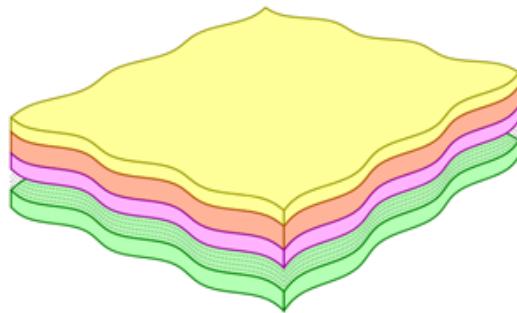


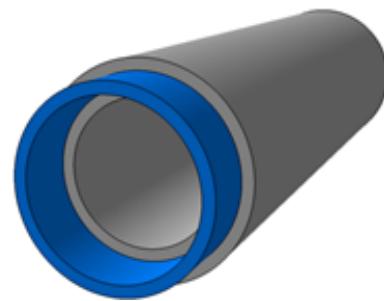
## Transient Field Computation in multilayered structures



[PhD Pierrick MORA dec. 2015]

[MORA et al. (2016) Ultrasonics]

[KAUSEL et al. (1992) J. Eng. Mech.]



[PhD Aditya KRISHNA sept. 2020]

**Asumptions:** infinite structures invariant in two directions

Convenient for:

- A localized source emitting a short signal
- Immersed and Embedded plates and pipes
- 2D and 3D cases

**Can be compared to:**

- $k_n(\omega, \nu)$ -modal methods

- $\omega_n(\mathbf{k})$ -modal methods

[KAUSEL (1994) IJNME] [DUCASSE et al. (2014) Wave Motion]

Complementary approaches

[MORA (2021) Wave Motion]

# Outline

## I. The FOURIER-FOURIER-LAPLACE method

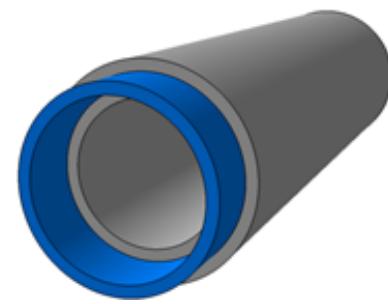
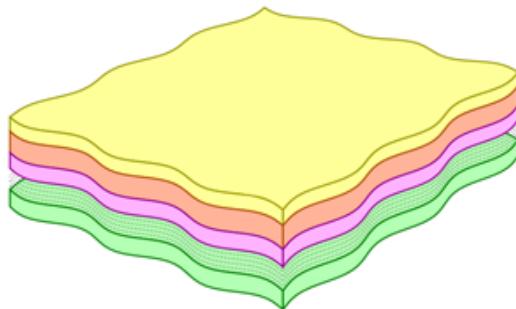
## II. Computation steps and results

## III. Additional tools

- ▷ Graphical user interface (under development)
- ▷ Mode computation

## IV. Towards hybrid methods

## TraFiC – I. The FOURIER-FOURIER-LAPLACE method



Time  $t \rightarrow$  complex Laplace variable  $s$

Space:

- ▷ material properties invariant with respect to  
 $\mathbf{x} = (x, y)$
- ▷ Horizontal position  $\mathbf{x} \rightarrow$  Horizontal wavevector  $\mathbf{k}$
- ▷ Computation in the  $(\mathbf{k}, z, s)$ -domain
- ▷ ODEs with respect to the vertical position  $z$
- ▷ material properties invariant with respect to  $\theta$  and  $z$
- ▷ Axial position  $z \rightarrow$  ax. wavenumber  $k$
- ▷ Azimuthal position  $\theta \rightarrow$  az. wavenumber  $n$
- ▷ Computation in the  $(r, n, k, s)$ -domain
- ▷ ODEs with respect to the radial position  $r$

## TraFiC – I. The FOURIER-FOURIER-LAPLACE method

- ▷ The displacement vector  $\tilde{\mathbf{U}}(z)$  satisfies in each plane layer ( $\mathbf{n}$  unit vertical vector,  $\mathbb{I}$  identity matrix):

$$(\mathbf{n} \diamond \mathbf{n}) \tilde{\mathbf{U}}''(z) - i [(\mathbf{n} \diamond \mathbf{k}) + (\mathbf{k} \diamond \mathbf{n})] \tilde{\mathbf{U}}'(z) - [(k \diamond k) + \rho s^2 \mathbb{I}] \tilde{\mathbf{U}}(z) = -\tilde{\mathbf{F}}(z) \quad (1)$$

$$\text{Stress vector in the } z\text{-direction: } \tilde{\Sigma}_z(z) = (\mathbf{n} \diamond \mathbf{n}) \tilde{\mathbf{U}}'(z) - i (\mathbf{n} \diamond \mathbf{k}) \tilde{\mathbf{U}}(z) \quad (2)$$

- ▷ The displacement vector  $\tilde{\mathbf{U}}(r)$  satisfies in each tubular layer:

$$\begin{aligned} \mathbb{T} = & \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} & (\mathbf{n}_r \diamond \mathbf{n}_r) \tilde{\mathbf{U}}''(r) + \\ & \left\{ -i k (\mathbf{n}_r \diamond \mathbf{n}_z + \mathbf{n}_z \diamond \mathbf{n}_r) + \right. \\ & \frac{1}{r} \left[ (\mathbf{n}_r \diamond \mathbf{n}_r) - i ((\mathbf{n}_r \diamond \mathbf{n}_\theta) (n \mathbb{I} + i \mathbb{T}) + (n \mathbb{I} + i \mathbb{T}) (\mathbf{n}_\theta \diamond \mathbf{n}_r)) \right] \left. \right\} \tilde{\mathbf{U}}'(r) - \\ & \left\{ [\rho s^2 \mathbb{I} + k^2 (\mathbf{n}_z \diamond \mathbf{n}_z)] + \right. \\ & \frac{k}{r} \left[ i (\mathbf{n}_r \diamond \mathbf{n}_z) + (n \mathbb{I} + i \mathbb{T}) (\mathbf{n}_\theta \diamond \mathbf{n}_z) + (\mathbf{n}_z \diamond \mathbf{n}_\theta) (n \mathbb{I} + i \mathbb{T}) \right] + \\ & \left. \frac{1}{r^2} (n \mathbb{I} + i \mathbb{T}) (\mathbf{n}_\theta \diamond \mathbf{n}_\theta) (n \mathbb{I} + i \mathbb{T}) \right\} \tilde{\mathbf{U}}(r) = -\tilde{\mathbf{F}}(r) \end{aligned} \quad (3)$$

$$\text{Radial stress: } \tilde{\Sigma}_r(r) = (\mathbf{n}_r \diamond \mathbf{n}_r) \tilde{\mathbf{U}}'(r) - i \left[ \frac{1}{r} (\mathbf{n}_r \diamond \mathbf{n}_\theta) (n \mathbb{I} + i \mathbb{T}) + k (\mathbf{n}_r \diamond \mathbf{n}_z) \right] \tilde{\mathbf{U}}(r) \quad (4)$$

## TraFiC – I. The FOURIER-FOURIER-LAPLACE method

Exact solutions of Eq. (1) without volumic source in each plane layer:  $\forall z$ ,  $z_{\beta-1} < z < z_\beta$ , six partial waves:

$$\begin{aligned}\tilde{\mathbf{U}}(z) = & \underbrace{\sum_{i=1}^3 a_{\beta,i} \exp[-i \kappa_{\beta,i} (z - z_\beta)] \mathbf{p}_{\beta,i}}_{\text{upgoing waves}} + \\ & \underbrace{\sum_{i=4}^6 a_{\beta,i} \exp[-i \kappa_{\beta,i} (z - z_{\beta-1})] \mathbf{p}_{\beta,i}}_{\text{downgoing waves}}.\end{aligned}\quad (5)$$

(General anisotropy)

### Sources at interfaces:

$$\Delta \tilde{\mathbf{U}}(z_\beta) = \Phi_\beta \quad \text{or/and} \quad \Delta \tilde{\Sigma}_z(z_\beta) = \Psi_\beta \quad (6)$$

- Fluid layers, with two partial waves only, are also included in TraFiC

Exact solutions of Eq. (3) without volumic source in each tubular layer:  $\forall r$ ,  $r_{\beta-1} < r < r_\beta$ , six partial waves:

$$\begin{aligned}\tilde{\mathbf{U}}(r) = & \underbrace{\sum_{i=1}^3 a_{\beta,i} \mathbf{I}_{n,\beta,i}(\eta_{\beta,i} r)}_{\text{ingoing waves}} + \\ & \underbrace{\sum_{i=4}^6 a_{\beta,i} \mathbf{K}_{n,\beta,i}(\eta_{\beta,i} r)}_{\text{outgoing waves}}.\end{aligned}\quad (7)$$

(functions including modified Bessel functions and normalization by exponentials)

(Limitation: transversely isotropy with axial symmetry)

### Sources at interfaces:

$$\Delta \tilde{\mathbf{U}}(r_\beta) = \Phi_\beta \quad \text{or/and} \quad \Delta \tilde{\Sigma}_r(r_\beta) = \Psi_\beta \quad (8)$$

## TraFiC – I. The FOURIER-FOURIER-LAPLACE method

### About the use of the Laplace transform

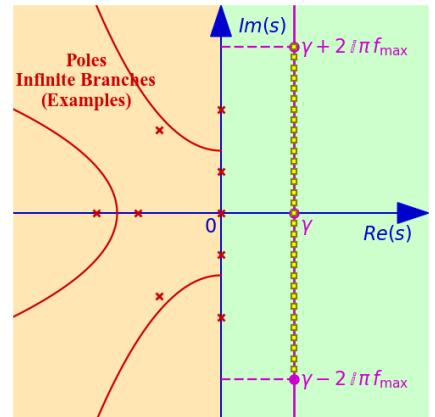
LAPLACE transform:  $H(s) = \int_0^\infty h(t) e^{-st} dt$ .

Bromwich-Mellin Formula:

$$\forall \gamma > 0, h(t) = e^{\gamma t} \int_{-\infty}^{+\infty} H(\gamma + 2i\pi f) e^{2i\pi f t} df. \quad (9)$$

$f \mapsto H(\gamma + 2i\pi f)$  is the Fourier Transform of the signal  $t \mapsto h(t) e^{-\gamma t}$ .

**1D time grid:**  $t_m = m \delta t$ ,  $0 \leq m < 2 N_t$ , duration  $d = 2 N_t \delta t$



The *FFT* can be used while both the Nyquist-Shannon criterion and its dual are satisfied:

[Cooley & Tukey (1965)] [Phinney (1965)]

- Band-limited spectrum:  $\forall f > f_{\max} = \frac{1}{2\delta t}, H(\gamma + 2i\pi f) \approx 0$
- Finite duration:  $\forall t, t \notin [0, d[, h(t) e^{-\gamma t} \approx 0$     (*exponential window method* [KAUSEL *et al.* (1992) J. Eng. Mech.] )

## TraFiC – II. Computation steps and results

### 1) Dimensioning the problem: time and space grids

Duration of interest  $d$  and highest frequency  $f_{\max}$   $\implies$  Time grid with  $\gamma$  and  $\delta f$

Highest speed and source location  $\implies$  space of interest **Beware of space periodization!**

Space of interest and highest wavenumbers  $\implies$  space grid (1D or 2D)

**2D space grid:**

$$\begin{cases} x_i = i \delta x, \quad -N_x < i \leq N_x, \quad \text{Period: } 2 N_x \delta x ; \\ y_j = j \delta y, \quad -N_y < j \leq N_y, \quad \text{Period: } 2 N_y \delta y . \end{cases} \iff \begin{cases} k_{x,i} = i \delta k_x, \quad -N_x < i \leq N_x, \quad k_{x,\max} = \pi / \delta x ; \\ k_{y,j} = j \delta k_y, \quad -N_y < j \leq N_y, \quad k_{y,\max} = \pi / \delta y . \end{cases}$$

or

$$\begin{cases} \theta_i = i / (\pi N_\theta), \quad -N_\theta < i \leq N_\theta, \quad \text{Period: } 2 \pi ; \\ z_j = j \delta z, \quad -N_z < j \leq N_z, \quad \text{Period: } 2 N_z \delta z . \end{cases} \iff \begin{cases} n, \quad -N_\theta < n \leq N_\theta ; \\ k_{z,j} = j \delta k_z, \quad -N_z < j \leq N_z, \quad k_{z,\max} = \pi / \delta z . \end{cases}$$

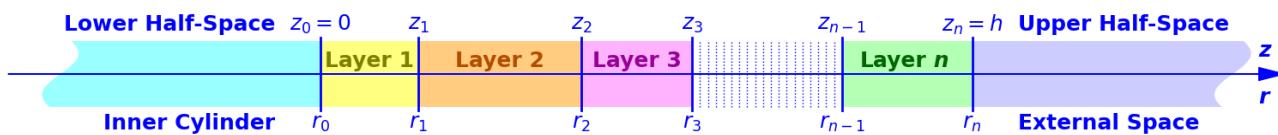
**Object-oriented programming in *Python*:** Grid classes include numerical LAPLACE and FOURIER transforms (direct and inverse), zero-padding...

## TraFiC – II. Computation steps and results

### 2) GREEN functions: computation and storage

In the FFL domain, each computation for a given  $(\mathbf{k}, s)$  is independent of the others

$\Rightarrow$  Massively Parallel Computation



- Normal wavenumbers and polarizations computed and stored once and for all
- One given direction of excitation at one interface  $\Rightarrow$  GREEN function, characterized by the coefficients of the partial waves.

### 3) Field computation

Components of the excitation

*Compromise between CPU and memory*

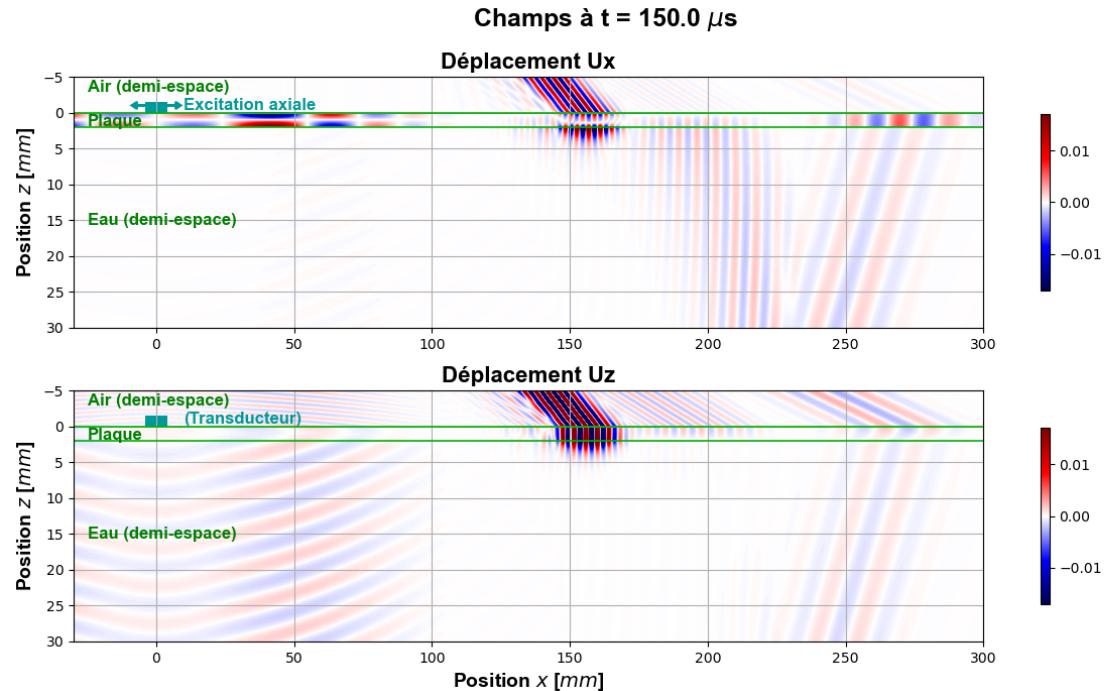
$\xrightarrow{\text{FFL}}$  Linear combination of the GREEN functions  
 $\downarrow$  Coefficients of the total wave  
 One file for each pair (field, normal position)  $\xleftarrow[\text{FFL}^{-1}]{}$   $\downarrow$  Selected fields for selected normal position

### 4) Post-processing

Signals, snapshots, ... by using zero-padding

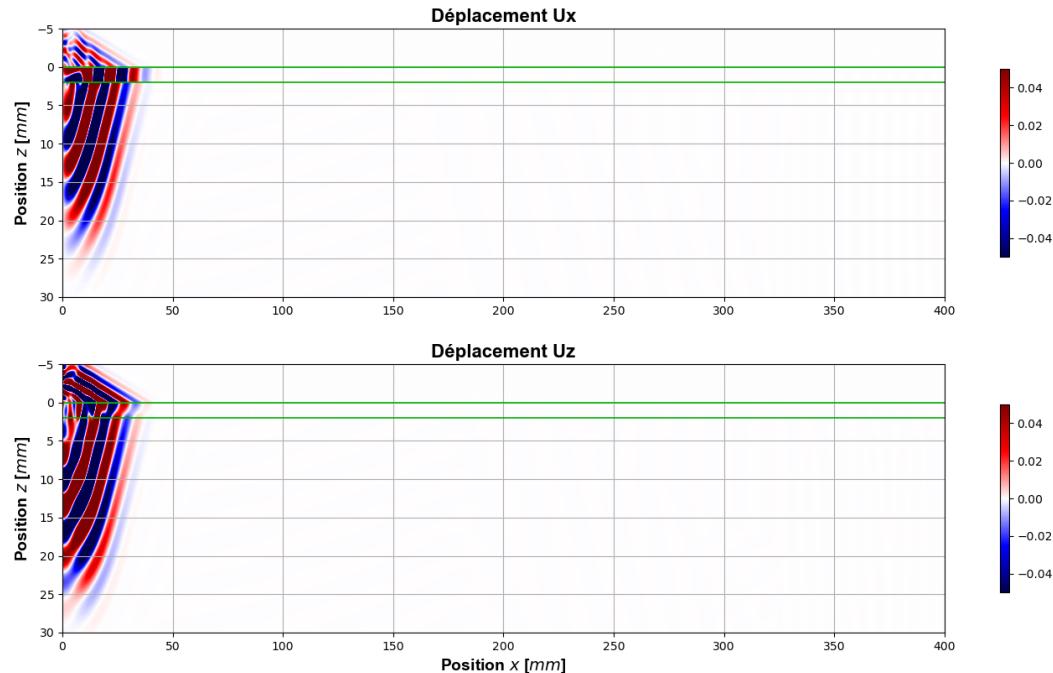
## TraFiC – II. Computation steps and results

Semi-immersed nylon plate of 2.26 mm thickness



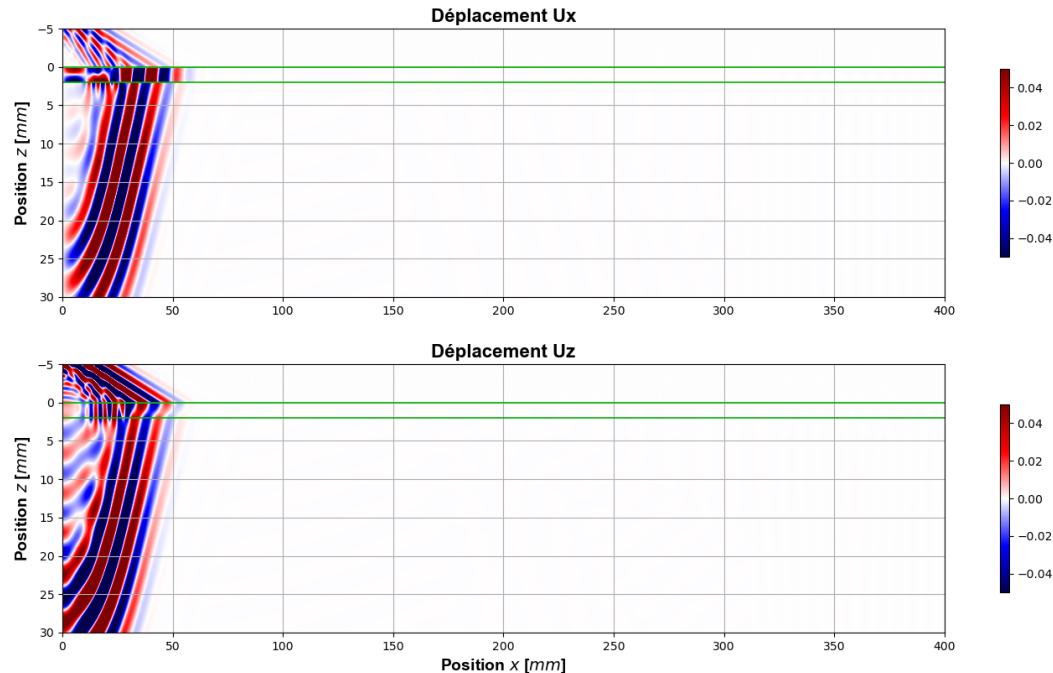
## TraFiC – II. Computation steps and results

Champs à  $t = 010.0 \mu\text{s}$

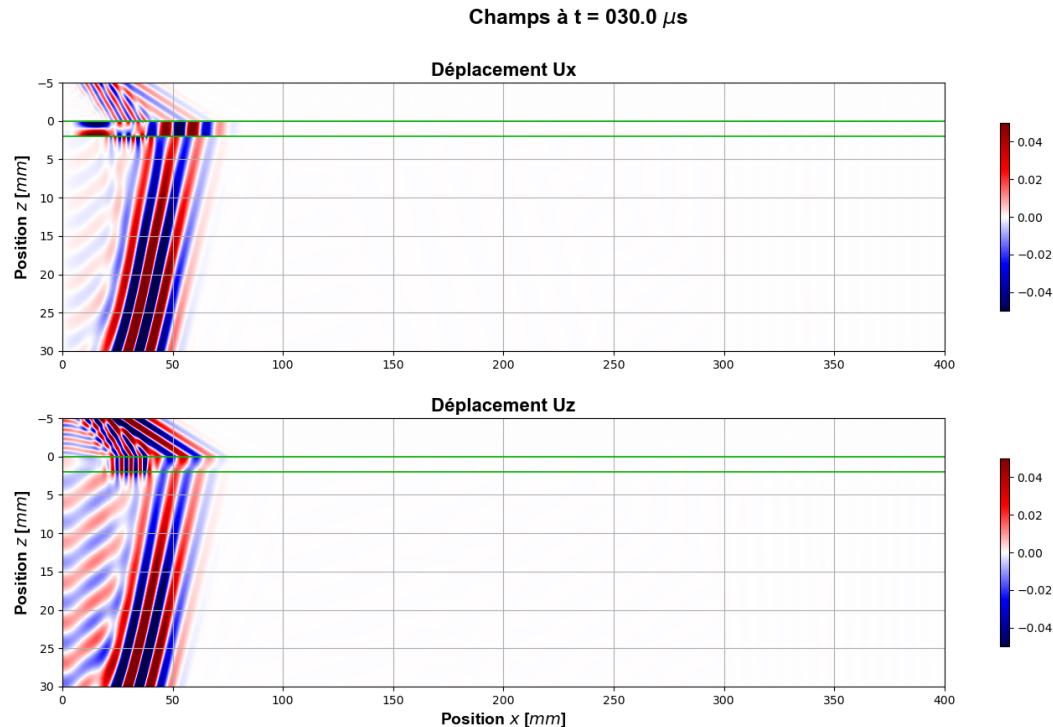


## TraFiC – II. Computation steps and results

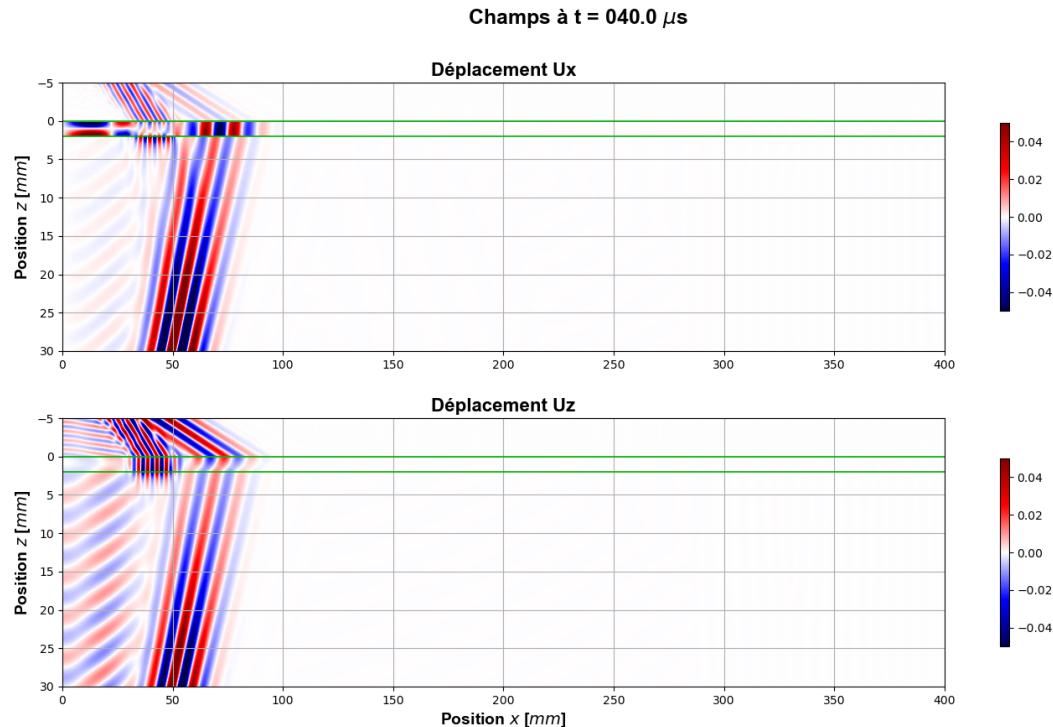
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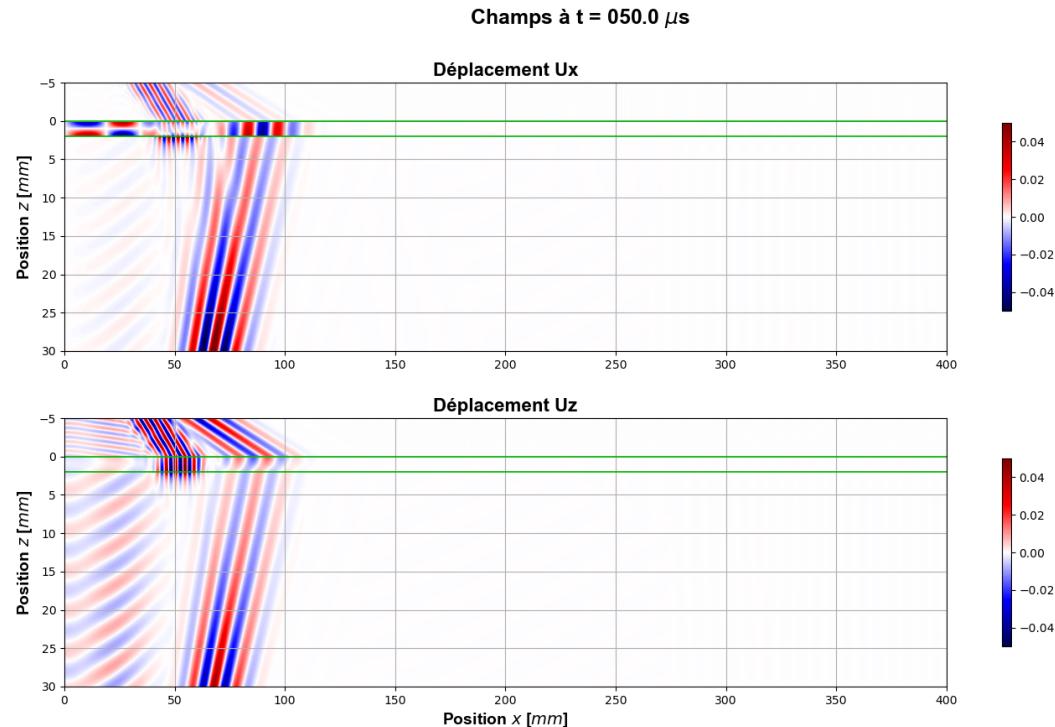
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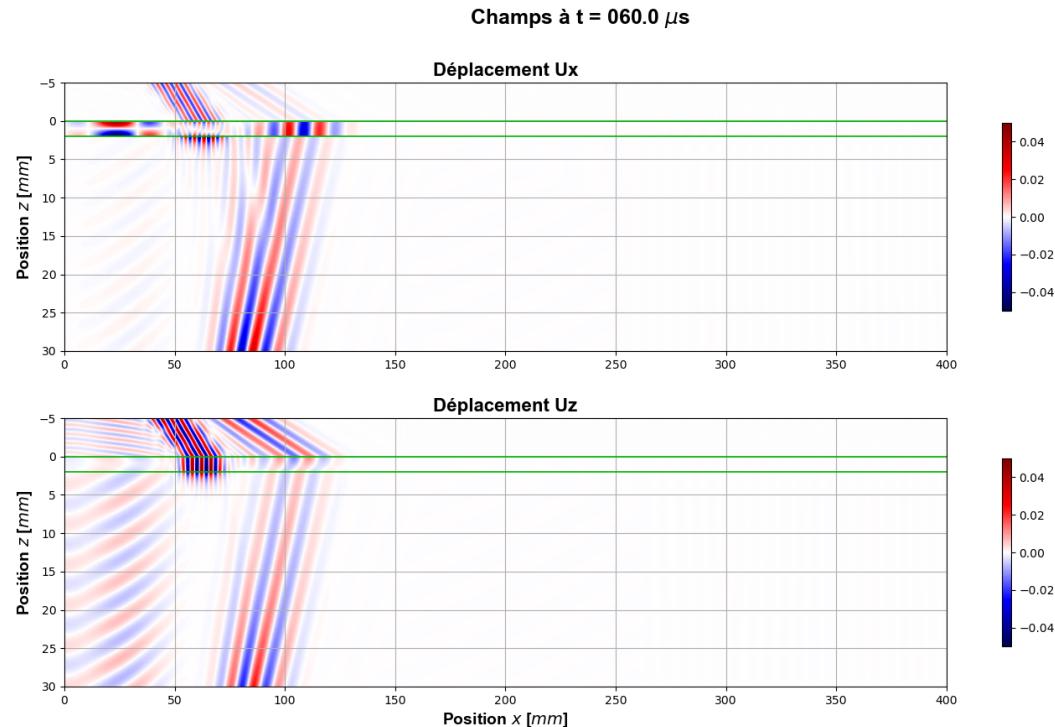
## TraFiC – II. Computation steps and results



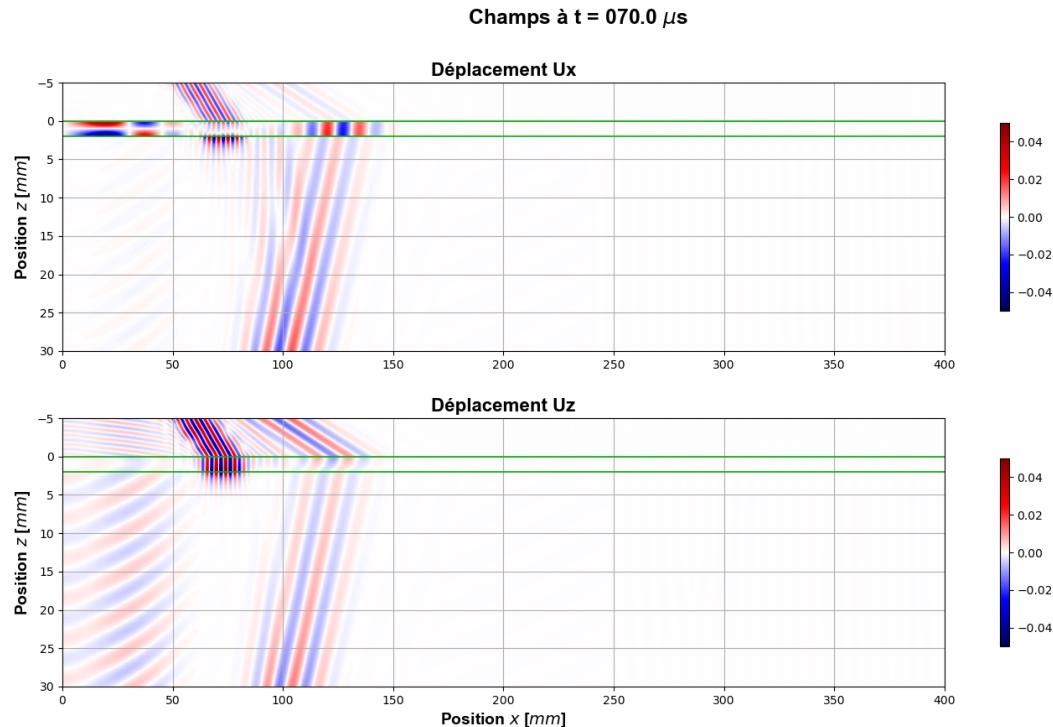
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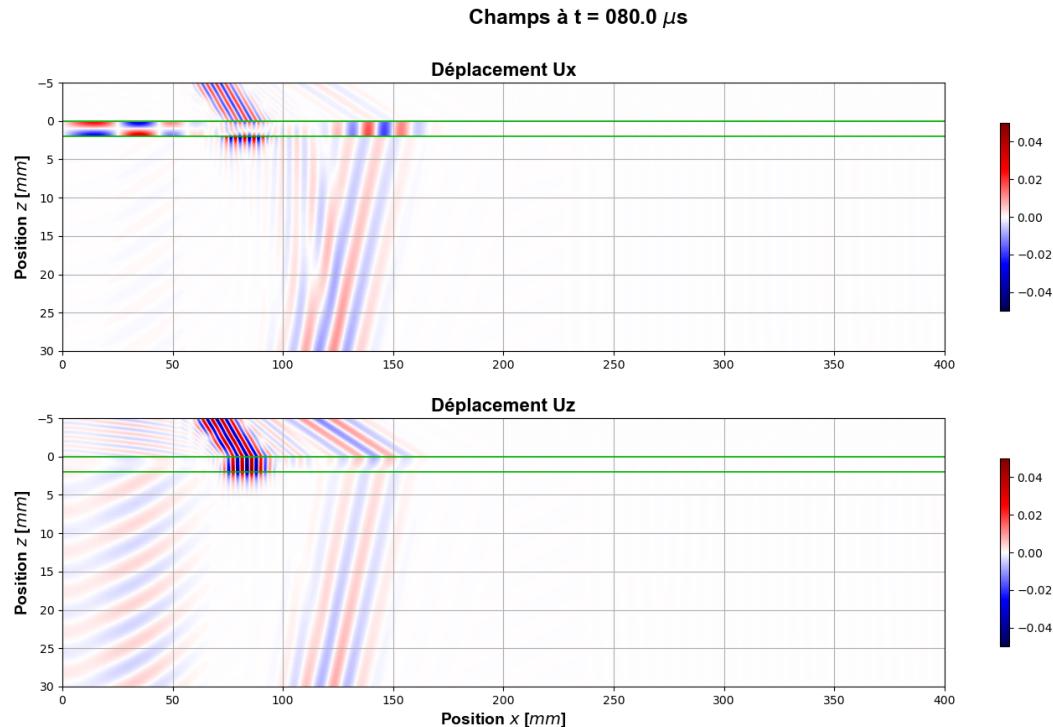
## TraFiC – II. Computation steps and results



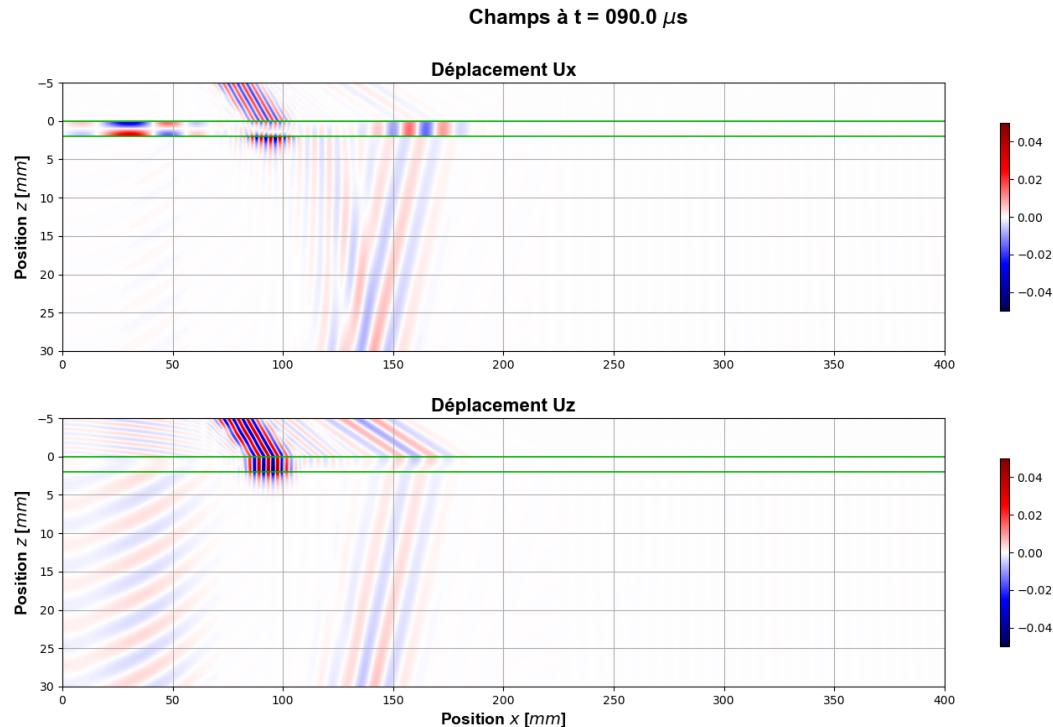
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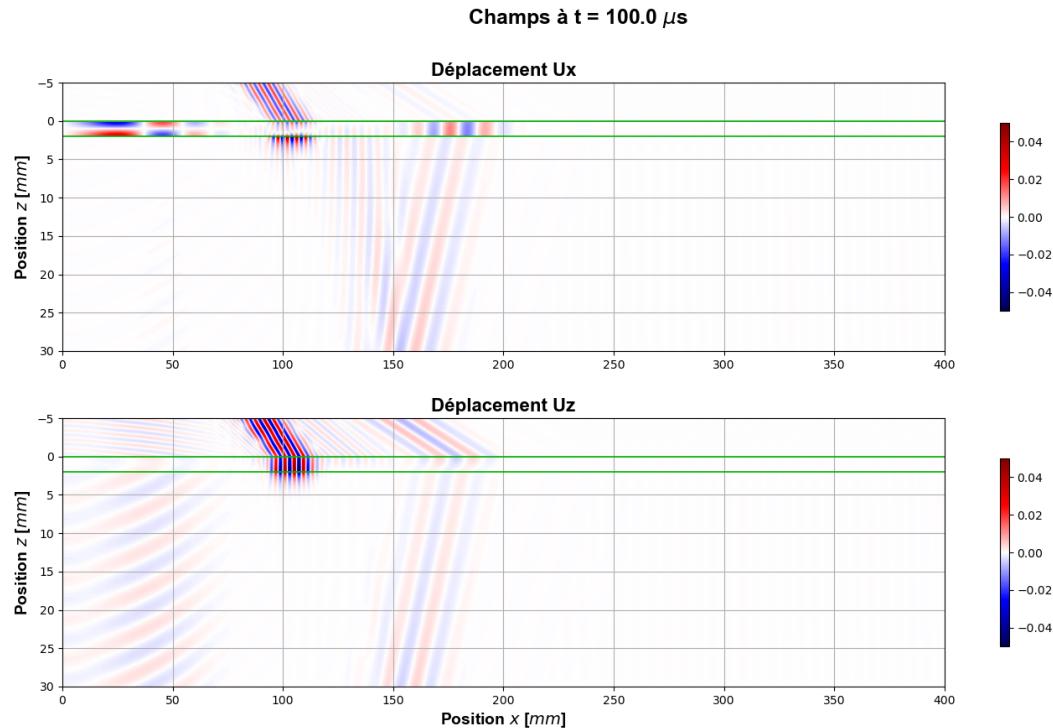
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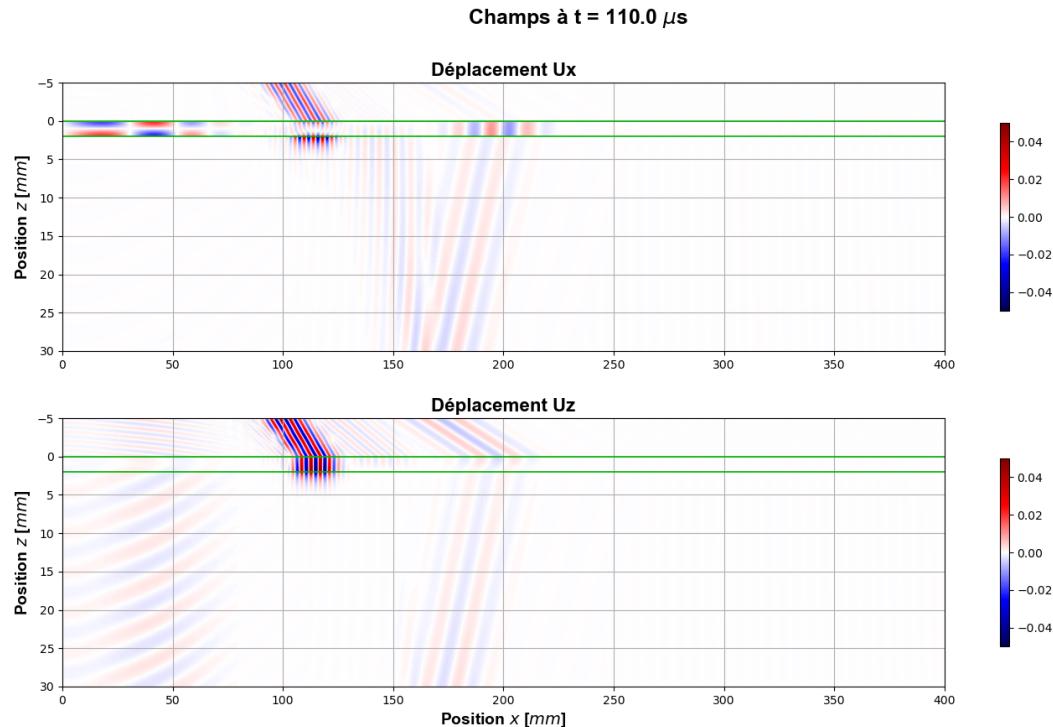
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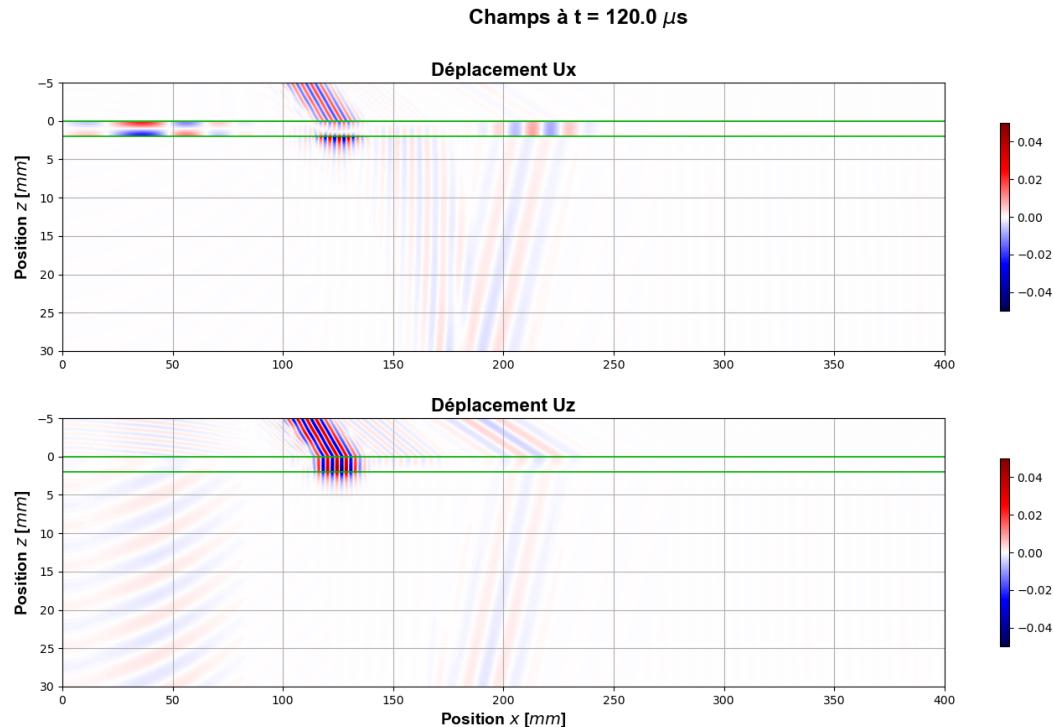
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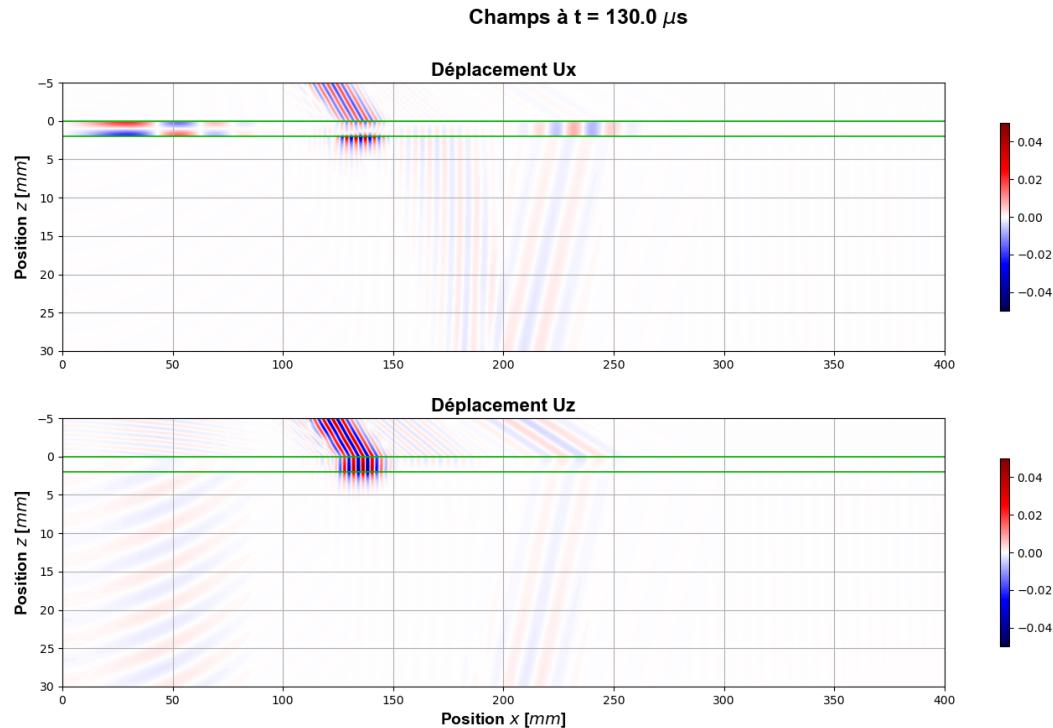
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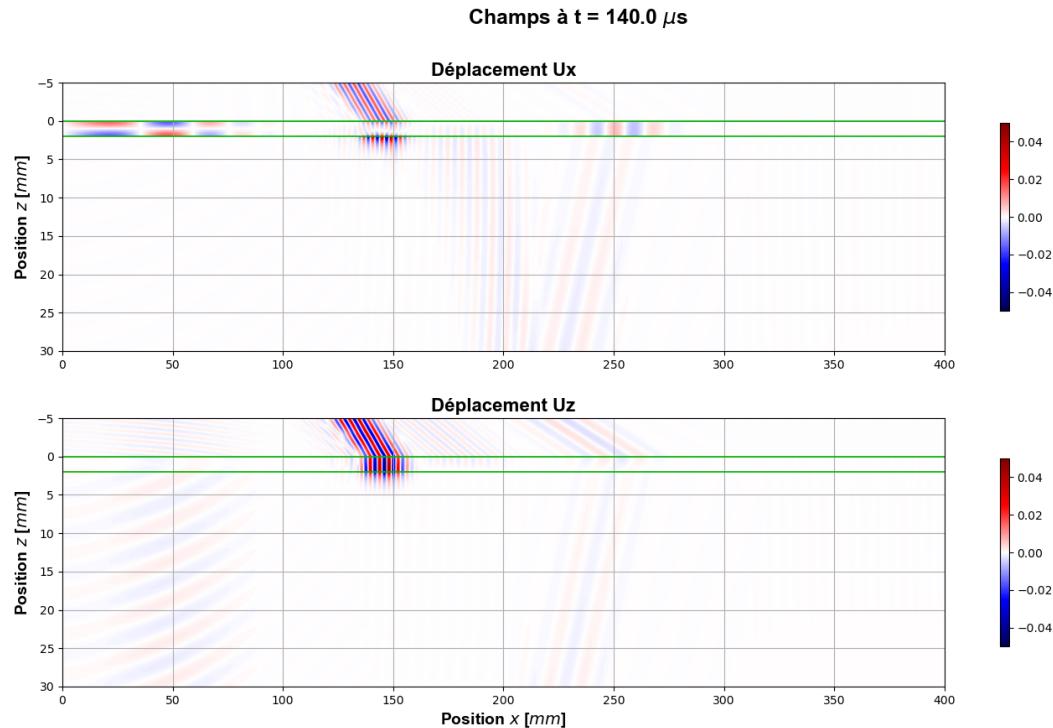
## TraFiC – II. Computation steps and results



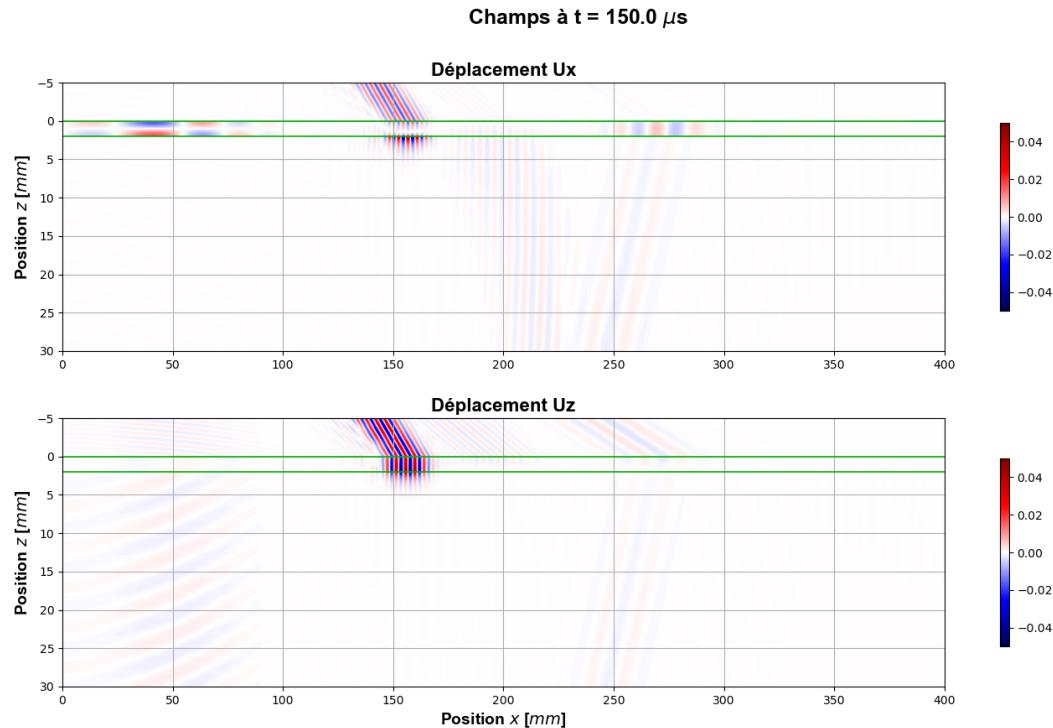
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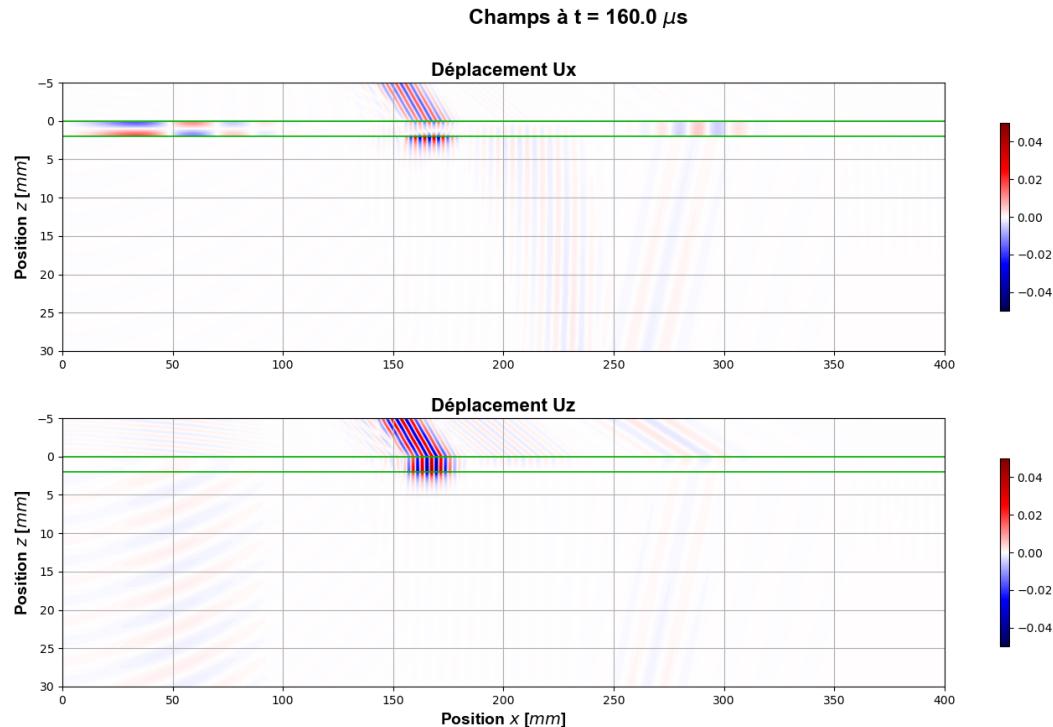
## TraFiC – II. Computation steps and results



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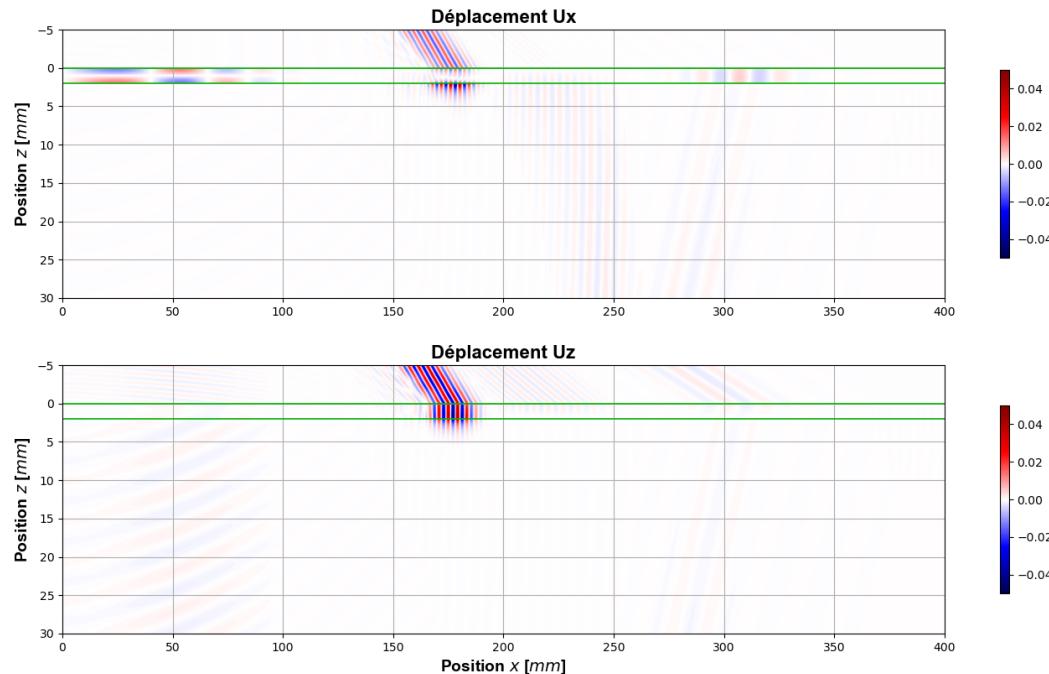


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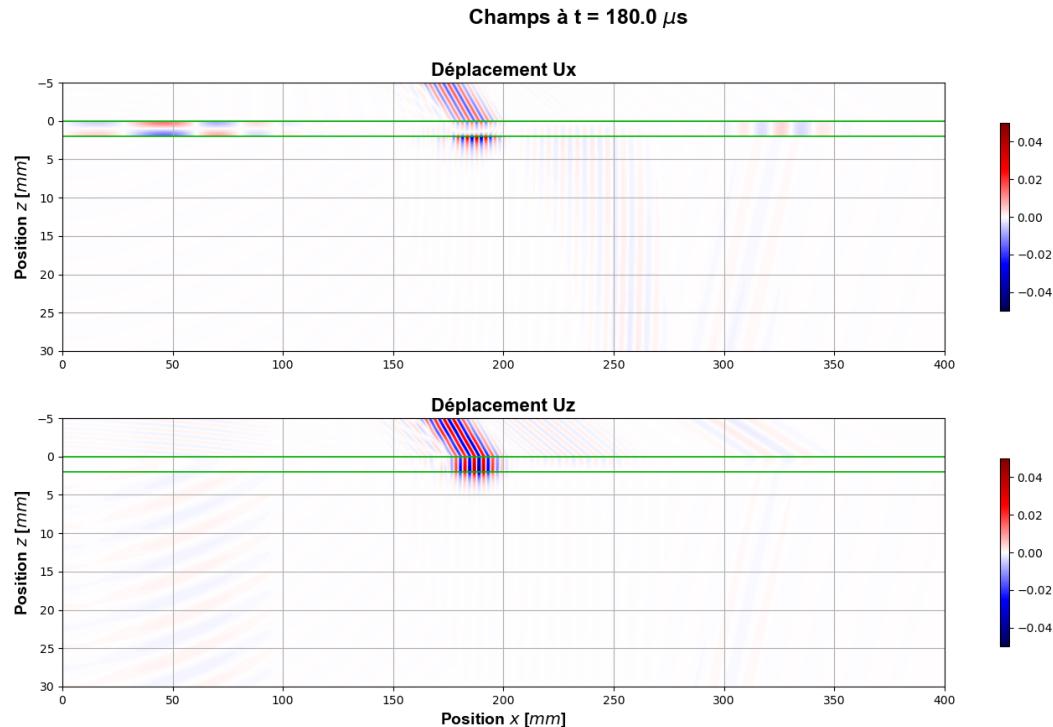


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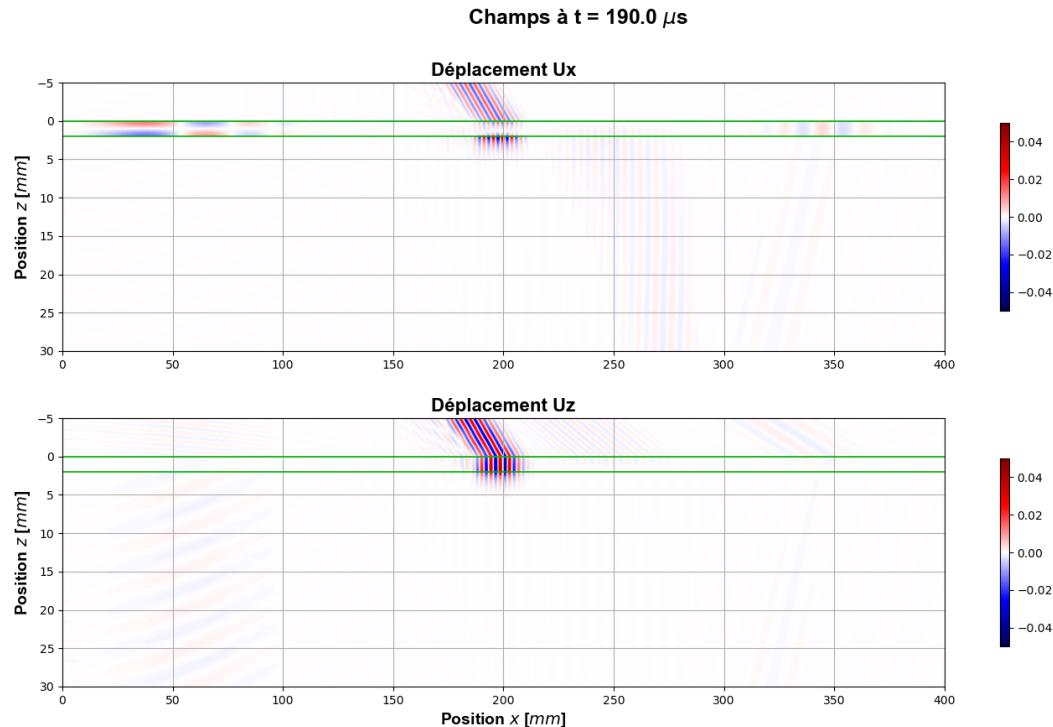
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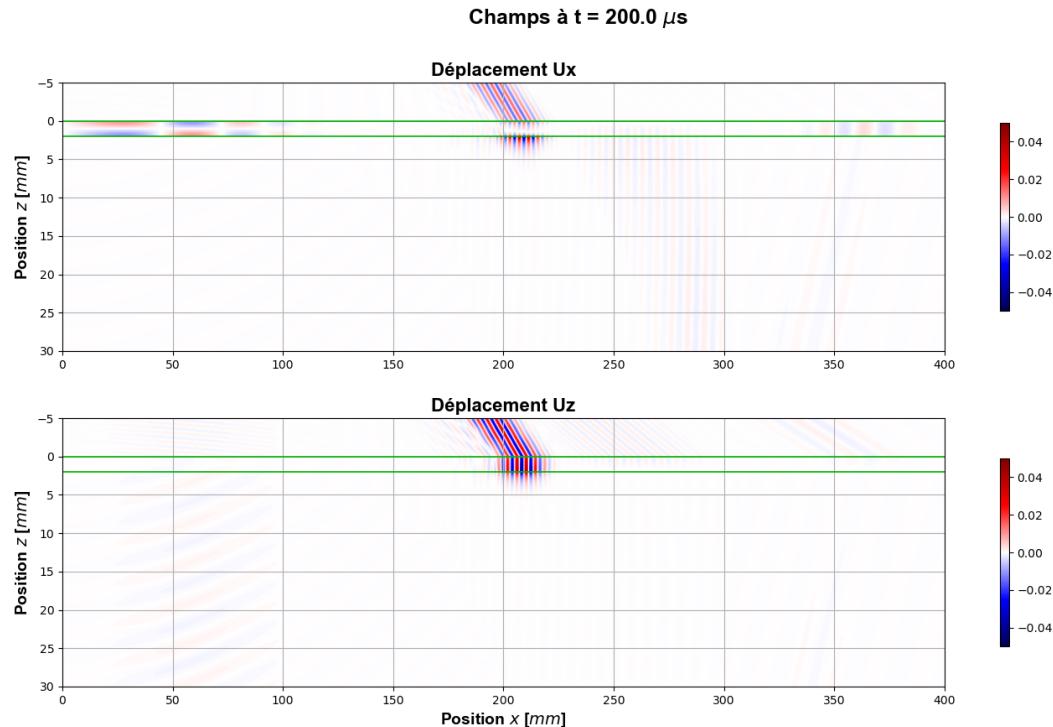
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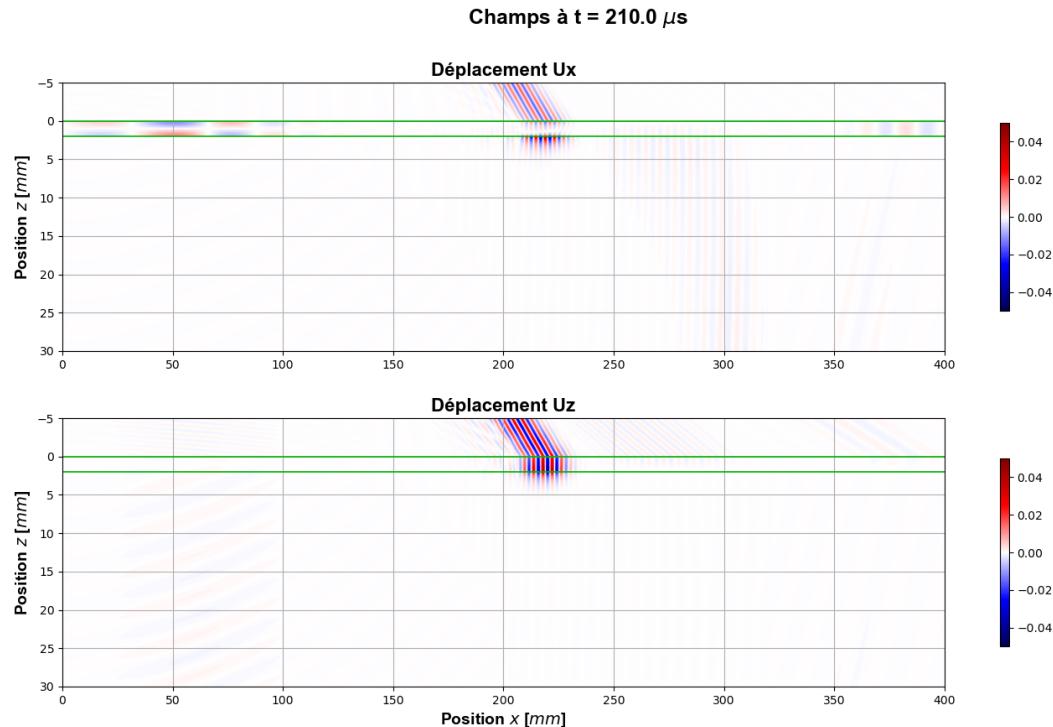
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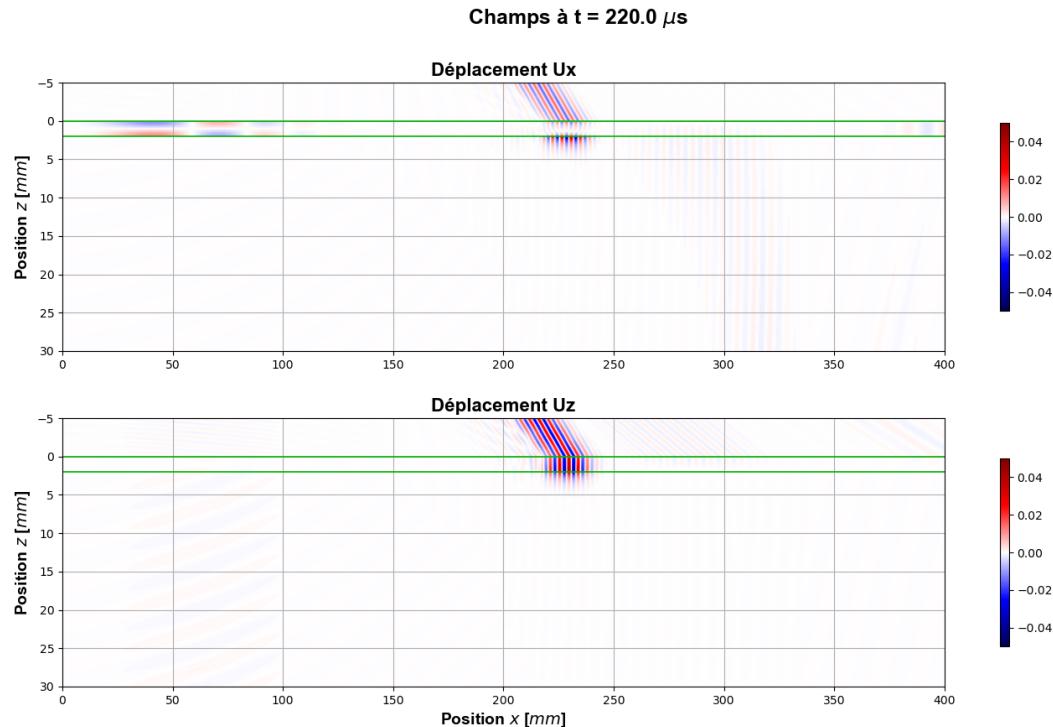
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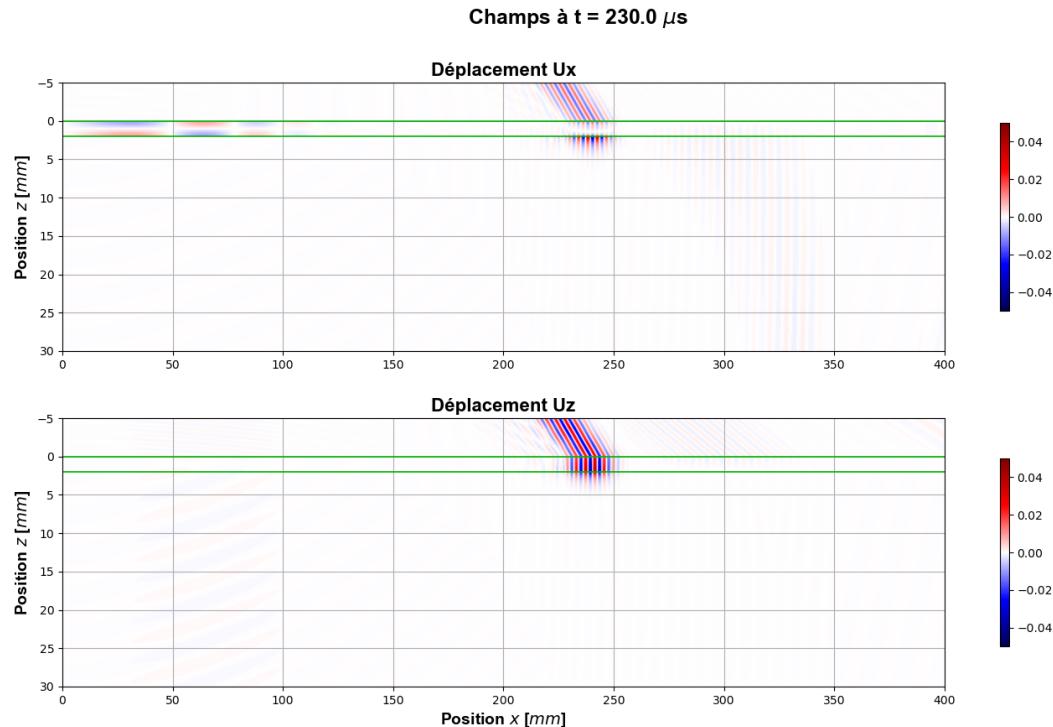
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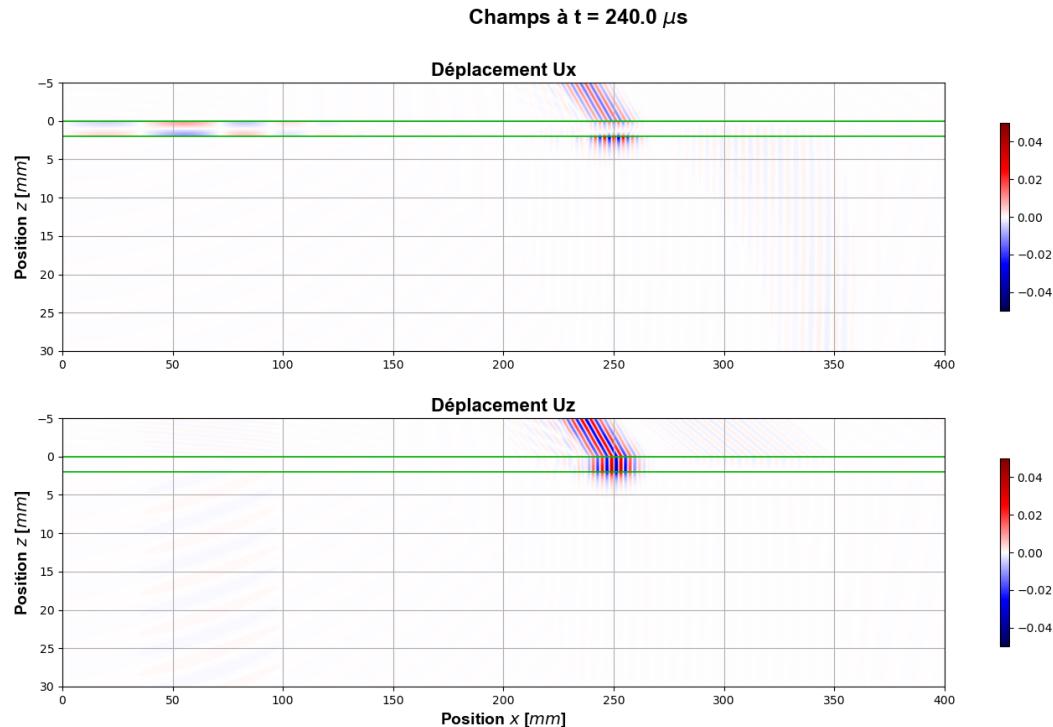
## TraFiC – II. Computation steps and results



## TraFiC – II. Computation steps and results

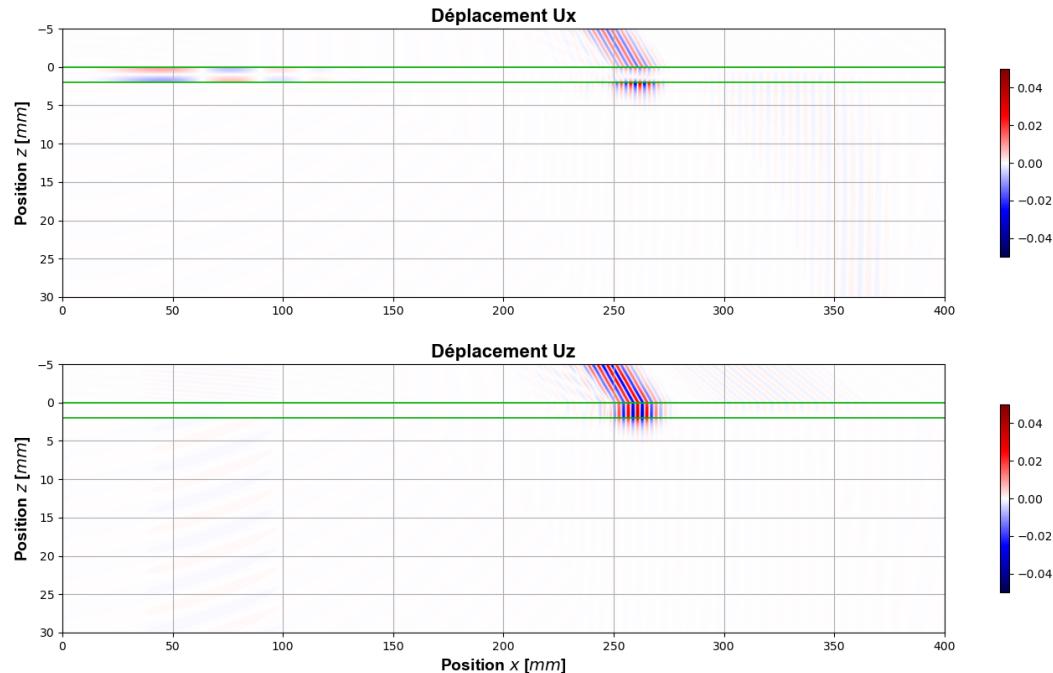


## TraFiC – II. Computation steps and results



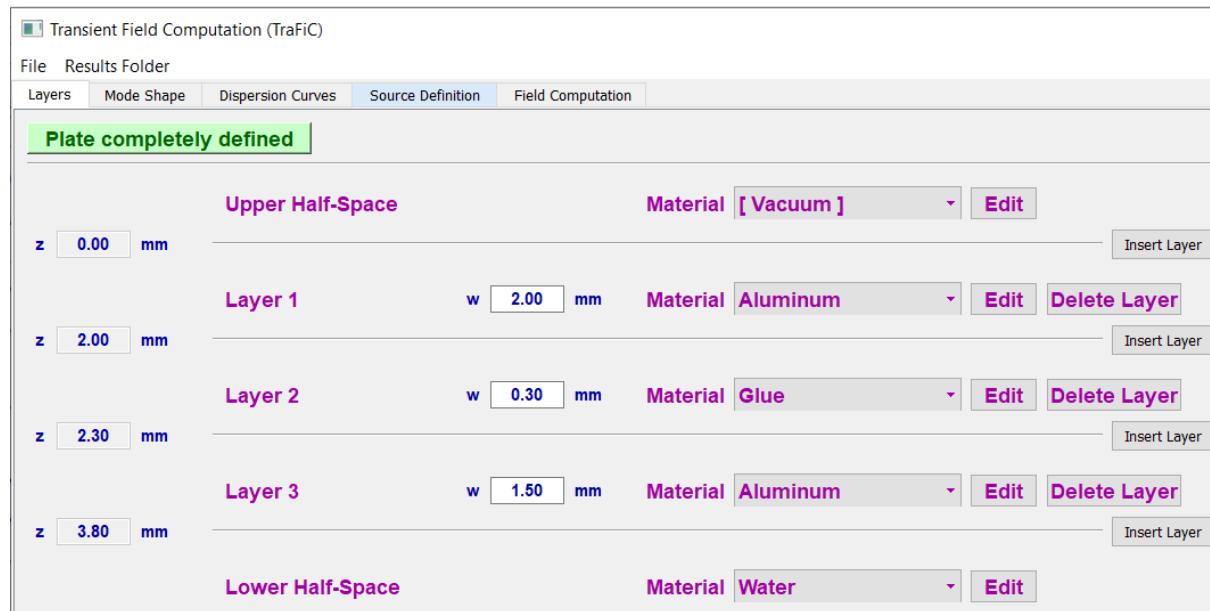
## TraFiC – II. Computation steps and results

Champs à  $t = 250.0 \mu\text{s}$



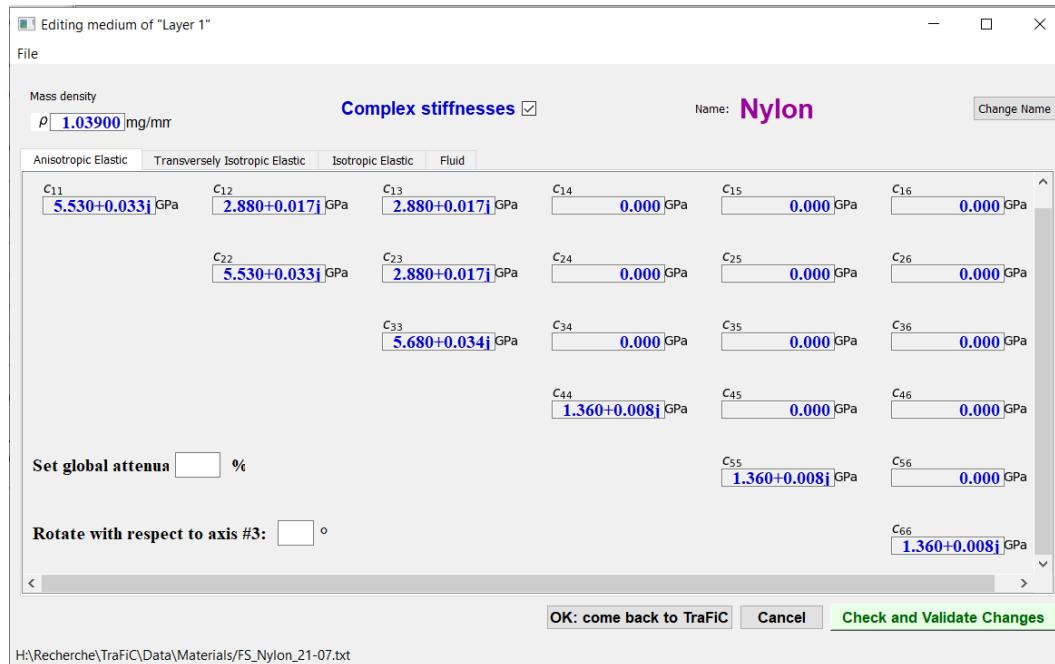
## TraFiC – III. Additional tools

### GUI (under development)



## TraFiC – III. Additional tools

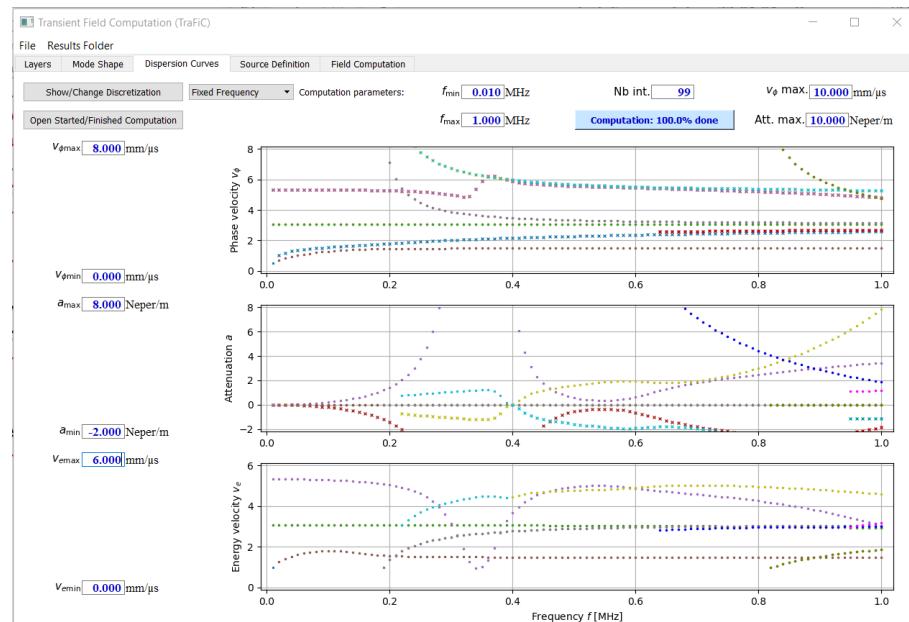
### GUI (under development)



## TraFiC – III. Additional tools

### Mode computation of immersed multilayer plates

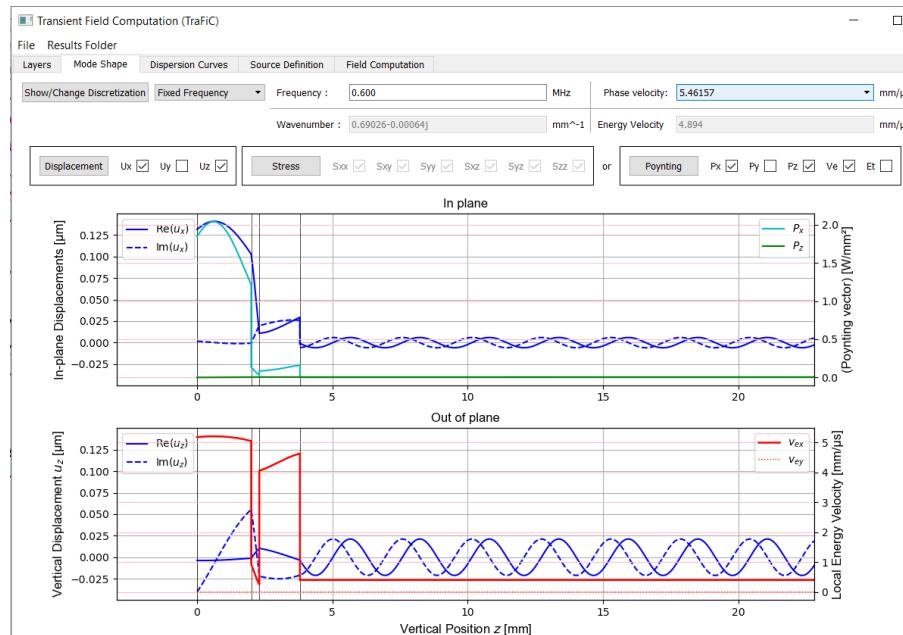
[Ducasse & Deschamps, *Mode computation of immersed multilayer plates by solving an eigenvalue problem*, to be published in Wave Motion]



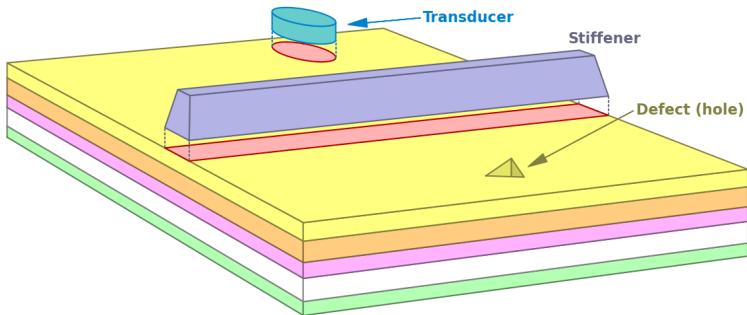
## TraFiC – III. Additional tools

### Mode computation of immersed multilayer plates

[Ducasse & Deschamps, *Mode computation of immersed multilayer plates by solving an eigenvalue problem*, to be published in Wave Motion]



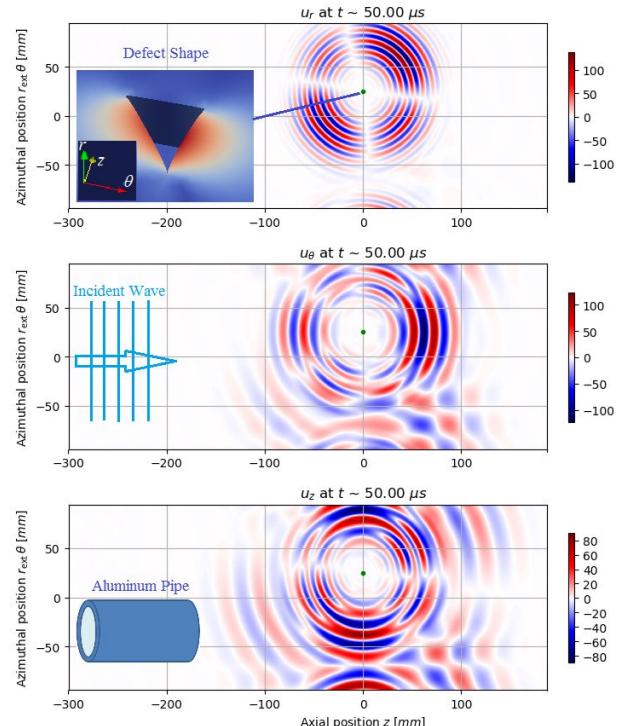
## TraFiC – IV. Towards hybrid computation



A PhD work by **Romain Kubecki** (Ing. MatMéca) is in progress, subject to funding (CEA + DGA-AID):  
**Development of hybrid numerical methods for the diffraction of ultrasonic waves by obstacles on the surface of laminated structures, and application to non-destructive testing**

Coll. POEMS/CEA-List/ $I_2M$ -Apy

Domain Decomposition & Asymptotic Methods



Asymptotic Method, coll. Marc BONNET (POEMS)