

# 雅克比矩阵推导

$$f(\theta^{acc}) = \|g\|^2 - \|a\|^2$$

$$\frac{\partial f}{\partial \theta^{acc}} \because \|g\|^2 \text{ 与 } \theta^{acc} \text{ 无关}$$

$$\therefore \frac{\partial f}{\partial \theta^{acc}} = -\frac{\partial \|a\|^2}{\partial \theta^{acc}} = -\frac{\partial \|a\|^2}{\partial a} \cdot \frac{\partial a}{\partial \theta^{acc}}$$

$$= -2a^T \cdot \frac{\partial a}{\partial \theta^{acc}}$$

其中  $a = (I - \alpha) K_a (A - ba)$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ S_{ayx} & 0 & 0 \\ S_{azx} & S_{azy} & 0 \end{bmatrix} * \begin{bmatrix} \frac{1}{K_{ax}} & \frac{1}{K_{ay}} & \frac{1}{K_{az}} \end{bmatrix} * \begin{bmatrix} A - \begin{bmatrix} b_{ax} \\ b_{ay} \\ b_{az} \end{bmatrix} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{K_{ax}} & \frac{1}{K_{ay}} & \frac{1}{K_{az}} \\ -S_{ayx} \frac{1}{K_{ax}} & \frac{1}{K_{ay}} & \frac{1}{K_{az}} \\ -S_{azx} \frac{1}{K_{ax}} & -S_{azy} \frac{1}{K_{ay}} & \frac{1}{K_{az}} \end{bmatrix} * \begin{bmatrix} A - \begin{bmatrix} b_{ax} \\ b_{ay} \\ b_{az} \end{bmatrix} \end{bmatrix}$$

设  $S_{ayx} = -S_{ayx} \quad S_{azx} = -S_{azx} \quad S_{azy} = -S_{azy}$

$$K_{ax} = \frac{1}{K_{ax}} \quad K_{ay} = \frac{1}{K_{ay}} \quad K_{az} = \frac{1}{K_{az}} \quad (1)$$

其  $A = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}$  为提供的数据

下面用sympy推导的[here](#)

a矩阵

$$\begin{bmatrix} K_{ax} (Ax - b_{ax}) \\ K_{ax} S_{ayx} (Ax - b_{ax}) + K_{ay} (Ay - b_{ay}) \\ K_{ax} S_{azx} (Ax - b_{ax}) + K_{ay} S_{azy} (Ay - b_{ay}) + K_{az} (Az - b_{az}) \end{bmatrix}$$

对Sayx求偏导

$$\begin{bmatrix} 0 \\ K_{ax} (Ax - b_{ax}) \\ 0 \end{bmatrix}$$

对Sazx求偏导

$$\begin{bmatrix} 0 \\ 0 \\ Kax\left(Ax-bax\right) \end{bmatrix}$$

对Sazy求偏导

$$\begin{bmatrix} 0 \\ 0 \\ Kay\left(Ay-bay\right) \end{bmatrix}$$

对Kax求偏导

$$\begin{bmatrix} Ax-bax \\ Sayx\left(Ax-bax\right) \\ Sazx\left(Ax-bax\right) \end{bmatrix}$$

对Kay求偏导

$$\begin{bmatrix} 0 \\ Ay-bay \\ Sazy\left(Ay-bay\right) \end{bmatrix}$$

对Kaz求偏导

$$\begin{bmatrix} 0 \\ 0 \\ Az-baz \end{bmatrix}$$

对bax求偏导

$$\begin{bmatrix} -Kax \\ -KaxSayx \\ -KaxSazx \end{bmatrix}$$

对bay求偏导

$$\begin{bmatrix} 0 \\ -Kay \\ -KaySazy \end{bmatrix}$$

对baz求偏导

$$\begin{bmatrix} 0 \\ 0 \\ -Kaz \end{bmatrix}$$

$$J_f = -2a^T \cdot \left[ \frac{\partial a}{\partial S_{ayx}} \quad \frac{\partial a}{\partial S_{azx}} \quad \frac{\partial a}{\partial S_{azy}} \quad \frac{\partial a}{\partial K_{ax}} \quad \frac{\partial a}{\partial K_{ay}} \quad \frac{\partial a}{\partial K_{az}} \quad \frac{\partial a}{\partial b_{ax}} \quad \frac{\partial a}{\partial b_{ay}} \quad \frac{\partial a}{\partial b_{az}} \right]$$

## 代码实现

ppt中提供的公式与imu\_tk论文中提供的公式不同，下面按照代码中TODO进行处理。  
在calibration.h 中是这样定义的

```
mis_mat_ <<  _T(1)  , -mis_yz , mis_zy ,
              mis_xz ,  _T(1)  , -mis_zx ,
              -mis_xy , mis_yx ,  _T(1)  ;

scale_mat_ <<  s_x ,  _T(0) ,  _T(0) ,
               _T(0) ,  s_y ,  _T(0) ,
               _T(0) ,  _T(0) ,  s_z ;

bias_vec_ <<  b_x , b_y , b_z ;
```

下面对代码进行修改:

[here](#)

```
// acc_calib_params[0] = init_acc_calib_.misYZ();
// acc_calib_params[1] = init_acc_calib_.misZY();
// acc_calib_params[2] = init_acc_calib_.misZX();

acc_calib_params[0] = init_acc_calib_.misXZ();
acc_calib_params[1] = init_acc_calib_.misXY();
acc_calib_params[2] = init_acc_calib_.misYX();
```

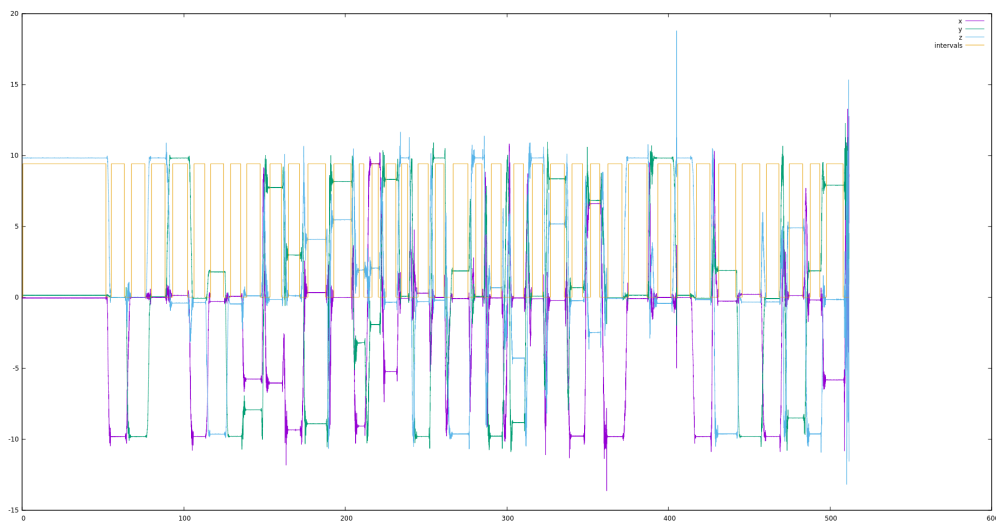
[here](#)

```
0, 0, 0,
min_cost_calib_params[0],
min_cost_calib_params[1],
min_cost_calib_params[2],
```

here

```
// mis_yz, mis_zy, mis_zx:
_T2(0), _T2(0), _T2(0),
// mis_xz, mis_xy, mis_yx:
params[0], params[1], params[2],
```

效果如下:



加速度结果

```
Accelerometers calibration: Better calibration obtained using threshold multipli
er 6 with residual 0.120131
Misalignment Matrix
      1      -0      0
-0.00354989      1     -0
-0.00890444 -0.0213032      1
Scale Matrix
0.00241267      0      0
      0 0.00242659      0
      0      0 0.00241232
Bias Vector
33124.2
33275.2
32364.4

Accelerometers calibration: inverse scale factors:
414.478
412.102
414.538
```

磁力计结果

```
Total                                0.3064

Termination:                        CONVERGENCE (Function tolerance reached. |cost
_change|/cost: 1.098767e-11 <= 1.000000e-06)

Gyroscopes calibration: residual 0.00150696
Misalignment Matrix
      1 0.00927517 0.00990014
0.00507442      1 -0.0322229
 0.0162201 -0.0239393      1
Scale Matrix
0.000209338      0      0
      0 0.000209834      0
      0      0 0.000209664
Bias Vector
32777.1
32459.8
32511.8

Gyroscopes calibration: inverse scale factors:
4776.96
4765.68
4769.53
```