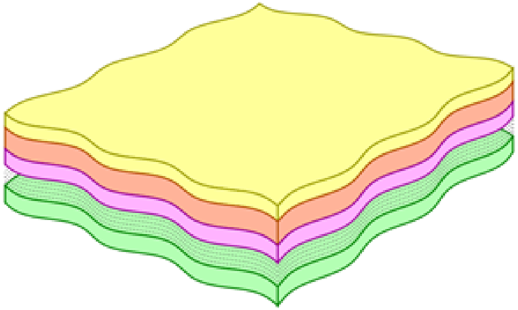
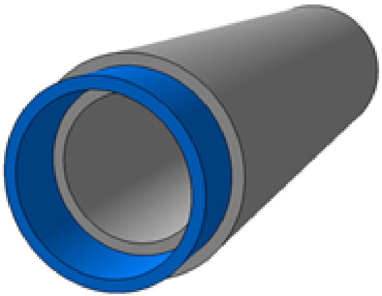
Transient Field Computation in multilayered structures





[PhD Pierric Mora dec. 2015]

[Mora *et al.* (2016) Ultrasonics]

[Kausel *et al.* (1992) J. Eng. Mech.]

[PhD Aditya Krishna sept. 2020]

Asumptions: infinite structures invariant in two directions

Convenient for:

- A localized source emitting a short signal
- Immersed and Embedded plates and pipes
- 2D and 3D cases

Can be compared to:

- $k_n(\omega, \nu)$-modal methods
- $\omega_n(\mathbf{k})$-modal methods

[Kausel (1994) IJNME] [Ducasse *et al.* (2014) Wave Motion]

Complementary approaches

[Mora (2021) Wave Motion]

Outline

I. **The** FOURIER-FOURIER-LAPLACE **method**

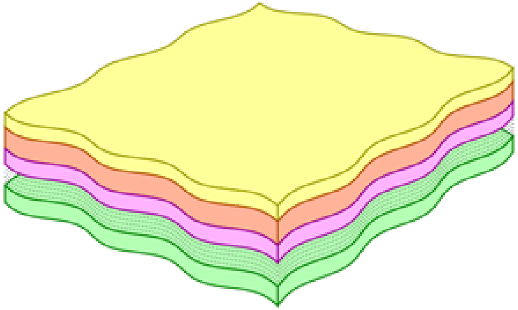
II. **Computation steps and results**

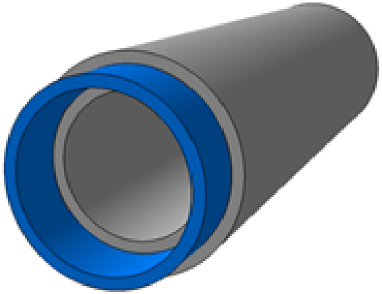
III. **Additional tools**

- ▷ Graphical user interface (under development)
- ▷ Mode computation

IV. **Towards hybrid methods**

TraFiC – I. The Fourier-Fourier-Laplace method





Time $t \to$ complex Laplace variable s

Space:

- material properties invariant with respect to $\mathbf{x} = (x, y)$
- Horizontal position $\mathbf{x} \to$ Horizontal wavevector $\mathbf{k}$
- Computation in the $(\mathbf{k}, z, s)$-domain
- ODEs with respect to the vertical position z

Space:

- material properties invariant with respect to θ and z
- Axial position $z \to$ ax. wavenumber k
Azimuthal position $\theta \to$ az. wavenumber n
- Computation in the (r, n, k, s)-domain
- ODEs with respect to the radial position r

TraFiC – I. The Fourier-Fourier-Laplace method

▷ The displacement vector $\tilde{\mathbf{U}}(z)$ satisfies in each plane layer ($\mathbf{n}$ unit vertical vector, $\mathbb{I}$ identity matrix):

$$(\mathbf{n}\diamond\mathbf{n})\,\tilde{\mathbf{U}}''(z) - \mathrm{i}\,\left[(\mathbf{n}\diamond\mathbf{k}) + (\mathbf{k}\diamond\mathbf{n})\right]\,\tilde{\mathbf{U}}'(z) - \left[(\mathbf{k}\diamond\mathbf{k}) + \rho\, s^2\,\mathbb{I}\right]\,\tilde{\mathbf{U}}(z) = -\tilde{\mathbf{F}}(z) \tag{1}$$

$$\text{Stress vector in the } z\text{-direction:}\qquad \tilde{\mathbf{\Sigma}}_z(z) = (\mathbf{n}\diamond\mathbf{n})\,\tilde{\mathbf{U}}'(z) - \mathrm{i}\,(\mathbf{n}\diamond\mathbf{k})\,\tilde{\mathbf{U}}(z) \tag{2}$$

▷ The displacement vector $\tilde{\mathbf{U}}(r)$ satisfies in each tubular layer: $[(\mathbf{a}\diamond\mathbf{b})_{im} = a_i\, c_{ij\ell m}\, b_m]$

$$\left[\mathbb{T} = \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}\right]$$

$$\begin{aligned}
&(\mathbf{n}_r\diamond\mathbf{n}_r)\,\tilde{\mathbf{U}}''(r) \;+ \\
&\Big\{-\mathrm{i}\,k\,(\mathbf{n}_r\diamond\mathbf{n}_z + \mathbf{n}_z\diamond\mathbf{n}_r) \;+ \\
&\frac{1}{r}\left[\,(\mathbf{n}_r\diamond\mathbf{n}_r) - \mathrm{i}\,\left(\,(\mathbf{n}_r\diamond\mathbf{n}_\theta)\,(n\,\mathbb{I}+\mathrm{i}\,\mathbb{T}) + (n\,\mathbb{I}+\mathrm{i}\,\mathbb{T})\,\,(\mathbf{n}_\theta\diamond\mathbf{n}_r)\,\right)\right]\Big\}\,\tilde{\mathbf{U}}'(r) \;- \\
&\Big\{\left[\,\rho\, s^2\,\mathbb{I} + k^2\,(\mathbf{n}_z\diamond\mathbf{n}_z)\,\right] + \\
&\frac{k}{r}\left[\,\mathrm{i}\,(\mathbf{n}_r\diamond\mathbf{n}_z) + (n\,\mathbb{I}+\mathrm{i}\,\mathbb{T})\,\,(\mathbf{n}_\theta\diamond\mathbf{n}_z) + (\mathbf{n}_z\diamond\mathbf{n}_\theta)\,\,(n\,\mathbb{I}+\mathrm{i}\,\mathbb{T})\,\right] + \\
&\frac{1}{r^2}\,(n\,\mathbb{I}+\mathrm{i}\,\mathbb{T})\,\,(\mathbf{n}_\theta\diamond\mathbf{n}_\theta)\,\,(n\,\mathbb{I}+\mathrm{i}\,\mathbb{T})\Big\}\,\tilde{\mathbf{U}}(r) = -\tilde{\mathbf{F}}(r)
\end{aligned} \tag{3}$$

$$\text{Radial stress:}\qquad \tilde{\mathbf{\Sigma}}_r(r) = (\mathbf{n}_r\diamond\mathbf{n}_r)\,\tilde{\mathbf{U}}'(r) - \mathrm{i}\,\left[\frac{1}{r}\,(\mathbf{n}_r\diamond\mathbf{n}_\theta)\,\,(n\,\mathbb{I}+\mathrm{i}\,\mathbb{T}) + k\,(\mathbf{n}_r\diamond\mathbf{n}_z)\right]\,\tilde{\mathbf{U}}(r) \tag{4}$$

TraFiC – I. The Fourier-Fourier-Laplace method

Exact solutions of Eq. (1) without volumic source in each plane layer: $\forall z\,,\ z_{\beta-1} < z < z_\beta\,,$ six partial waves:

$$\tilde{\mathbf{U}}(z) = \underbrace{\sum_{i=1}^{3} a_{\beta,i}\,\exp[-\mathbb{i}\,\kappa_{\beta,i}\,(z-z_\beta)]\,\mathbf{p}_{\beta,i}+}_{\text{upgoing waves}} \underbrace{\sum_{i=4}^{6} a_{\beta,i}\,\exp[-\mathbb{i}\,\kappa_{\beta,i}\,(z-z_{\beta-1})]\,\mathbf{p}_{\beta,i}\,.}_{\text{downgoing waves}} \tag{5}$$

(General anisotropy)

Sources at interfaces:

$$\Delta\tilde{\mathbf{U}}(z_\beta) = \boldsymbol{\phi}_\beta \quad \text{or/and} \quad \Delta\tilde{\boldsymbol{\Sigma}}_z(z_\beta) = \boldsymbol{\psi}_\beta \tag{6}$$

Exact solutions of Eq. (3) without volumic source in each tubular layer: $\forall r\,,\ r_{\beta-1} < r < r_\beta\,,$ six partial waves:

$$\tilde{\mathbf{U}}(r) = \underbrace{\sum_{i=1}^{3} a_{\beta,i}\,\mathbf{I}_{n,\beta,i}(\eta_{\beta,i}\,r)+}_{\text{ingoing waves}} \underbrace{\sum_{i=4}^{6} a_{\beta,i}\,\mathbf{K}_{n,\beta,i}(\eta_{\beta,i}\,r)\,.}_{\text{outgoing waves}} \tag{7}$$

(functions including modified Bessel functions and normalization by exponentials)

(Limitation: transversely isotropy with axial symmetry)

Sources at interfaces:

$$\Delta\tilde{\mathbf{U}}(r_\beta) = \boldsymbol{\phi}_\beta \quad \text{or/and} \quad \Delta\tilde{\boldsymbol{\Sigma}}_r(r_\beta) = \boldsymbol{\psi}_\beta \tag{8}$$

- Fluid layers, with two partial waves only, are also included in TraFiC

TraFiC – I. The Fourier-Fourier-Laplace method

About the use of the Laplace transform

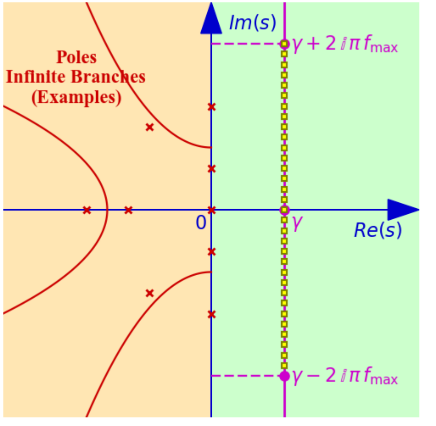
Laplace transform: $H(s) = \int_0^\infty h(t)\,\mathbb{e}^{-s\,t}\,\mathrm{d}t$.

Bromwich-Mellin Formula:

$$\forall \gamma > 0\,,\ h(t) = \mathbb{e}^{\gamma\,t} \int_{-\infty}^{+\infty} H(\gamma + 2\,\mathbb{i}\,\pi\,f)\,\mathbb{e}^{2\,\mathbb{i}\,\pi\,f\,t}\,\mathrm{d}f\ . \tag{9}$$

$f \mapsto H(\gamma + 2\,\mathbb{i}\,\pi\,f)$ is the Fourier Transform of the signal $t \mapsto h(t)\,\mathbb{e}^{-\gamma\,t}$.

1D time grid: $t_m = m\,\delta t$, $0 \leqslant m < 2\,N_t$, duration $d = 2\,N_t\,\delta t$



The *FFT* can be used while both the Nyquist-Shannon criterion and its dual are satisfied:

[Cooley & Tukey (1965)] [Phinney (1965)]

- Band-limited spectrum: $\forall f > f_{\max} = \dfrac{1}{2\,\delta t}\,,\ H(\gamma + 2\,\mathbb{i}\,\pi\,f) \approx 0$
- Finite duration: $\forall t\,,\ t \notin [0, d[\,,\ h(t)\,\mathbb{e}^{-\gamma\,t} \approx 0$ (*exponential window method* [Kausel *et al.* (1992) J. Eng. Mech.])

TraFiC – II. Computation steps and results

1) Dimensioning the problem: time and space grids

Duration of interest d and highest frequency $f_{\max}$ $\Longrightarrow$ Time grid with γ and δf

Highest speed and source location $\Longrightarrow$ space of interest **Beware of space periodization!**

Space of interest and highest wavenumbers $\Longrightarrow$ space grid (1D or 2D)

2D space grid:

$$\left\{\begin{array}{lll} x_i = i\,\delta x, & -N_x < i \leqslant N_x, & \text{Period: } 2\,N_x\,\delta x\ ; \\ y_j = j\,\delta y, & -N_y < j \leqslant N_y, & \text{Period: } 2\,N_y\,\delta y\ . \end{array}\right. \Longleftrightarrow \left\{\begin{array}{lll} k_{x\,i} = i\,\delta k_x, & -N_x < i \leqslant N_x, & k_{x\,\max} = \pi/\delta x\ ; \\ k_{y\,j} = j\,\delta k_y, & -N_y < j \leqslant N_y, & k_{y\,\max} = \pi/\delta y\ . \end{array}\right.$$

or

$$\left\{\begin{array}{lll} \theta_i = i/(\pi\,N_\theta), & -N_\theta < i \leqslant N_\theta, & \text{Period: } 2\,\pi\ ; \\ z_j = j\,\delta z, & -N_z < j \leqslant N_z, & \text{Period: } 2\,N_z\,\delta z\ . \end{array}\right. \Longleftrightarrow \left\{\begin{array}{lll} n, & -N_\theta < n \leqslant N_\theta & ; \\ k_{z\,j} = j\,\delta k_z, & -N_z < j \leqslant N_z, & k_{z\,\max} = \pi/\delta z\ . \end{array}\right.$$

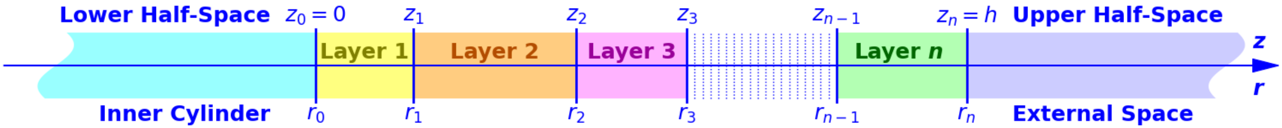
Object-oriented programming in *Python*: Grid classes include numerical Laplace and Fourier transforms (direct and inverse), zero-padding. . .

TraFiC – II. Computation steps and results

2) Green functions: computation and storage

In the FFL domain, each computation for a given $(\mathbf{k}, s)$ is independent of the others

$\Longrightarrow$ **Massively Parallel Computation**



- Normal wavenumbers and polarizations computed and stored once and for all
- One given direction of excitation at one interface $\Longrightarrow$ Green function, characterized by the coefficients of the partial waves.

3) Field computation

Compromise between CPU and memory

Components of the excitation $\underset{\text{FFL}}{\Longrightarrow}$ Linear combination of the Green functions

$\Downarrow$ Coefficients of the total wave

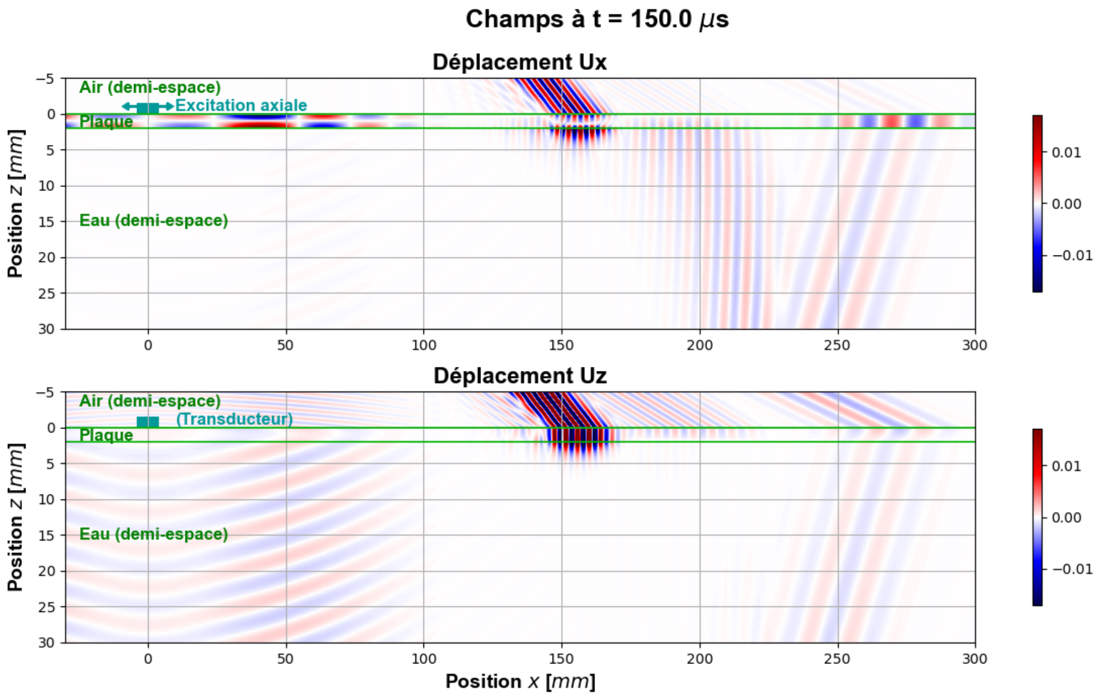
One file for each pair (field, normal position) $\underset{\text{FFL}^{-1}}{\Longleftarrow}$ $\Downarrow$ Selected fields for selected normal position

4) Post-processing

Signals, snapshots, . . . by using zero-padding

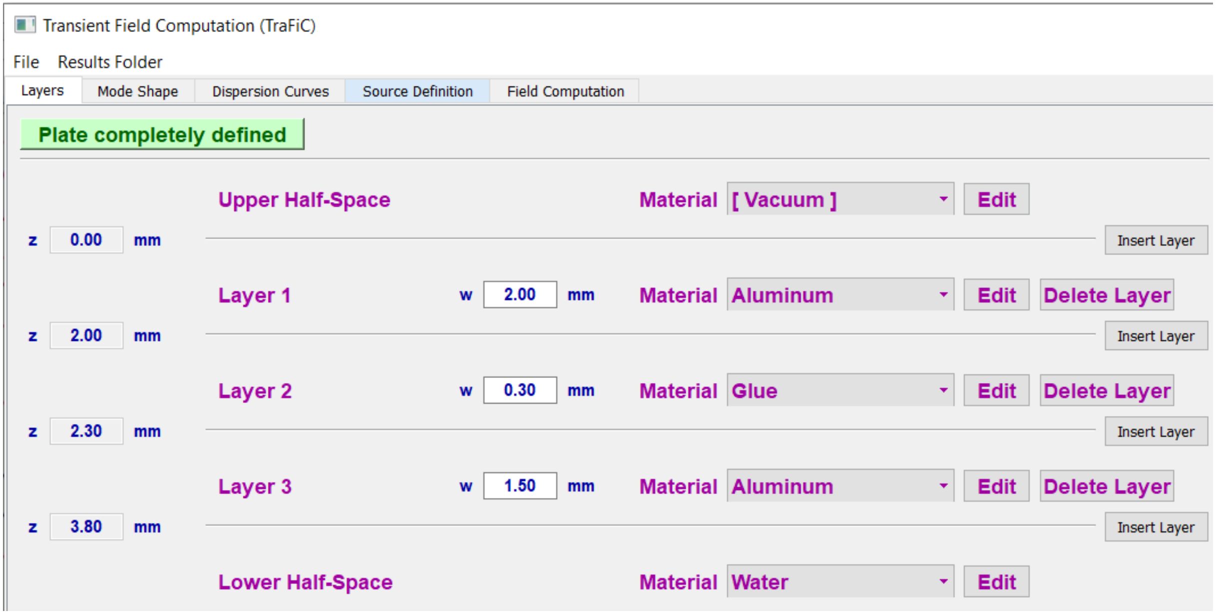
TraFiC – II. Computation steps and results

Semi-immersed nylon plate of 2.26 mm thickness



TraFiC – III. Additional tools

GUI (under development)



TraFiC – III. Additional tools

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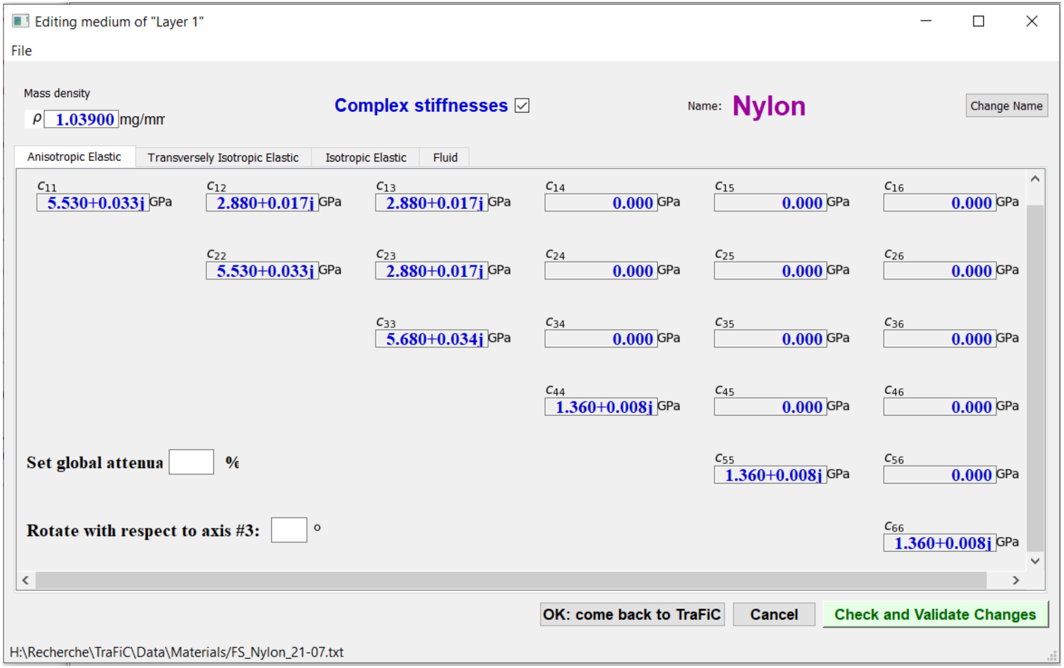
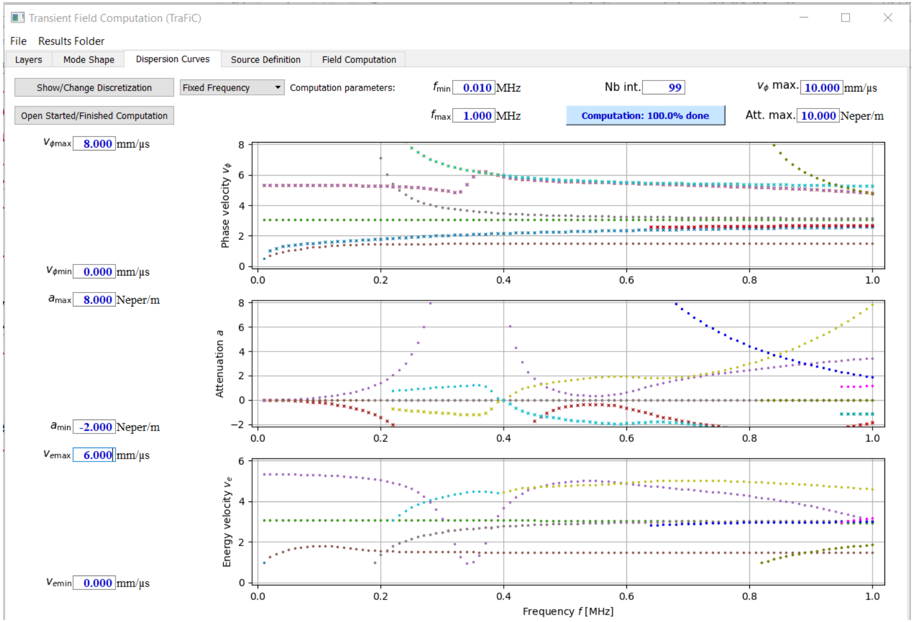


TraFiC – III. Additional tools

Mode computation of immersed multilayer plates

[Ducasse & Deschamps, *Mode computation of immersed multilayer plates by solving an eigenvalue problem*, to be published in Wave Motion]



TraFiC – III. Additional tools

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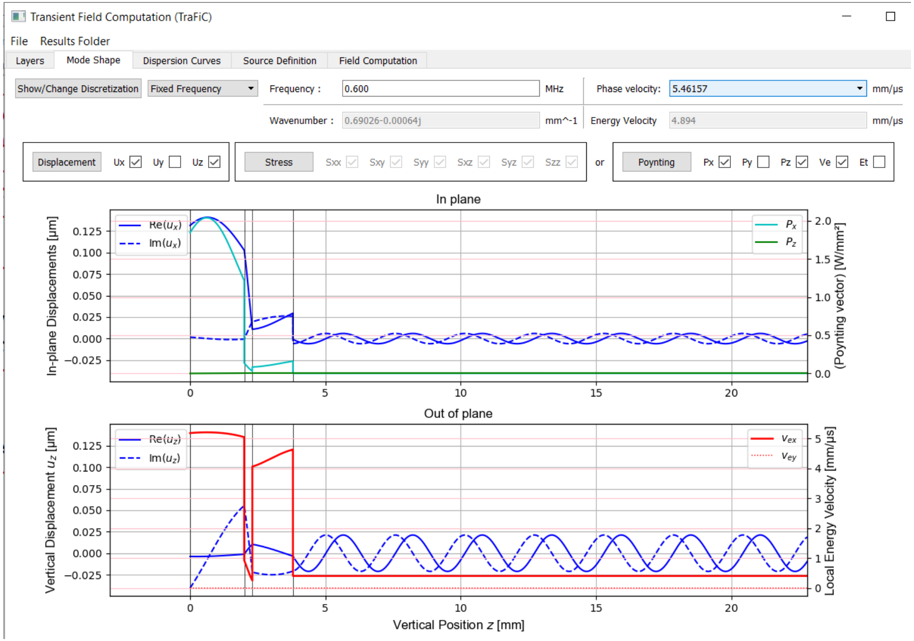
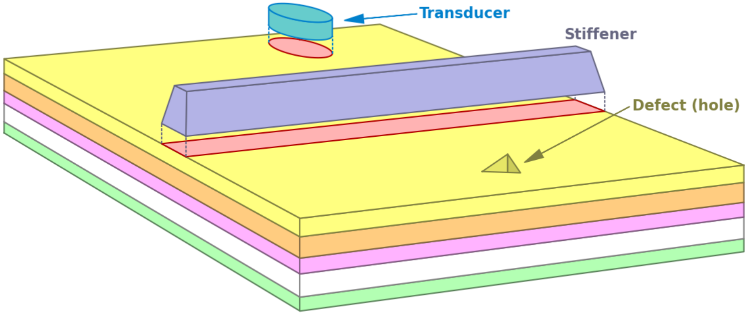


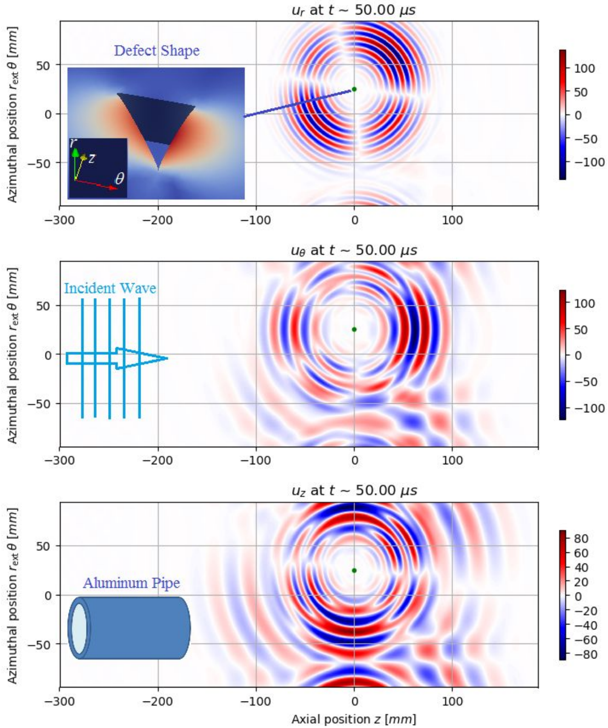
TraFiC – IV. Towards hybrid computation



A PhD work that should start soon, subject to funding (CEA + DGA-AID): **Development of hybrid numerical methods for the diffraction of ultrasonic waves by obstacles on the surface of laminated structures, and application to non-destructive testing**

Coll. POEMS/CEA-List/I$_2$M-Apy

Domain Decomposition & Asymptotic Methods



Asymptotic Method, coll. Marc Bonnet (POEMS)