CS2109s - Tutorial 2

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Sep 6, 2023

Annoucements

Important admin

1. Attendance Marking will be done at the end of the lesson via the QR code.
2. Random checks will be performed.
3. Slight changes to the bonus rubrics:
    3.1 To be eligible for the bonus, you have to fulfill every other segment.
    3.2 This change will be effective from this tutorial's bonus. (last week is not affected)
4. Survey Results
5. If you are going to submit late for PS, it would help if you can inform me early!

Tidbits from tutorials

1. Error in Trace in Q4a – Fixed in the uploaded tutorial slides.
    - The error is in Priority Queue in my code.
2. The characteristic of Semi-Dynamic is the agent being time constraint or having some constraint that is not consistent. Due to this then it would need to be factored into the performance metric. One example was the Chess with time constraint in class. Another possible is a part-picking robot that has time or speed constraint in some states (ie. for safety).

Problem Set 1

1. There are many who confused Left/Right for the M&C question; be really careful.
2. Environment formulation, actions described in prose, listing it is recommended.
3. Environment formulation, thinking in terms of implementation.
4. Argument is unclear - learn how to articulate your argument
    4.1 Moisture in the air > air cools on cold surface > water vapour condenses to water > gravity causes water to drip

Question 1 [G]

In case you havn't: https://www.google.com/logos/2010/pacman10-i.html

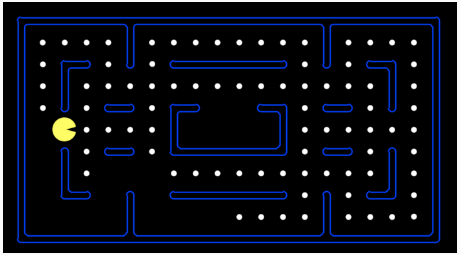
Figure 1: Pac-Man Simplified

- **State representation**: Position of Pac-Man and the positions of the uneaten pellets
- **Initial State**: Filled grid entirely with pellets
- **Goal State**: No pellets left in the grid
- **Action**: Moving up/down/left/right
- **Transition Model**: Updating the position of Pac-Man and eating pellet (if applicable)
- **Cost function**: 1 for each action taken

Give a non-trivial (whats a trivial heuristic?) admissible heuristic for this problem.

Recap

- What is the intuition behind A* Search?
- What is an admissible heuristic?
- What is a consistent heuristic?

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- What is a consistent heuristic?

Summary

- A* Search: $f(N) = g(N) + h(N)$
- Admissible heuristic: $h(N) \leq h^*(N)$
- Consistent heuristic: $h(N) \leq c(N, N') + h(N')$

Where,

- $f(.)$ - Evaluation Function.
- $g(.)$ - Cost Function from start to current node.
- $h(.)$ - Estimated cost from current node to goal.
- $c(N, N')$ - Action cost from N to N'.

Answer

- Trival
  - h_1 : Number of pellets left at any point in time.
- Non-Trival
  - h_2 : The Maximum among all Manhattan distances from each remaining pellet to the current position of Pac-Man.
    - Admissible, $\max_{p \in P}$ path over all pellets ensure that $\leq h^*$. ie. furthest
  - h_3 : The average over all Euclidean distances from each remaining pellet to the current position of Pac-Man.
    - Admissible, $h_3 \leq h_2 \leq h^*$

Or.. Relax the problem - Pac-Man can pass through walls:

- With less restrictions, we can take short cuts (ie. adding edges between states)
- Optimal solution to the original is a solution to the relaxed, but may not be optimal
- We can find the optimal for relaxed problem on the path by taking short cuts
- Cost of optimal to the relaxed is always lower than the original's
- making it an admissible heuristic for the original problem.

Question 2

Given a graph $G = (V, E)$ where,

- each node v_n having coordinates (x_n, y_n),
- each edge (v_i, v_j) having weight equals to the distance between v_i and v_j, and
- a unique goal node v_g with coordinates (x_g, y_g),

$$h_{SLD}(n) = \sqrt{(x_n - x_g)^2 + (y_n - y_g)^2}$$
$$h_1(n) = max\{|x_n - x_g|, |y_n - y_g|\}$$
$$h_2(n) = |x_n - x_g| + |y_n - y_g|$$

a. Is $h_1(n)$ an admissible heuristic?
b. Is $h_2(n)$ an admissible heuristic?
c. Which heuristic function would you choose for A* search? And why?

Recap

- What is the intuition behind A* Search?
- How to decide admissibility?
- How is a heuristic *useful*, how to decide which to use?
- What is the h^* optimal heuristic?
- What is a potential downfall of choosing an optimal heuristic?
- Admissible heuristic: $h(N) \leq h^*(N)$

Answer

a. Admissible, proof:
   - h_{SLD} is admissible - $h^*(n) \geq h_{SLD}(n) = \sqrt{(x_n - x_g)^2 + (y_n - y_g)^2}$
     - $\sqrt{(x_n - x_g)^2 + (y_n - y_g)^2} \geq \sqrt{(x_n - x_g)^2} = |x_n - x_g|$
     - $\sqrt{(x_n - x_g)^2 + (y_n - y_g)^2} \geq \sqrt{(y_n - y_g)^2} = |y_n - y_g|$
   - $h^*(n) \geq h_{SLD}(n) = \sqrt{(x_n - x_g)^2 + (y_n - y_g)^2} \geq h_1(n) = max\{|x_n - x_g|, |y_n - y_g|\}$

b. Not admissible, counter example:
   - Consider a graph where v_s has coordinates $(0, 0)$ and v_g has coordinates $(1, 1)$
   - $h^*(v_s) = \sqrt{2} < h_2(v_s) = 2$.

c. $h_{SLD}(n)$:
   - $h_2(n)$ is not admissible, so we may not get an optimal solution if we use it
   - $h_{SLD}(n) \geq h_1(n)$ so $h_{SLD}(n)$ dominates $h_1(n)$
   - will incur less search cost (on average) if we use $h_{SLD}(n)$.
   - ***Intuition***: *Choose the heuristic that is closest to* h^*

Question 3 [G]

a. Given that a heuristic h is such that $h(G) = 0$, where G is any goal state, prove that if h is consistent, then it must be admissible.
b. Give an example of an admissible heurstic function that is not consistent.

Recap

- Admissible heuristic: $h(N) \leq h^*(N)$
- Consistent heuristic: $h(N) \leq c(N, N') + h(N')$

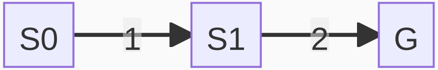
Answer 3a

Intuition: Show on the number of action required to reach the goal from n to goal G.

Let the no. of actions k to be required to reach from n_k to G on optimal path $P_{n_k \to G}$.
Node n_k is k steps away from G, ie. $k = 3 \implies P_{n_3 \to G} : n_3 \to n_2 \to n_1 \to G$.

- **Base**: 1 action; i.e. node n_1 is one step away from G.
  - Since consistent, $h(n_1) \leq c(n_1, G) + h(G)$ and $h(G) = 0$
  - $\implies h(n_1) \leq c(n_1, G) = h^*(n_1) \implies$ admissible.
- **Inductive**: Assume for $k-1$ actions, path $P_{n_{k-1} \to G}, h(n_{k-1}) \leq h^*(n_{k-1})$.
  - Since consistent, $h(n_k) \leq c(n_k, n_{k-1}) + h(n_{k-1})$
  - $\implies h(n_k) \leq c(n_k, n_{k-1}) + h^*(n_{k-1}) = h^*(n_k) \implies$ admissible.

Answer 3b

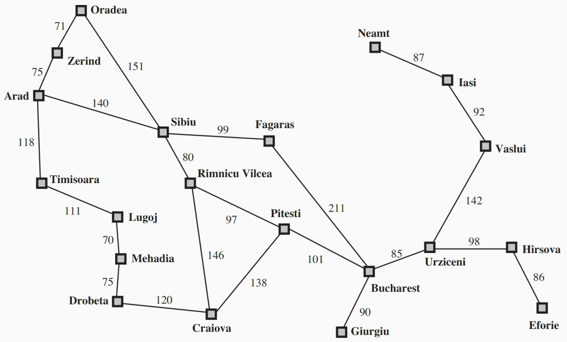

$$\begin{tikzcd} S0 \arrow[r, "1"] & S1 \arrow[r, "2"] & G \end{tikzcd}$$

s	$S0$	$S1$	G
$h(s)$	3	1	0
$h^*(s)$	3	2	0
$h(s) \leq h^*(s)$	T	T	T

Heuristic h is

- Admissible - $\forall s : h(s) \leq h^*(s)$
- Not consistent - $3 = h(s_0) > c(s_0, s_1) + h(s_1) = 2$

Question 4 [G]



Values of h_{SLD} - straight-line distances to Bucharest

Arad	366
Bucharest	0
Craiova	160
Drobeta	242
Eforie	161
Fagaras	176
Giurgiu	77
Hirsova	151
Iasi	226
Lugoj	244
Mehadia	241
Neamt	234
Oradea	380
Pitesti	100
Rimnicu Vilcea	193
Sibiu	253
Timisoara	329
Urziceni	80
Vaslui	199
Zerind	374

Figure 2: Graph of Romania.

Considering, $h(n) = |h_{SLD}(\text{Craiov}) - h_{SLD}(n)|$

a. Trace A* search (`TREE-SEARCH`) by expanding the search trees, showing (g, h, f)

b. Prove that $h(n)$ is an admissible heuristic.

Answer 4a

The tuple in each node denotes (g, h, f) are shown along the trace:

```
Fagaras (0, 16, 16)
|-- Bucharest (211, 160, 371)
+-- Sibiu (99, 93, 192)
    |-- Arad (239, 206, 445)
    |-- Fagaras (198, 16, 214)
    |   |-- Bucharest (409, 160, 569)
    |   +-- Sibiu (297, 93, 390)
    |-- Oradea (250, 220, 470)
    +-- Rimnicu_Vilcea (179, 33, 212)
        |-- Craiova (325, 0, 325)
        |-- Pitesti (276, 60, 336)
        +-- Sibiu (259, 93, 352)
```

Answer 4b

Let D be the straight-line distance between n and Craiova:

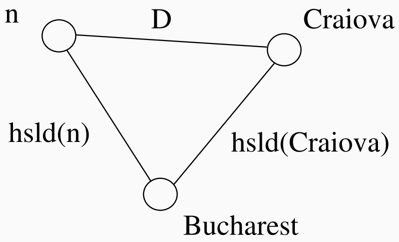


Figure 3: Triangle Inequality Example

- By the Triangle Inequality, D must be at least as much as the difference of the 2 other sides, ie. $D \geq |h_{SLD}(Craiova) - h_{SLD}(n)| = h(n)$.
- But we also know that $h^*(n) \geq D \geq h(n)$, so $h^*(n) \geq h(n)$, and
- $h(n)$ is admissible.

Note: The definition of triangle inequality is, sum of the lengths of any two sides must be greater than or equal to the the length of the third side.

Question 5

Using the example graph,

a. Show that A* using graph search returns a non-optimal solution path from start S to G when using an admissible but inconsistent $h(n)$.
b. Then, show that tree search will return the optimal solution with the same heuristic.

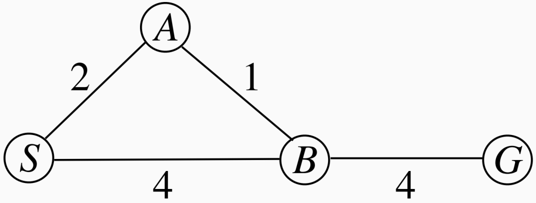


Figure 4: Example graph; We assume that $h(G) = 0$.

Table 2: Inconsistent heuristic; $h(S) > h(B) + 4$

$.$	S	B	A	G
$h(.)$	7	0	3	0
h^*	7	4	5	0

Answer 5a

Intuition: We may miss out on some shorter paths that we mistakenly think are long after reaching the node as the heuristic function violates the triangle inequality.

The tuple in each node denotes (g, h, f) are shown along the trace:

```
S (0, 7, 7)
|--  A (2, 3, 5)
|   |-- [X] B (3, 0, 3)
|   +-- [X] S (4, 7, 11)
+--  B (4, 0, 4)
    |--  A (5, 3, 8)
    |   |-- [X] B (6, 0, 6)
    |   +-- [X] S (7, 7, 14)
    |--  G (8, 0, 8)
    +-- [X] S (8, 7, 15)
```

Frontier

```
S(7-)
B(4-S) A(5-S)
A(5-S) A(8-SB) G(8-SB)
A(8-SB) G(8-SB)
G(8-SB)
```

Answer 5b

```
S (0, 7, 7)
|-- A (2, 3, 5)
|   |-- B (3, 0, 3)
|   |   |-- A (4, 3, 7)
|   |   |   |-- B (5, 0, 5)
|   |   |   |   |-- A (6, 3, 9)
|   |   |   |   |-- G (9, 0, 9)
|   |   |   |   +-- S (9, 7, 16)
|   |   |   +-- S (6, 7, 13)
|   |   |-- G (7, 0, 7)
|   |   +-- S (7, 7, 14)
|   +-- S (4, 7, 11)
+-- B (4, 0, 4)
    |-- A (5, 3, 8)
    |-- G (8, 0, 8)
```

Frontier

```
S(7-)
B(4-S) A(5-S)
A(5-S) A(8-SB) G(8-SB) S(15-SB)
B(3-SA) A(8-SB) G(8-SB) S(11-SA) S(15-SB)
A(7-SAB) G(7-SAB) A(8-SB) G(8-SB) S(11-SA) S(14-SAB) S(15-SB)
B(5-SABA) G(7-SAB) A(8-SB) G(8-SB) S(11-SA) S(13-SABA) S(14-SAB) S(15-SB)
G(7-SAB) A(8-SB) G(8-SB) A(9-SABAB) G(9-SABAB) S(11-SA) S(13-SABA) S(14-SA
    S(15-SB) S(16-SABAB)
```

Question 6

Assume no negative cycles.

a. Would A* work with negative edge weights? If yes, prove it; otherwise, provide a counterexample.
b. What if there are negative cycles?

Answer 6

Recall that in the proof of A, *negative weights do not affect the optimality of A* as long as there are no negative cycles. Therefore, the same proof applies.

Bonus Question

To help you further your understanding, not compulsory; Work for Snack/EXP!

Tasks

1. Fork the repository https://github.com/eric-vader/CS2109s-2324s1-bonus
2. We will be first solving Question 5 using code, `astar(graph, inital_node, goal_test, heuristics, is_tree, is_update)` that returns the **best path** found:
   2.1 Able to solve 5a via `is_tree=False,is_update=False`
   2.2 Able to solve 5b via `is_tree=True,is_update=False`
3. Some code have been implemented for you; You can reuse the PQ or use another.
4. You should print the frontier and explored at the beginning of the loop.

To claim your snack & EXP, show me your forked repository and your code's output.

Buddy Attendance Taking

Take Attendance for your buddy: https://forms.gle/Ckkq639TNwWEx3NT6

1. Random checks will be conducted - `python ../checks.py TG0`

Figure 5: Buddy Attendance