

# MATHEMATICS

## FOR BASIC 5



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# Strand 1: Number

Sub-strand 1: Counting, Representation & Cardinality

Welcome, young mathematicians! In this section, we will explore big numbers, all the way up to one million! We will learn how to count them, show them in different ways, and understand what each digit means. Get ready for an exciting journey into the world of large numbers!

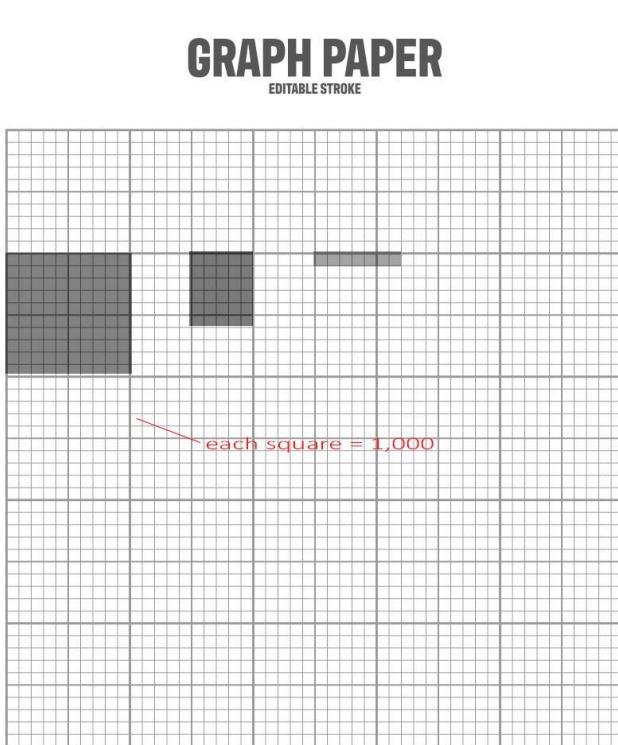
## Topic 1: Modeling Number Quantities Up to 1,000,000

We can use different tools to help us understand how much a number represents.

### 1. Using Graph Sheets:

Imagine a small square on a graph sheet represents 1,000 units. To show a larger number, we would shade many of these squares.

**Example:** To model the number 137,000, if each 1cm × 1cm square



represents 1,000 units, we would need to shade 137 of these squares.

**Question 1.1:** If each square on a graph sheet represents 1,000, how many squares would you need to shade to represent 250,000?

## 2. Using Multi-Base Blocks:

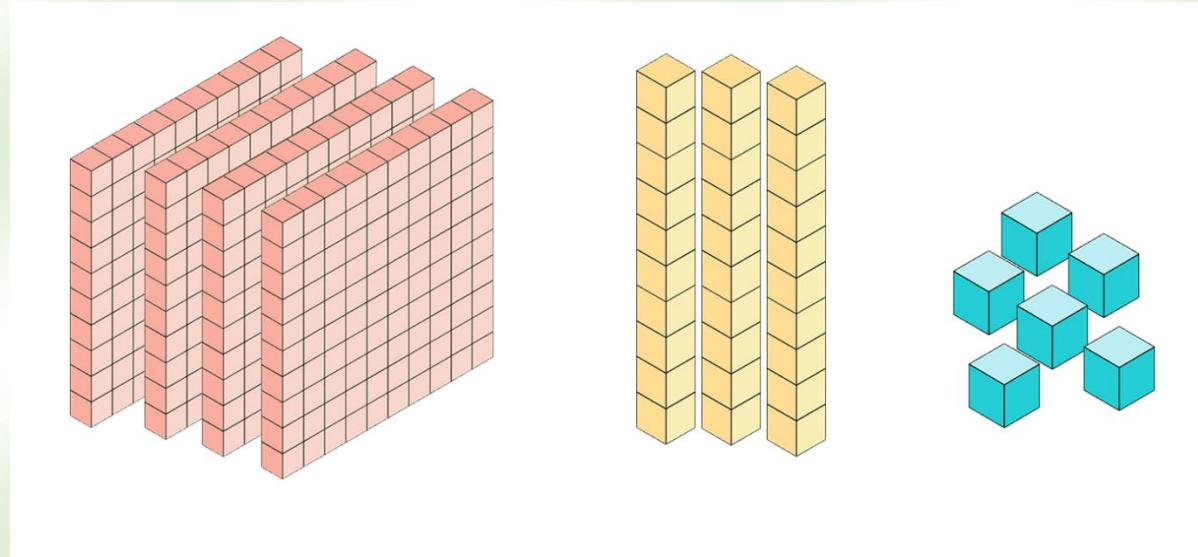
Let's say:

- A large cube = 1,000 units
- A rod made of 10 large cubes =  $10 \times 1,000 = 10,000$  units
- A flat made of 10 rods of 10 large cubes =  $10 \times 10,000 = 100,000$  units
- A large block made of 10 flats of 10 rods of 10 large cubes =  $10 \times 100,000 = 1,000,000$  units

**Example:** How would we model 436,000 using these blocks?

- We need 4 flats, each representing 100,000:  $4 \times 100,000 = 400,000$
- We need 3 rods, each representing 10,000:  $3 \times 10,000 = 30,000$
- We need 6 large cubes, each representing 1,000:  $6 \times 1,000 = 6,000$

So, to model 436,000, we would use 4 flats, 3 rods, and 6 large cubes.



**Question 1.2:** How would you model the number 720,000 using multi-base blocks (flats, rods, and large cubes)?

### 3. Using Token Currency Notes:

Let's imagine we have special play money: ₦10 notes, ₦100 notes, and ₦500 notes. How can we use these to show a certain amount up to ₦10,000?

**Example:** How many of each note would you need to model ₦23,480?

- Thousands: To get to ₦23,000, we can use 23 of the ₦1000 notes (if we had them), or we can think about it with our notes:
  - ₦500 notes:  $23,000 \div 500 = 46$  notes
- Hundreds: We need ₦400 more, which is four ₦100 notes.
- Tens: We need ₦80 more, which is eight ₦10 notes.

So, ₦23,480 can be modeled with forty-six ₦500 notes, four ₦100 notes, and eight ₦10 notes.

**Question 1.3:** Using ₦100 and ₦10 notes, how would you model an amount of ₦1,350?

## Topic 2: Reading and Writing Numbers Up to 1,000,000

Numbers can be written in figures (like 2,524,513) and in words (like two million, five hundred and twenty-four thousand, five hundred and thirteen).

### 1. Place Value:

To read and write large numbers, we need to understand place value. For a number up to 1,000,000, we have the following places:

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
?	?	?	?	?	?	?

**Example:** In the number 2,524,513:

- The digit 2 is in the millions place, so it represents 2,000,000.
- The digit 5 is in the hundred thousands place, so it represents 500,000.
- The digit 2 is in the ten thousands place, so it represents 20,000.
- The digit 4 is in the thousands place, so it represents 4,000.
- The digit 5 is in the hundreds place, so it represents 500.
- The digit 1 is in the tens place, so it represents 10.
- The digit 3 is in the ones place, so it represents 3.

### 2. Expanded Form:

We can write a number by adding the value of each of its digits.

**Example:** The expanded form of 2,524,513 is:

$$(2 \times 1,000,000) + (5 \times 100,000) + (2 \times 10,000) + (4 \times 1,000) + (5 \times 100) + (1 \times 10) + (3 \times 1) = 2,000,000 + 500,000 + 20,000 + 4,000 + 500 + 10 + 3$$

### 3. Reading Numbers:

We read large numbers by grouping the digits into threes from the right. Each group has a name (ones, thousands, millions, etc.).

**Example:** The number 2,524,513 is read as "two million, five hundred and twenty-four thousand, five hundred and thirteen."

Millions	Hundred Thousands	Ten Thousands	Thousands	Hundreds	Tens	Ones
2	5	2	4	5	1	3

**Question 2.1:** Write the number 805,321 in words.

**Question 2.2:** Write "one million, twenty thousand, and fifty" in figures.

**Question 2.3:** What is the value of the digit 7 in the number 476,190?

### Topic 3: Identifying Numbers in a Number Chart

A number chart can help us see the relationship between numbers.

**Example:** Consider this number chart (in multiples of 1,500):

10,000	11,500	13,000	14,500	16,000	17,500
20,000	21,500	23,000	24,500	26,000	27,500
30,000	31,500	33,000	34,500	36,000	37,500
40,000	41,500	43,000	44,500	46,000	47,500
50,000	51,500	53,000	54,500	56,000	57,500
60,000	61,500	63,000	64,500	66,000	67,500

If we pick the number 33,000, we can see the numbers around it:

- To the left: 31,500 (1,500 less)
- To the right: 34,500 (1,500 more)
- Above: 23,000 (10,000 less)
- Below: 43,000 (10,000 more)

10,000	11,500	13,000	14,500	16,000	17,500
20,000	21,500	23,000	24,500	26,000	27,500
30,000	31,500	33,000	34,500	36,000	37,500
40,000	41,500	43,000	44,500	46,000	47,500
50,000	51,500	53,000	54,500	56,000	57,500
60,000	61,500	63,000	64,500	66,000	67,500

**Question 3.1:** In the number chart above, what number is directly to the right of 23,000?

**Question 3.2:** What number is directly below 13,000 in the chart?

**Question 3.3:** If there was a number to the left of 10,000 in the same pattern, what would it be?

## Topic 4: Comparing and Ordering Whole Numbers Up to 100,000

We use symbols to compare numbers:

- '**>**' means "greater than"
- '**<**' means "less than"
- '**=**' means "equal to"

To compare numbers, we start by looking at the digits in the highest place value.

**Example 1:** Compare 132,734 and 132,635.

Both numbers have the same digits in the hundred thousands, ten thousands, and thousands places (132,000). So, we look at the hundreds place. 7 hundreds is greater than 6 hundreds. Therefore,  
**132,734>132,635.**

**Example 2:** Order the numbers 1020, 1025, 2673, 2873 in ascending order (from smallest to largest).

Comparing them, we get: 1020, 1025, 2673, 2873.

**Example 3:** What can you say about the numbers 102,345 and 102,534?

- 102,345 is less than 102,534 ( $102,345 < 102,534$ ).
- 102,534 is greater than 102,345 ( $102,534 > 102,345$ ).
- The hundred thousands, ten thousands, and thousands digits are the same.
- In the hundreds place, 3 is less than 5.
- The numbers are quite close, with a difference of  $102,534 - 102,345 = 189$ . So, 102,345 is almost 200 less than 102,534.

**Question 4.1:** Use the symbols  $>$ ,  $<$ , or  $=$  to compare the following pairs of numbers: a) 56,789 and 56,987 b) 100,000 and 99,999

**Question 4.2:** Arrange the following numbers in descending order (from largest to smallest): 23,450; 23,054; 24,350; 24,035.

## Topic 5: Rounding Whole Numbers Up to 100,000

Rounding helps us to estimate numbers. We can round to the nearest ten, hundred, thousand, or ten thousand.

### Rules for Rounding:

1. Identify the place value you are rounding to.
2. Look at the digit to its right.
3. If the digit to the right is 5 or more, round up (add 1 to the digit in the rounding place).
4. If the digit to the right is less than 5, round down (keep the digit in the rounding place the same, and change all digits to the right to zero).

**Example 1:** Round 129,500 to the nearest 10,000.

- The ten thousands place has a '2'.
- The digit to its right (thousands place) is '9'.
- Since 9 is 5 or more, we round up the '2' to '3'.
- All digits to the right become zero.
- So, 129,500 rounded to the nearest 10,000 is 130,000.

**Example 2:** Round 19,100 to the nearest 1,000.

- The thousands place has a '9'.
- The digit to its right (hundreds place) is '1'.
- Since 1 is less than 5, we round down. The '9' stays the same, and digits to the right become zero.
- So, 19,100 rounded to the nearest 1,000 is 19,000.

### Rounding Up and Rounding Down (for Estimation):

Sometimes, we talk about rounding up (making the number slightly larger) or rounding down (making it slightly smaller) for estimations.

Consider the number 214,765:

Round to the Nearest	Round Up	Round Down	Round Off (Standard Rounding)
Ten	214,770	214,760	214,770
Hundred	214,800	214,700	214,800
Thousand	215,000	214,000	215,000

**Question 5.1:** Round the number 78,345 to the nearest thousand.

**Question 5.2:** Round the number 45,678 to the nearest hundred.

**Question 5.3:** What is 62,987 rounded up to the nearest ten thousand?  
What is it rounded down to the nearest ten thousand?

## Topic 6: Skip Counting Forwards and Backwards

Skip counting means counting by adding or subtracting the same number each time. We can skip count by 500s, 1000s, and other amounts.

### Example 1: Skip counting forwards in 500s

If we start at 15,290 and skip count by 500s, we get:

15,290, 15,790, 16,290, 16,790, 17,290, 17,790, 18,290, ...

Notice that we add 500 each time.

### Example 2: Skip counting forwards in 1000s

If we start at 31,285 and skip count by 1000s, we get:

31,285, 32,285, 33,285, 34,285, 35,285, 36,285, ...

Here, we add 1000 each time.

### Example 3: Identifying errors in skip counting

Let's skip count by 50s starting from 2,000:

2000, 2050, 2100, 2150, 2250, 2300, ...

What number is missing? You're right, 2200 is missing!

**Question 6.1:** Skip count forwards in 1000s starting from 45,670. Write down the next five numbers.

**Question 6.2:** Skip count backwards in 500s starting from 88,000. Write down the next four numbers.

**Question 6.3:** Here is a skip count by 200s: 12,500, 12,700, 12,900, 13,000, 13,300. What is the mistake?

## Topic 7: Understanding Roman Numerals up to C (100)

Roman numerals are a system of writing numbers using letters. Here are some basic Roman numerals you need to know:

- I = 1
- V = 5
- X = 10
- L = 50
- C = 100

Other numbers are made by combining these letters.

### Rules for Roman Numerals:

1. When a symbol appears after a larger (or equal) symbol, it is added. For example, VI =  $5 + 1 = 6$ , and XII =  $10 + 1 + 1 = 12$ .
2. When a smaller symbol appears before a larger symbol, it is subtracted. For example, IV =  $5 - 1 = 4$ , and IX =  $10 - 1 = 9$ .
3. You cannot have more than three of the same symbol in a row (like III = 3, but we don't write IIII for 4).
4. V, L, and C are usually not repeated.

### Example:

- III = 3
- IV = 4
- VI = 6
- IX = 9
- X = 10
- XV = 15
- XX = 20
- XXX = 30
- XL = 40 ( $50 - 10$ )
- L = 50
- LX = 60 ( $50 + 10$ )

- $XC = 90$  ( $100 - 10$ )
- $C = 100$

<b>1 I</b>	<b>21 XXI</b>	<b>41 XLI</b>	<b>61 LXI</b>	<b>81 LXXXI</b>
<b>2 II</b>	<b>22 XXII</b>	<b>42 XLII</b>	<b>62 LXII</b>	<b>82 LXXXII</b>
<b>3 III</b>	<b>23 XXIII</b>	<b>43 XLIII</b>	<b>63 LXIII</b>	<b>83 LXXXIII</b>
<b>4 IV</b>	<b>24 XXIV</b>	<b>44 XLIV</b>	<b>64 LXIV</b>	<b>84 LXXXIV</b>
<b>5 V</b>	<b>25 XXV</b>	<b>45 XLV</b>	<b>65 LXV</b>	<b>85 LXXXV</b>
<b>6 VI</b>	<b>26 XXVI</b>	<b>46 XLVI</b>	<b>66 LXVI</b>	<b>86 LXXXVI</b>
<b>7 VII</b>	<b>27 XXVII</b>	<b>47 XLVII</b>	<b>67 LXVII</b>	<b>87 LXXXVII</b>
<b>8 VIII</b>	<b>28 XXVIII</b>	<b>48 XLVIII</b>	<b>68 LXVIII</b>	<b>88 LXXXVIII</b>
<b>9 IX</b>	<b>29 XXIX</b>	<b>49 XLIX</b>	<b>69 LXIX</b>	<b>89 LXXXIX</b>
<b>10 X</b>	<b>30 XXX</b>	<b>50 L</b>	<b>70 LXX</b>	<b>90 XC</b>
<b>11 XI</b>	<b>31 XXXI</b>	<b>51 LI</b>	<b>71 LXXI</b>	<b>91 XCI</b>
<b>12 XII</b>	<b>32 XXXII</b>	<b>52 LII</b>	<b>72 LXXII</b>	<b>92 XCII</b>
<b>13 XIII</b>	<b>33 XXXIII</b>	<b>53 LIII</b>	<b>73 LXXIII</b>	<b>93 XCIII</b>
<b>14 XIV</b>	<b>34 XXXIV</b>	<b>54 LIV</b>	<b>74 LXXIV</b>	<b>94 XCIV</b>
<b>15 XV</b>	<b>35 XXXV</b>	<b>55 LV</b>	<b>75 LXXV</b>	<b>95 XCV</b>
<b>16 XVI</b>	<b>36 XXXVI</b>	<b>56 LVI</b>	<b>76 LXXVI</b>	<b>96 XCVI</b>
<b>17 XVII</b>	<b>37 XXXVII</b>	<b>57 LVII</b>	<b>77 LXXVII</b>	<b>97 XCVII</b>
<b>18 XVIII</b>	<b>38 XXXVIII</b>	<b>58 LVIII</b>	<b>78 LXXVIII</b>	<b>98 XCVIII</b>
<b>19 XIX</b>	<b>39 XXXIX</b>	<b>59 LIX</b>	<b>79 LXXIX</b>	<b>99 XCIX</b>
<b>20 XX</b>	<b>40 XL</b>	<b>60 LX</b>	<b>80 LXXX</b>	<b>100 C</b>

**Question 7.1:** What Hindu-Arabic number does the Roman numeral X represent?

**Question 7.2:** What Roman numeral represents the number 50?

**Question 7.3:** Identify the Roman numerals among these: 12, VII, 45, XL, 100, CC.

## Topic 8: Counting and Converting Between Hindu-Arabic and Roman Numerals

Now let's practice converting between our usual numbers (Hindu-Arabic) and Roman numerals.

### Example 1: Converting Hindu-Arabic to Roman Numerals

Let's convert 24 to a Roman numeral:

- 20 is XX
- 4 is IV
- So, 24 is XXIV.

Let's convert 60:

- 50 is L
- 10 is X
- So, 60 is LX.

Let's convert 94:

- 90 is XC
- 4 is IV
- So, 94 is XCIV.

### Example 2: Converting Roman Numerals to Hindu-Arabic

Let's convert LVI to a Hindu-Arabic number:

- L is 50
- V is 5
- I is 1
- So,  $LVI = 50 + 5 + 1 = 56$ .

Let's convert XCIX:

- XC is 90 ( $100 - 10$ )
- IX is 9 ( $10 - 1$ )
- So, XCIX =  $90 + 9 = 99$ .

## Hindu-Arabic

Hindu-Arabic	Roman Numeral
1	I
2	II
3	III
4	IV
5	V
6	VI
10	VII
50	IX
100	L
500	C
1000	D

**Question 8.1:** Convert the Hindu-Arabic number 35 to a Roman numeral.

**Question 8.2:** Convert the Roman numeral LXXXVII to a Hindu-Arabic number.

**Question 8.3:** Count from 1 to 10 using Roman numerals.

## Topic 9: Identifying Factors of Whole Numbers 1 - 100

Factors of a number are whole numbers that divide exactly into that number without leaving a remainder.

### Example 1: Factors of 24 using arrays

Imagine we have 24 objects. We can arrange them in different rectangular arrays:

- 1 row of 24:  $1 \times 24 = 24$ . So, 1 and 24 are factors.
- 2 rows of 12:  $2 \times 12 = 24$ . So, 2 and 12 are factors.
- 3 rows of 8:  $3 \times 8 = 24$ . So, 3 and 8 are factors.
- 4 rows of 6:  $4 \times 6 = 24$ . So, 4 and 6 are factors.

The factors of 24 are  $\{1, 2, 3, 4, 6, 8, 12, 24\}$ .

**Question 9.1:** What are the factors of 12? Use arrays in your mind or draw them if it helps.

**Question 9.2:** Is 5 a factor of 30? Why or why not?

**Question 9.3:** Find all the factors of 16.

## Topic 10: Generating and Identifying Prime and Composite Numbers

- A **prime number** is a whole number greater than 1 that has exactly two factors: 1 and itself.
- A **composite number** is a whole number greater than 1 that has more than two factors.
- The number 1 is neither prime nor composite.

### Example 1: Finding factors to identify prime and composite numbers

Number	Factors	Number of Factors	Prime or Composite
1	1	1	Neither
2	1, 2	2	Prime
3	1, 3	2	Prime
4	1, 2, 4	3	Composite
5	1, 5	2	Prime
6	1, 2, 3, 6	4	Composite

### Example 2: Sieve of Eratosthenes (a way to find prime numbers)

1. Write down numbers from 1 to 100.
2. Cross out 1 (not prime).
3. Circle 2 (first prime), and then cross out all other multiples of 2 (4, 6, 8, ...).
4. Circle the next number that is not crossed out, which is 3 (next prime), and cross out all other multiples of 3 (6, 9, 12, ...).
5. Continue this process. The numbers you circle will be the prime numbers.

1	2	3	3	5	6	X	X
9	10	3	12	13	14	15	16
27	24	21	13	18	19	20	20
31	32	33	34	25	26	27	28
51	52	53	54	55	56	57	50
61	62	63	64	65	66	77	78
81	72	73	74	75	76	77	80
91	82	83	84	85	86	87	90
91	92	93	94	95	96	97	100

**Question 10.1:** Is the number 17 prime or composite? List its factors.

**Question 10.2:** Give three examples of composite numbers between 1 and 30.

**Question 10.3:** What is the smallest prime number?

## Topic 11: Identifying Even and Odd Numbers

Even numbers can be arranged perfectly into pairs (a "twos array"), with no single item left over. Odd numbers, when arranged in pairs, will always have one item left over.

### Example 1: Using twos arrays

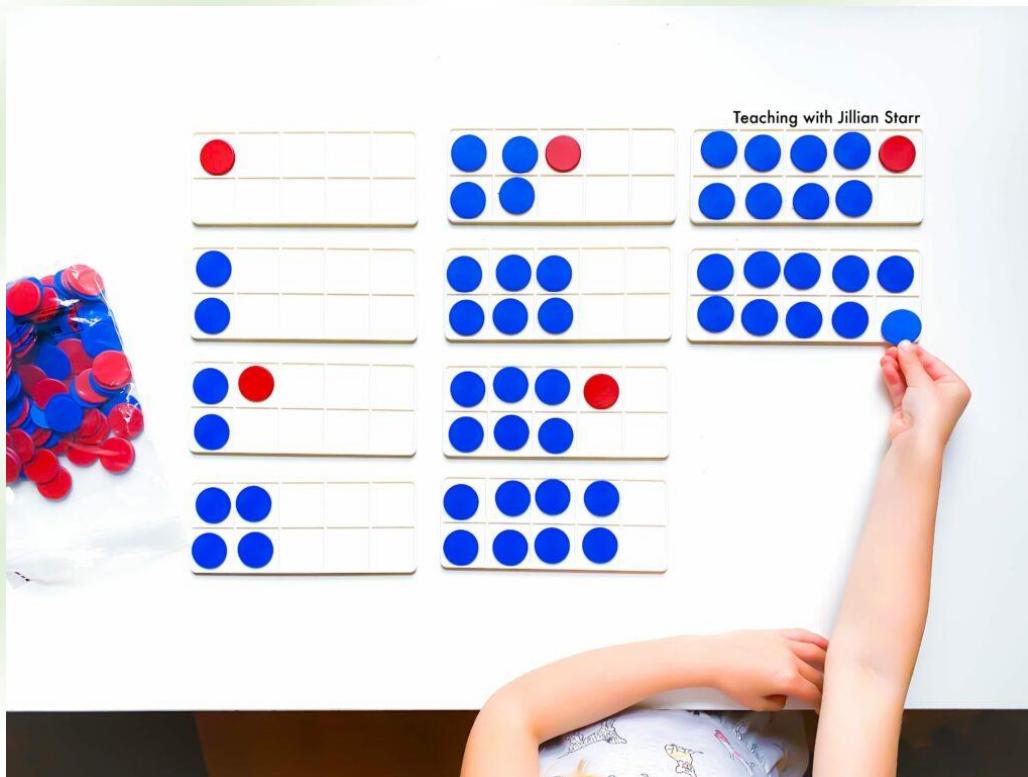
Imagine arranging some counters in rows of two:

- **4 counters:** (···) (2 rows of 2, no leftover. 4 is even.)
- **5 counters:** ..... (2 rows of 2, with 1 leftover. 5 is odd.)

Even numbers end in 0, 2, 4, 6, or 8. Odd numbers end in 1, 3, 5, 7, or 9.

## Example 2: Skip counting

- Skip counting by twos starting from 1 gives odd numbers: 1, 3, 5, 7, 9, ...
- Skip counting by twos starting from 2 gives even numbers: 2, 4, 6, 8, 10, ...



**Question 11.1:** Is the number 18 even or odd? Explain using the idea of a twos array.

**Question 11.2:** List all the odd numbers between 20 and 30.

**Question 11.3:** If you have 37 sweets, can you share them equally between two friends so that each gets a whole number of sweets? Why or why not?

## Topic 12: Determining the Highest Common Factor (HCF)

The Highest Common Factor (HCF) of two or more numbers is the largest number that divides exactly into each of them. We can use prime factorization to find the HCF.

### 1. Prime Factorization using a Factor Tree:

To find the prime factors of a number, we can use a factor tree. We break down the number into its factors until we are left with only prime numbers.

**Example:** Prime factors of 24:

$$\begin{array}{c} 24 \\ / \backslash \\ 2 \quad 12 \\ / \backslash \\ 2 \quad 6 \\ / \backslash \\ 2 \quad 3 \end{array}$$

*So, the prime factors of 24 are  $2 \times 2 \times 2 \times 3$ .*

### 2. Finding HCF using Prime Factorization:

To find the HCF of two numbers (e.g., 36 and 48):

**Prime factors of 36:**

$$\begin{array}{c} 36 \\ / \backslash \\ 2 \quad 18 \\ / \backslash \\ 2 \quad 9 \\ / \backslash \\ 3 \quad 3 \end{array}$$

$$36 = 2 \times 2 \times 3 \times 3$$

**Prime factors of 48:**

$$\begin{array}{c} 48 \\ / \backslash \\ 2 \quad 24 \\ / \backslash \\ 2 \quad 12 \\ / \backslash \\ 2 \quad 6 \\ / \backslash \\ 2 \quad 3 \end{array}$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

Now, we identify the common prime factors and multiply them:

Common factors are 2 (appears twice in both) and 3 (appears once in both).

$$\text{HCF}(36, 48) = 2 \times 2 \times 3 = 12.$$

$$\begin{array}{r}
 36 \\
 / \backslash \\
 2 \ 18 \\
 / \backslash \\
 2 \ 9 \\
 / \backslash \\
 3 \ 3
 \end{array}$$

$36 = 2 \times 2 \times 3 \times 3$

$$\begin{array}{r}
 48 \\
 / \backslash \\
 2 \ 24 \\
 / \backslash \\
 2 \ 12 \\
 / \backslash \\
 2 \ 6 \\
 / \backslash \\
 2 \ 3
 \end{array}$$

$$48 = 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{Common prime factors} = \{2, 3\}$$

**Question 12.1:** Find the prime factorization of 20.

**Question 12.2:** Use prime factorization to find the HCF of 12 and 18.

**Question 12.3:** Find the HCF of 10, 15, and 20 using prime factorization.

## Topic 13: Recognizing the Relationship Between Factors and Multiples

Factors and multiples are related to each other.

- If a number 'a' is a factor of a number 'b', then 'b' is a multiple of 'a'.

### Example 1: Even and Odd Numbers

- Even numbers are multiples of 2 (e.g., 4 is a multiple of 2 because  $2 \times 2 = 4$ ). 2 is a factor of every even number.
- Odd numbers are not multiples of 2.

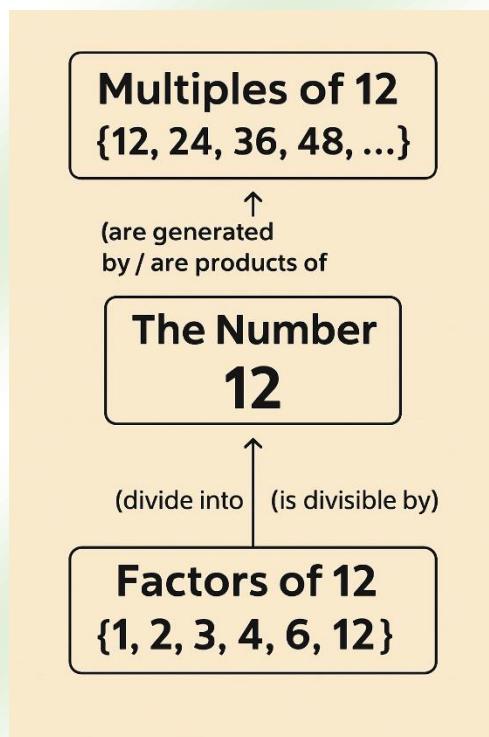
### Example 2: Multiples of 4 and 6

- Multiples of 4 are: 4, 8, 12, 16, 20, 24, ... (4 is a factor of all these numbers).
- Multiples of 6 are: 6, 12, 18, 24, 30, ... (6 is a factor of all these numbers).
- Notice that 12 and 24 are common multiples of 4 and 6.

### Example 3: Perfect Numbers

A perfect number is equal to the sum of its factors (excluding the number itself). For example, the factors of 6 (excluding 6) are 1, 2, and 3. Their sum is  $1+2+3=6$ . So, 6 is a perfect number.

Here's a diagram illustrating the relationship between factors and multiples, using the number 12 as an example:



**Question 13.1:** Is 7 a factor of 21? Is 21 a multiple of 7?

**Question 13.2:** List the first five multiples of 8. What are some factors of these multiples?

**Question 13.3:** The factors of 28 (excluding 28) are 1, 2, 4, 7, and 14. Is 28 a perfect number? Show your working.

**Sub-strand 2: Operations**

Now, we will learn some clever ways to multiply and divide numbers in our heads, and how to multiply larger numbers.

### **Topic 14: Mental Math Strategies for Multiplication and Division**

There are many tricks we can use to solve multiplication and division problems quickly in our minds!

#### **Mental Math Strategies:**

- **Skip counting from a known fact:**
  - If you know  $5 \times 7 = 35$ , then  $6 \times 7 = 35 + 7 = 42$ , and  $7 \times 7 = 42 + 7 = 49$ .
  - If you know  $8 \times 8 = 64$ , then  $7 \times 8 = 64 - 8 = 56$ , and  $6 \times 8 = 56 - 8 = 48$ .
- **Doubling:**
  - To find  $8 \times 3$ , think:  $4 \times 3 = 12$ , so  $8 \times 3 = 12 + 12 = 24$ .
- **Patterns when multiplying by 9:**
  - For  $9 \times 6$ , think:  $10 \times 6 = 60$ , then  $60 - 6 = 54$ .
  - For  $7 \times 9$ , think:  $7 \times 10 = 70$ , then  $70 - 7 = 63$ .
- **Repeated doubling:**
  - If  $2 \times 6 = 12$ , then  $4 \times 6 = 12 + 12 = 24$ , and  $8 \times 6 = 24 + 24 = 48$ .
- **Repeated halving:**
  - For  $60 \div 4$ , think:  $60 \div 2 = 30$ , and  $30 \div 2 = 15$ .
- **Relating division to multiplication:**
  - For  $64 \div 8$ , think: " $8 \times \underline{\hspace{1cm}}$  = 64?". The answer is 8.

**Question 14.1:** Use skip counting from  $4 \times 6 = 24$  to find  $6 \times 6$ .

**Question 14.2:** Use the doubling strategy to find  $6 \times 4$  (think of  $3 \times 4$ ).

**Question 14.3:** Use the pattern for multiplying by 9 to find  $9 \times 8$ .

## Topic 15: More Mental Math Strategies for Multiplication

Let's learn some more quick ways to multiply in our heads.

- **Annexing zeros (multiplying by multiples of 10, 100, 1000):**
  - To find  $3 \times 200$ , think:  $3 \times 2 = 6$ ,
  - then add two zeros to get 600.
- **Halving and doubling:**
  - To find  $32 \times 5$ ,
  - we can do  $16 \times 10 = 160$  (halve 32 and double 5).
- **Using the distributive property:**

To find  $6 \times 18$ ,

- think:  $6 \times (10 + 8)$
- $= (6 \times 10) + (6 \times 8)$
- $= 60 + 48 =$
- 108.

To find  $29 \times 7$ ,

- think:  $(30 - 1) \times 7$
- $= (30 \times 7) - (1 \times 7)$
- $= 210 - 7$
- 203.

- **Multiplying by one and zero:**
  - Any number multiplied by one is the number itself (e.g.,  $15 \times 1 = 15$ ).
  - Any number multiplied by zero is zero (e.g.,  $15 \times 0 = 0$ ).

**Question 15.1:** Use the annexing zeros strategy to find  $7 \times 300$ .

**Question 15.2:** Use halving and doubling to find  $25 \times 8$ .

**Question 15.3:** Use the distributive property to find  $8 \times 12$ .

## Topic 16: Multiplying Multi-Digit Numbers by 2-Digit Numbers

When we multiply larger numbers, we can use different methods to keep our work organized.

### 1. Expand and Box Method (Partial Decomposition):

Let's multiply  $448 \times 2$ :

X	400	40	8
2	800	80	16

$$448 \times 2 = 800 + 80 + 16 = 896.$$

Now, let's try  $25 \times 32$ :

$$25 = 20 + 5 \quad 32 = 30 + 2$$

x	30	2
20	600	40
5	150	10

$$25 \times 32 = 600 + 40 + 150 + 10 = 800.$$

### 2. Column or Vertical Method:

$$\begin{array}{r}
 & 32 \\
 \times & 25 \\
 \hline
 & 160 \quad (5 \times 32) \\
 + & 640 \quad (20 \times 32) \\
 \hline
 & 800
 \end{array}$$

### 3. Distributive Property:

$$25 \times 32$$

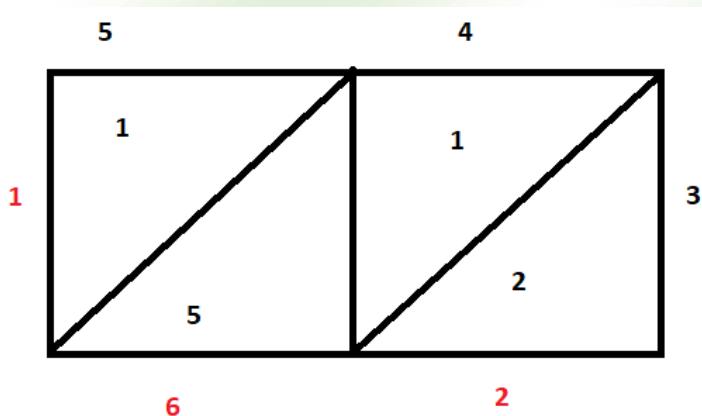
$$= 25 \times (30 + 2)$$

$$= (25 \times 30) + (25 \times 2)$$

$$= 750 + 50 = 800.$$

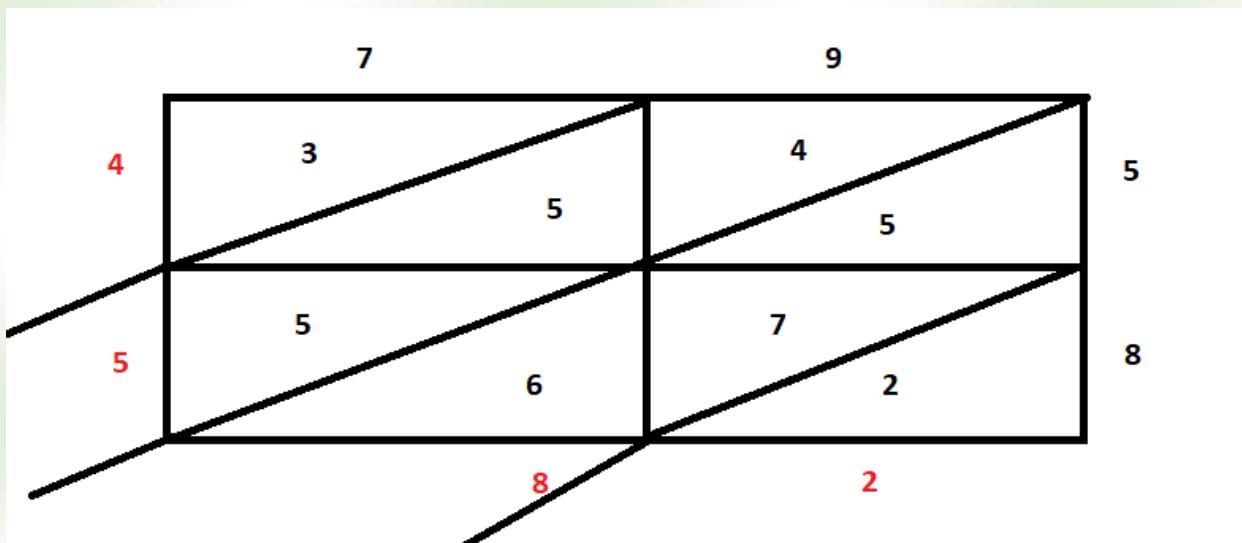
### 5. Lattice Method:

For  $54 \times 3$ :



Read the result along the diagonals: 1,  $5+1 = 6$  and 2.  
So,  $54 \times 3 = 162$ .

For  $79 \times 58$ :



Reading diagonals from right to left:

$$2, 5+7+6=18 \text{ (carry 1)},$$

$$4+5+5+1=15 \text{ (carry 1)},$$

$$3+1=4.$$

Result: 4582.

**Question 16.1:** Use the column method to multiply  $123 \times 12$ .

**Question 16.2:** Use the expand and box method to multiply  $35 \times 21$ .

**Question 16.3:** Choose any method to multiply  $67 \times 45$ .

## Topic 17: Basic Division Facts

Division is splitting a number into equal groups. Knowing our multiplication facts helps us with division!

### Understanding Division:

$12 \div 3 = ?$  means "How many groups of 3 are there in 12?". The answer is 4, because  $3 \times 4 = 12$ .

### Divisibility Rules (Quick Checks):

- **Divisible by 3:**

The sum of the digits is divisible by 3

e.g., for 123,  $1+2+3=6$ ,

and 6 is divisible by 3,

so 123 is divisible by 3.

- **Divisible by 4:**

The last two digits form a number divisible by 4

e.g., for 516, 16 is divisible by 4,

so 516 is divisible by 4.

- **Divisible by 6:**

The number is divisible by both 2 (even) and 3

(sum of digits divisible by 3).

- **Divisible by 9:**

The sum of the digits is divisible by 9

(e.g., for 279,  $2+7+9=18$ ,

and 18 is divisible by 9,

so 279 is divisible by 9).

$\times$	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9
2	2	4	6	8	10	12	14	16	18
3	3	6	9	12	15	18	21	24	27
4	4	8	12	16	20	24	28	32	36
5	5	10	15	20	25	30	35	40	45
6	6	12	18	24	30	36	42	48	54
7	7	14	21	28	35	42	49	56	63
8	8	16	24	32	40	48	56	64	72
9	9	18	27	36	45	54	63	72	81

**Question 17.1:** What is  $36 \div 6$ ? How does this relate to a multiplication fact?

**Question 17.2:** Is 45 divisible by 9? Use the divisibility rule to check.

**Question 17.3:** What is  $56 \div 7$ ?

## Topic 18: Dividing 3-Digit Numbers by 1-Digit Numbers

We can use long division to divide larger numbers.

**Example:** Divide 864 by 2

$$\begin{array}{r}
 432 \\
 2) \overline{864} \\
 -8 \\
 \hline
 0 \\
 -4 \\
 \hline
 0
 \end{array}$$

So,  $864 \div 2 = 432$ . We repeatedly subtract parts of the dividend by multiplying the divisor.

**Question 18.1:** Divide 693 by 3 using long division.

**Question 18.2:** If 5 friends share 755 sweets equally, how many sweets does each friend get?

## Topic 19: Solving Multi-Step Word Problems

Sometimes, a word problem needs more than one step to solve. We need to read carefully and decide which operations to use and in what order.

**Example:** Ama had 25 mangoes. She bought 15 more. Then, she shared all the mangoes equally among 5 friends. How many mangoes did each friend get?

**Step 1: Find the total number of mangoes.**  $25+15=40$  mangoes.

**Step 2: Divide the total number of mangoes by the number of friends.**  $40\div 5=8$  mangoes per friend.

**Question 19.1:** Kofi sold 30 oranges on Monday and 45 oranges on Tuesday. If he sells them in packs of 5, how many packs did he sell in total?

**Question 19.2:** A baker made 120 cookies. He ate 12 and then packed the rest into boxes of 8. How many boxes did he use?

## Topic 20: Understanding Integers

Integers are whole numbers, including positive numbers, negative numbers, and zero (... -2, -1, 0, 1, 2 ...). We can use a number line to help us add and subtract integers.

### Adding Integers:

- To add a positive integer, move to the right on the number line.
- To add a negative integer, move to the left on the number line.

### Examples:

1.  $9+(-4)=5$  (Start at 9, move 4 steps to the left)
2.  $-8+4=-4$  (Start at -8, move 4 steps to the right)
3.  $-3+(-5)=-8$  (Start at -3, move 5 steps to the left)

## Subtracting Integers:

Subtracting an integer is the same as adding its opposite.  $a - b = a + (-b)$   
 $a - (-b) = a + b$

### Examples:

9.  $-5 - 1 = -5 + (-1) = -6$  (Start at -5, move 1 step to the left)
10.  $-2 - 1 = -2 + (-1) = -3$  (Start at -2, move 1 step to the left)
11.  $2 - 6 = 2 + (-6) = -4$  (Start at 2, move 6 steps to the left)

**Question 20.1:** Use a number line to find the value of  $-7 + 3$ .

**Question 20.2:** Use a number line to find the value of  $4 - (-2)$ .

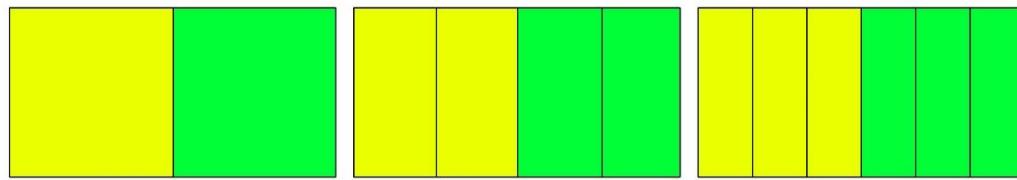
**Question 20.3:** The temperature was  $5^{\circ}\text{C}$  and it dropped by  $8^{\circ}\text{C}$ . What is the new temperature?

### Sub-strand 3: Fractions

Fractions help us talk about parts of a whole. Let's learn how to work with them!

#### Topic 21: Equivalent Fractions

Equivalent fractions are fractions that have different numerators and denominators but represent the same value.



**How to find equivalent fractions:** Multiply or divide both the numerator and the denominator by the same non-zero number.

**Example:** Consider the fraction  $\frac{4}{3}$ .

- Multiply by 2:  $2 \times \frac{4}{3} = \frac{8}{6}$ . So,  $\frac{4}{3}$  and  $\frac{8}{6}$  are equivalent.
- Multiply by 3:  $3 \times \frac{4}{3} = \frac{12}{9}$ . So,  $\frac{4}{3}$ ,  $\frac{8}{6}$ , and  $\frac{12}{9}$  are all equivalent.

To compare, add, or subtract fractions with different denominators, we often need to find equivalent fractions with a common denominator, preferably the Lowest Common Denominator (LCD).

**Example:** Find equivalent fractions for  $\frac{3}{4}$ ,  $\frac{5}{6}$  and  $\frac{7}{10}$  with a common denominator.

The prime factors of the denominators are:  $4=2\times2$ ,  $6=2\times3$ ,  $10=2\times5$

The LCD is found by taking the highest power of each prime factor:

$$2\times2\times3\times5 = 4\times3\times5 = 60.$$

Now, let's find the equivalent fractions with a denominator of 60:

- $\frac{3}{4} = 15 \times \frac{3}{4} = \frac{45}{60}$  (Since  $60 \div 4 = 15$ )
- $\frac{5}{6} = 10 \times \frac{5}{6} = \frac{50}{60}$  (Since  $60 \div 6 = 10$ )
- $\frac{7}{10} = 6 \times \frac{7}{10} = \frac{42}{60}$  (Since  $60 \div 10 = 6$ )

**Question 21.1:** Write two fractions that are equivalent to  $\frac{1}{3}$ .

**Question 21.2:** Find the equivalent fraction of  $\frac{2}{5}$  with a denominator of 15.

**Question 21.3:** What is the LCD of  $\frac{1}{21}$  and  $\frac{3}{8}$ ? Use it to write equivalent fractions.

## Topic 22: Comparing and Ordering Fractions

To compare fractions, we can use a common denominator or convert them to decimals or percentages.

### Method 1: Using a Common Denominator (LCD)

**Example:** Which is larger,  $\frac{5}{6}$  or  $\frac{3}{4}$ ?

The LCD of 6 and 4 is 12.

- $\frac{5}{6} = 2 \times \frac{5}{6} = \frac{10}{12}$
- $\frac{3}{4} = 3 \times \frac{3}{4} = \frac{9}{12}$

Since  $\frac{10}{12} > \frac{9}{12}$ , we know that  $\frac{5}{6} > \frac{3}{4}$ .

### Method 2: Converting to Decimals

**Example:** Compare  $\frac{5}{6}$ ,  $\frac{3}{4}$ , and  $\frac{2}{3}$ .

- $\frac{5}{6} = 0.83$
- $\frac{3}{4} = 0.75$
- $\frac{2}{3} = 0.67$

Ordering them from largest to smallest:  $\frac{5}{6} > \frac{3}{4} > \frac{2}{3}$ .

**Question 22.1:** Which is smaller,  $\frac{2}{5}$  or  $\frac{3}{7}$ ? Show your working.

**Question 22.2:** Order the following fractions from smallest to largest:  $\frac{1}{2}$ ,  $\frac{2}{3}$ ,  $\frac{3}{4}$

## Topic 23: Adding and Subtracting Like Fractions (One Denominator a Multiple of the Other)

When adding or subtracting fractions where one denominator is a multiple of the other, we can easily find a common denominator.

### Example 1: Addition

Add  $\frac{1}{3}$  and  $\frac{2}{6}$ .

The LCD of 3 and 6 is 6 (since 6 is a multiple of 3).

Convert  $\frac{1}{3}$  to an equivalent fraction with a denominator of 6:

$$\frac{1}{3} = 2 \times \frac{1}{3} = \frac{2}{6}$$

$$\text{Now, add: } \frac{2}{6} + \frac{2}{6} = \frac{4}{6}$$

$$\text{Simplify the result: } \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$$

### Example 2 : Subtraction

Subtract  $\frac{1}{8}$  from  $\frac{3}{4}$

The LCD of 4 and 8 is 8.

Convert  $\frac{3}{4}$  to an equivalent fraction with a denominator of 8:  $\frac{3}{4} = 2 \times \frac{3}{4} = \frac{6}{8}$

$$\text{Now, subtract: } \frac{6}{8} - \frac{1}{8} = \frac{6-1}{8} = \frac{5}{8}$$

**Question 23.1:** Add  $\frac{1}{4} + \frac{3}{8}$ .

**Question 23.2:** Subtract  $\frac{2}{9}$  from  $\frac{5}{3}$ .

## Topic 24: Adding and Subtracting Fractions Greater Than One (Improper or Mixed)

We can add and subtract mixed numbers (fractions greater than one) by either working with them directly or by converting them to improper fractions.

### Example 1: Adding Like Mixed Fractions

Add  $2\frac{1}{3} + 3\frac{2}{3}$ .

Add the whole numbers:  $2+3 = 5$ . Add the fractions:  $\frac{1}{3} + \frac{2}{3} = \frac{1+2}{3} = \frac{3}{3} = 1$ .

Total:  $5+1=6$ .

### Example 2: Subtracting Like Mixed Fractions (Converting to Improper)

Subtract  $2\frac{1}{3}$  from  $3\frac{2}{3}$ .

Convert to improper fractions:  $3\frac{2}{3} = \frac{(3 \times 3) + 2}{3} = \frac{9+2}{3} = \frac{11}{3}$ .

$$2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{6+1}{3} = \frac{7}{3}.$$

$$\text{Subtract: } \frac{11}{3} - \frac{7}{3} = \frac{11-7}{3} = \frac{4}{3}.$$

Convert back to a mixed number:  $\frac{4}{3} = 1\frac{1}{3}$ .

### Example 3: Adding Mixed Fractions with Different Denominators

Add  $2\frac{1}{3} + 3\frac{2}{5}$ .

Convert to improper fractions:  $2\frac{1}{3} = \frac{(2 \times 3) + 1}{3} = \frac{7}{3}$        $3\frac{2}{5} = \frac{(5 \times 3) + 2}{5} = \frac{17}{5}$

Find the LCD of 3 and 5, which is 15.

Convert to equivalent fractions with denominator 15:

$$\frac{7}{3} = \frac{7 \times 5}{3 \times 5} = \frac{35}{15}$$

$$\frac{17}{5} = \frac{17 \times 3}{5 \times 3} = \frac{51}{15}$$

$$\text{Add: } \frac{35}{15} + \frac{51}{15} = \frac{35+51}{15} = \frac{86}{15}$$

Convert back to a mixed number:  $\frac{86}{15} = 5\frac{11}{15}$  (since  $86 \div 15 = 5$  with a remainder of 11).

**Question 24.1:** Add 152+251.

**Question 24.2:** Subtract 141 from 321.

**Question 24.3:** Calculate 241+132.

## Topic 25: Multiplying a Whole Number by a Fraction

Multiplying a whole number by a fraction means finding that fraction of the whole number, or repeated addition of the fraction.

### Example 1: Repeated Addition

$5 \times \frac{2}{3}$  means adding  $\frac{2}{3}$  five times:  $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ .

### Example 2: Whole Number by a Mixed Fraction

$3 \times 2\frac{2}{3}$  can be rewritten as  $3 \times (2 + \frac{2}{3})$ .

1. **Multiply the whole number by the whole part:**  $3 \times 2 = 6$
2. **Multiply the whole number by the fractional part:**  $3 \times \frac{2}{3} = \frac{3 \times 2}{3} = \frac{6}{3} = 2$
3. **Add the results:**  $6 + 2 = 8$

**Question 25.1:** Calculate  $4 \times 53$ .

**Question 25.2:** Find the value of  $2 \times 121$ .

## Topic 26: Multiplying a Fraction by a Whole Number

Multiplying a fraction by a whole number is the same as multiplying a whole number by a fraction. It can be thought of as finding a fraction "of" a quantity.

**Example 1:**  $\frac{2}{3} \times 5$  means  $\frac{2}{3}$  of 5.

Imagine 5 sheets of paper. We need to find  $\frac{2}{3}$  of each. Each sheet gives us  $\frac{2}{3}$ . For 5 sheets, we have  $5 \times \frac{2}{3} = \frac{10}{3} = 3\frac{1}{3}$ .

### Example 2: Mixed Fraction by a Whole Number

Multiply  $4\frac{4}{5} \times 5$ .

**Convert the mixed fraction to an improper fraction:**

$$4\frac{4}{5} = \frac{(4 \times 5) + 4}{5} = \frac{20 + 4}{5} = \frac{24}{5}.$$

**Multiply the improper fraction by the whole number:**

$$\frac{24}{5} \times 5 = \frac{24 \times 5}{5 \times 1}$$

**Now, multiply the numerators and the denominators:**

$$= \frac{120}{5}$$

**Simplify the result:** =24

**Question 26.1:** What is  $41 \times 7$ ?

**Question 26.2:** Calculate 32 of 6.

#### Sub-strand 4: Decimals

Decimals are another way to represent parts of a whole, often based on powers of ten.

#### Topic 27: Representing Decimals

Decimals help us write fractions with denominators that are powers of 10 (like 10, 100, 1000).

#### Place Value Chart for Decimals:

Whole Numbers	Decimal Point	Fractions
Hundreds	.	Tenths, Hundredths, Thousandths

- **Tenths** ( $\frac{1}{10}$  or 0.1): One part out of ten equal parts.
- **Hundredths** ( $\frac{1}{100}$  or 0.01): One part out of one hundred equal parts.
- **Thousandths** ( $\frac{1}{1000}$  or 0.001): One part out of one thousand equal parts

## Example 1: Place Value Chart

Consider the number 3.145:

Ones	.	Tenths	Hundredths	Thousands
3	.	1	4	5

This represents  $3 + \frac{1}{10} + \frac{4}{100} + \frac{5}{1000}$ .

As a fraction, this would be  $\frac{3145}{1000}$ .

## Example 2: Converting Fractions to Decimals

- $\frac{3}{8}$  To convert this to a decimal, we can perform the division

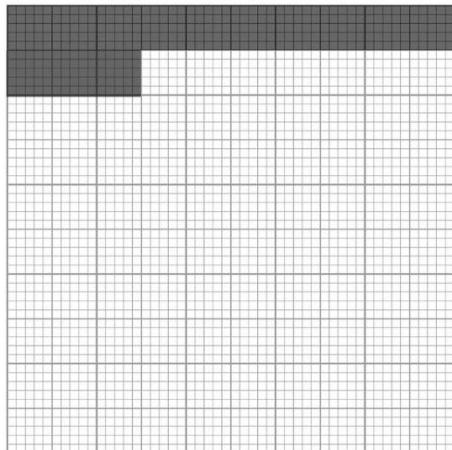
$$3 \div 8 = 0.375. \text{ So,}$$

$$\frac{3}{8} = 0.375 = \frac{375}{1000}$$

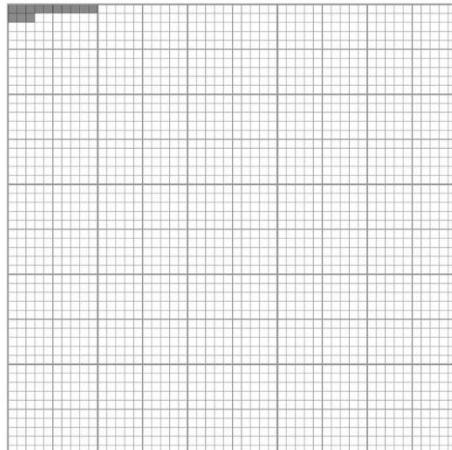
## Example 3: Modeling Decimals with Graph Sheets

To model 0.13 on a graph sheet:

- Imagine a large square (e.g., 10x10) represents 1 whole.
- 0.1 means shading 1 strip of 10 small squares ( $\frac{1}{10}$  of the large square).
- 0.03 means shading 3 small squares ( $\frac{3}{100}$  of the large square).
- 0.003 means shading a tiny portion representing 3 thousandths (hard to show precisely on a simple grid but conceptual).



$$\frac{13}{100}$$



$$\frac{13}{1000}$$

**Question 27.1:** Write the decimal 0.6 as a fraction.

**Question 27.2:** Write the fraction  $\frac{25}{100}$  as a decimal.

**Question 27.3:** How would you represent 0.007 using the idea of parts of a thousand?

## Topic 28: Comparing and Ordering Decimal Fractions

To compare and order decimals, we can compare the digits in each place value position from left to right.

### Example 1: Comparing Decimals

Which is greater, 0.758 or 0.73?

Align the decimal points:

0.758  
 0.730 (We can add a zero to make the number of decimal places the same)

Comparing place by place:

- The ones place is the same (0).
- The tenths place is the same (7).
- In the hundredths place, 5 is greater than 3.

So,  $0.758 > 0.73$ .

### Example 2: Comparing a Mixture of Fractions and Decimals

Order  $0.758$ ,  $\frac{5}{8}$ , and  $73\%$  from least to largest.

First, convert them all to decimals:

- $0.758 = 0.758$
- $\frac{5}{8} = 0.625$  (since  $5 \div 8 = 0.625$ )
- $73\% = \frac{73}{100} = 0.73$

Now, compare the decimals:  $0.625 < 0.73 < 0.758$

So, the order from least to largest is  $85$ ,  $73\%$ ,  $0.758$ .

**Question 28.1:** Use the symbols  $<$ ,  $>$ ,  $=$  to compare 0.235 and 0.253.

**Question 28.2:** Order the following from largest to smallest: 0.6, 21, 0.58.

## Topic 29: Rounding Decimals

Rounding decimals is similar to rounding whole numbers.

- **Rounding to the nearest tenth:** The rounded number will have one digit after the decimal point. Look at the digit in the hundredths place to decide whether to round up or down.
- **Rounding to the nearest hundredth:** The rounded number will have two digits after the decimal point. Look at the digit in the thousandths place to decide whether to round up or down.

### Rule for Rounding:

1. Identify the place value you are rounding to.
2. Look at the digit immediately to its right.
3. If that digit is 5 or more, round up the digit in the rounding place.
4. If that digit is less than 5, keep the digit in the rounding place the same.
5. Drop the digits to the right of the rounding place.

### Example 1: Rounding to the nearest tenth

- Round 0.38 to the nearest tenth: The tenths digit is 3. The hundredths digit is 8 (which is 5 or more), so we round up. 0.38 rounded to the nearest tenth is 0.4.
- Round 4.085 to the nearest tenth: The tenths digit is 0. The hundredths digit is 8 (which is 5 or more), so we round up. 4.085 rounded to the nearest tenth is 4.1.
- Round 56.584 to the nearest tenth: The tenths digit is 5. The hundredths digit is 8 (which is 5 or more), so we round up. 56.584 rounded to the nearest tenth is 56.6.

## Example 2: Rounding to the nearest hundredth

- Round 18.096 to the nearest hundredth: The hundredths digit is 9. The thousandths digit is 6 (which is 5 or more), so we round up. 18.096 rounded to the nearest hundredth is 18.10.
- Round 30.084 to the nearest hundredth: The hundredths digit is 8. The thousandths digit is 4 (which is less than 5), so we keep it the same. 30.084 rounded to the nearest hundredth is 30.08.

**Question 29.1:** Round 0.72 to the nearest tenth.

**Question 29.2:** Round 3.456 to the nearest hundredth.

**Question 29.3:** Convert the fraction 32 to a decimal (to three decimal places) and then round it to the nearest hundredth.

## Topic 30: Adding and Subtracting Decimals Using Models

We can use place value understanding and models to add and subtract decimals.

### Example 1: Addition

Add 0.645 and 0.39.

Think of this in terms of thousandths:  $0.645 = \frac{645}{1000}$        $0.39 = \frac{390}{1000}$

Adding them:  $\frac{645}{1000} + \frac{390}{1000} = \frac{645+390}{1000} = \frac{1035}{1000} = 1.035$ .

Alternatively, we can line up the decimal points and add as if they were whole numbers:

$$\begin{array}{r}
 0.645 \\
 + 0.390 \\
 \hline
 1.035
 \end{array}$$

**Example 2: Subtraction**

Subtract 0.395 from 0.6.

Think of this in terms of thousandths:

$$0.6 = \frac{6}{10} = \frac{600}{1000}$$

$$0.395 = \frac{395}{1000}$$

$$\text{Subtracting them: } \frac{600}{1000} - \frac{395}{1000} = \frac{600-395}{1000} = \frac{205}{1000} = 0.205.$$

Alternatively, line up the decimal points and subtract:

$$\begin{array}{r} 0.600 \\ - 0.395 \\ \hline 0.205 \end{array}$$

**Question 30.1:** Use the fraction method to add 0.25 and 0.4.

**Question 30.2:** Use the column method to subtract 0.123 from 0.5.

**Topic 31: Multiplying a Decimal by a Whole Number Using Models**

Multiplying a decimal by a whole number is like repeated addition.

**Example 1:  $0.4 \times 10$** 

This means adding 0.4 ten times:

$$0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 + 0.4 = 4.0.$$

Alternatively, convert to fractions:  $0.4 = \frac{4}{10}$ .

$$\frac{4}{10} \times 10 = \frac{4 \times 10}{10} = \frac{40}{10} = 4.$$

**Example 2:** Multiply 0.2 by 3.

This means  $0.2+0.2+0.2=0.6$ .

Using fractions:  $0.2=\frac{1}{5}$ .  $\frac{1}{5} \times 3 = \frac{3}{5} = 0.6$ .

A general rule: Multiply the numbers as if they were whole numbers, and then place the decimal point in the product so that it has the same number of decimal places as the original decimal number.

**Question 31.1:** Use repeated addition to find the value of  $0.15 \times 4$ .

**Question 31.2:** Convert to fractions to calculate  $0.7 \times 5$ .

## Sub-strand 5: Percentages

Let's take a closer look at percentages and how we use them!

### Topic 32: Determining Percentage of a Quantity

**Understanding Percentage:** A percentage is a fraction out of 100. The symbol '%' means "per cent," which translates to "out of one hundred."

#### Finding the Percentage of a Quantity:

To find a percentage of a number, we follow these steps:

1. **Convert the percentage to a fraction:** Divide the percentage by 100.
  - o Example:  $30\% = \frac{30}{100}$
  - o Example:  $75\% = \frac{75}{100}$
2. **Multiply the fraction by the quantity:** Multiply the fraction you just found by the number you want to find the percentage of.
  - o Example: Find 40% of 60.
    - Convert 40% to a fraction:  $\frac{40}{100}$
    - Multiply by 60:  $\frac{40}{100} \times 60 = \frac{40 \times 60}{100} = \frac{2400}{100} = \frac{24}{1} = 24$ .
    - So, 40% of 60 is 24.

#### "Vice Versa" - Finding What Percentage One Quantity is of Another:

To find what percentage a part is of a whole:

1. **Form a fraction:** Create a fraction where the numerator is the part and the denominator is the whole.

- Example: What percentage is 15 of 50? The fraction is  $\frac{15}{50}$ .
- 2. Convert the fraction to a percentage:** Multiply the fraction by 100%.
- $\frac{15}{50} \times 100\% = \frac{1500}{50}\% = 30\%$ .
  - So, 15 is 30% of 50.

**Question 32.1 (Detailed):** What is 35% of 80? Show each step.

**Question 32.2 (Detailed):** 12 is what percentage of 40? Show your working.

### Topic 33: Benchmark Percentages for Estimation

#### Benchmark Percentages and Their Fractional Equivalents:

These are useful for quick calculations and estimations:

Percentage	Fraction
10%	$\frac{1}{10}$
20%	$\frac{2}{10}$
25%	$\frac{4}{10}$
50%	$\frac{1}{2}$
75%	$\frac{3}{4}$
100%	1

#### Using Benchmarks to Estimate:

Let's estimate 70% of 60 using benchmarks.

- We know 50% of 60 is  $\frac{1}{2} \times 60 = 30$ .

- We know 20% of 60 is  $\frac{1}{5} \times 60 = 12$ .
- So, 70% is 50%+20%, which is approximately  $30+12 = 42$ .

To verify: 70% of 60 =  $\frac{70}{100} \times 60 = \frac{4200}{100} = 42$ . Our estimation was accurate in this case!

**(Indication for Image: A table or chart displaying benchmark percentages and their corresponding fractions.)**

**Question 33.1 (Detailed):** What fraction is equivalent to 75%? Use this to find 75% of 36.

**Question 33.2 (Detailed):** Estimate 30% of 50 using benchmark percentages. Show your estimation steps.

## Topic 34: Percent in Real-Life Applications

### Real-World Uses of Percentages:

- **Discounts:** "Save 15% today!"
- **Grades:** "You scored 95% on the quiz."
- **Interest Rates:** "The savings account has a 2% annual interest rate."
- **Surveys:** "60% of people prefer chocolate ice cream."

### Solving a Real-Life Problem:

A pair of shoes costs ₦90. There is a 20% discount. How much money do you save? What is the new price?

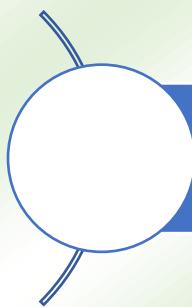
1. **Calculate the discount:** 20% of ₦90 =  $\frac{20}{100} \times 90 = \frac{1800}{100} = ₦18$ . You save ₦18.

2. **Calculate the new price:** New price = Original price - Discount  
New price = ₦90 - ₦18 = ₦72. The new price of the shoes is ₦72.

(Indication for Image: Photos of real-life scenarios where percentages are used, like a shop with a sale sign, a report card, etc.)

**Question 34.1 (Detailed):** A phone with an original price of ₦200 is on sale for 30% off. What is the sale price?

**Question 34.2 (Detailed):** In a class of 50 students, 80% passed a test. How many students passed the test?



## Strand 2: Algebra

### Sub-strand 1: Patterns and Relationships

Patterns are all around us, and in mathematics, we can often find rules that describe these patterns.

#### Topic 35: Extending Patterns

A pattern is a sequence that repeats or changes according to a specific rule. We can use concrete materials (like blocks or counters) or just numbers to work with patterns.

##### Example 1: Skip Counting

Skip counting is a type of pattern.

- Skip counting by 20: 20, 40, 60, 80, ... (Each number is 20 more than the one before it).
- Skip counting by 500: 500, 1000, 1500, ... (Each number is 500 more than the one before it).

##### Example 2: Patterns with Shapes

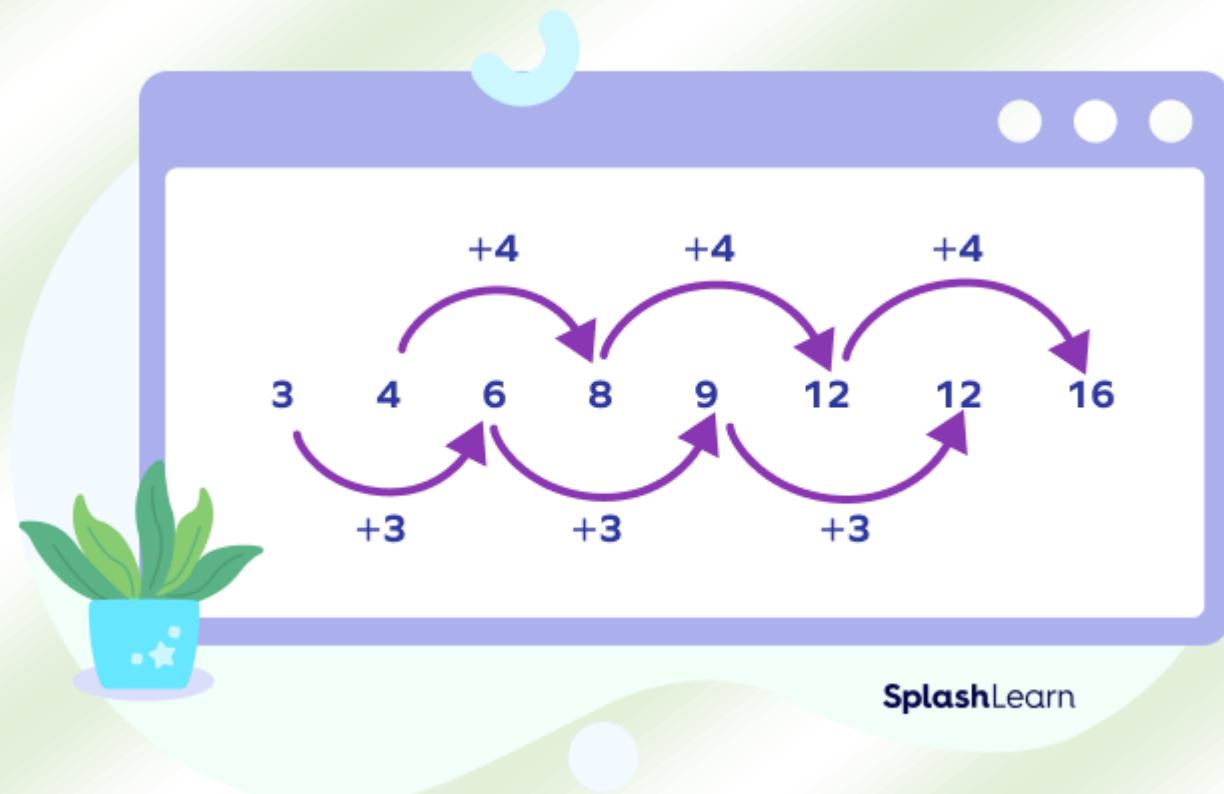
Imagine a pattern of squares where each step adds more squares:

Step 1: □ (1 square)

Step 2: □□□ (3 squares total, added 2)

Step 3: □□□□□□ (6 squares total, added 3)

To extend this, we would likely add a row of 4 squares to get 10 total in Step 4. The number of squares added each time increases by one (2, then 3, then 4).



**Question 35.1:** Extend the number pattern: 5, 10, 15, 20, \_\_\_, \_\_\_. Explain the rule.

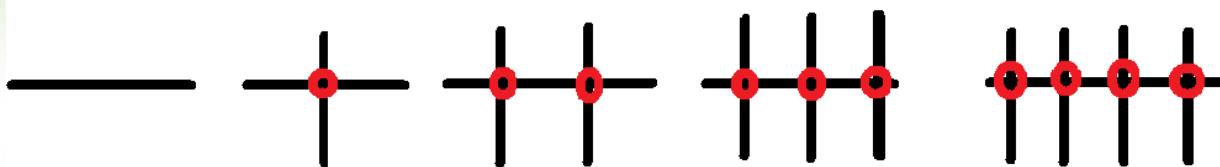
**Question 35.2:** If a pattern starts with one dot, then three dots, then five dots, how many dots would be in the next two steps?

### Topic 36: Describing Pattern Rules

To understand a pattern fully, we need to be able to describe its rule using mathematical language.

## Example 1: Intersecting Lines

Number of lines	Number of Intersections
1	0
2	1
3	2
4	3
5	4



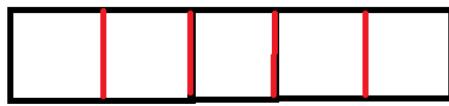
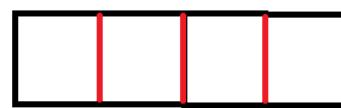
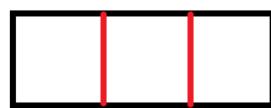
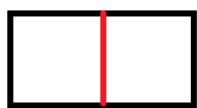
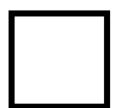
The number of intersections is one less than the number of lines.

## Example 2: Tiles and Joints

Imagine square tiles placed in a row. Joints are where the tiles meet.

Number of tiles	Number of joints
1	0
2	1
3	2
4	3
5	4

The number of joints is one less than the number of tiles.



**Question 36.1:** Describe the rule for the pattern: 2, 4, 6, 8, ... using mathematical language.

**Question 36.2:** Look at the pattern: 1, 3, 7, 15, ... Describe how each number relates to the previous one.

## Topic 37: Predicting Subsequent Elements

Once we know the rule for a pattern, we can use it to predict what the next terms will be.

### Example 1: Number Pattern

Consider the pattern: 3, 6, 9, 12, ...

The rule seems to be "add 3 to the previous term."

To find the next two terms:

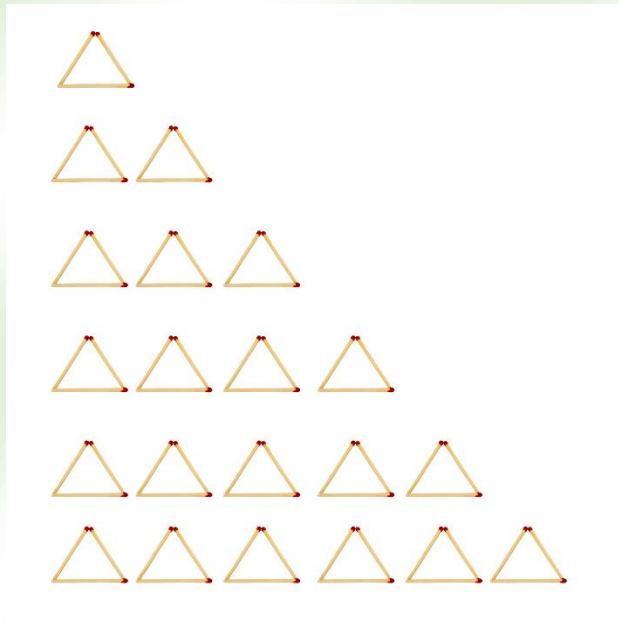
- $12+3=15$
- $15+3=18$

So, the next two terms are 15 and 18.

## Example 2: Geometric Pattern (Matchsticks and Triangles)

Number of triangles	Number of match sticks
1	3
2	6
3	9
4	12
5	?
n	?

The rule is that the number of matchsticks is 3 times the number of triangles. For 5 triangles, we would predict  $3 \times 5 = 15$  matchsticks. For 'n' triangles, we would predict  $3 \times n$  matchsticks.



**Question 37.1:** The pattern is 2, 5, 8, 11, ... What are the next two terms? What is the rule?

**Question 37.2:** Using the table above for matchsticks and triangles, how many matchsticks would be used for the 9th geometric pattern?

## Topic 38: Visual Representation to Verify Predictions

Drawing a pattern can help us see if our predictions are correct.

### Example 1: Matchsticks and Triangles (Continued)

Pattern Number (triangles)	Number of match sticks
1	3
2	6
3	9
4	12
5	?

We predicted that for 5 triangles, we'd need 15 matchsticks. Let's visualize:

Pattern 1:  $\triangle$  (3 sticks)

Pattern 2:  $\triangle\triangle$  (6 sticks)

Pattern 3:  $\triangle\triangle\triangle$  (9 sticks)

Pattern 4:  $\triangle\triangle\triangle\triangle$  (12 sticks)

Pattern 5:  $\triangle\triangle\triangle\triangle\triangle$  (15 sticks)

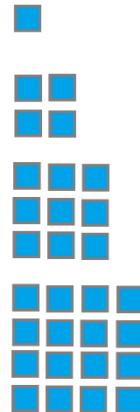
Our visual representation confirms our prediction. The rule is indeed to multiply the number of triangles by 3 to get the number of matchsticks.

## Example 2: Matchsticks and Squares

Pattern Number (side length)	Number of small squares
1	1
2	4
3	9
4	?

The number of small squares is the pattern number multiplied by itself (squared). For pattern number 4, we predict  $4 \times 4 = 16$  small squares.

Let's visualize:



**Question 38.1:** The pattern is 1, 4, 9, ...

Predict the next term. Draw a visual representation to support your prediction (think about squares of dots, using  $\cdot$ ).

**Question 38.2:** Using the matchstick square pattern (where the pattern number is the side length), how many matchsticks would be needed for the 3rd pattern? (Hint: draw it using lines for matchsticks).

## Topic 39: Solving Problems Using Pattern Rules

We can apply our knowledge of pattern rules to find missing terms or continue sequences.

**Example 1:** Describe the pattern rule and determine the next three elements:

(i) 1, 10, 7, 70, 67, 670, ...

\* Rule: Multiply by 10, then subtract 3.

\* Next three elements:  $670 - 3 = 667$ ,  $667 \times 10 = 6670$ ,  $6670 - 3 = 6667$ .

\* Sequence continues: 1, 10, 7, 70, 67, 670, **667**, **6670**, **6667**, ...

(ii) 10, 12, 16, 22, 30 ...

\* Rule: Add 2, then add 4, then add 6, then add 8... (adding consecutive even numbers).

\* Next three elements:  $30 + 10 = 40$ ,  $40 + 12 = 52$ ,  $52 + 14 = 66$ .

\* Sequence continues: 10, 12, 16, 22, 30, **40**, **52**, **66**, ...

(iii) 50, 48, 47, 45, 44 ...

\* Rule: Subtract 2, then subtract 1, then subtract 2, then subtract 1... (alternating subtraction).

\* Next three elements:  $44 - 2 = 42$ ,  $42 - 1 = 41$ ,  $41 - 2 = 39$ .

\* Sequence continues: 50, 48, 47, 45, 44, **42**, **41**, **39**, ...

**Example 2:** Describe the pattern rule and determine the next three elements:

(i) 0.25, 0.5, 0.75, \_\_\_, \_\_\_, \_\_\_ \* Rule: Add 0.25.

\* Next three elements: 1.00, 1.25, 1.50.

(ii) 2.50, 5, 7.50, \_\_\_, \_\_\_, \_\_\_ \* Rule: Add 2.50.

\* Next three elements: 10.00, 12.50, 15.00.

(iii) 64, 32, 16, \_\_, \_\_, \_\_ \* Rule: Divide by 2.

\* Next three elements: 8, 4, 2.

(iv) 900, 450, 225, \_\_, \_\_, \_\_ \* Rule: Divide by 2.

\* Next three elements: 112.5, 56.25, 28.125.

**Question 39.1:** Describe the rule and find the next two terms in the pattern: 1, 4, 7, 10, ...

**Question 39.2:** What are the next three numbers in the sequence: 100, 90, 80, ...?

## Topic 40: Identifying Errors in Patterns

Sometimes, a number in a sequence might not follow the established pattern. We need to identify and explain these errors.

**Example:** Shika's marbles: 2, 4, 6, 8, 12, ...

The pattern seems to be adding 2 to each bag.

- Bag 1: 2 marbles
- Bag 2:  $2+2=4$  marbles
- Bag 3:  $4+2=6$  marbles
- Bag 4:  $6+2=8$  marbles
- Bag 5 should have  $8+2=10$  marbles, but it has 12.
- The error is in the fifth bag; it should have 10 marbles instead of 12.

Bags	1	2	3	4	5	6	7
Marbles	2	4	6	8	12	?	?

**Question 40.1:** In the pattern 5, 10, 15, 25, 30, what is the error? Explain why.

**Question 40.2:** The sequence is 1, 3, 5, 7, 11. Is 11 the correct next term? Why or why not?

### Topic 41: Writing Rules in Words and Algebra

We can describe pattern rules using both words and algebraic expressions.

**Example:**

Term/Input (n)	Result/Output (A)	Rule in words	Rule in Algebra
1	7	7 times n	$7n$
2	14	7 time 2	$7(2)$
3	21	7 time 3	$7(3)$

Here, the rule is: to get the output, multiply the input (n) by 7. Algebraically, this is  $7n$ .

Let's look at another:

Term/Input (n)	Result/Output (B)	Rule in words	Rule in Algebra
1	0	4 times one less than n	$4(n-1)$
2	4	4 times 2 less than 1	$4(2-1)$
3	8	4 times 3 less than 1	$4(3-1)$
4	12	4 times 4 less than 1	$4(4-1)$

Here, the rule is: subtract 1 from the input ( $n$ ), then multiply by 4.  
Algebraically, this is  $4(n-1)$ .

**Question 41.1:** Complete the table for the following pattern: Output is 1 more than 3 times the input.

Term/Input (n)	Result/Output (C)	Rule in words	Rule in Algebra
1	4	1 more than 3 times n	$1+3n$
2	7		
3	10		

**Question 41.2:** Write a rule in words and in algebra for the pattern in Result/Output D: 5, 9, 13, ...

### Topic 42: Describing Relationships in Tables or Charts

Tables and charts can show how two sets of numbers are related. We can find a mathematical expression to describe this relationship.

**Example:** Boxed Lunches

Number of students	Cost of lunch (€)
1	3
2	6
3	9
4	12
5	15
?	90

(i) Pattern: The cost of lunch increases by ₦3 for each additional student. The cost is 3 times the number of students.

(ii) If the cost is ₦90, and the cost per student is ₦3, then the number of students is  $90 \div 3 = 30$ . So, 30 students went on the trip.

**Question 42.1:** Describe the relationship in the following table using a mathematical expression.

Input	Output
1	5
2	7
3	9
4	11

**Question 42.2:** Using the table above, what would be the output if the input is 10?

## Sub-strand 2: Algebraic Expressions

Algebraic expressions are mathematical phrases that use numbers, letters (representing unknown numbers), and operation signs.

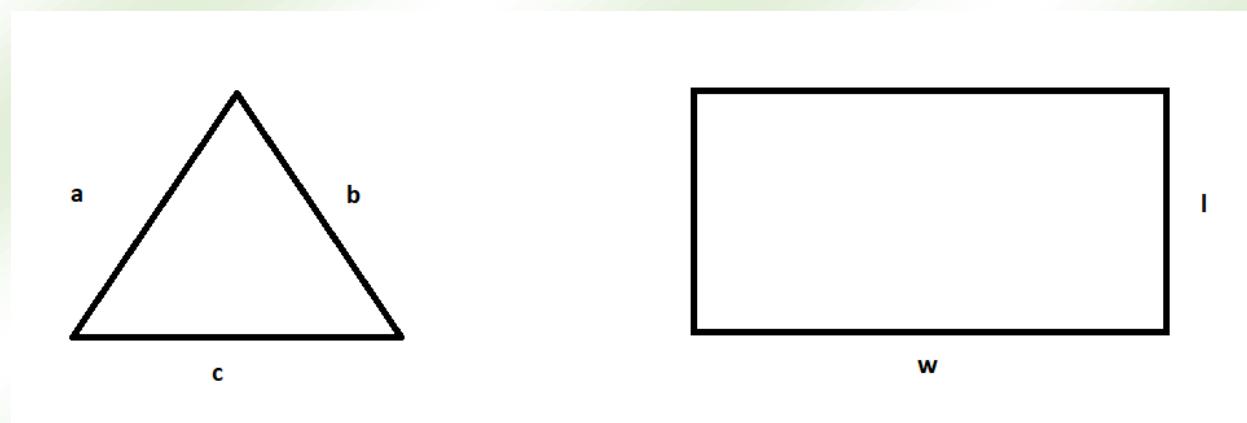
### Topic 43: Understanding Algebraic Expressions

Algebraic expressions are like sentences in the language of mathematics. They combine numbers, variables (letters), and operations.

#### Examples of writing algebraic expressions from word problems:

1. **Sum of 8 and s:** This means we add 8 and s. The algebraic expression is  $8+s$ .
2. **8 times the sum of c and 7:** First, find the sum of c and 7, which is  $(c+7)$ . Then multiply this sum by 8. The algebraic expression is  $8(c+7)$ .
3. **Take away 4 from m:** This means we subtract 4 from m. The algebraic expression is  $m-4$ .
4. **Subtract 4 from 7 times g:** First, find 7 times g, which is  $7g$ . Then subtract 4 from this. The algebraic expression is  $7g-4$ .

#### Examples of writing algebraic expressions for the perimeter of shapes:



For a triangle with sides  $a$ ,  $b$ , and  $c$ , the perimeter (the total distance around the outside) is  $a + b + c$ .

For a rectangle with length  $l$  and width  $w$ , the perimeter is  $l + w + l + w$ , which can be written as  $2l + 2w$  or  $2(l + w)$ .

**Question 43.1:** Write an algebraic expression for "y divided by 6".

**Question 43.2:** Write an algebraic expression for "5 less than twice x".

## Topic 44: Simplifying Algebraic Expressions

**(Indicator from provided content: Simplify basic algebraic expressions by grouping like terms.)**

**Like terms** are terms that have the same variable raised to the same power. We can combine like terms by adding or subtracting their coefficients (the numbers in front of the variables). Constant terms (numbers without variables) are also like terms.

**Examples of simplifying algebraic expressions:**

1.  $m+5m$ : These are like terms (both have  $m$  to the power of 1). Combine the coefficients:  $1m+5m = (1+5)m=6m$ .
2.  $b+(-3b)+b$ : All terms have  $b$  to the power of 1. Combine coefficients:  $1b-3b+1b = (1-3+1)b = -1b$ , which is written as  $-b$ .
3.  $-7n+6n$ : Both terms have  $n$  to the power of 1. Combine coefficients:  $-7n+6n = (-7+6)n = -1n$ , which is written as  $-n$ .
4.  $9w-4-10w$ : We have terms with  $w$  and constant terms. Combine the  $w$  terms:  $9w-10w = (9-10)w = -1w = -w$ . The constant term is  $-4$ . So the simplified expression is  $-w-4$ .
5.  $8w+5w$ : Both terms have  $w$  to the power of 1. Combine coefficients:  $8w+5w = (8+5)w = 13w$ .

6.  $-m+9-5m$ : Combine the m terms:  $-1m-5m = (-1-5)m = -6m$ .  
 The constant term is +9. So the simplified expression is  $-6m+9$ .

$-8+4x+ -x - 6 + 12$	
$x$	constants
$4x$	-8
$-x$	-6
	+12
$3x$	-2

MANEUVERING THE MIDDLE

**Question 44.1:** Simplify the expression:  $3a+7a-2a$ .

**Question 44.2:** Simplify the expression:  $-4y+6-y+3$ .

### Topic 45: Substituting Values into Algebraic Expressions

**Substitute a value for an unknown into an expression and correctly calculate the answer.)**

To evaluate an algebraic expression, we replace the variables with given numerical values and then perform the operations.

#### Examples of substitution:

- Find the value of  $z+7d$  when  $z=7$  and  $d=4$ . Substitute the values:  
 $7+7(4) = 7+28 = 35$ .

2. Find the value of  $4-5s+3b$  when  $s=6$  and  $b=2$ . Substitute the values:  $4-5(6)+3(2) = 4-30+6 = -26+6 = -20$ .
3. Find the value of  $8k+d$  when  $k=2$  and  $d=3$ . Substitute the values:  $8(2)+3 = 16+3 = 19$ .
4. Find the value of  $-7-2b+6-3r$  when  $b=3$  and  $r=4$ .

Substitute the values:  $-7-2(3)+6-3(4)$

$$= -7-6+6-12$$

$$= -13+6-12$$

$$= -7-12$$

$$= -19.$$

5. Find the value of  $7(5f-3n)-8$  when  $n=3$  and  $f=7$ .

Substitute the values:  $7(5(7)-3(3))-8$

$$= 7(35-9)-8$$

$$= 7(26)-8$$

$$= 182-8$$

$$= 174.$$

6. Find the value of  $-5d-k+7$  when  $k=14$  and  $d=5$ .

Substitute the values:  $-5(5)-14+7$

$$= -25-14+7$$

$$= -39+7$$

$$= -32.$$

**Question 45.1:** Find the value of  $2x+3y$  when  $x=5$  and  $y=2$ .

**Question 45.2:** Find the value of  $a^2-b$  when  $a=4$  and  $b=6$ .

**Sub-strand 3: Variables and Equations**

Equations are mathematical statements that show that two expressions are equal. They often contain variables, which are letters representing unknown numbers.

**Topic 46: Expressing Problems as Equations**

We can translate word problems into equations to help us solve them. We use a letter (a variable) to stand for the unknown quantity.

**Example 1:**

(i) The cost of two pens is ₦15. If one costs ₦5.50, what is the cost of the other pen?

\* Let  $x$  be the cost of the other pen.

\* The equation is:  $5.50+x = 15$ .

(ii) The product of two numbers is 120. If one of the numbers is 24, what is the other number?

\* Let  $y$  be the other number.

\* The equation is:  $24 \times y = 120$ .

**Example 2: Ama's Plant Growth**

Ama's plant is initially 5cm tall and grows 2cm a day.

(i) Write a mathematical sentence (equation) that represents the height of the plant after  $d$  days.  
 \* Let  $h$  be the height of the plant in cm.  
 \* After  $d$  days, the plant will have grown  $2 \times d$  cm.  
 \* The initial height was 5 cm.  
 \* The equation is:  $h=5+2d$ .

(ii) What will the height of the plant be after 20 days?

\* Substitute  $d=20$  into the equation:  $h = 5+2(20)$

$$=5+40$$

$$=45 \text{ cm.}$$

(iii) How many days will the height take to reach 75cm?

\* Substitute  $h=75$  into the equation:  $75 = 5+2d$ .

Complete the table:

Days (d)	1	2	3	4	5
Height (h)	$5+2(1)=7 \text{ cm}$	$5+2(2)=9 \text{ cm}$	$5+2(3)=11 \text{ cm}$	$5+2(4)=13 \text{ cm}$	$5+2(5)=15 \text{ cm}$

**Question 46.1:** Write an equation for the problem: "A number plus 7 equals 19." Use the variable  $n$  for the number.

**Question 46.2:** Kwame had some mangoes. He gave 8 to his friend and now has 12 left. Write an equation to find how many mangoes Kwame started with. Use the variable  $m$ .

## Topic 47: Solving One-Step Equations

Solving an equation means finding the value of the variable that makes the equation true. For one-step equations, we often use the idea of inverse operations to isolate the variable.

### Example 1: Using Concrete Materials (Balance Scale)

Imagine a balance scale. An equation is like a balanced scale where both sides have equal weight. To solve for the unknown, we need to get the unknown by itself on one side while keeping the scale balanced.

Consider the equation  $p+3=7$ .

- We can represent  $p$  as an unknown number of blocks and add 3 blocks to one side of the scale.
- The other side has 7 blocks.
- To find  $p$ , we need to remove the 3 blocks from the side with  $p$ . To keep the scale balanced, we must also remove 3 blocks from the other side.
- This leaves  $p$  by itself on one side and  $7-3=4$  blocks on the other side.
- So,  $p=4$ .

### **Example 2: Solving Symbolically**

Consider the equation  $x-5=12$ . To isolate  $x$ , we need to undo the subtraction of 5. The inverse operation of subtracting 5 is adding 5. Add 5 to both sides of the equation:  $x-5+5=12+5$   $x=17$

Consider the equation  $3y=15$  (which means  $3 \times y = 15$ ). To isolate  $y$ , we need to undo the multiplication by 3. The inverse operation of multiplying by 3 is dividing by 3. Divide both sides of the equation by 3:

$$\frac{3y}{3} = \frac{15}{3}$$

$$y=5$$

Consider the equation  $\frac{z}{4}=6$ . To isolate  $z$ , we need to undo the division by 4. The inverse operation of dividing by 4 is multiplying by 4. Multiply both sides of the equation by 4:

$$\frac{z}{4} \times 4 = 6 \times 4$$

$$= \frac{z}{4} \times 4 = 6 \times 4$$

$$= \frac{z}{1} = 24$$

$$Z = 24$$

**Question 47.1:** Solve the equation  $a+8=15$  symbolically.

**Question 47.2:** Solve the equation  $2b=18$  symbolically.

### Topic 48: Creating Problems for Equations

We can create word problems or stories that match a given equation.

**Example 1:** Create two different stories for the equation  $5+k=9$ .

- **Story 1 (Addition):** "Ama had 5 oranges. Her friend gave her  $k$  more oranges. Now Ama has 9 oranges in total. How many oranges did her friend give her?"
- **Story 2 (Combining quantities):** "There are 5 red balloons and  $k$  blue balloons at a party. In total, there are 9 balloons. How many blue balloons are there?"

### Example 2: Solving a Puzzle with Equations

Imagine a puzzle like this:

$$\clubsuit + 2 = 7$$

$$\clubsuit + \diamond = 10$$

From the first equation, we can find the value of  $\clubsuit$ :

$$\clubsuit = 7 - 2 = 5.$$

Now we can substitute the value of  $\clubsuit$  into the second equation:

$$5 + \diamond = 10$$

$$\diamond = 10 - 5 = 5.$$

So,  $\clubsuit = 5$  and  $\diamond = 5$ .

$$\text{Strawberry} + \text{Strawberry} + \text{Strawberry} = 36$$

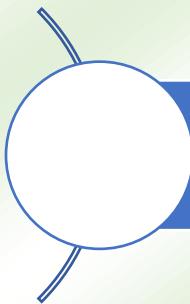
$$\text{Watermelon} + \text{Watermelon} + \text{Watermelon} = 57$$

$$\text{Mango} + \text{Mango} + \text{Mango} = 9$$

$$\text{Mango} + \text{Watermelon} + \text{Strawberry} + \text{Watermelon} = ?$$

**Question 48.1:** Create a word problem for the equation  $x - 3 = 10$ .

**Question 48.2:** Create a word problem for the equation  $4y = 20$ .



## Strand 3: Geometry and Measurement

### Sub-strand 1: Lines and Shapes

We'll be looking at different types of four-sided shapes called quadrilaterals and their properties.

#### Topic 49: Properties of Squares and Rectangles

Let's start by looking closely at squares and rectangles.

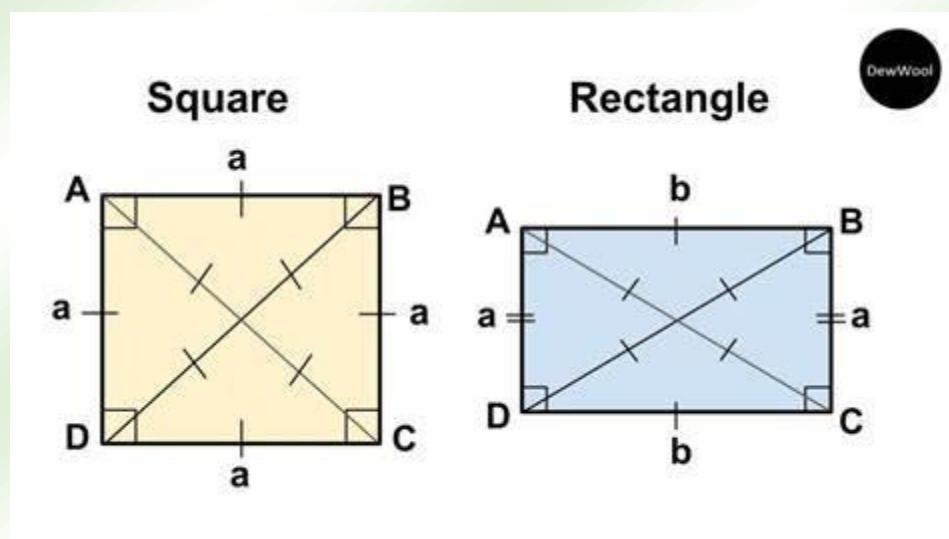
##### Rectangle:

- Has **four sides**.
- **Opposite sides are equal** in length.
- **Opposite sides are parallel**.
- Has **four right angles** (90 degrees).

##### Square:

- Has **four sides**.
- **All sides are equal** in length.
- **Opposite sides are parallel**.
- Has **four right angles** (90 degrees).

Notice that a square is a special type of rectangle where all the sides are equal.



**Question 49.1:** What is one property that both a square and a rectangle share?

**Question 49.2:** What is a property that a square has but a rectangle does not necessarily have?

### Topic 50: Investigating Properties with Paper Folding

We can use paper folding to discover some interesting properties of squares and rectangles.

#### Activity:

1. Take a cut-out of a rectangle. Fold it so that two opposite sides meet perfectly. What do you notice about the lengths of these sides?
2. Unfold it. Now fold it so that the other two opposite sides meet perfectly. What do you notice about their lengths?
3. Fold a corner of the rectangle. Can you see a right angle? How many right angles does a rectangle seem to have?
4. Now take a cut-out of a square. Fold it so that two opposite sides meet. Do the same for the other pair of opposite sides. What do you notice about the lengths of all the sides?

5. Fold a corner of the square. Is it also a right angle?

**Diagonals:** A diagonal is a line segment that joins two non-adjacent vertices (corners) of a shape.

1. On your square, fold it from one corner to the opposite corner to create a diagonal. Do the same for the other pair of opposite corners. Where do the diagonals seem to meet? Do they seem to be the same length?
2. Repeat this with your rectangle. Do its diagonals also seem to bisect (cut in half) each other? Do they seem to be the same length?

Let's fill in a table based on our investigations:

Properties	Rectangle	Square
All sides are congruent	No	Yes
Opposite sides are congruent	Yes	Yes
Opposite sides are parallel	Yes	Yes
All angles are right angles	Yes	Yes
Opposite angles are congruent	Yes	Yes
Diagonals bisect each other	Yes	Yes
Diagonals are congruent	Yes	Yes
Diagonals meet at right angles	No	Yes

**(Indication for Image: Step-by-step diagrams showing how to fold a square and a rectangle to investigate their sides, angles, and diagonals.)**

**Question 50.1:** Based on the paper folding, do the diagonals of a rectangle always have the same length?

**Question 50.2:** Based on the paper folding, do the diagonals of a square always meet at right angles?

## Topic 51: Regular Polygons

A **regular polygon** is a polygon where all sides are of equal length AND all interior angles are of equal measure.

**How to prove a polygon is regular:**

1. **Measuring:** You can measure all the sides. If they are all the same length, that's one condition met. Then, you can measure all the angles. If they are all the same size, then the polygon is regular.
2. **Folding and Superimposing:** For some shapes, you can fold them in ways that show if sides and angles match up. If they all match up perfectly through folding, the polygon is likely regular.

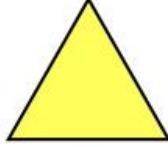
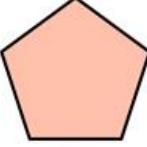
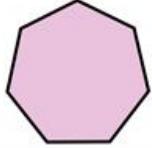
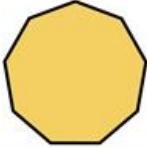
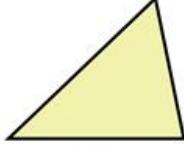
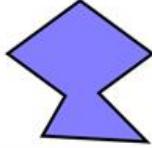
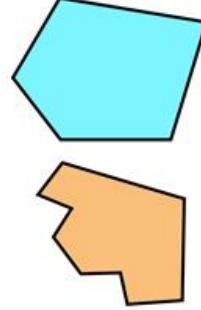
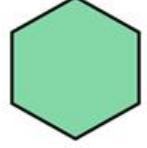
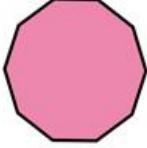
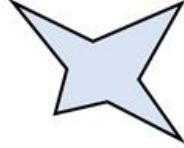
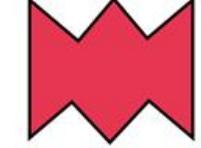
**Examples:**

- A **square** is a regular quadrilateral because all its sides are equal, and all its angles are right angles. If you fold a square along its diagonals or through the midpoints of opposite sides, the parts will perfectly overlap.
- An **equilateral triangle** (a triangle with all sides equal) is also regular because all its angles are also equal (60 degrees each).

A **rectangle** is generally NOT regular because, although all its angles are equal (right angles), its sides are not necessarily all equal.

A **rhombus** (a quadrilateral with all sides equal) is generally NOT regular because, although all its sides are equal, its angles are not necessarily all equal.

## Regular and Irregular Polygons

	Regular Polygons	Irregular Polygons
With odd number of sides	   	  
With even number of sides	   	   

**Question 51.1:** Is a regular triangle called an equilateral triangle? Why?

**Question 51.2:** Can a rectangle ever be a regular polygon? If so, when?

## Sub-strand 2: Measurement

We will learn how to estimate and calculate the perimeter and surface area of 2D shapes.

## Topic 52: Estimating and Calculating Perimeter

**Perimeter** is the total distance around the outside of a 2D shape.

**Estimating Perimeter:** We can use things we know the size of (referents) to estimate.

- A centimetre is about the width of your thumb.
- A metre is about the length of two big steps.

**Example:** Let's estimate the perimeter of your exercise book.

- Estimate the length in thumb widths (cm).
- Estimate the width in thumb widths (cm).
- Add them up and multiply by two (since there are two lengths and two widths).

Now, let's measure the actual length and width with a ruler and calculate the perimeter. Perimeter of a rectangle =  $2 \times (\text{length} + \text{width})$ . Compare your estimate to the calculated perimeter.

**Question 52.1:** Estimate the perimeter of your math set cover using your hand span as a referent. Then, measure it with a ruler and calculate the actual perimeter. How close was your estimate?

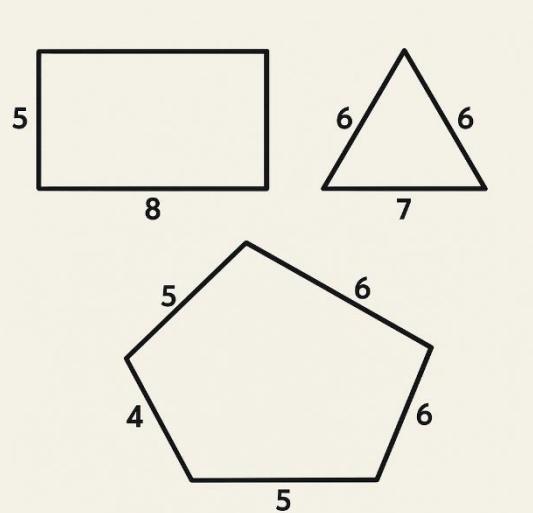
**Question 52.2:** Estimate the perimeter of the classroom floor using your pace steps as a referent.

## Topic 53: Calculating Perimeter of Given Shapes

To find the perimeter of any 2D shape, we simply add the lengths of all its sides.

**Example 1: Rectangle** A rectangle has sides of length 5 cm and 3 cm.  
 $\text{Perimeter} = 5 \text{ cm} + 3 \text{ cm} + 5 \text{ cm} + 3 \text{ cm} = 16 \text{ cm}$ . Alternatively,  $\text{Perimeter} = 2 \times (5 \text{ cm} + 3 \text{ cm}) = 2 \times 8 \text{ cm} = 16 \text{ cm}$ .

**Example 2: Triangle** A triangle has sides of length 4 m, 6 m, and 7 m.  
 $\text{Perimeter} = 4 \text{ m} + 6 \text{ m} + 7 \text{ m} = 17 \text{ m}$ .



**Question 53.1:** Calculate the perimeter of a square with sides of 8 cm.

**Question 53.2:** A pentagon has sides of length 2 m, 3 m, 2.5 m, 3.5 m, and 4 m. What is its perimeter?

## Topic 54: Calculating Surface Area of Given Shapes

It seems there might be a slight confusion in the term used here. For 2D shapes, we usually talk about **area**, which is the amount of surface the shape covers, not "surface area" (which is more commonly used for 3D shapes). Let's proceed with calculating the **area** of 2D shapes.

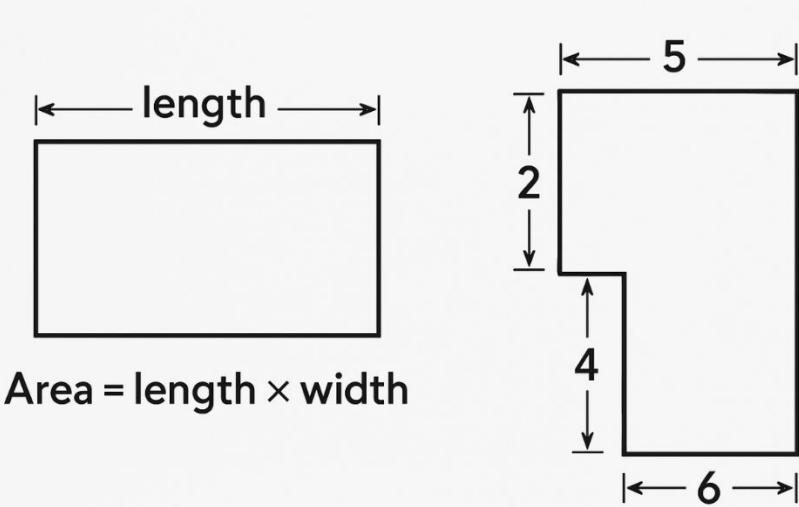
**Area of a Rectangle:** Area = length × width.

**Example:** A rectangle has a length of 7 cm and a width of 4 cm. Area =  $7 \text{ cm} \times 4 \text{ cm} = 28 \text{ square centimetres(cm}^2\text{)}$ .

**Area of a Compound Shape (made of rectangles):** To find the area of a compound shape made of rectangles, we can split it into simpler rectangles, find the area of each, and then add them together.

**Example:** Consider an L-shaped figure that can be split into two rectangles:

- Rectangle 1: length = 5 m, width = 2 m, Area =  $5 \text{ m} \times 2 \text{ m} = 10 \text{ m}^2$ .
- Rectangle 2: length = 3 m, width = 4 m, Area =  $3 \text{ m} \times 4 \text{ m} = 12 \text{ m}^2$ .
- Total Area =  $10 \text{ m}^2 + 12 \text{ m}^2 = 22 \text{ m}^2$ .



**Question 54.1:** Calculate the area of a rectangle with length 9 m and width 6 m.

**Question 54.2:** A compound shape is made of two rectangles. Rectangle A has dimensions 3 cm by 5 cm, and Rectangle B has dimensions 2 cm by 4 cm. What is the total area of the compound shape?

### Topic 55: Referents for Cubic Units

**Volume** is the amount of space a 3D object occupies. We measure volume in cubic units.

- A **cubic centimetre (cm<sup>3</sup>)** is the volume of a cube with sides that are 1 cm long. Think of a sugar cube as being roughly 1cm<sup>3</sup>.
- A **cubic metre (m<sup>3</sup>)** is the volume of a cube with sides that are 1 metre long. A large water tank (polytank) can be a good referent for something that can hold about 1m<sup>3</sup> of water.

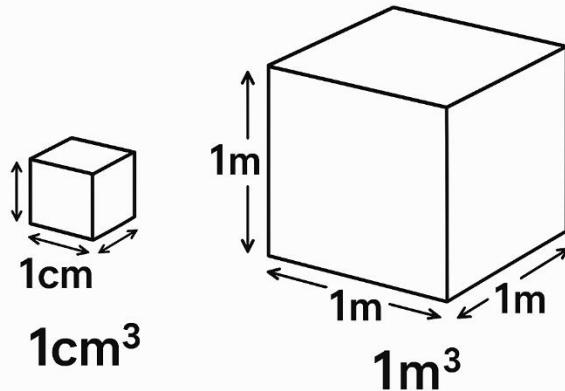
#### Relationship between cm<sup>3</sup> and m<sup>3</sup>:

Since 1 metre = 100 centimetres, then:

$$1\text{m}^3 = 1\text{m} \times 1\text{m} \times 1\text{m}$$

$$= (100\text{cm}) \times (100\text{cm}) \times (100\text{cm}) = 1,000,000\text{cm}^3.$$

$$\text{Also, } 1\text{cm}^3 = 1,000,000\text{m}^3.$$



**Question 55.1:** What everyday object can you use as a referent for approximately 1cm<sup>3</sup>?

**Question 55.2:** What is the volume in cm<sup>3</sup> of a cube with sides of 10 cm?

### Topic 56: Determining Volume by Counting Cubes

We can find the volume of a rectangular prism (box) by seeing how many unit cubes (1cm<sup>3</sup>) would fit inside it.

**Example:** Consider a box where the base can fit  $2 \times 7 = 14$  of 1cm<sup>3</sup> cubes, and the box is 3 cm high. This means we can stack 3 layers of these 14 cubes.

Total number of cubes =  $2 \times 7 \times 3 = 42$  cubes. So, the volume of the box is 42cm<sup>3</sup>.

**Formula for the Volume of a Box:** Volume = length × width × height (V=l×w×h)

**Question 56.1:** A box has a length of 4 cm, a width of 3 cm, and a height of 2 cm. What is its volume in cm<sup>3</sup>?

**Question 56.2:** If a box has a base that can hold 10 1cm<sup>3</sup> cubes and its height is 5 cm, what is its volume?

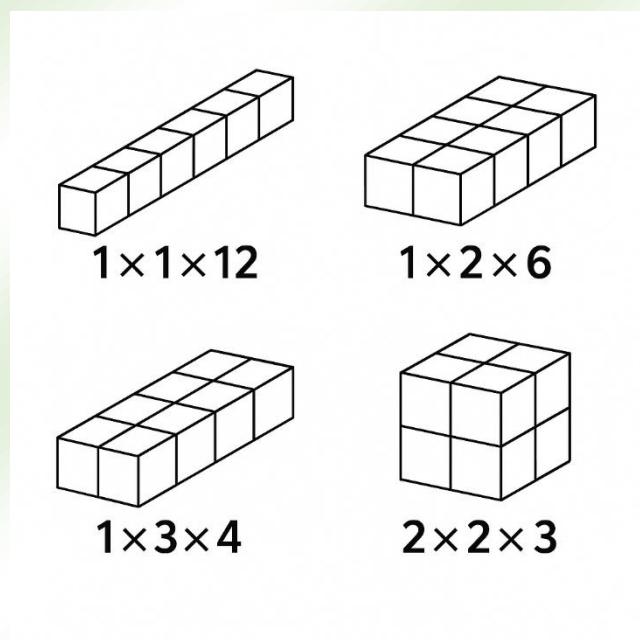
### Topic 57: Boxes with the Same Volume

Different boxes can have the same volume if their dimensions multiply to the same number.

**Example:** You have 12 cm<sup>3</sup> cubes. What are the possible dimensions (length × width × height) of boxes you can make using all these cubes?

- 1cm×1cm×12cm
- 1cm×2cm×6cm
- 1cm×3cm×4cm
- 2cm×2cm×3cm

These are some of the possible dimensions.



**Question 57.1:** You have 24 cm<sup>3</sup> cubes. Find two different sets of dimensions for a box you can make.

**Question 57.2:** Can a box with dimensions 2cm×5cm×3cm have the same volume as a box with dimensions 1cm×10cm×3cm? Explain.

### Topic 58: Understanding Capacity

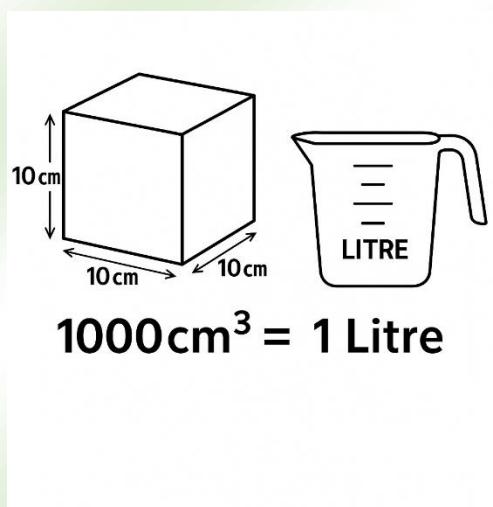
**Capacity** is the amount a container can hold. We often measure capacity in units like litres (L) and millilitres (mL).

#### Relationship between Volume and Capacity:

- 1cm<sup>3</sup> is equal to 1 millilitre (mL).
- A container with a volume of 1000cm<sup>3</sup> can hold 1000 mL, which is equal to 1 litre (L).

So, 1L=1000cm<sup>3</sup>.

We also know that 1m<sup>3</sup>=1,000,000cm<sup>3</sup>=1000L.



**Question 58.1:** A container has a volume of 500cm<sup>3</sup>. What is its capacity in millilitres? In litres?

**Question 58.2:** A bottle can hold 2 litres of water. What is the volume of the bottle in cm<sup>3</sup>?

**Sub-strand 3: Measurement - Angles**

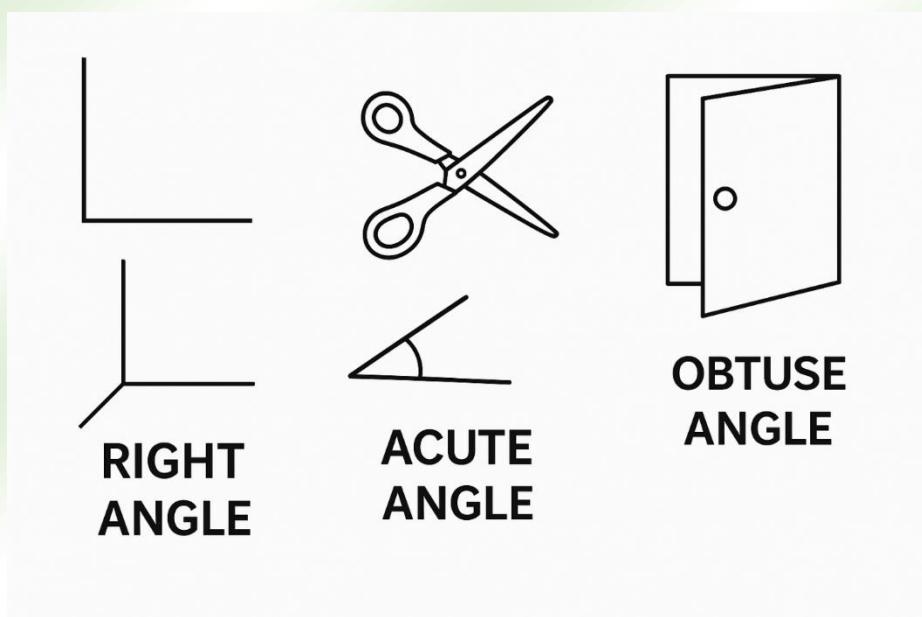
We will now explore angles and how to measure them.

**Topic 59: Identifying Angles**

An **angle** is formed when two lines or rays meet at a common endpoint called the vertex.

We can find angles all around us!

- **Right Angle:** A right angle looks like the corner of a square or a book. Examples in the environment include the corner of a wall, the corner of a table, or the angle where the hands of a clock are at 3:00 or 9:00.
- **Angles less than a right angle (Acute Angles):** These are "sharp" angles. Examples include the angle of a partially opened pair of scissors, the angle of a ramp, or the angle formed by the hands of a clock at 1:00.
- **Angles larger than a right angle (Obtuse Angles):** These are "wide" angles (but less than a straight line). Examples include the angle of a door that is more than halfway open, or the angle formed by the hands of a clock at 2:00.



**Question 59.1:** Give an example of an object in the classroom that forms a right angle.

**Question 59.2:** Give an example of an angle in the environment that is less than a right angle.

**Question 59.3:** Give an example of an angle in the environment that is larger than a right angle.

## Topic 60: Measuring and Classifying Angles

We use a **protractor** to measure angles in degrees ( $^\circ$ ).

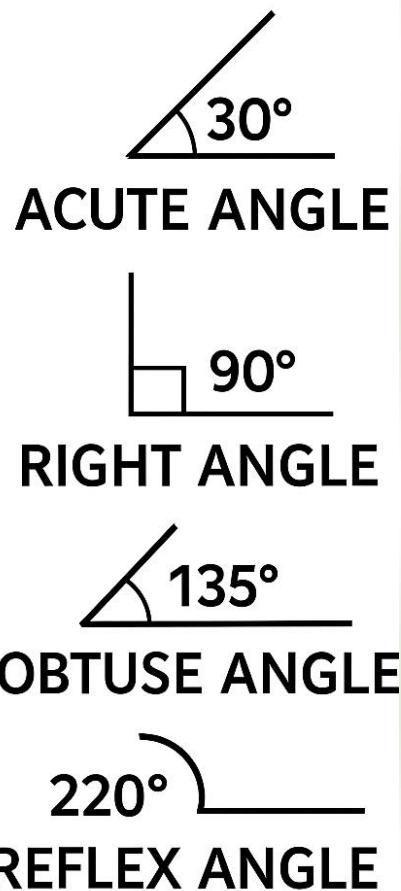
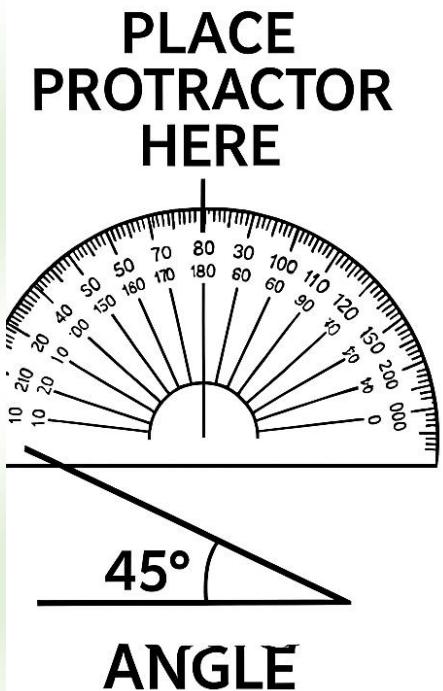
### How to use a protractor:

1. Place the center point of the protractor on the vertex of the angle.
2. Align the base line ( $0^\circ$  line) of the protractor with one of the rays of the angle.
3. Read the degree marking where the other ray of the angle crosses the protractor scale.

### Classification of Angles by Size:

- **Acute Angle:** Measures less than  $90^\circ$ .

- **Right Angle:** Measures exactly  $90^\circ$ .
- **Obtuse Angle:** Measures greater than  $90^\circ$  but less than  $180^\circ$ .
- **Straight Angle:** Measures exactly  $180^\circ$  (forms a straight line).
- **Reflex Angle:** Measures greater than  $180^\circ$  but less than  $360^\circ$ .



**Question 60.1:** What type of angle measures  $60^\circ$ ?

**Question 60.2:** An angle measures  $110^\circ$ . What type of angle is it?

**Activity:**

Use a protractor to measure the angles on a worksheet and classify each one as acute, right, or obtuse. You can also try drawing angles of specific measures, like  $30^\circ$ ,  $90^\circ$ , and  $135^\circ$ .

We will learn how to describe the position and movement of objects using cardinal points and explore transformations of 2D shapes.

## Topic 61: Describing Position Using Cardinal Points

The **cardinal points** are North (N), South (S), East (E), and West (W). We use these to describe the direction of one place or object relative to another.

### Example 1:

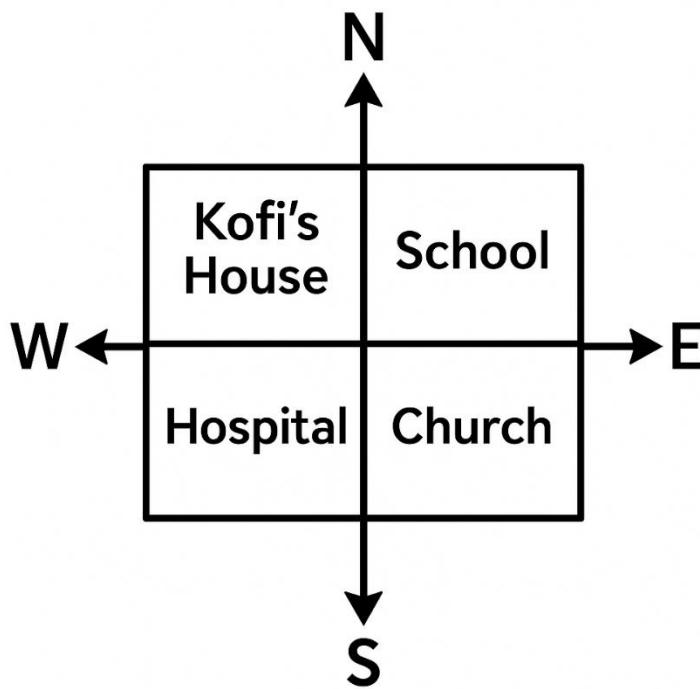
If point P is to the west of point T, it means if you are at T, you would need to travel west to get to P. If point P is to the north of point A, it means if you are at A, you would need to travel north to get to P.

### Example 2:

Imagine a simple map or grid. If Kofi's house is our starting point:

- The school is East of Kofi's house means you would travel east from Kofi's house to reach the school.
- The hospital is North of Kofi's house means you would travel north from Kofi's house to reach the hospital.

**Example 3:** Giving directions on a grid. "From Kofi's house, move 2 squares to the east and 2 squares to the north to get to the church." This describes a path using the cardinal directions.



**Question 61.1:** If a market is south of a school, describe the position of the school relative to the market using cardinal points.

**Question 61.2:** Give directions from a point labeled 'X' to a point labeled 'Y' on a simple grid using north, south, east, and west.

### Topic 62: Identifying Reflections of 2D Shapes

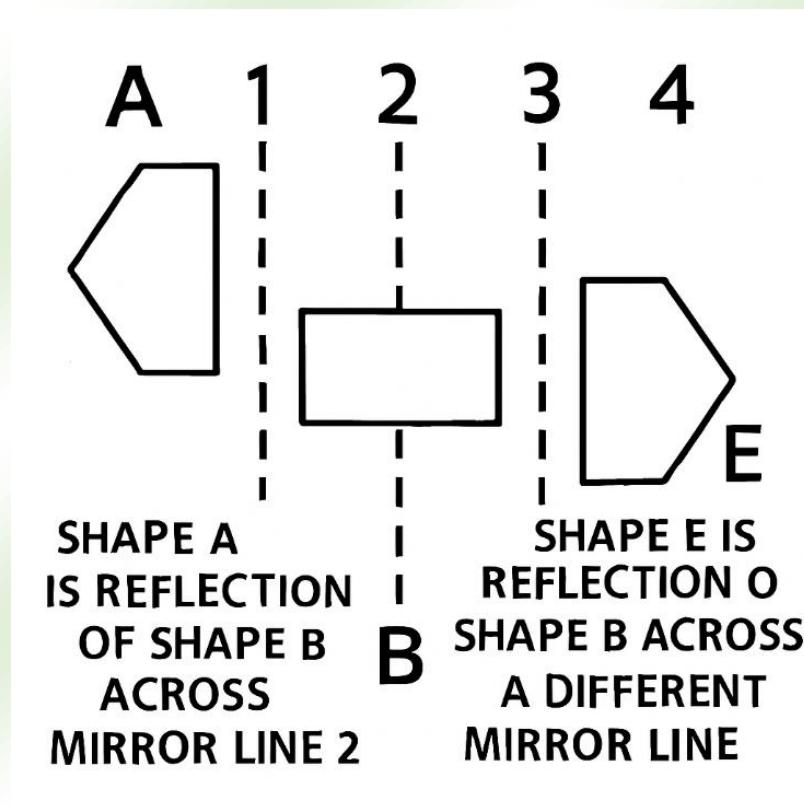
A **reflection** is a transformation that flips a shape over a line, called the **mirror line**. The reflected shape (the image) is a mirror image of the original shape (the object).

#### Properties of Reflection:

- The image is the same size and shape as the object.
- The distance from each point on the object to the mirror line is the same as the distance from the corresponding point on the image to the mirror line.
- The orientation of the image is reversed (it's flipped).

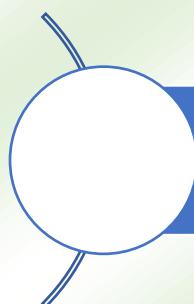
#### Example:

Imagine a shape 'B' and a mirror line. Its reflection 'E' will be on the opposite side of the mirror line, the same distance away.



**Question 62.1:** If you reflect the letter 'T' vertically, what image do you get?

**Question 62.2:** Look at a shape and its reflection across a horizontal line. What can you say about the points on the shape and their corresponding points on the image relative to the mirror line?



## Strand 4: Data

Sub-strand 1: Data Collection, Organisation, Presentation, Interpretation and Analysis

We will learn about different types of data and how to present and understand it.

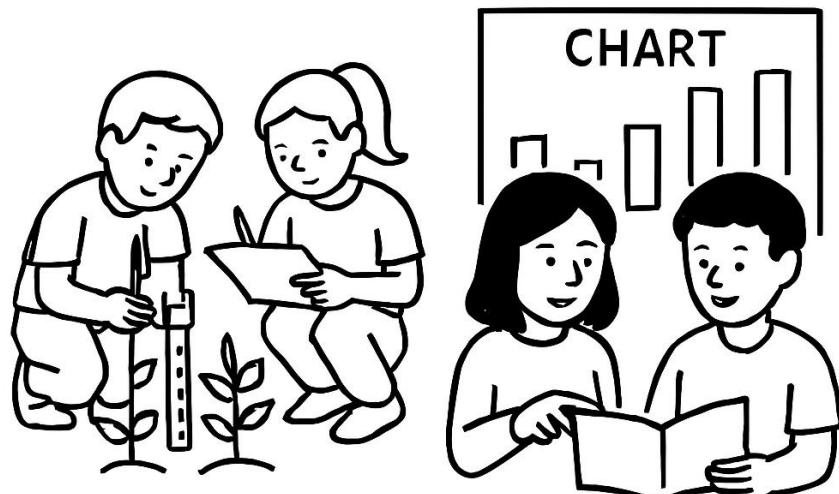
### Topic 63: First-hand vs. Second-hand Data

- **First-hand data** is information that you collect yourself. For example, if your class measures the heights of all the students, that height data is first-hand for your class.
- **Second-hand data** is information that has already been collected and organized by someone else. For example, if you use the rainfall data that another class collected, that data is second-hand for you.

#### Example:

Mrs. Acquaye's class measured the rainfall for 5 days. For Mrs. Acquaye's class, this rainfall data is **first-hand**. However, if another

# COLLECTING DATA

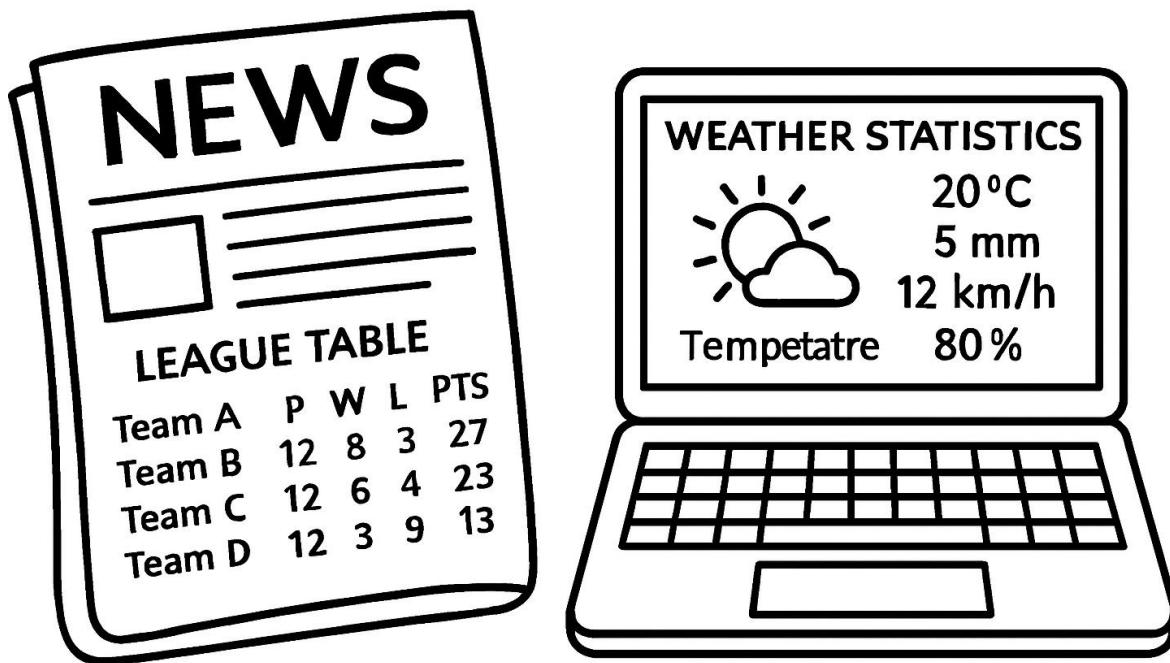


class uses this same rainfall data for their lesson, it becomes **second-hand** data for them.

We can find second-hand data in many places:

- **Newspapers:** League tables for sports, price lists, weather reports.
- **Magazines:** Survey results, statistics on various topics.
- **Internet:** Data on websites about population, sales figures, research findings.

**Example:** A league table in a newspaper shows second-hand data because the newspaper collected the results of many matches that other teams played.



**Question 63.1:** What is the main difference between first-hand and second-hand data?

**Question 63.2:** Give an example of second-hand data you might find on the internet.

### Topic 64: Constructing and Interpreting Double Bar Graphs

A **double bar graph** is used to compare two sets of data side-by-side. It has:

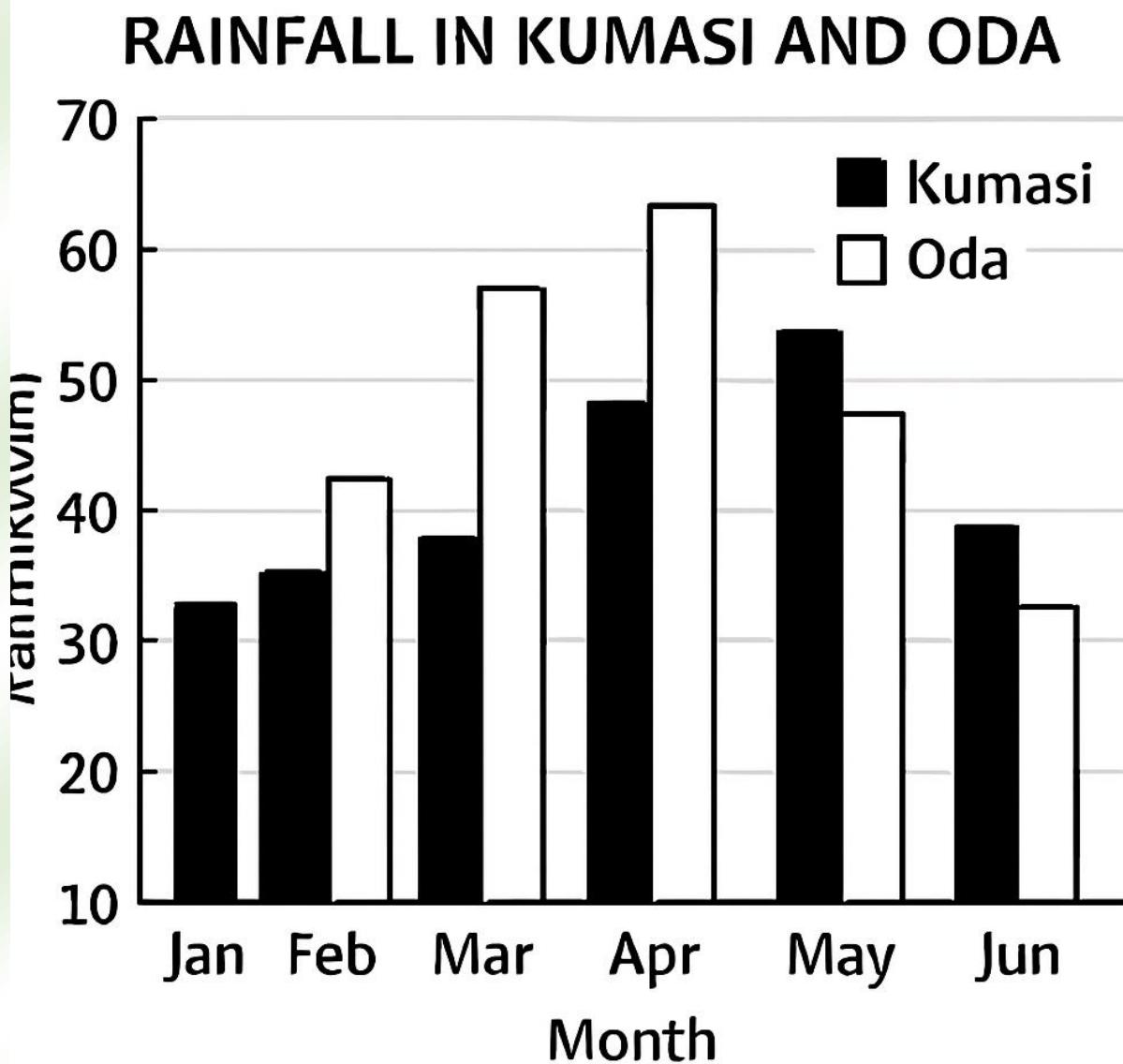
- A **title** that tells what the graph is about.
- **Labelled axes:** A horizontal axis and a vertical axis, each labelled with what they represent.
- **Bars** representing the amounts for each category, with pairs of bars (one for each set of data) next to each other.
- A **key or legend** to show which bar represents which set of data.

**Example:** Rainfall in mm for Kumasi and Oda (January to June)

Month	Kumasi (mm)	Oda (mm)
January	5	3
February	10	10
March	15	13
April	20	25
May	50	40
June	45	50

To draw a double bar graph for this data:

1. Draw a horizontal axis (for months) and a vertical axis (for rainfall in mm).
2. Label the months along the horizontal axis.
3. Choose a scale for the vertical axis (e.g., each division represents 5 mm).
4. For each month, draw two bars side-by-side, one for Kumasi's rainfall and one for Oda's rainfall, according to the data.
5. Create a key to show which colour or pattern represents Kumasi and which represents Oda.
6. Add a title to the graph (e.g., "Rainfall Comparison: Kumasi vs. Oda").



**Question 64.1:** What does a double bar graph help us to do?

**Question 64.2:** Look at the example data. In which month was the rainfall in Oda higher than in Kumasi?

## Topic 65: Interpreting Double Bar Graphs

Once a double bar graph is drawn, we can read and interpret it to answer questions and draw conclusions about the data.

**Example:** Let's use the rainfall graph for Kumasi and Oda again.

Using the graph, we can answer questions like:

- In which month did Kumasi have the highest rainfall? (Look for the tallest bar for Kumasi).
- What was the difference in rainfall between Kumasi and Oda in March? (Compare the heights of the two bars for March).
- Were there any months where Kumasi and Oda had the same amount of rainfall? (Look for bars of equal height for both cities in the same month).

### Example Questions from the provided content:

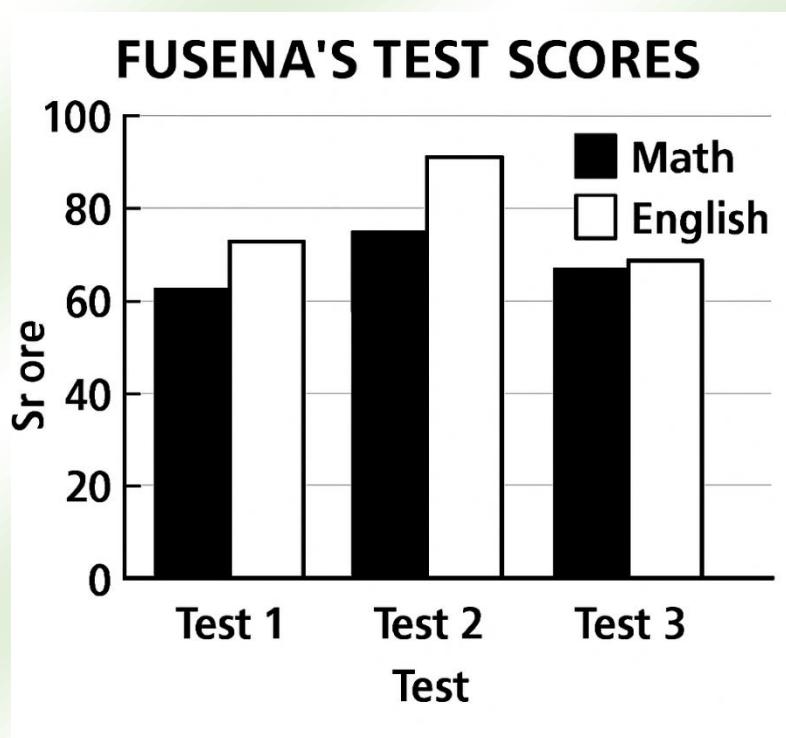
Let's say we have a table of test scores for Fusena in Math and English:

Test	Math (%)	English (%)
1	70	80
2	75	75
3	65	85

If we drew a double bar graph, we could answer:

- In which test was Fusena's worst performance? (Look for the lowest bar).

- How did Fusena perform in Math compared to English across the tests?



**Question 65.1:** Using the imagined rainfall graph, in which month did Oda have the least rainfall?

**Question 65.2:** Using the imagined Fusena's test score graph, in which test did she perform better in Math than English?

### Topic 66: Identifying Double Bar Graphs in Media

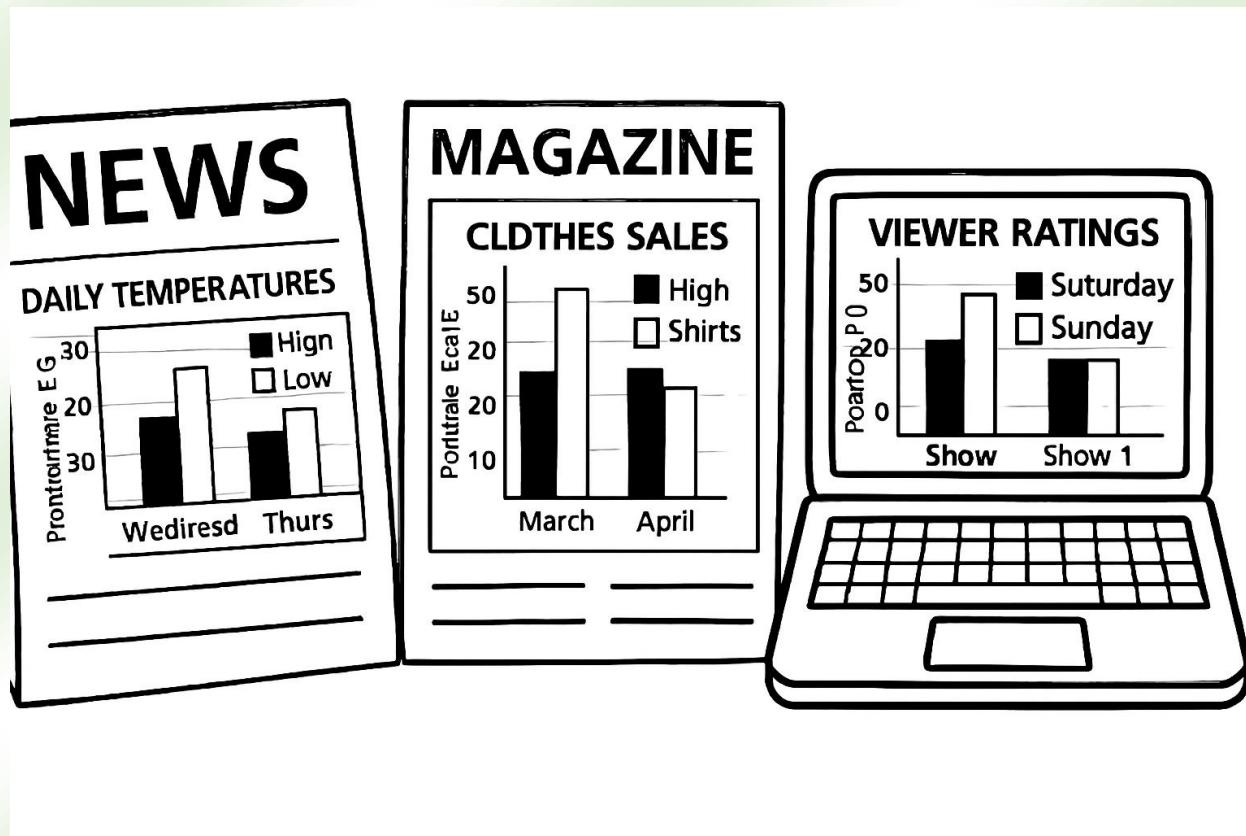
Double bar graphs are a common way to present and compare data in various forms of media.

- You might see them in **newspapers** comparing things like sales of two products over several months or the performance of two sports teams across a season.
- **Magazines** could use them to compare survey results from different groups or changes in statistics over time.

- The **internet** is full of double bar graphs on websites showing all sorts of comparative data, from economic trends to website traffic.

### Activity:

Try to find an example of a double bar graph in a newspaper, a magazine, or online. Describe what the graph is comparing and what conclusions you can draw from it.



**Question 66.1:** Where might you typically find a double bar graph comparing the sales of two different brands of cars over the last year?

**Question 66.2:** Why do you think double bar graphs are useful for presenting information in newspapers and on the internet?

**Sub-strand 2: Chance (Probability)**

We will learn to describe how likely it is for certain events to happen.

**Topic 67: Describing Likelihood**

We can use words to describe how likely an outcome is:

- **Impossible:** The outcome cannot happen.
  - Example: Rolling a 7 on a standard 6-sided die is impossible.
- **Possible:** The outcome can happen. This can be further described as:
  - **Likely:** The outcome has a good chance of happening.
  - **Unlikely:** The outcome does not have a good chance of happening.
  - Example: Rolling a 1 on a standard 6-sided die is possible.
  - Example: Picking a red ball from a bag with mostly red balls is possible (likely).
  - Example: Picking a blue ball from a bag with only a few blue balls and many other colours is possible (unlikely).
- **Certain:** The outcome will definitely happen.
  - Example: Rolling a number from 1 to 6 on a standard 6-sided die is certain.



**IMPOSSIBLE   POSSIBLE   CERTAIN**

**Question 67.1:** Describe the likelihood of picking a yellow sweet from a bag containing only yellow sweets.

**Question 67.2:** Describe the likelihood of a bird speaking English.

**Question 67.3:** Describe the likelihood of rolling an even number on a standard 6-sided die.

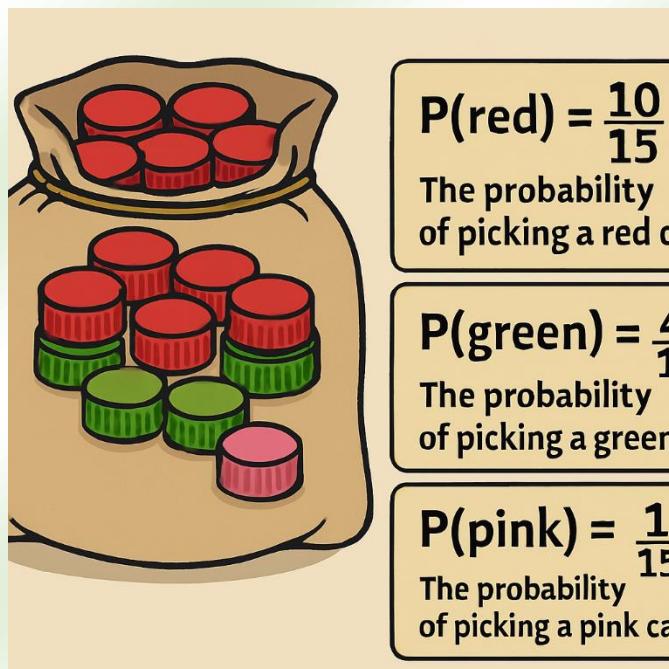
### Topic 68: Designing Probability Experiments

We can set up simple experiments to see these likelihoods in action.

**Example Experiment:** A bag contains 10 red, 4 green, and 1 pink bottle top. If you pick one without looking:

1. Picking a black bottle top is **impossible** because there are no black bottle tops in the bag.
2. Picking a red bottle top is **possible (likely)** because there are the most red bottle tops.
3. Picking a pink bottle top is **possible (unlikely)** because there is only one pink bottle top.

4. Picking a red or green or pink bottle top is **certain** because these are the only colours in the bag.



**Activity:** Design a simple experiment (like using coloured counters or drawing cards) where one outcome is impossible, one is certain, and at least one is possible (either likely or unlikely).

**Question 68.1:** Design an experiment using a spinner with different coloured sections where picking blue is a certain outcome.

**Question 68.2:** Design an experiment using a standard deck of playing cards where picking a heart is a possible (unlikely) outcome.

## Topic 69: Conducting Probability Experiments

Let's try a simple experiment and see what happens.

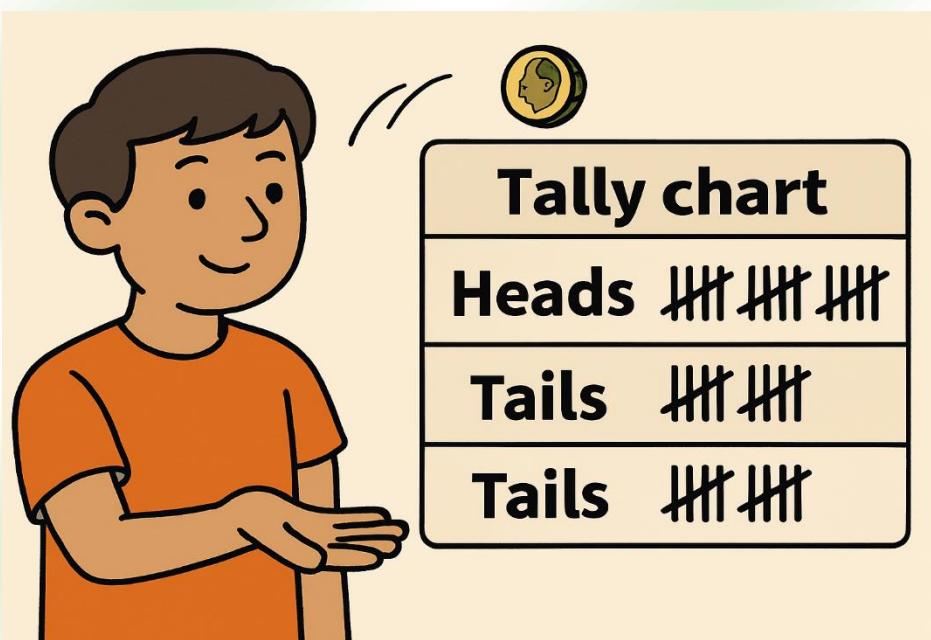
### Experiment: Tossing a Coin

- Predict the outcome of tossing a coin three times. What do you expect to happen (e.g., how many heads, how many tails)?
- Toss a coin three times and record the result (e.g., Head, Tail, Head).
- Did the outcome match your prediction? Explain why it might or might not have.

Now, let's repeat this many times!

In pairs, toss a coin, say, 20 times. Each time, record whether it lands on Heads (H) or Tails (T). After 20 tosses, count how many times you got Heads and how many times you got Tails.

You might find that you get roughly equal numbers of heads and tails, but it's not always exactly the same. This is because each coin toss is independent, and the outcome has an element of chance. However, over many trials, the results tend to even out.



**Question 69.1:** If you toss a coin 10 times, would you expect to get exactly 5 heads and 5 tails? Why or why not?

**Question 69.2:** What happens to the results of a probability experiment as you repeat it more and more times?