Problem 1: Determinants can be floppy

Problem Statement

Consider a non-singular matrix $\mathbf{A} \in \mathbb{R}^{n\times n}$, given by

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{pmatrix}$$

Let us compute its determinant using the following formula:

$$\det(\mathbf{A}) \coloneqq \sum_{k=1}^{n} (-1)^k a_{1k} \det(\widehat{\mathbf{A}}^{1k})$$

where $\widehat{\mathbf{A}}^{1k} \in \mathbb{R}^{(n-1)\times(n-1)}$ is the matrix obtained by removing the first row and k-th column of $\mathbf{A}$.

a

Suppose for $\mathbf{A} \in \mathbb{R}^{n\times n}$, computing $\det(\mathbf{A})$ using the above formula takes y_n flops. Find the relationship between y_n and y_{n-1}. Assume that negating a number does not require a flop.

b

Use your recurrence relation in (a) to prove that $n! \leq y_n \leq \frac{1}{2}(n+1)!$ for each $n \geq 5$. Here, $z! = z(z-1)(z-2)\ldots 1$ denotes the factorial of z.

c

Estimate how much time your computer takes for one flop. Please submit your code.

d

Solution

a

b

c

Problem 2: It Schur is Nice

Problem Statement

a

b

c

Solution

a

b

c

d

Problem 3: A Plethora of Inversion Methods

Problem Statement

Solution