

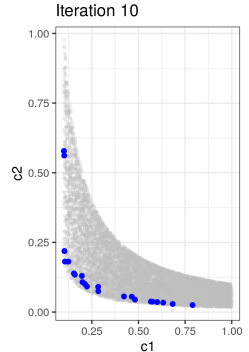
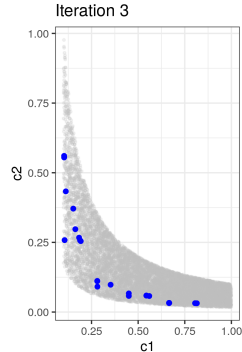
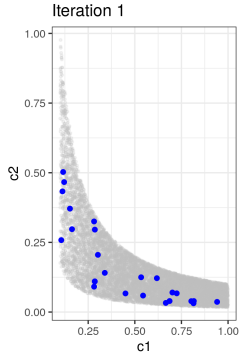
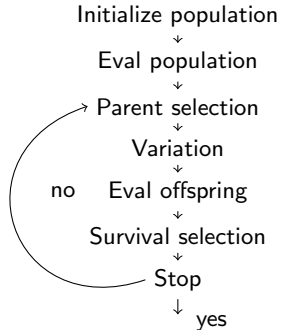
Multi-criteria Optimization

Evolutionary Approaches

Bernd Bischl Frank Hutter Lars Kotthoff
Marius Lindauer Joaquin Vanschoren

A-posteriori methods and evolutionary algorithms I

Evolutionary multi-objective algorithms (EMOAs) evolve a diverse population over time to approximate the Pareto front.



Algorithm 1 Basic EA template loop

- 1: Init and eval population $\mathcal{P}_0 \subset \Lambda$ with $|\mathcal{P}| = \mu$
 - 2: $t \leftarrow 0$
 - 3: **repeat**
 - 4: Select parents and generate offspring \mathcal{Q}_t with $|\mathcal{Q}_t| = \lambda$
 - 5: Select μ survivors \mathcal{P}_{t+1}
 - 6: $t \leftarrow t + 1$
 - 7: **until** Stop criterion fulfilled
-

- Note that (as in the EA lecture unit) we are using somewhat non-standard notation here.
- Nearly all steps in the above template work also for EMOAs but both parent and survival selection are now less obvious. How do we rank under multiple objectives?

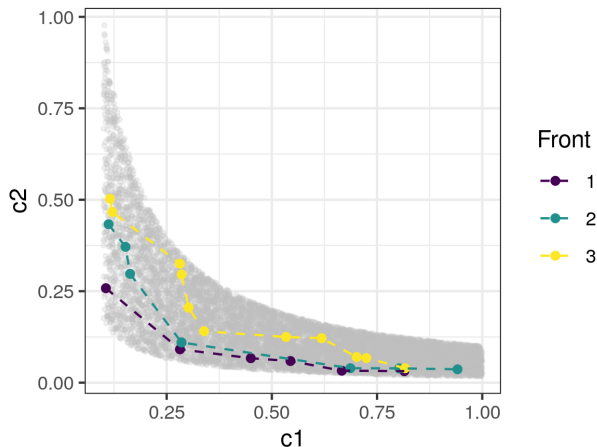
The **non-dominated sorting genetic algorithm (NSGA-II)** was published by [Dep et al. 2002].

- Follows a $(\mu + \lambda)$ strategy.
- All previously discussed variation strategies can be used; the original paper uses tournament selection, polynomial mutation and simulated binary crossover.
- Parent and survival selection rank candidates by
 - ① **Non-dominated sorting** as main criterion
 - ② **Crowding distance assignment** as tie breaker

NSGA-II: non-dominated sorting I

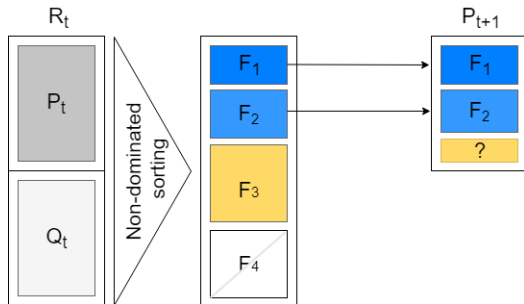
NDS partitions an objective space set into fronts $\mathcal{F}_1 \prec \mathcal{F}_2 \prec \mathcal{F}_3 \prec \dots$

- \mathcal{F}_1 is non-dominated, each $\lambda \in \mathcal{F}_2$ is dominated, but only by points in \mathcal{F}_1 , each $\lambda \in \mathcal{F}_3$ is dominated, but only by points in \mathcal{F}_1 and \mathcal{F}_2 , and so on.
- We can easily compute the partitioning by computing all non-dominated points \mathcal{F}_1 , removing them, then computing the next layer of non-dominated points \mathcal{F}_2 , and so on.



NSGA-II: non-dominated sorting II

How does survival selection now work? We fill μ places one by one with $\mathcal{F}_1, \mathcal{F}_2, \dots$ until a front can no longer **fully** survive (here: \mathcal{F}_3).

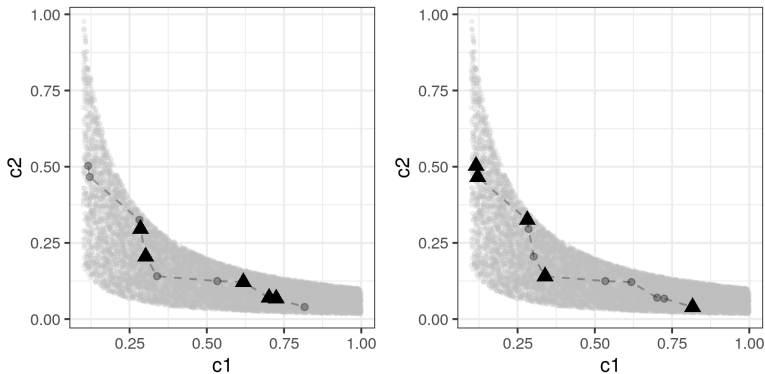


Which individuals survive from \mathcal{F}_3 ? \rightarrow **crowding sort**

NB: the same principle to rank individuals is applied in tournament selection in parent selection.

NSGA-II: crowding distance I

Idea: Add *good* representatives of front \mathcal{F}_3 , define this as points of "low density" in c-space.



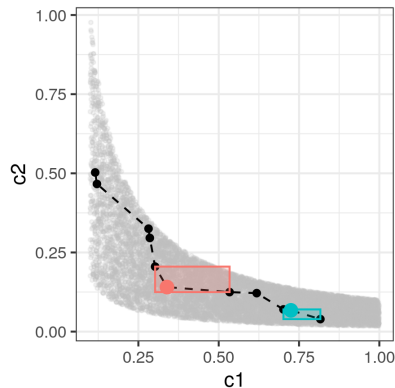
Left: Not good, points very close together. Right: better.

NSGA-II: crowding distance II

For each objective c_j

- Sort points by c_j
- Normalize scores to $[0,1]$
- Assign border points (which have score 0 or 1) a CD of ∞ (they should always be selected, if possible)
- Each point gets a distance score, which is the distance between its 2 next-neighbors w.r.t. the sorting of c_j

For each point, all of its m distance scores are summed up (or averaged) and points are ranked w.r.t. to this overall score.



Red: Point with high CD. Blue: Low CD.

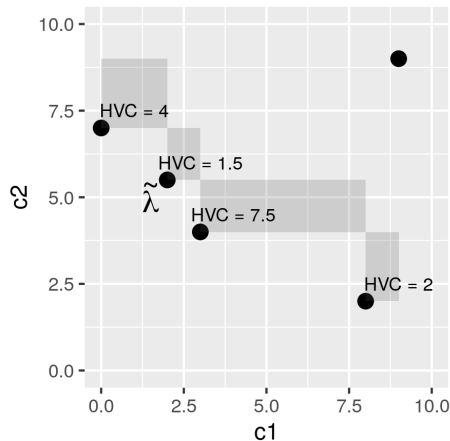
Selection criteria: contribution to the hypervolume I

SMS-EMOA

(S-Metric-Selection-EMOA) [Beume et al. 2007] is a $(\mu + 1)$ EMOA and evaluates fitness of an individual $\lambda \in \mathcal{P} \subset \Lambda$ based on its contribution to the dominated HV:

$$\Delta s(\lambda, \mathcal{P}) = S(\mathcal{P}, R) - S(\mathcal{P} \setminus \{\lambda\}, R).$$

- Dark rectangles: HV contribution of dots.
- Grey point: reference point.
- The HVC contribution is the volume of space that is dominated only by λ , and nothing else.
- $\tilde{\lambda}$ has lowest S-metric contribution.



Algorithm 2 SMS-EMOA

- 1: Generate start population \mathcal{P}_0 of size μ
 - 2: $t \leftarrow 0$
 - 3: **repeat**
 - 4: Generate **one** individual \mathbf{q} by recombination and mutation of \mathcal{P}_t
 - 5: $\{\mathcal{F}_1, \dots, \mathcal{F}_k\} \leftarrow \text{NDS}(\mathcal{P}_t \cup \{\mathbf{q}\})$
 - 6: $\tilde{\boldsymbol{\lambda}} \leftarrow \operatorname{argmin}_{\boldsymbol{\lambda} \in \mathcal{F}_k} \Delta s(\boldsymbol{\lambda}, \mathcal{F}_k)$
 - 7: $\mathcal{P}_{t+1} \leftarrow (\mathcal{P}_t \cup \{\mathbf{q}\}) \setminus \{\tilde{\boldsymbol{\lambda}}\}$
 - 8: $t \leftarrow t + 1$
 - 9: **until** Termination criterion fulfilled
-

- L5: the set of temporary $(\mu + 1)$ individuals is partitioned by NDS into k fronts $\mathcal{F}_1, \dots, \mathcal{F}_k$.
- L6-7: In last front, find $\tilde{\boldsymbol{\lambda}} \in \mathcal{F}_k$ with smallest HV contribution - and kill it.
- Fitness of an individual is mainly the rank of its front and HV contribution as tie-breaker.