing and Graphing Questionnaire Data

After collecting data using a questionnaire and organizing it in a table, the next steps are to analyze the data and present it visually using graphs.

Step 1: Create Frequency Tables

A **frequency table** shows how many times each answer occurs for a particular question. We can use tallies to help count the occurrences.

Example: Using the class survey data from the previous topic, let's create a frequency table for "Favourite drink":

Favourite drink	**Tally**	**Frequency**
Coke	II	2
Fanta	III	3
Sprite	I	1

And for "Shoe size":

Shoe size	**Tally**	**Frequency**
6	III	3
7	II	2

Step 2: Draw Graphs

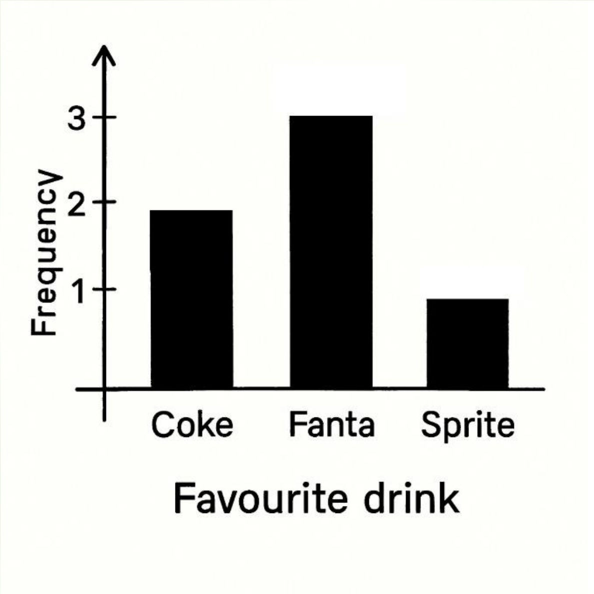
Favourite drink	Tally	Frequency
Coke	II	2
Fanta	III	3
Sprite	I	1

Shoe size	Tally	Frequency
6	III	3
7	II	2

We can use bar graphs or line graphs to represent the data from the frequency tables.

- **Bar graphs** are good for comparing frequencies of different categories (like favourite drinks or shoe sizes).
- **Line graphs** are more suitable for showing trends over time or continuous data (though we discussed that shoe size is discrete, a line graph *could* technically be drawn, but a bar graph is more typical).

Example: A bar graph for "Favourite drink" would have the drinks on one axis and the frequency on the other, with bars of different heights representing the number of students who chose each drink.



Step 3: Draw Conclusions and Solve Problems

By looking at the graphs, we can draw conclusions and solve problems.

Example: From the "Favourite drink" bar graph, we can see that Fanta is the most popular drink among the surveyed students. If we were buying drinks for a class party based on this data, we might buy more Fanta.

Activity: Using the data you collected on favorite snacks, create a frequency table and then draw a bar graph to represent the data. What is the most popular snack in your class based on your survey?

Example from the content:

Let's consider situation (a): "The type of drinks to buy for a class party."

1. **Questionnaire:** Ask students: "What is your favourite drink?" (Provide options like Coke, Fanta, Sprite, Water, etc.)
2. **Collect Data:** Record their responses.
3. **Frequency Table:** Count how many students chose each drink.
4. **Bar Graph:** Draw a bar graph showing the number of votes for each drink.
5. **Conclusion/Problem Solving:** The drink with the highest bar is the most popular, and you would likely buy more of that for the party.

Question 50.1: What type of graph is most suitable for displaying the frequency of different shoe sizes in a class? Why?

Question 50.2: After conducting a survey on favourite fruits and creating a bar graph, how would you determine which fruit is the most liked?

Sub-strand 2: Chance or Probability

We will learn to identify possible outcomes of probability experiments and determine theoretical and experimental probabilities.

Topic 51: Possible Outcomes and Theoretical Probability

Probability is the measure of how likely an event is to occur.

The **possible outcomes** are all the things that can happen in a probability experiment.

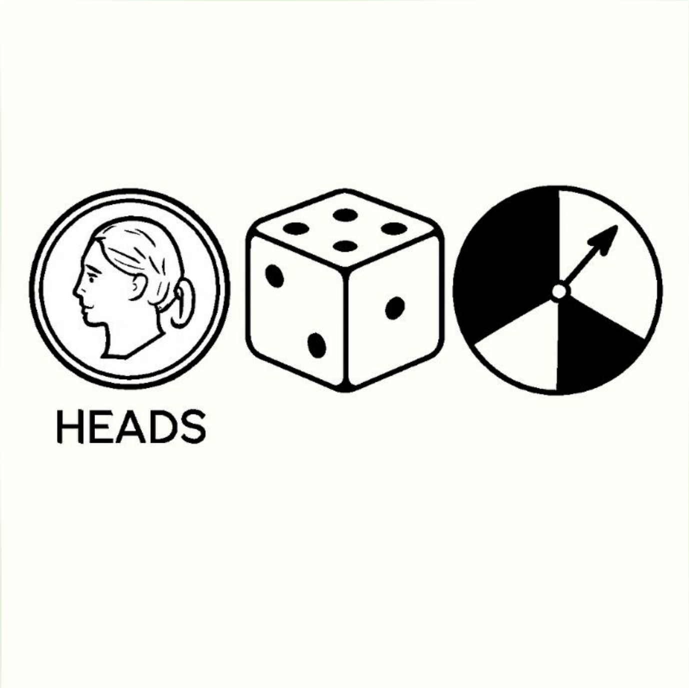
Theoretical probability is what we expect to happen based on the nature of the experiment. It is calculated as:

Theoretical Probability of an event=Total number of possible outcomesNumber of favourable outcomes

Examples:

1. **Tossing a coin:**
   - Possible outcomes: Head (H), Tail (T) - Total 2 outcomes.
   - Theoretical probability of getting a Head: $\frac{1}{2}$.
   - Theoretical probability of getting a Tail: $\frac{1}{2}$.
2. **Rolling a standard 6-sided die:**
   - Possible outcomes: 1, 2, 3, 4, 5, 6 - Total 6 outcomes.
   - Theoretical probability of rolling a 2: $\frac{1}{6}$.
   - Theoretical probability of rolling a number greater than 4 (i.e., 5 or 6): $\frac{2}{6}=\frac{1}{3}$.
   - Theoretical probability of rolling a 1 or a 3: $\frac{2}{6}=\frac{1}{3}$.
3. **Spinning a spinner with 4 equal sectors (Red, Blue, Green, Yellow):**
   - Possible outcomes: Red, Blue, Green, Yellow - Total 4 outcomes.

- Theoretical probability of landing on Blue: $\frac{1}{4}$.



Activity: What are the possible outcomes when you roll an 8-sided die (numbered 1 to 8)? What is the theoretical probability of rolling a 5?

Topic 52: Experimental Probability

Experimental probability is what actually happens when we conduct a probability experiment many times. It is calculated as:

Experimental Probability of an event = Total number of trials Number of times the event occurs

Example: If you toss a coin 100 times and get Heads 42 times, the experimental probability of getting a Head is $\frac{42}{100}$=0.42.

The theoretical probability of getting a Head is $\frac{1}{2}$= 0.5. Notice that the experimental probability might not be exactly the same as the theoretical probability, especially with a small number of trials. However, as the number of trials increases, the experimental probability tends to get closer to the theoretical probability.

Activity: In a group, toss a coin 50 times and record the number of heads and tails. Calculate the experimental probability of getting a head. How does it compare to the theoretical probability?

Topic 53: Comparing Theoretical and Experimental Probability

As we discussed, theoretical probability is what we expect, and experimental probability is what we observe.

Example from the content:

If in 100 coin tosses, Heads appeared 42 times, the experimental probability of Heads is $\frac{42}{100}$ =0.42, while the theoretical probability is $\frac{50}{100}$=0.5.

For rolling a die 100 times, we can look at the frequency of each outcome in a table. Let's say the results were:

Outcome	Frequency
1	15
2	18
3	16
4	17
5	19
6	15
Total	100

- Experimental probability of rolling a 2: $\frac{18}{100} = 0.18$. Theoretical probability: $\frac{1}{6} \approx 0.167$.
- **1. Rolling a number greater than 4 (i.e., 5 or 6)**
- Experimental count: **19 + 15 = 34**
  Total trials: **100**
- **Experimental probability:**
- $\frac{34}{100} = 0.34 = \frac{17}{50}$
- **Theoretical probability:**
  There are **2 favourable outcomes (5, 6)** out of **6**.
- $\frac{2}{6} = \frac{1}{3} \approx 0.333$
-

- **2. Rolling a 1 or a 3**
- Experimental count: **15 + 16 = 31**
  Total trials: **100**
- **Experimental probability:**

- $\frac{31}{100} = 0.31$
- **Theoretical probability:**
  There are **2 favourable outcomes (1, 3)** out of **6**.
- $\frac{2}{6} = \frac{1}{3} \approx 0.333$
  As you can see, the experimental probabilities are close to the theoretical probabilities, and they tend to get closer with more trials.

Activity: Combine the results from all the groups in your class for the coin toss experiment (from the previous activity). Calculate the overall experimental probability of getting a head. Is it closer to the theoretical probability of 0.5 than your individual group's result?

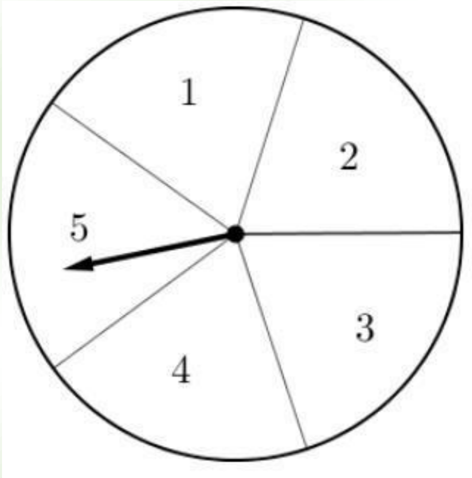
Question 53.1: What is the difference between theoretical and experimental probability?

Question 53.2: If you flip a fair coin 10 times and get 7 heads, what is the experimental probability of getting a head? What is the theoretical probability?

Topic 54: Predicting Probability Using Theoretical Probability

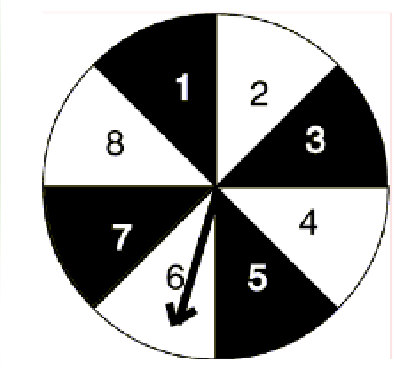
We can use theoretical probability to predict how likely an outcome is before we even conduct an experiment.

Example 1: Spinning a 5-sector spinner (numbered 1 to 5)



- Possible outcomes: 1, 2, 3, 4, 5 (Total 5 outcomes)
- Theoretical probability of pinning a 2: $\frac{1}{5}$
- Theoretical probability of pinning a number greater than 4 (i.e., 5): $\frac{1}{5}$
- Theoretical probability of pinning a 1 or a 3: $\frac{2}{5}$

Example 2: Spinning an 8-sector spinner (numbered 1 to 8)



- Possible outcomes: 1, 2, 3, 4, 5, 6, 7, 8 (Total 8 outcomes)
- Theoretical probability of pinning a 2: $\frac{1}{8}$
- Theoretical probability of pinning a number greater than 4 (i.e., 5, 6, 7, or 8): $\frac{4}{8} = \frac{1}{2}$
- Theoretical probability of pinning a 1 or a 3: $\frac{2}{8} = \frac{1}{4}$

Activity: What is the theoretical probability of rolling an even number on a standard 6-sided die?

Topic 55: Experimental Probability Approaching Theoretical Probability

The **Law of Large Numbers** states that as the number of trials in a probability experiment increases, the experimental probability of an event gets closer and closer to its theoretical probability.

Example: If we combine the results of multiple groups spinning a 5-sector spinner 100 times each, the overall experimental probabilities for pinning a 2, a number greater than 4, or a 1 or 3 will likely be closer to their theoretical probabilities (51, 51, and 52 respectively) than the results from a single group's 100 spins. The same would apply to the 8-sector spinner.

Activity: Discuss why combining the results from the whole class might give a better estimate of the true probability than your individual group's experiment.

Question 55.1: Explain in your own words why the experimental probability might be different from the theoretical probability in a small number of trials.

Question 55.2: What happens to the experimental probability as you repeat an experiment many, many times?