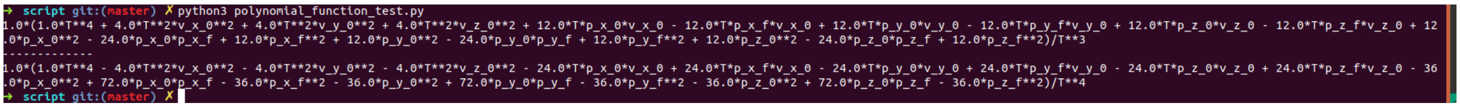
STEP1

here

STEP2

使用sympy简化J及计算J的导数here

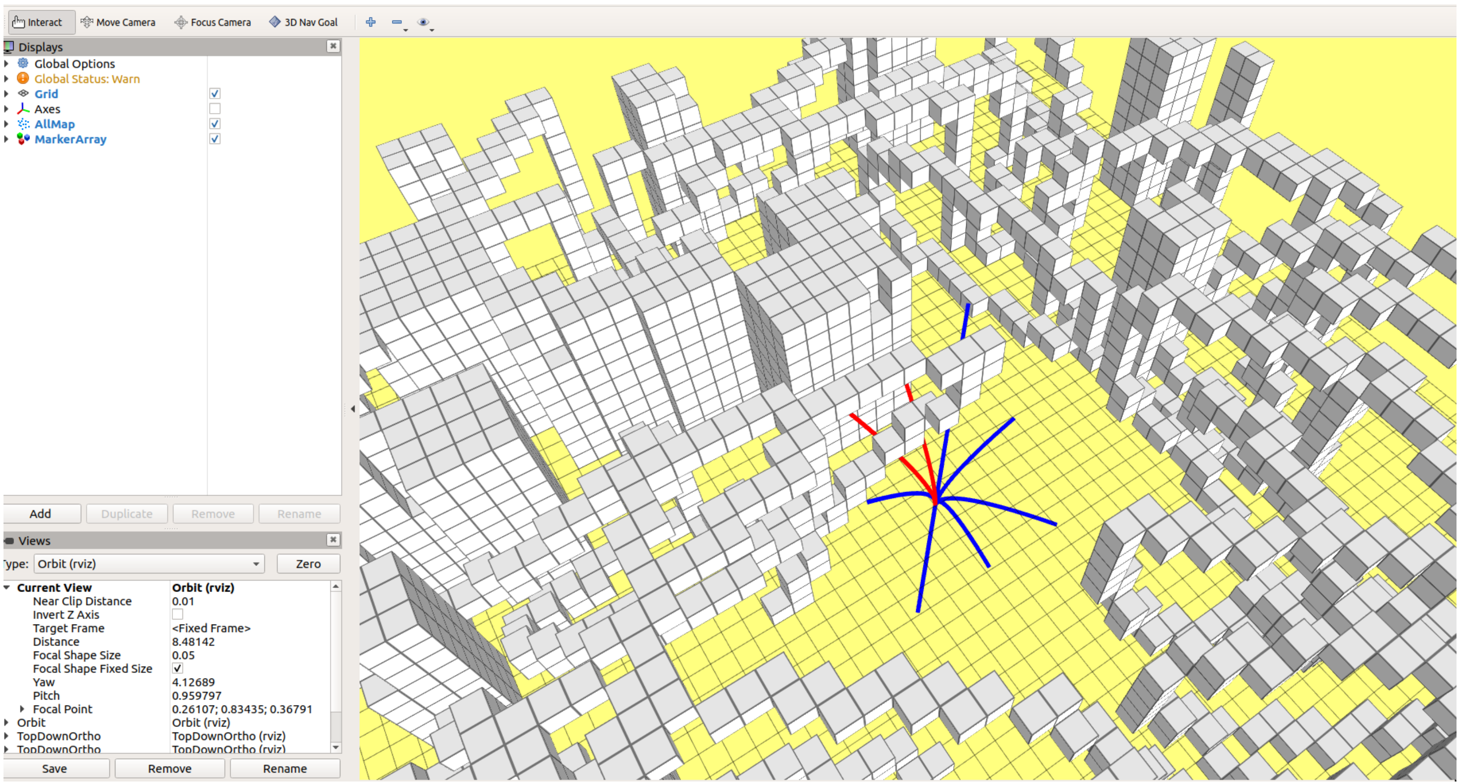
演算结果:



```
→ script git:(master) ✗ python3 polynomial_function_test.py
1.0*(1.0*T**4 + 4.0*T**2*v_x_0**2 + 4.0*T**2*v_y_0**2 + 4.0*T**2*v_z_0**2 + 12.0*T*p_x_0*v_x_0 - 12.0*T*p_x_f*v_x_0 + 12.0*T*p_y_0*v_y_0 - 12.0*T*p_y_f*v_y_0 + 12.0*T*p_z_0*v_z_0 - 12.0*T*p_z_f*v_z_0 + 12
.0*p_x_0**2 - 24.0*p_x_0*p_x_f + 12.0*p_x_f**2 + 12.0*p_y_0**2 - 24.0*p_y_0*p_y_f + 12.0*p_y_f**2 + 12.0*p_z_0**2 - 24.0*p_z_0*p_z_f + 12.0*p_z_f**2)/T**3
-------------
1.0*(1.0*T**4 - 4.0*T**2*v_x_0**2 - 4.0*T**2*v_y_0**2 - 4.0*T**2*v_z_0**2 - 24.0*T*p_x_0*v_x_0 + 24.0*T*p_x_f*v_x_0 - 24.0*T*p_y_0*v_y_0 + 24.0*T*p_y_f*v_y_0 - 24.0*T*p_z_0*v_z_0 + 24.0*T*p_z_f*v_z_0 - 36
.0*p_x_0**2 + 72.0*p_x_0*p_x_f - 36.0*p_x_f**2 - 36.0*p_y_0**2 + 72.0*p_y_0*p_y_f - 36.0*p_y_f**2 - 36.0*p_z_0**2 + 72.0*p_z_0*p_z_f - 36.0*p_z_f**2)/T**4
→ script git:(master) ✗
```

J导数求根及J值计算here

效果如下



个人一些总结

关于如果P给定，V不给定情况

minimum principle

$$\dot{s}^*(t) = f(s^*(t), u^*(t)), \quad given: s^*(0) = s(0)$$

$\lambda(t)$ is the solution of:

$$\dot{\lambda}(t) = -\nabla_s H(s^*(t), u^*(t), \lambda(t))$$

with the boundary condition of:

$$\lambda(T) = -\nabla h(s^*(T))$$

and the optimal control input is:

$$u^*(t) = \arg\min_{u(t)} H(s^*(t), u(t), \lambda(t))$$

Optimal Boundary Value Problem (OBVP)

- Previous slides are about fixed final state problem.
- How about the final state is (partially)-free?
  - Did you notice where is the boundary condition?

$$\lambda(t) = -\nabla h(s^*(t))$$

For fixed final state problem:

$$h(s(T)) = \begin{cases} 0, & if \quad s = s(T) \\ \infty, & otherwise \end{cases}$$ Not differentiable

So we discard this condition, and use given $x(T)$ to directly solve for unknown variables

$$s^*(t) = \begin{bmatrix} \frac{\alpha}{120}t^5 + \frac{\beta}{24}t^4 + \frac{\gamma}{6}t^3 + \frac{a_0}{2}t^2 + v_0 t + p_0 \\ \frac{\alpha}{24}t^4 + \frac{\beta}{6}t^3 + \frac{\gamma}{2}t^2 + a_0 t + v_0 \\ \frac{\alpha}{6}t^3 + \frac{\beta}{2}t^2 + \gamma t + a_0 \end{bmatrix}$$

For (partially)-free final state problem:

$$given \quad s_i(T), \ i \in I$$

We have boundary condition for other costate:

$$\lambda_j(T) = \frac{\partial h(s^*(T))}{\partial s_j}, \quad for \ j \neq i$$

Then we solve this problem again.

按照公式

$$\lambda_j = \frac{\partial h(S^*(T))}{\partial S_j}, for j! = i$$

所以对于V来说$\lambda(T) = 0$就是偏导的极值,即代码推到中的c)中关于V的部分为0。公式就修改为

$$\begin{bmatrix} \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 \\ 0 & 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 \\ 2T & 0 & 0 & 2 & 0 & 0 \\ 0 & 2T & 0 & 0 & 2 & 0 \\ 0 & 0 & 2T & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

因公式编辑有不同md解析有不用情况,故截图

$$\begin{bmatrix} \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 & 0 \\ 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 & 0 \\ 0 & 0 & \frac{1}{6}T^3 & 0 & 0 & \frac{1}{2}T^2 \\ 2T & 0 & 0 & 2 & 0 & 0 \\ 0 & 2T & 0 & 0 & 2 & 0 \\ 0 & 0 & 2T & 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{bmatrix} = \begin{bmatrix} \Delta P_x \\ \Delta P_y \\ \Delta P_z \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

然后求解α, β,看在码了这么久公式份上，如果理解有误，请及时指正。