Montclair State University

Department of Mathematical Sciences

Comprehensive Exam Study Guide – Operations Research

The following list of topics is not meant to be comprehensive. The exam may cover topics that are not listed on this study guide but were covered in the corresponding course or covered in prerequisite undergraduate courses. The sample questions are merely representative of content that may be covered on the exam.

Topics

Graphical solutions of linear programming problems and sensitivity analysis

Convex sets and the feasible set of a linear programming problem

Modeling with linear programming (applications)

Simplex Method

Special cases that arise in solving a linear programming problem: multiple solutions, no solution, degeneracy, unbounded objective function value

Duality theory

The transportation problem and a streamlined simplex method for the transportation problem

Game Theory (two-person zero-sum games): Find a pure strategy solution, if it exists, and state the expected payoff value; find a mixed (randomized) strategy solution using the graphical method; and formulate an equivalent linear programming problem.

References

Taha, Hamdy A, *Operations Research An Introduction*, Brooks/Cole

Winston, Wayne L., *Operations Research: Applications and Algorithm*, Brooks/Cole

Hillier, Frederick S. and Lieberman, Gerald J., *Introduction to Operations Research*, McGraw Hill Education

Sample Problems

1. Use the graphical method to find all solutions of the linear programming problem:

$$\begin{aligned}
&\text{Minimize } z = x + 2y \\
&\text{subject to} \quad \begin{array}{rcrcr} x & - & 3y & \leq & 6 \\ 2x & + & 4y & \geq & 8 \\ x & - & 3y & \geq & -6 \end{array} \\
&\qquad\qquad x \geq 0, \quad y \geq 0
\end{aligned}$$

2. Let A be an $m \times n$ matrix of real numbers, $\boldsymbol{b}$ a column vector with m entries of real numbers, and $\boldsymbol{x}$ a column vector with n entries of real numbers. Prove that the set $S = \{\boldsymbol{x} : A\boldsymbol{x} \leq \boldsymbol{b} \text{ and } \boldsymbol{x} \geq \boldsymbol{0}\}$, is a convex set.

3. Set up the following problem as a complete linear programming problem. Do not attempt to solve the linear programming problem.

   The McGraw Chemical Company uses nitrates, phosphates, and potash in the manufacture of three types (A, B, C) of chemical fertilizers. McGraw has 1200 tons of nitrates, 2000 tons of phosphates and 1500 tons of potash. The table below gives the number of tons of each chemical in each container of fertilizer.

	A	B	C
Nitrates	0.05	0.05	0.08
Phosphates	0.10	0.08	0.12
Potash	0.05	0.08	0.12

   The unit profits for each container of fertilizer are in the following table:

A	B	C
8.0	11.40	6.20

   There are back orders for fertilizer B so that at least 8000 containers of B must be produced. How many containers of each type should be produced for a maximum total profit?

4. Use the Simplex method in tabular form to solve the following problem.

$$\begin{aligned}
&\text{Maximize } z = 5x_1 + x_2 \\
&\text{subject to} \quad \begin{array}{rcrcr} 2x_1 & + & x_2 & \leq & 6 \\ x_1 & - & x_2 & \leq & 0 \end{array} \\
&\qquad\qquad x_1 \geq 0, \quad x_2 \geq 0
\end{aligned}$$

5. Write the dual problem of the linear programming problem in problem 4 above and find the solution to the dual problem.

6. For the transportation model in the following table:

	Destination			
	1	2	3	Supply

	1	\$10	\$4	\$2	**8**
Source	2	\$2	\$3	\$4	**5**
	3	\$1	\$2	\$0	**6**
Demand		**7**	**6**	**6**	

(a) Write the linear programming problem for finding the minimum transportation cost. Do not attempt to solve the linear programming problem.

(b) Use the least-cost method to find the starting solution for the Transportation Algorithm (a streamlined simplex method for the transportation problem).

(c) Beginning with the starting solution in Part (b), find the optimal solution using the Transportation Algorithm.