1702956 - Eric Akira Sobrinho Hamabata
Atividade Semana 2 Cálculo

1) a) Derivada de $f(x) = x^5$

$$f'(c) = \lim_{x \to c} \frac{f(x) - f(c)}{x - c}$$

$$\lim_{x \to c} \frac{x^5 - c^5}{x - c} = \lim_{x \to c} \frac{\cancel{(x-c)}(x^4 + x^3c + x^2c^2 + xc^3 + c^4)}{\cancel{x-c}}$$

$$f'(c) = \lim_{x \to c} x^4 + x^3c + x^2c^2 + xc^3 + c^4$$

$$f'(c) = c^4 + c^3c + c^2c^2 + cc^3 + c^4$$

$$f'(c) = c^4 + c^4 + c^4 + c^4 + c^4$$

$$f'(c) = 5c^4$$

Resposta 1 a)

$$\boxed{f'(x) = 5x^4}$$

b)

$$\frac{y - f(x_0)}{x - x_0} = f'(x_0)$$

$x_0 = -1$

$f(x_0) = -1$

$f'(x_0) = 5x^4 = 5 \cdot (-1)^4 = 5$

$$\frac{y - (-1)}{x - (-1)} = 5$$

$y + 1 = 5(x + 1)$

$y = 5x + 5 - 1$

$y = 5x + 4$

Resposta 1 b)

função da reta tangente em $(-1, -1)$ = $f(x) = 5x + 4$

2) $f(x)=\sqrt[4]{x}$

$f'(x)=x^{\frac{1}{4}}$ Regra do Tombo $f(x)=x^{a} \to f'(x)=a.x^{a-1}$

$f'(x)=\frac{1}{4}.x^{\frac{1}{4}-1}$

$\boxed{f'(x)=\frac{1}{4}.x^{\frac{-3}{4}}}$; $x>0$

Resposta 2 a)

$f'(x)=\frac{1}{4}.x^{\frac{-3}{4}}$

$\frac{y-f(x_0)}{x-x_0}=f'(x_0)$

$\frac{y-2^{2}}{x-2^{8}}=2^{-8}$

$y-2^{2}=2^{-8}(x-2^{8})$

$y-4=\frac{1}{256}(x-256)$

$y=\frac{x}{256}+4-1$

$\boxed{y=\frac{x}{256}+3}$

$x_0=256$

$f(x_0)=\sqrt[4]{256}$

$f(x_0)=4$

$f(x_0)=2^{2}$

$f'(x_0)=\frac{1}{4}.256^{-\frac{3}{4}}$

$f'(x_0)=\frac{1}{4}.\sqrt[4]{256^{-3}}$

$\frac{1}{4}.\sqrt[4]{\frac{1}{256^{3}}}$

$\frac{1}{4}.\sqrt[4]{\frac{1}{(4^{4})^{3}}}$

$\frac{1}{4}.\frac{1}{\sqrt[4]{(2^{4})^{3}}}$

$\frac{1}{4}.\frac{1}{\sqrt[4]{2^{24}}}$

$\frac{1}{4}.\frac{1}{2^{6}}=\frac{1}{2^{8}}=2^{-8}$

Resposta 2 b)

$y=f(x)=\frac{x}{256}+3$

$y=\frac{x}{256}+3$ $\quad \sqrt[4]{256,32}$

$y=\frac{256,32}{256}+3=1,00125+3$

$\boxed{y\cong 4,00125}$

Resposta 2 c)

$y\cong 4,00125$

3) a) $f(x) = x^7 \operatorname{sen} x$ $\quad g(x) = x^7 \quad g'(x) = 7x^6$

$h(x) = \operatorname{sen} x \quad h'(x) = \cos x$

Regra do Produto

$f'(x) = g'(x) \cdot h(x) + g(x) \cdot h'(x)$

$f'(x) = 7x^6 \operatorname{sen} x + x^7 \cos x$

Resposta 3a)

$f'(x) = 7x^6 \cdot \operatorname{sen} x + x^7 \cos x$

b) $f(x) = \cos x \cdot \log_{10} x$ $\quad g(x) = \cos x \quad g'(x) = -\operatorname{sen} x$

Regra Produto

$h(x) = \log_{10} x \quad h'(x) = \frac{1}{x \ln 10}$

$f'(x) = g'(x) h(x) + g(x) \cdot h'(x)$

$f'(x) = -\operatorname{sen} x \cdot \log_{10} x + \cos x \cdot \frac{1}{x \ln 10}$

Resposta 3b)

$f'(x) = -\operatorname{sen} x \cdot \log_{10} x + \cos x \cdot \frac{1}{x \ln 10}$

c) $f(x) = \frac{2x^2+1}{x+5}$ $\quad g(x) = 2x^2+1 \quad g'(x) = 4x$

$h(x) = x+5 \quad h'(x) = 1$

Regra quociente

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{(h(x))^2}$$

$$f'(x) = \frac{4x(x+5) - (2x^2+1) \cdot 1}{(x+5)^2} = \frac{4x^2+20x-2x^2-1}{x^2+10x+25} = \boxed{\frac{2x^2+20x+1}{x^2+10x+25}}$$

$$\frac{4x(x+5)}{(x+5)^2} - \frac{2x^2+1}{(x+5)^2}$$

$$\frac{4x}{x+5} - \frac{2x^2+1}{(x+5)^2}$$

Resposta 3c)

$$f'(x) = \frac{2x^2+20x+1}{x^2+10x+25}$$

d) $f(x) = \dfrac{x^2}{tg\,x}$

$g(x) = x^2 \quad g'(x) = 2x$

$h(x) = tg\,x \quad h'(x) = sec^2 x$

Regra quociente

$$f'(x) = \frac{g'(x) \cdot h(x) - g(x) \cdot h'(x)}{(h(x))^2} = f'(x) = \frac{2x \cdot tg\,x - x^2 \cdot sec^2 x}{(tg\,x)^2}$$

$$f'(x) = \frac{2x}{tg\,x} - \frac{x^2\, sec^2 x}{(tg\,x)^2}$$

Resposta 3d)

$$f'(x) = \frac{2x}{tg\,x} - \frac{x^2 \cdot sec^2 x}{(tg\,x)^2}$$