Quick tips before trying to solve the OLG model.

1- Determine whether the zero steady state is possible:

if f(0) = 0 (or, equivalently, w(0) = 0), then the zero (autarky) steady state exists. It may be stable or unstable, but it exists.

Cobb-Douglas production:

```
dAmZZBmsejYq[k_] = i   α
```

In[ ]:=

```
dAmZZBmsejYq[╋] = ╋
```

```
╋
```

CES production with complementary inputs: $\rho<0$

Important remark: verify the exact functional form of the production function, in particular how the exponents are written.

In[ ]:=

```
dACQ[k_] = (α * i   ρ + (╋ - α))   (╋ / ρ)
Jgkgr[dACQ[i]┌ i → ╋┌
qqsknrgmlq → {ρ < ╋┌ α > ╋┌ α < ╋}]
```

```
╋
```

If f(0) > 0, then the zero (autarky) steady states does not exist.

```
Jgkgr[dACQ[i]┌ i → ╋┌
qqsknrgmlq → {ρ > ╋┌ α > ╋┌ α < ╋}]
```

```
(╋ - α)^‡
```

2- Determine whether the interest rate appears in the savings function:

if log-utility, the interest rate does not appear.

```
snpgkcJmeSrgjgrw[x_] = ╋/ v
```

```
Qmjtc[snpgkcJmeSrgjgrw[u - q] ==
  β * P * snpgkcJmeSrgjgrw[P * q]⟮ q]
```

$$\left\{\left\{q \to \frac{u\ \beta}{\text{╋} + \beta}\right\}\right\}$$

Other functional forms: CIES, R appears in the savings function:

```
snpgkcAGCQ[x_] = v   (-╋/ σ)
```

In[ ]:=
```
Qmjtc[snpgkcAGCQ[u - q] == β * P * snpgkcAGCQ[P * q]⟮
  q]
```

$$\left\{\left\{q \to \frac{P^{\sigma}\ u\ \beta^{\sigma}}{P + P^{\sigma}\ \beta^{\sigma}}\right\}\right\}$$

3 - Pick your own model!

In[2]:=
```
d[k_⟮ α _⟮ ρ _] = Gd[ρ == ╋⟮ i   α⟮ (α * i   ρ + (╋ - α))   (╋/ ρ)]
s[x_⟮ σ _] = Gd[σ == ╋⟮ Jme[v]⟮ v   (╋ - ╋/ σ) / (╋ - ╋/ σ)]
udslargml[k_⟮ α _⟮ ρ _] = d[i⟮ α⟮ ρ] - B[d[i⟮ α⟮ ρ]⟮ i] * i
Pdslargml[k_⟮ α _⟮ ρ _] = B[d[i⟮ α⟮ ρ]⟮ i]
```

In[6]:=
```
qYtgleq[w_⟮ R_⟮ σ _⟮ β _]⟯ =
  q /╬ Qmjtc[B[s[w - q⟮ σ]⟮ q] + β * B[s[R * q⟮ σ]⟮ q] == ╋⟮ q][[╋]]
```

In[7]:=
```
bcpgtYrgtc[k1_, k_, α_, β_, σ_, ρ_, n_] :=
 -B[
   k1 -
    ╋/@(╋ + n) *
     (qYtgleq[u, P, σ, β] /.
       {u → udslargml[k, α, ρ],
        P → Pdslargml[k1, α, ρ]}), k] /
  B[
   k1 -
    ╋/@(╋ + n) *
     (qYtgleq[u, P, σ, β] /.
       {u → udslargml[k, α, ρ],
        P → Pdslargml[k1, α, ρ]}), k1]
```

In[8]:=
```
bcp[sols_, i_, α_, β_, σ_, ρ_, n_] :=
 Jgkgr[bcpgtYrgtc[i╋, i╋, α, β, σ, ρ, n],
  {i╋, i╋} → {i /.. sols[[i]], i╋ → i /.. sols[[i]]},
  Bgpcargml → {-Dpmk Zmtc-, -Dpmk Zmtc-}]
```

In[9]:=
```
qrYZgjgrw[α_, β_, σ_, ρ_, n_] :=
 (Gd[
   Jclerf[LQmjtc[i == ╋/(╋ + n) * qYtgleq[u, P, σ, β] //
       {u → udslargml[i, α, ρ], P → Pdslargml[i, α, ρ]},
      i, PcYjq]] == ╋, qmjq = {{i → ╋}},
   qmjq = LQmjtc[i == ╋/(╋ + n) * qYtgleq[u, P, σ, β] //
      {u → udslargml[i, α, ρ], P → Pdslargml[i, α, ρ]},
     i, PcYjq]],
  Dmp[g = ╋, g ≤ Jclerf[qmjq],
   Npglr[-                    QrcYbw qrYrc -, g,
    - AYngrYj jctcj, - i //. qmjq[[g]], ¬ BcpgtYrgtc= -,
    bcp[qmjq, g, α, β, σ, ρ, n],
    Gd[-╋ < bcp[qmjq, g, α, β, σ, ρ, n] < ╋, - QrYZjc -,
     - SlqrYZjc -]],
   g++])
```

In[10]:=
```
kwnpmepYk[α_, β_, σ_, ρ_, n_] := (Npglr[
   -Npmbsargml dslargml,    d(i)= -, d[i, α, ρ], - l -,
   -Srgjgrw dslargml,       s(a)= -, s[a, σ], - l -,
   -QYtgleq dslargml,       q(u( -, QsZqapgnr[i, r],
   -), -, QsZqsncpqapgnr[-i -, -r+╋-, -| -], -))= -,
   qYtgleq[u, P, σ, β], - l -,
   -QsZqrgrsrgle,           q(u( -, QsZqapgnr[i, r],
   -), -, QsZqsncpqapgnr[-i -, -r+╋-, -| -], -))= -,
   qYtgleq[u, P, σ, β] //
    {u → udslargml[i, α, ρ], P → Pdslargml[i, α, ρ]}, - l -,
   -QrcYbw QrYrcq,          Qmjtc -,
   QsZqapgnr[-i -, -r+╋-], - == ╋/(╋+l)q(u( -,
   QsZqapgnr[i, r], -), -,
```

```
QsZqsncpqapgnr[-i-┌ -r+╋-┌ -┘ -]┌ -))-┌ - l-┌
-   Ugrf rfc nYpYkcrcpq┐  -┌
Osgcr[
 Gd[
  Jclerf[LQmjtc[i == ╋/(╋ + n)*qYtgleq[u┌ P┌ σ┌ β] /╬
      {u → udslargml[i┌ α┌ ρ]┌
       P → Pdslargml[i┌ α┌ ρ]}┌ i┌ PcYjq]] == ╋┌
  {{i → ╋}}┌
  LQmjtc[i == ╋/(╋ + n)*qYtgleq[u┌ P┌ σ┌ β] /╬
    {u → udslargml[i┌ α┌ ρ]┌ P → Pdslargml[i┌ α┌ ρ]}┌
   i┌ PcYjq]]]┌ - l-┌
 -QrYZgjgrw┐              Amknsrc rfc bcpgtYrgtc
  md  -┌ QsZqapgnr[-i-┌ -r+╋-]┌
 - ugrf pcqncar rm  -┌ QsZqapgnr[-i-┌ -r-]┌
 - Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc-┌ - l-┌
 -   Ugrf rfc nYpYkcrcpq┐  -]┘
Npglr[Osgcr[qrYZgjgrw[α┌ β┌ σ┌ ρ┌ n]]]┘
qmjqQgknjc = i /╬ qmjq┘
Gd[KYv[qmjqQgknjc] == ╋┌ kYvi = ╋╬·┌
 kYvi = ╋╬╋* KYv[qmjqQgknjc]]┘
Osgcr[AmlrmspNjmr[
  {i╋ == i┌
   i╋ == ╋/(╋ + n)*
     (qYtgleq[u┌ P┌ σ┌ β] /╬
       {u → udslargml[i┌ α┌ ρ]┌
        P → Pdslargml[i╋┌ α┌ ρ]})}┌ {i┌ ╋┌ kYvi}┌
  {i╋┌ ╋┌ kYvi}]])
```

In[ ]:= `kwnpmepYk[╋╬·┌ ╋╬═┌ ╋┌ ╋┌ ╋]`

Npmbsargml dslargml┐ $d(i)=i^{\blacksquare\blacksquare}$

Srgjgrw dslargml┐ $s(a)=\blacksquare\sqrt{a}$

QYtgleq dslargml┐ $q(u(i_r)\urcorner i'_{r+\blacksquare})=\frac{\blacksquare\,\blacksquare P\,u}{\blacksquare\,\blacksquare+\blacksquare\,\blacksquare P}$

QsZqrgrsrgle┐ $q(u(i_r)\urcorner i'_{r+\blacksquare})=\frac{\blacksquare\,\blacksquare}{\blacksquare\,\blacksquare+\frac{\blacksquare\,\blacksquare}{i\,\blacksquare\,\blacksquare}}$

QrcYbw QrYrcq┐ Qmjtc $i_{r+\blacksquare}$ == $\blacksquare/(\blacksquare+l)q(u(i_r)\urcorner i'_{r+\blacksquare})$

Ugrf rfc nYpYkcrcpq┐ $\{\{i\to\blacksquare\blacksquare\}\urcorner\{i\to\blacksquare\blacksquare\blacksquare:\blacksquare\blacksquare\blacksquare\|=\}\}$

QrYZgjgrw┐ Amknsrc rfc bcpgtYrgtc md $i_{r+\blacksquare}$

ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc

Ugrf rfc nYpYkcrcpq┐

QrcYbw qrYrc ┼

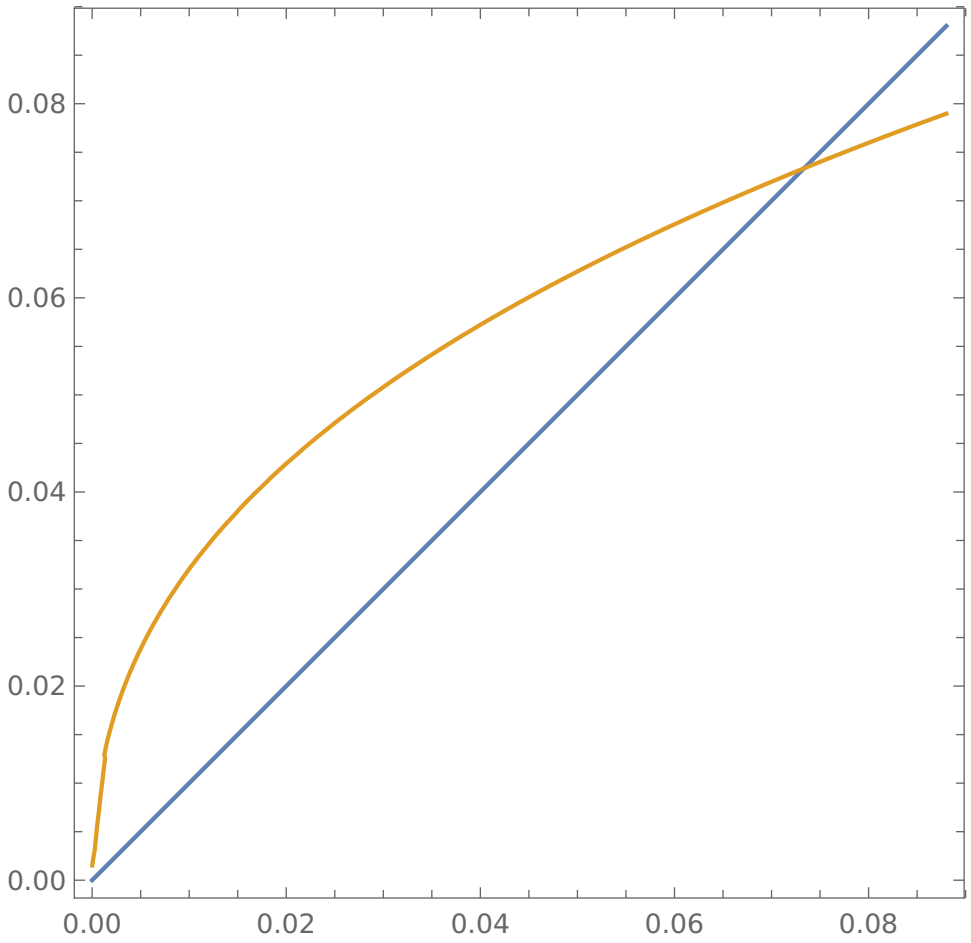
┌ AYngrYj jctcj┐ ┼╫┐ BcpgtYrgtc=∞ SlqrYZjc

QrcYbw qrYrc ┼┌ AYngrYj jctcj┐

┼╫┼:┼┼┼=┐ BcpgtYrgtc=┼╫┈┈┼┆::┼ QrYZjc

Lsjj

Out[ ]=



In[ ]:= kwnpmepYk[

Npmbsargml dslargml┐ $d(i)=\frac{1}{\sqrt{1^{2}+\frac{1^{2}}{i}}}$

Srgjgrw dslargml┐ $s(a)=\sqrt{a}$

QYtgleq dslargml┐ $q(u(i_r)\urcorner i'_{r+1}))=\frac{1\,P\,u}{1^{2}+1\,P}$

QsZqrgrsrgle┐ $q(u(i_r)\urcorner$

$$i'_{r+1}))=\frac{=\left(\frac{1}{\sqrt{1^{2}+\frac{1^{2}}{i}}}-\frac{1^{2}}{\left(1^{2}+\frac{1^{2}}{i}\right)^{1/1}\,i}\right)}{\left(1^{2}+\frac{=}{\left(1^{2}+\frac{1^{2}}{i}\right)^{1/1}\,i}\right)\left(1^{2}+\frac{1^{2}}{i}\right)^{1/1}\,i}$$

QrcYbw QrYrcq┐ Qmjtc i_{r+1} == $1/(1+1)q(u(i_r)\urcorner i'_{r+1}))$

Ugrf rfc nYpYkcrcpq┐ {{i → 1}}

QrYZgjgrw┐ Amknsrc rfc bcpgtYrgtc md i_{r+1}

ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc

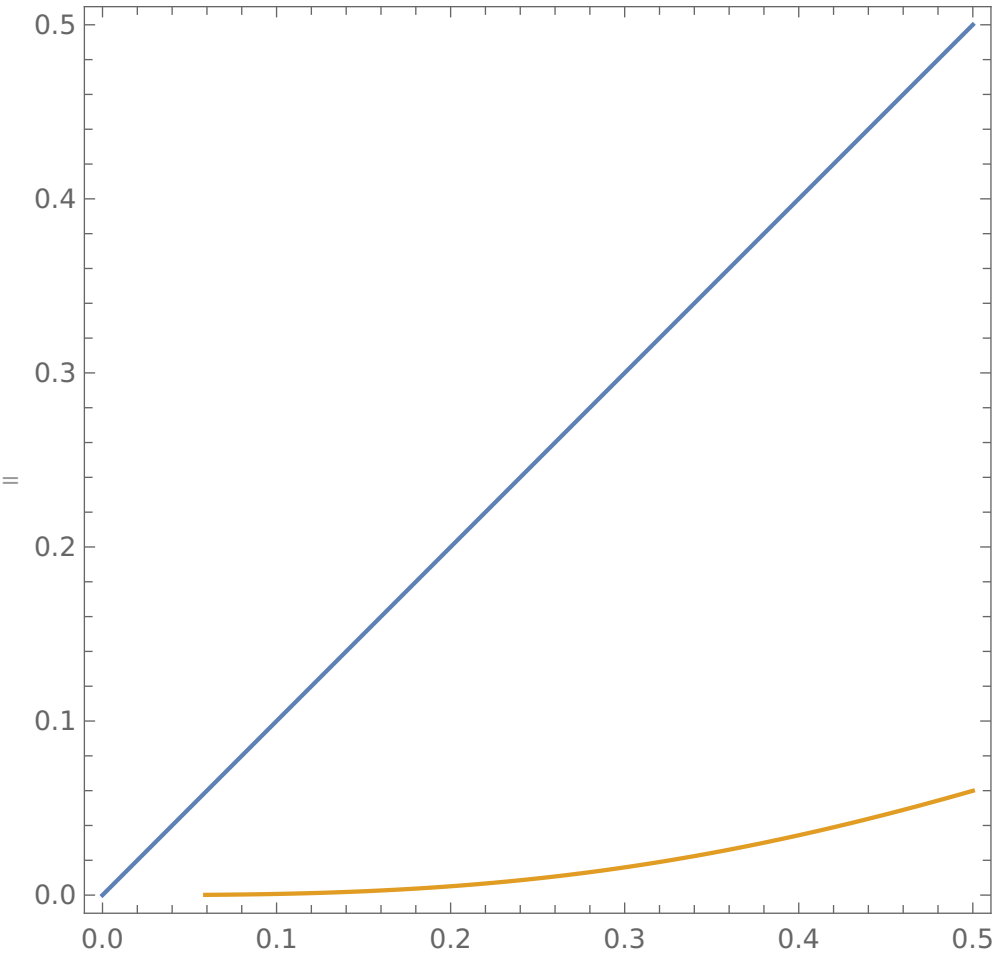
Ugrf rfc nYpYkcrcpq┐

QrcYbw qrYrc 1

⌜ AYngrYj jctcj┐ 1┐ BcpgtYrgtc=1 QrYZjc

Lsjj

Out[ ]=

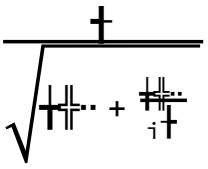


In[ ]:= kwnpmepYk[▒▒▒▒▒▒▒▒]

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

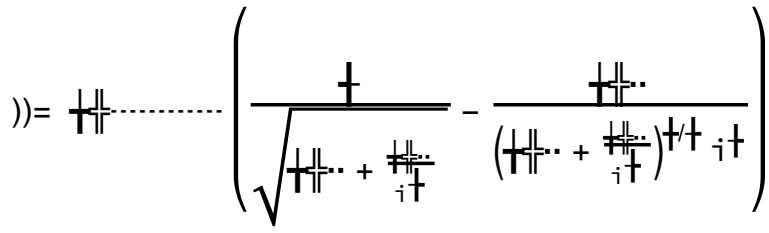
Npmbsargml dslargml┐ d(i)=



Srgjgrw dslargml┐ s(a)=Jme[a]

QYtgleq dslargml┐ q(u(i$_r$)⌠i$^{|}_{r+}$))= ⋯⋯ u

QsZqrgrsrgle┐ q(u(i$_r$)⌠i$^{|}_{r+}$



QrcYbw QrYrcq┐ Qmjtc i$_{r+}$ == /(+l)q(u(i$_r$)⌠i$^{|}_{r+}$))

Ugrf rfc nYpYkcrcpq┐ {{i →}}

QrYZgjgrw┐ Amknsrc rfc bcpgtYrgtc md i$_{r+}$

ugrf pcqncar rm i$_r$ Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc

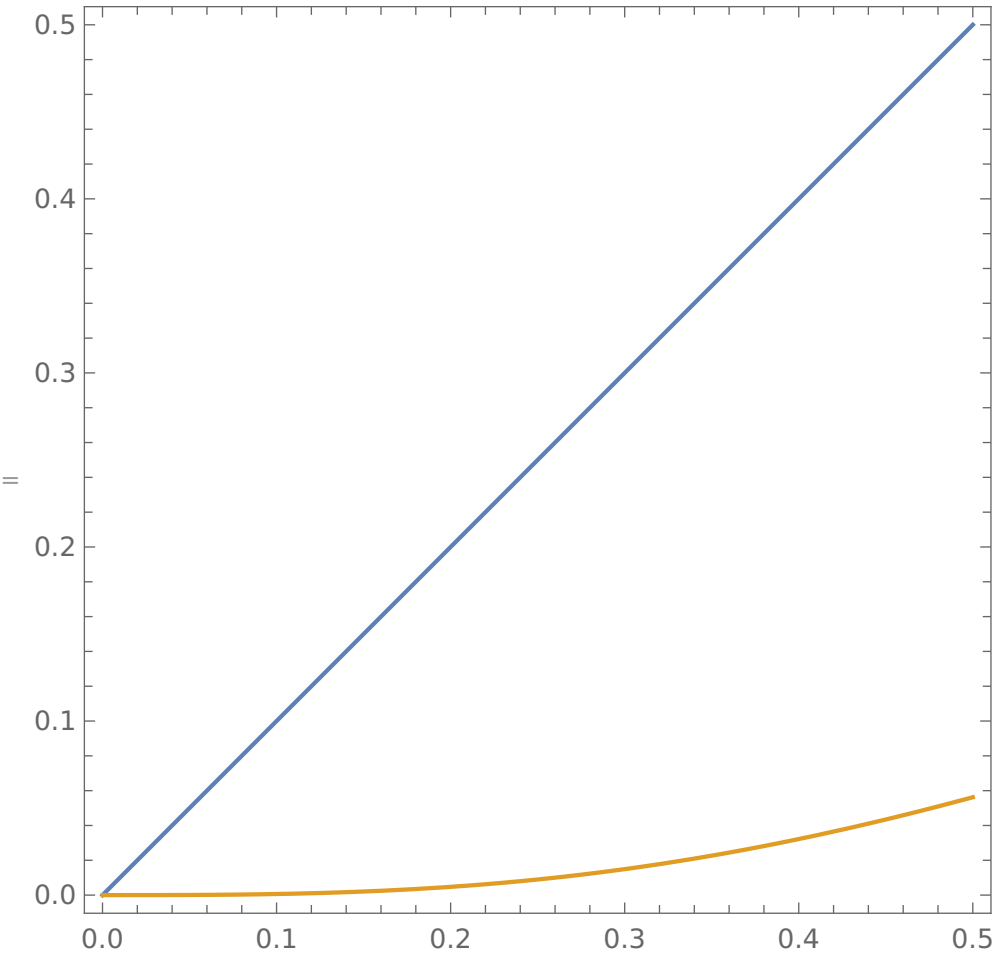
Ugrf rfc nYpYkcrcpq┐

QrcYbw qrYrc

⌠ AYngrYj jctcj┐ ┐ BcpgtYrgtc= QrYZjc

Lsjj

Out[ ]=



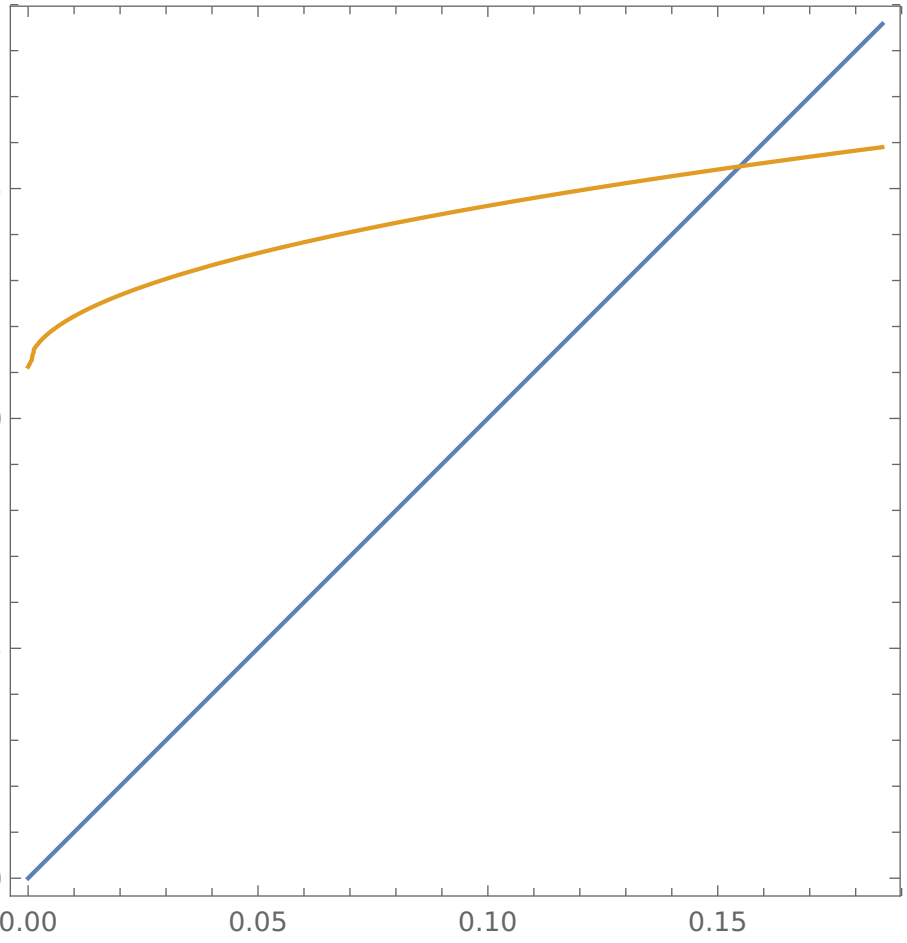
In[ ]:= kwnpmepYk[▒]

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

```
Npmbsargml dslargml┐      d(i)=(┼╫·· + ┼╫·· i┼╫··)┼╫
Srgjgrw dslargml┐         s(a)=Jme[a]
QYtgleq dslargml┐         q(u(i_r)╭i|_r+┼))= ┼╫·····u
QsZqrgrsrgle┐             q(u(i_r)╭i|_r+┼))=
 ┼╫·····((┼╫·· + ┼╫·· i┼╫··)┼╫ - ┼╫·· (┼╫·· + ┼╫·· i┼╫··)┼╫ i┼╫··)
QrcYbw QrYrcq┐            Qmjtc i_r+┼ == ┼/(┼+l)q(u(i_r)╭i|_r+┼))
   Ugrf rfc nYpYkcrcpq┐   {{i → ┼╫┼···=┼┼}}
QrYZgjgrw┐                Amknsrc rfc bcpgtYrgtc md i_r+┼
  ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc
   Ugrf rfc nYpYkcrcpq┐
                          QrcYbw qrYrc ┼╭ AYngrYj jctcj┐
 ┼╫┼···=┼┼┐ BcpgtYrgtc=┼╫┼··┼┼== QrYZjc
Lsjj
```

Out[ ]=



In[ ]:= **kwnpmepYk[■■■, ■■■, ■■■]**

Npmbsargml dslargml┐ d(i)=(■■■ + ■■■ i■■■)■■■

Srgjgrw dslargml┐ s(a)=■ √a

QYtgleq dslargml┐ q(u(i_r)┌i_{r+■}^{|}))= ■■■ P u / ■■■ + ■■■ P

QsZqrgrsrgle┐ q(u(i_r)┌i_{r+■}^{|}))=

$$\frac{\text{■■■}(\text{■■■}+\text{■■■}\,i\,\text{■■■})\,\text{■■■}\,((\text{■■■}+\text{■■■}\,i\,\text{■■■})\,\text{■■■}-\text{■■■}\,(\text{■■■}+\text{■■■}\,i\,\text{■■■})\,\text{■■■}\,i\,\text{■■■})}{\left(\text{■■■}+\frac{\text{■■■}(\text{■■■}+\text{■■■}\,i\,\text{■■■})\,\text{■■■}}{i\,\text{■■■}}\right)i\,\text{■■■}}$$

QrcYbw QrYrcq┐ Qmjtc $i_{r+■}$ == ■/(■+l)q(u(i_r)┌$i_{r+■}^{|}$))

Ugrf rfc nYpYkcrcpq┐ {{i → ■■■=■■■}}

QrYZgjgrw┐ Amknsrc rfc bcpgtYrgtc md $i_{r+■}$

ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc

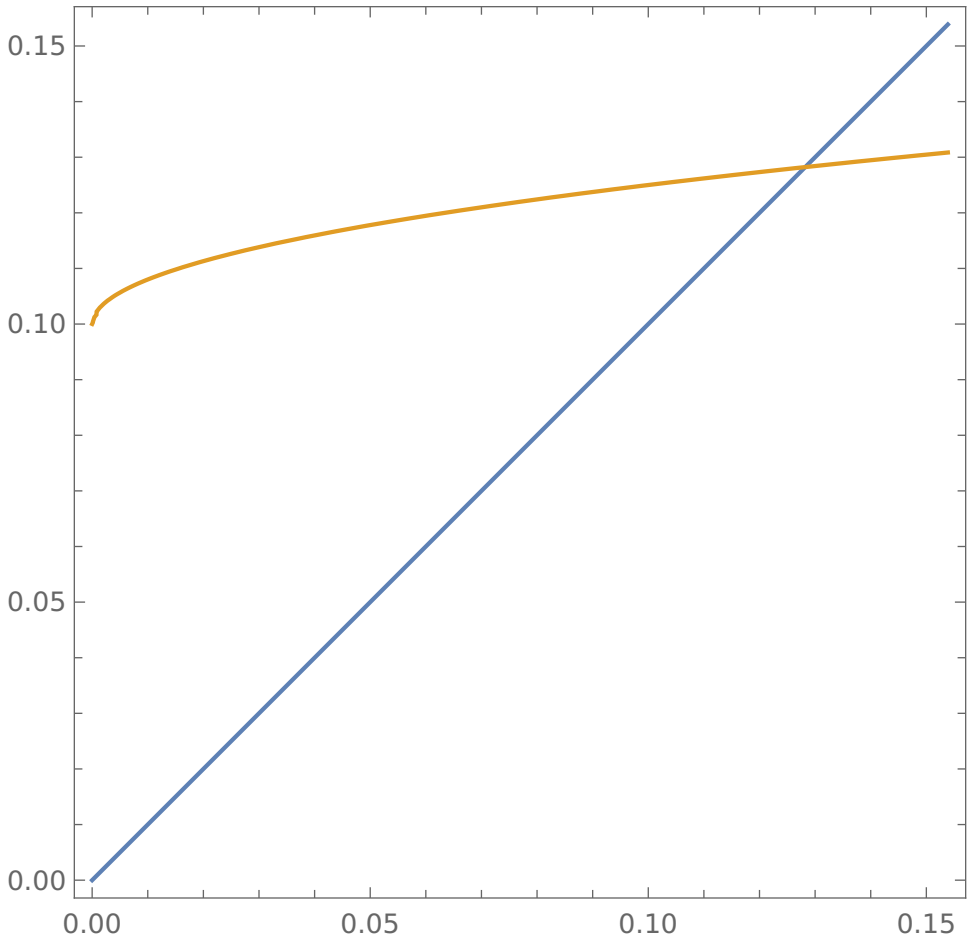
Ugrf rfc nYpYkcrcpq┐

QrcYbw qrYrc ■┌ AYngrYj jctcj┐

■■■=■■■┐ BcpgtYrgtc=■■■:■= QrYZjc

Lsjj

Out[ ]=



In[ ]:= kwnpmepYk[

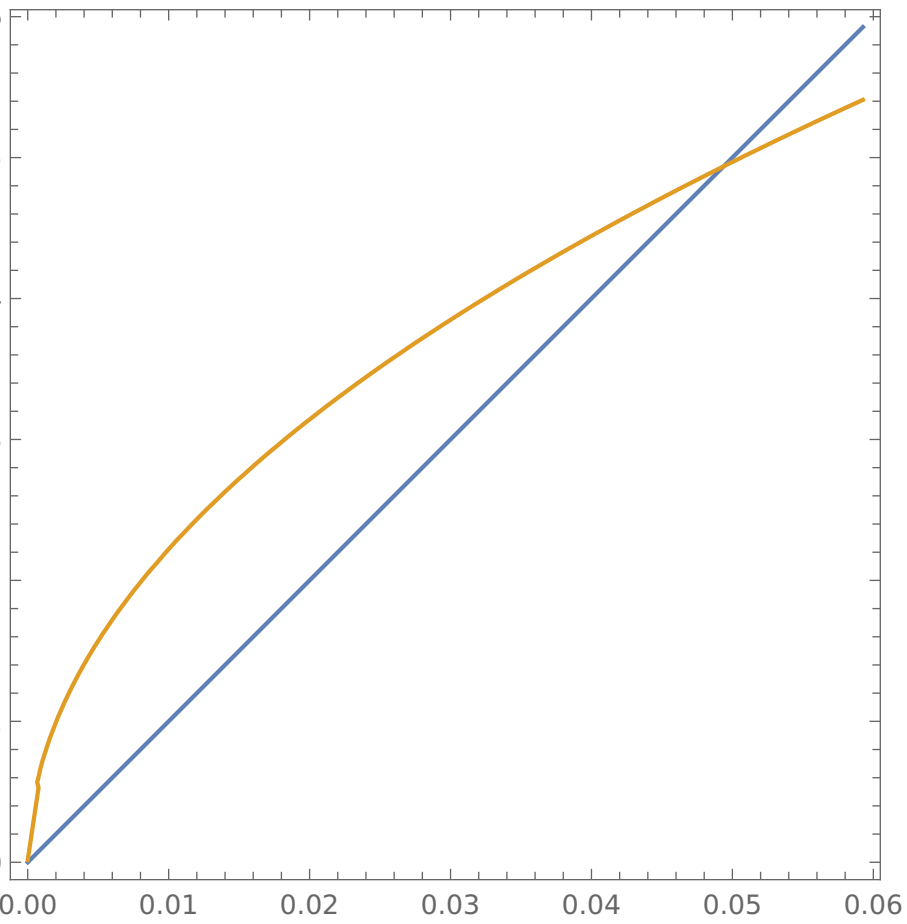
Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

```
Npmbsargml dslargml┐     d(i)=i╋╋..
Srgjgrw dslargml┐        s(a)=Jme[a]
QYtgleq dslargml┐        q(u(i_r)╭i|_r+╋))= ╋╋┈┈┈┈┈┈┈ u
QsZqrgrsrgle┐            q(u(i_r)╭i|_r+╋))= ╋╋┣┣┣┣┣┣ i╋╋..
QrcYbw QrYrcq┐           Qmjtc i_r+╋ == ╋/(╋+l)q(u(i_r)╭i|_r+╋))
   Ugrf rfc nYpYkcrcpq┐ {{i → ╋╋}╭{i → ╋╋┣..‖╋═╋:}}
QrYZgjgrw┐                Amknsrc rfc bcpgtYrgtc md i_r+╋
  ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc
   Ugrf rfc nYpYkcrcpq┐

                    QrcYbw qrYrc ╋
  ╭ AYngrYj jctcj┐ ╋╋┐ BcpgtYrgtc=∞ SlqrYZjc
                    QrcYbw qrYrc ╋
  ╭ AYngrYj jctcj┐ ╋╋┣..‖╋═╋:┐ BcpgtYrgtc=╋╋.. QrYZjc
Lsjj
```

Out[ ]=



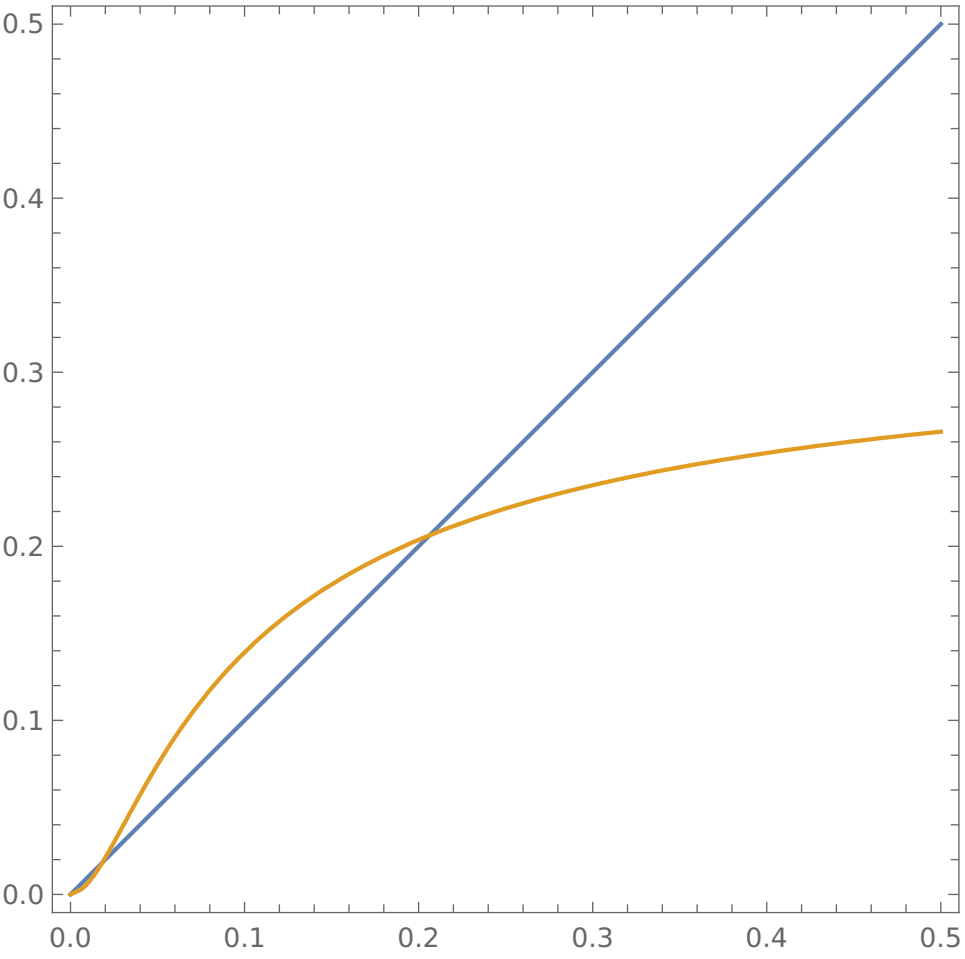
In[32]:= **kwnpmepYk[╋╬╋╮╋╬═╮╋╮–╋╮╋]**

```
Npmbsargml dslargml╮      d(i)= ╋ / (╋╬‖ + ╪╬╪_i)
Srgjgrw dslargml╮         s(a)=╋ √a
QYtgleq dslargml╮         q(u(i_r)╭i'_r+╋))= (╋¦╬ P u) / (╋··╬ + ╋¦╬ P)
QsZqrgrsrgle╮             q(u(i_r
 )╭i'_r+╋))= ╋╬¦ ( ╋/(╋╬‖+╪╬╪_i) − ╪╬╪/((╋╬‖+╪╬╪_i)^╋ i) ) / ( (╋··╬ + ╪╬¦/((╋╬‖+╪╬╪_i)^╋ i╋)) (╋╬‖ + ╪╬╪_i)^╋ i╋ )
QrcYbw QrYrcq╮            Qmjtc i_r+╋ == ╋/(╋+l)q(u(i_r)╭i'_r+╋))
   Ugrf rfc nYpYkcrcpq╮   {{i → ╋╬╋╋═╋··╋}╭{i → ╋╬╋╋¦╋╋··}}
QrYZgjgrw╮               Amknsrc rfc bcpgtYrgtc md i_r+╋
  ugrf pcqncar rm i_r Ylb ctYjsYrc Yr cYaf qrcYbw qrYrc
   Ugrf rfc nYpYkcrcpq╮
```

Out[32]= Zmprcb

In[33]:=
```
AmlrmspNjmr[
 {i╋ == i╭
  i╋ ==
   ╋/ (╋ + ╋)*
    (qYtgleq[u╭ P╭ ╋╭ ╋╬═] ╬
      {u → udslargml[i╭ ╋╬╋╭ -╋]╭
       P → Pdslargml[i╋╭ ╋╬╋╭ -╋]})}╭ {i╭ ╋╭ kYvi}╭
 {i╋╭ ╋╭ kYvi}]
```

Out[33]=



In[34]:=
```
Cvnmpr[-vYknjcq╬nbd -╭ CtYjsYrgmlLmrcZmmi[]]
```