Solution:

- 1. Please find the formulation of the optimization problem as well as the output from Python attached below.
- 2. For more analytical parts, please find the code file named 'code.ipynb'. This file includes all the code, explanations and relevant outputs step by step.
- 3. Some key steps and outputs are included in the following:
 - a. Formulate the optimization problem

Let
$$X_{ij} = \begin{cases} 1, & item \ i \ is \ at \ price \ j \\ 0, & item \ i \ is \ not \ at \ price \ j \end{cases}$$
 P_{ij} represents the price j for item i D_{ij} represents the demand for item i at price j

Max
$$\sum \sum X_{ij} P_{ij} D_{ij}$$

s.t. $\sum_{j} X_{ij} = 1$
 $\sum \sum X_{ij} P_{ij} = k$
 $X_{ij} \in \{0,1\}$

The possible values of prices are given, and k is derived based on the given price options. Demand is predicted based on the trained regression tree using historical data.

b. Implement the optimization problem

Break down the above problem into two models – one for event A items, and the other for event B items. And then sum up.

i) Result for event A items:

```
when k=87.97:
when k=77.97:
Objective Function Value: 12339.52 Objective Function Value: 14233.42
                                    x11: -0
x11: 1
x12: -0
                                    x12: 1
                                    x13: -0
x13: -0
x14: -0
                                    x14: -0
                                    x21: -0
x21: 1
x22: -0
                                    x22: 1
x23: -0
                                    x23: -0
x24: -0
                                    x24: 0
x31: 1
                                    x31: 1
x32: -0
                                    x32: -0
x33: -0
                                    x33: -0
x34: -0
                                    x34: -0
when k=82.97:
                                    when k=92.97:
Objective Function Value: 11299.20 Objective Function Value: 14713.42
x11: 1
                                    x11: -0
x12: 0
                                    x12: 1
x13: -0
                                    x13: -0
x14: -0
                                    x14: -0
x21: -0
                                    x21: -0
x22: 1
                                    x22: 1
x23: -0
                                    x23: -0
x24: -0
                                    x24: -0
x31: 1
                                    x31: -0
x32: -0
                                    x32: 1
x33: -0
                                    x33: -0
x34: -0
                                    x34: -0
                                      when k=107.97:
when k=97.97:
                                      Objective Function Value: 14885.37
Objective Function Value: 15243.12
                                      x11: -0
x11: -0
                                      x12: -0
x12: -0
                                      x13: -0
x13: -0
                                      x14: 1
x14: 1
                                      x21: -0
x21: -0
                                      x22: -0
x22: 1
                                      x23: -0
x23: -0
                                      x24: 1
x24: -0
                                      x31: 1
x31: 1
                                      x32: -0
x32: -0
x33: -0
                                      x33: -0
x34: -0
                                      x34: -0
when k=102.97:
                                      when k=112.97:
Objective Function Value: 14213.08
                                      Objective Function Value: 15628.45
x11: -0
                                      x11: 0
x12: -0
                                      x12: 0
x13: 1
                                      x13: -0
x14: -0
                                      x14: 1
x21: -0
                                      x21: 0
x22: -0
                                      x22: -0
x23: -0
                                      x23: -0
x24: 1
                                      x24: 1
x31: 1
                                      x31: 0
x32: -0
                                      x32: 1
x33: -0
                                      x33: -0
x34: -0
                                      x34: -0
```

```
when k=117.97:
Objective Function Value: 14477.93
x11: 0
x12: 0
x13: -0
x14: 1
x21: 0
x22: 0
x23: -0
x24: 1
x31: 0
x32: 0
x33: 1
x34: 0
when k=122.97:
Objective Function Value: 12445.76
x11: 0
x12: 0
x13: 0
x14: 1
x21: 0
x22: 0
x23: 0
x24: 1
x31: 0
x32: 0
x33: 0
```

From the above, when $X_{14} = X_{24} = X_{32} = 1$, the objective function reaches maximum.

So when we set the price for item A to \$40.99, the price for item C to \$40.99, the price for item D to \$30.99, the maximum revenue for event A items is \$15628.45.

ii) Result for event B items:

```
when k=91.98:
Objective Function Value: 9548.87
x11: 1
x12: 0
x21: 1
x22: 0
when k=96.98:
Objective Function Value: 9618.49
x11: 1
x12: 0
x21: -0
x22: 1
when k=101.98:
Objective Function Value: 10587.02
x11: 0
x12: 1
x21: 0
```

From the above, when $X_{12} = X_{22} = 1$, the objective function reaches maximum.

So when we set the price for item B to \$50.99, the price for item E to \$50.99, the maximum revenue for event B items is \$10587.02.

iii) Conclusion:

The pricing decision and the maximum revenue are as follows:

item#	price
A	\$40.99
В	\$50.99
С	\$40.99
D	\$30.99
Е	\$50.99

The maximum revenue is \$26215.47 (\$15628.45+\$10587.02).