Some but not all of the questions will be graded. Usually model answers will be provided for all of them.

1. Curves

Question 1.1. Let C be a projective non-singular curve, D a divisor on C of degree $d > 0$ such that $\mathcal{L}(D)$ is base point-free of dimension r. Let $\varphi : C \to \mathbb{P}^r$ be the morphism associated to D

(a) Projecting from a point $P \notin C$ induces a morphism $\varphi_P : C \to \mathbb{P}^{r-1}$. Show that this morphism is associated to subseries of $\mathcal{L}(D)$

(b) Projecting from a point $P \in C$ induces a rational map $\varphi_P : C - \{P\} \to \mathbb{P}^{r-1}$. Show that it extends to a morphism $\overline{\varphi_P} : C \to= \mathbb{P}^{r-1}$. Identify a linear series that this morphism is associated to.

Question 1.2. (a) Show that any two effective divisor of degree d in $\mathbb{P}^1$ are linearly equivalent.

(b) Let C be a projective non-singular curve, D a divisor on C of degree $d > 0$ and such that $l(D) = \dim \mathcal{L}(D) = d + 1$. Show that $C = \mathbb{P}^1$ Hint consider the morphism $C \to \mathbb{P}^d$ associated to D. Reduce the question to the case $d = 1$ by projecting from points on C.

(c) Show that if C is a projective non-singular curve that is not isomorphic to $\mathbb{P}^1$, then for any $d > 1$ there are effective divisors of degree d that are not linearly equivalent.

Question 1.3. Let C be the twisted cubic parameterized by (s^3, s^2t, st^2, t^3).

(a) Show that the projection of the curve from the point $(1, 0, 0, 0)$ onto the plane $X_0 = 0$ is a conic.

(b) Show that the projection from the point $(0, 1, 0, 0)$ onto the plane $X_1 = 0$ is a cuspidal cubic