ECE 530 (Fall 2023) **Instructor:** S. Bose

Homework 2

Assigned: Sep 7 Due: Sep 14

Instructions: *You are encouraged to discuss with your peers. You need to, however, turn in your own solutions. Mention the names of your collaborators. Crediting other classmates does not take away any points from you. Any submissions made after but within one week of the due date will be marked on 50% of the total points. You don't need to submit after that.*

Problem 1. A closer look at Broyden's method. [10+5 points]

The sequence of Jacobian surrogates in Broyden's method is given by

$$\mathbf{J}^k := \mathbf{J}^{k-1} + \frac{1}{\|\Delta\mathbf{x}^k\|_2^2}\left[\Delta\mathbf{F}^k - \mathbf{J}^{k-1}\Delta\mathbf{x}^k\right]\left[\Delta\mathbf{x}^k\right]^\top, \tag{1}$$

where $\Delta\mathbf{F}^k := \mathbf{F}(\mathbf{x}^k) - \mathbf{F}(\mathbf{x}^{k-1})$ and $\Delta\mathbf{x}^k := \mathbf{x}^k - \mathbf{x}^{k-1}$.

(a) For $\mathbf{A} \in \mathbb{R}^{n\times n}$, $\mathbf{u} \in \mathbb{R}^n$, $\mathbf{v} \in \mathbb{R}^n$, the Sherman-Morrison formula states that

$$\left(\mathbf{A} + \mathbf{u}\mathbf{v}^\top\right)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{u}\mathbf{v}^\top\mathbf{A}^{-1}}{1 + \mathbf{v}^\top\mathbf{A}^{-1}\mathbf{u}}.$$

Using the Sherman-Morrison formula and (1), prove that $\mathbf{B}^k := \left[\mathbf{J}^k\right]^{-1}$ satisfies

$$\mathbf{B}^k = \mathbf{B}^{k-1} + \frac{\Delta\mathbf{x}^k - \mathbf{B}^{k-1}\Delta\mathbf{F}^k}{\left[\Delta\mathbf{x}^k\right]^\top\mathbf{B}^{k-1}\Delta\mathbf{F}^k}\left[\Delta\mathbf{x}^k\right]^\top\mathbf{B}^{k-1}.$$

Comment why this recurrence relation in $\mathbf{B}^k$ is useful for the quasi-Newton method.

(b) Recall that Broyden's idea iteratively solves

$$\underset{\mathbf{J}^k\in\mathbb{R}^{n\times n}}{\text{minimize}}\ \|\mathbf{J}^k - \mathbf{J}^{k-1}\|_F,\ \text{subject to } \mathbf{J}^k\Delta\mathbf{x}^k = \Delta\mathbf{F}^k, \tag{2}$$

to define the sequence of Jacobian surrogates $\left\{\mathbf{J}^k\right\}$. Broyden suggested another idea that directly defines a sequence $\left\{\mathbf{C}^k\right\}$ of surrogates for the Jacobian inverses by iteratively solving

$$\underset{\mathbf{C}^k\in\mathbb{R}^{n\times n}}{\text{minimize}}\ \|\mathbf{C}^k - \mathbf{C}^{k-1}\|_F,\ \text{subject to } \mathbf{C}^k\Delta\mathbf{F}^k = \Delta\mathbf{x}^k. \tag{3}$$

This is the so-called *Broyden's bad method*. Compute $\mathbf{C}^k$ in terms of $\mathbf{C}^{k-1}, \Delta\mathbf{x}^k, \Delta\mathbf{F}^k$. Comment if you think Broyden's bad method is really that bad!
Hint: Exploit the similarity between (2) and (3).

Problem 2. Solving 'em power flows. **[5+5 points]**

This problem explores the use of Newton Rhapson (NR) method to solve the power flow problem. The file `PowerFlowNR.m` runs the NR iteration with a flat start for the three-bus example discussed in class. Refer to the notes for the details of the example. The file `computeJacobian_HW.m` computes the Jacobian matrix. *Copy these files to the same folder!* For this exercise, you can use (and modify) the provided Matlab files or you can write your own code from scratch. Use a tolerance level of $\varepsilon = 10^{-10}$ for the terminating condition. Make sure that the code provided converges to

$$\begin{pmatrix} \theta_2^* \\ \theta_3^* \\ v_3^* \end{pmatrix} = \begin{pmatrix} -0.0101 \\ -0.0635 \\ 0.9816 \end{pmatrix}.$$

(a) Discuss what you think would constitute a *physically meaningful* solution to the power flow equations. Qualitatively discuss how you would enforce a solver to produce such a meaningful solution.

(b) By varying the starting point for the actual NR iteration, explore if the solution provided above is the unique solution to the power flow problem for this example. If not, comment on whether this other solution you obtained is physically meaningful.