Comprehensive Examination Study Guide

Note: *You will be able to choose questions/problems on the exam. You will be required to answer some pedagogy questions and some mathematics content questions. This study guide provides questions and problems that are representative of the type of content that can be expected in the two-hour examination. Note that questions may combine concepts from more than one course. Students who had their course work several years ago should be familiar with the current literature and issues in Mathematics Education.*

Pedagogy Questions

1. *CCSS-M and Mathematical Practices*: Name and describe two of the Common Core State Standards Mathematical Practices. Use a sample problem to illustrate your explanation.

2. *Problem-solving and inquiry-based learning*: Pick a specific topic in the middle school or high school curriculum. Briefly describe how that topic is taught in a traditional setting. Then, describe how you might adapt the instruction to reflect a model that includes the problem solving processes.

3. *Use of manipulatives and technology*: Cite at least 3 examples of how manipulative experiences in the mathematics classroom can facilitate students learning of the concepts of number and space.

4. *Current issues in Math Ed*: Pick a current issue in mathematics education such as state standards, tracking, teacher evaluation, or high-stakes standardized testing. Write two arguments in reference to the issue, one from the affirmative position and one from the negative position. In each of your arguments, specifically address the perspective of students, teachers, and administrators.

5. *Administration and Supervision*: A mathematics supervisor needs to motivate teachers to use a variety of question formats to check for student understanding. These question types could be used for do now, classwork, homework, or assessment. Discuss three different question formats beyond the traditional ones. Give an example of each that is appropriate for your grade level(s).

Mathematics Content Questions

1. In 1957, Georg Polya published the renowned book How to Solve It. In the book, he outlines a four-stage approach to problem solving:

   i. Understand the problem
   ii. Devise a plan
   iii. Carry out the plan; and
   iv. Look back.

   Since the 1950s numerous researchers have critiqued Polya's model of the problem solving process. Offer at least two critiques of Polya's model.

2. A deck of Italian playing cards contains 40 cards (as opposed to 52 cards for a deck of American cards). The Italian deck contains four suits: coins, swords, cups, and clubs, and ten values: one through seven, soldier, horse, and king. How much higher is the probability of drawing a pair of kings from a deck of Italian playing cards than from a deck of American cards? Show work.

3. You are a traveling salesperson for a local company. Determine the number of unique roundtrips (URTs) if you need to visit:

   a. 10 cities (including your starting city)

   b. 25 cities (including your starting city)

   c. The capitals of the lower 48 states (including your starting city)

4. Use induction to show that for all $n > 1$, $1(1!) + 2(2!) + 3(3!) + \; + n(n!) = (n + 1)! - 1$

5. The given conditional statement is of the form $p \rightarrow q$ where p and q are the statements:
   p: Anne got a new muffler
   q: Anne drove her Mustang in the parade
   Form each of the following of the given statement [using the (word) statements]:

   - Converse
   - Inverse
   - Contrapositive

   Of the converse, inverse, and contrapositive statements, list those that are equivalent to the original conditional statements.

6. How many ways are there to select a committee of 13 politicians chosen from a room full (read infinite supply) of indistinguishable Democrats, indistinguishable Republications, and indistinguishable Independents?

   a. If every party must have at least two members on the committee?

   b. If every party must have at least two members on the committee and no group may have a majority of the committee members?

7. You are a consultant to a car rental company. You are trying to predict the number of rental cars that will end up in two cities, Orlando and Tampa, if their policy is such that cars can be rented in one location and dropped off in the other. The car company owns a total of 7000 cars. In analyzing historical records, you have already determined that 60% of the cars rented in Orlando are returned to Orlando, whereas 40% end up in Tampa. Of the cars rented from the Tampa office, 70% are returned to Tampa, whereas 30% end up in Orlando. Thus, you need to keep track of two values, O_n and T_n, the number of cars in Orlando and Tampa, respectively, at the end of day n.

   a. Suppose we start with 5000 cars in Orlando and 2000 cars in Tampa. Represent the above situation with a system of two difference equations.

   b. How many cars will be in each location after:

      i. 1 day?

      ii. 2 days?

iii. 3 days?

c. Find the equilibrium values for the system.

8. Our mathematics department has 10 classes that need to be scheduled in the spring. Below is a list of classes that cannot be scheduled at the same time due to teacher or enrollment constraints.

Class	**Cannot be scheduled with**
A	D I
B	D I J
C	E F I
D	A B F
E	H I
F	I C D
G	J
H	E I J
I	A B C E F H
J	B G H

a. Represent the data using a graph, where the vertices represent the classes and the edges represent the scheduling conflicts between classes.

b. Find the minimum number of time slots needed to schedule all of the classes.

c. Devise a schedule that would satisfy the scheduling requirements noted above. Is this solution unique? Why or why not? If the solution is not unique, provide an alternative schedule.

Good References

Ball, Deborah. (1992). Magical hopes: manipulatives and the reform of mathematics education. *American Educator.* Summer. pp. 14-18, 46-47.

Behrend, J. L. (2001). Are rules interfering with children's mathematical understanding? *Teaching Children Mathematics, 8*(1), 36-40.

Brown, L. C., & Seeley, C. L. (2010). Transitions from middle school to high school: Crossing the bridge. *Mathematics Teaching in the Middle School, 15*(6), 354-358.

Cho, H., & Lawrence, G. D. (2015). Building a positive mathematics community. *Mathematics Teaching in the Middle School, 20*(8), 499-502.

Cirillo, M., Herbel-Eisenmann, B., & Drake, C. (2009). Using curriculum to focus on understanding. *Mathematics Teaching in the Middle School, 15*(1), 51-56.

Clements, D. H. & McMillen, S. (1996). Rethinking "concrete" manipulatives. *Teaching Children Mathematics, 2*(5), 270-279.

Duckworth, Eleanor. (1972). *The having of wonderful ideas.* Harvard Educational Review, 42(2), 217-231.

Freer Weiss, D. M. (2006) Keeping it real: The rationale of keeping manipulatives in the middle grades. *Mathematics Teaching in the Middle School, 11*(5), 238-242.

Friel, S. N., & Markworth, K. A. (2009). A framework for analyzing geometric pattern tasks. *Mathematics Teaching in the Middle School, 15*(1), 25-33.

Hodges, T. E., Cady, J., & Collins, R. L. (2008). Fraction representation: The not-so-common denominator among textbooks. *Mathematics Teaching in the Middle School, 14*(2), 78-84.

Hollenbeck, R., & Fey, J. (2009). Technology and mathematics in the middle grades. *Mathematics Teaching in the Middle School, 14*(7), 430-435.

Horvath, et al. (2008). Middle-grades mathematics standards: Issues and implications. *Mathematics Teaching in the Middle School, 14*(5), 275-279.

Jacobs, V. R. & Kusiak, J. (2006). Got tools? Exploring children's use of mathematics tools during problem solving. *Teaching Children Mathematics, 12*(9), 470-477

Leitze, Annette. R. & Kitt, Nancy. (2000). Using homemade algebra tiles to develop algebra and prealgebra concepts. *Mathematics Teacher, 93*(6), 462-466, 520.

Ma, L. (1999). *Knowing and teaching elementary mathematics: Teachers' understanding of fundamental mathematics in China and the United States.* Mahwah, N.J.: Lawrence Erlbaum Associates.

Macintyre, T., & Hamilton, S. (2010). Mathematics learners and mathematics textbooks: a question of identity? Whose curriculum? Whose mathematics? *The Curriculum Journal, 21*(1), 3-23.

National Governors Association Center for Best Practices & Council of Chief State School Officers. (2010). *Common Core State Standards for Mathematics.* Washington, D.C.

Reys, B. J., & Bay-Williams, J. (2003). The role of textbooks in implementing the curriculum principle and the learning principle. *Mathematics Teaching in the Middle School, 9*(2), 120-122,124-125.

Simic-Muller, K. (2015). Social justice and proportional reasoning. *Mathematics Teaching in the Middle School, 21*(3), 162-168.

Stein, M. K. & Bovalino, J. W. (2001). Manipulatives: One piece of the puzzle. *Mathematics Teaching in the Middle School, 6*(6), 356-359.

Stein, M. K., & Kaufman, J. H. (2010). Selecting and supporting the use of mathematics curricula at scale. *American Educational Research Journal, 47* (3), 663-693.

Van de Walle, J. A., Karp, K. S., & Bay-Williams, J. M. (2012). *Elementary and middle school mathematics: Teaching developmentally* (8 ed.). Boston, MA: Pearson Education.

Webel, C. & Otten, S. (2015-2016). Teaching in a world with photomath. *Mathematics Teacher, 109*(5), 369-373.

Wiggins, G. & McTighe, J. (2005). *Understanding by Design.* Alexandria, VA: Association for Supervision and Curriculum Development (ASCD).