CS3243 Midterm

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Annoucements

1. Assignment 4 scores are now on Gradebook, please check.
2. Please check turnitin for feedback.

Good thinking question to further understand AC-3 algorithm

From student 1

If you did change the domains of the variables you need to recheck some constraints.

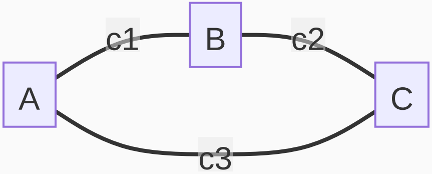


Figure 1: Fully connected situation, may need to check again.

From student 2

Can we consider bidirectional filtering instead of unidirectional filtering?

Previously from T05, Q3

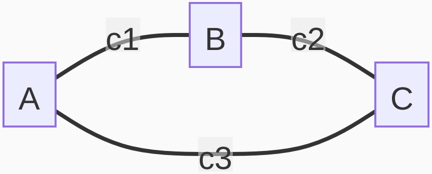


Figure 2: T05,Q3 constraint graph

Question 3a

$$A = \{1, 2\}, B = \{2, 4\}, C = \{1, 2, 3\}$$

Question 3b

$$(A, B, C) = (1, 2, 1)$$

Question 1 [Normal]

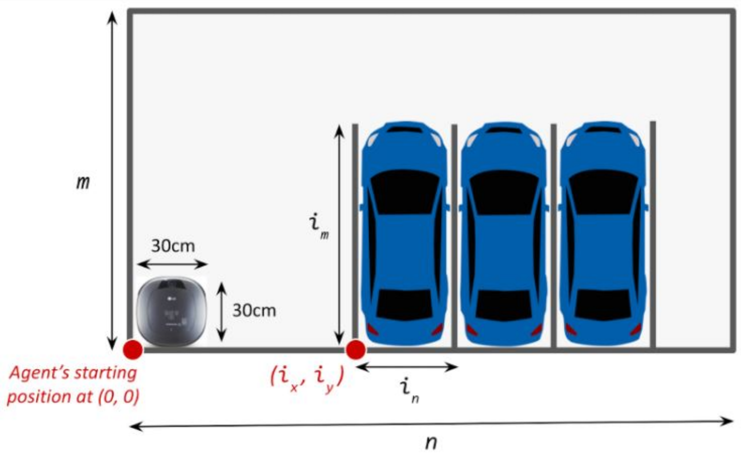


Figure 3: Floor cleaning illustration

Agent can move: Up, Down, Left and Right

Agent size: 30×30

Agent start: bottom left

Parking is orthogonal

Environment:

- Observable
- Single Agent
- Deterministic
- Static
- Continuous
- Sequential

Recap: How to forumulate?

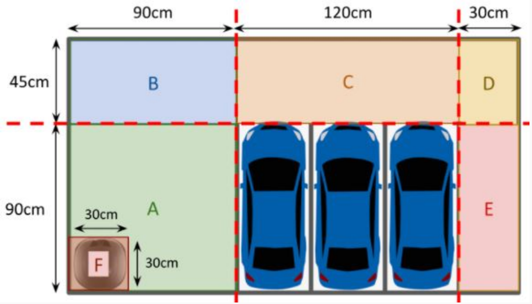


Figure 4: Floor cleaning solution

Answer

- **State representation**: Discretize into cells using GCD - A, B, C, D, E, F. Then each cells has coordinate and state - clean, not clean and occupied.
- **Initial state**: Fill cells with not clean or occupied from map.
- **Actions**: Up, Down, Left and Right upto cell size, Clean.
- **Transition Model**: Update position, Update State.
- **Step cost**: 1
- **Goal test**: If all cells are clean or occupied.

Question 2 [Normal/Hard]

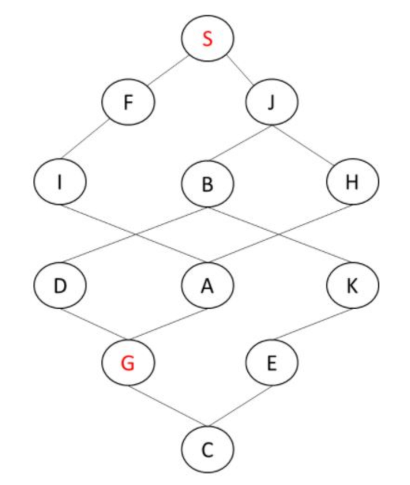


Figure 5: Q2 Graph

Question 2a

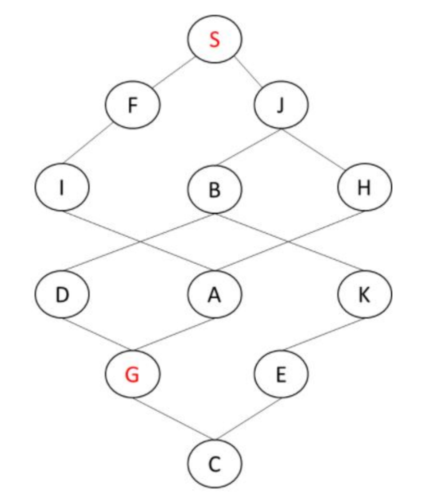
Trace the following algorithms:

i. BFS with limited-graph-based implementation and early goal test.
ii. BFS with limited-graph-based implementation and late goal test.
iii. DFS with limited-graph-based implementation

Question 2b

i. Trace the A* Search with limited-graph-based implementation
ii. Using A* search with limited-graph-based implementation

Question is asking for the order for the goal test performed - ‘in terms of when the goal test is performed’; if your sequence is on something else, it will be incorrect.



Answer 2a

i. Different answers due to goal check and reached check swapping in limited graph search v2:
   - S-F-J-I-B-H-A-D-K-A-G
   - S-F-J-I-B-H-A-D-K-G

ii. Same situation as above:
   - S-F-J-I-B-H-A-D-K-A-G
   - S-F-J-I-B-H-A-D-K-G

iii. S-J-H-A-I-F-G **[Trap: Graph is undirected!]**

Answer 2b

i. S-J-B-F-I-A-D-G

ii. S-F-I-A-G or S-J-B-D-G

Recap: Review all the relevant lecture notes.

Question 3 [Hard]

Let C^* be the cost of an optimal path. Assuming that every action costs at least some small positive constant ϵ. Explain why the time and space complexity of the UCS algorithm is given as: $O(b^{1+\lfloor C^*/\epsilon \rfloor})$.

Recap

- What is UCS?
- What is Geometric Series?
- How to compute limits?

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Recap

- What is UCS?
- What is Geometric Series?
- How to compute limits?

Answer
Assume at each state has actions b, at $k\epsilon$

1. Initial State: 1 node
2. At level $k = 1$: b nodes $\cdots$
3. At level $k = \lfloor C^*/\epsilon \rfloor$: $b^{\lfloor C^*/\epsilon \rfloor}$ nodes

$k = \lfloor C^*/\epsilon \rfloor + 1$ is generated for goal test.
Using geometric series sum: $a(\frac{1-r^{n+1}}{1-r})$
Then $a = 1, r = b, n = \lfloor C^*/\epsilon \rfloor + 1$:

$$\lim_{b \to \infty} \frac{1 - b^{\lfloor C^*/\epsilon \rfloor + 2}}{1 - b} \implies O(b^{1+\lfloor C^*/\epsilon \rfloor})$$

Question 4 [Normal]

Suppose you have two admissible heuristics, h_1 and h_2. You are also given an inadmissible heuristic h_3. You decide to create the following new heuristic functions.

Recap

- What is admissible?
- What are the 3 possibilities?
- How to prove admissible?
- How to prove inadmissible?

Question 4 - h4

$$h_4(n) = \min(\max(2h_1(n), h_3(n)), h_2(n))$$

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Answer

- $\max(2h_1(n), h_3(n))$ inadmissible
- $\min(\text{inadmissible}, \text{admissible})$ is admissible

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Answer

- $\min(h_2(n), h_3(n))$ is admissible
- max(admissible, admissible) is admissible

Question 4 - h6

$$h_6(n) = \min(h_3(n), \max(h_1(n), 1.1h_2(n)))$$

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Answer
Indeterminant, see 2 cases:

1. $h_1 = h_2 = 0, h_3 = \infty \implies h_6$ admissible
2. $h_1 = h_2 = h^*, h_3 = \infty \implies h_6$ inadmissible

Question 4 - h7

$$h_7(n) = \max((h_1(n) + h_2(n))/2, h_3(n))$$

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Question 4 - h7

$$h_7(n) = \max((h_1(n) + h_2(n))/2, h_3(n))$$

Answer

- $(h_1(n) + h_2(n))/2$ is admissible
- $\max(\text{admissible}, \text{inadmissible})$ is inadmissible

Question 5 [Hard]

Given a set of admissible heuristics: $h_1, h_2, h_3, \cdots, h_n$, define a new heuristic g where g is a function of the set of the heuristics that will result in A* expanding a minimal number of nodes while still guaranteeing admissibility.

Question 5 [Hard]

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Answer

- The heuristic that results in A* expanding minimal number of nodes is h^*.
- We are given that $\forall_s : h_i(s) \leq h^*(s)$

Idea is to get as close as possible to h^*; How can we get close given h_i?

- Take the max, consequently $g(s) = \max_{i \in [1,n]}(h_i(s)) \leq h^*(s)$

Draw a picture to visualize this.

Question 6 [Easy/Very Hard]

Wordle Game, we pirate the game to `UWordle` (unlimited guesses), modelled as follows:

- **State Space**: Each state is represented by the set of candidate words C_i.
- **Initial State**: Entire dictionary D.
- **Goal State**: The size of candidate words $|C| = 1$.
- **Action**: Pick a word in D.
- **Transition Model**: From the observation (sequence of colored tiles) after the action, we exclude all words in C_i that violate the new observed constraints to create C_{i+1}. Cost of every guess is 1.

P.S. Set by yours truely.

Recap

Review Tutorial 1.

Question 6.2a

Determine the environment characteristics of the above problem:

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Answer

Environment Characteristic	UWordle
Fully / Partially Observable	Partially Observable
Single / Multi-Agent	Single agent
Deterministic / Stochastic	Deterministic
Episodic / Sequential	Sequential
Static / Dynamic	Static
Discrete / Continuous	Discrete
Known / Unknown	Known

Very important to observe *Partially Observable* and *Known*. Known - whether the agent knows about the rules of the game.

Question 6.2b

Heuristic h_1 is such that for any s_i, $h_1(s_i) = k$, where k is a constant value. State and prove that, only for your chosen value of k, h_1 is admissible.

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Answer
$k = 0$, proof:

- $h_1(s_i) = k$
- $h^*(g) = 0, h^*(s_i) \geq 0$ [$\because$ goal state has the smallest value]
- $h_1(s_i) \leq h^*(s_i)$ [$\because$ admissibility]
- $k \leq h^*(s_i)$

The only valid k is 0.

Question 6.2c

1. Propose another distinctly different but useful[1] heuristic h_2 that dominates h_1:
    1.1 Explicitly state h_2,
    1.2 explain how h_2 can be computed, and
    1.3 prove dominance.
2. Discuss how h_2 is useful to a local search algorithm.
3. Admissibility.
    3.1 If your proposed heuristic is admissible, show a proof.
    3.2 If your proposed heuristic is not admissible, proof and justify why another admissible heuristic cannot be found.

Recap

- Environment is Partially Observable, unknowns: C_i, w_{target}, next observations
- What is a dominating relationship?

[1]Similar to Tutorial 3, Q1

Answer 6.2c.1 [Using Bayesian Statistics, Unknown Agent]

We can compute a function $f(\ell, C_i)$ which maps the letter to the number of times the letter ℓ is observed in each C_i. e.g. For $C_i = \{\text{hello}, \text{world}\}$, then f is given as follows:

ℓ	h	e	l	o	w	r	d
$f(\ell, C_i)$	1	1	3	2	1	1	1

Then $h_2(s_i)$ is defined over the occurance of the letters in word w over the $f(\ell, C_{i-1})$:

$$h_2(s_i) = \prod_{\ell \in w} \frac{f(\ell, C_{i-1})}{5 \times |C_{i-1}|}$$

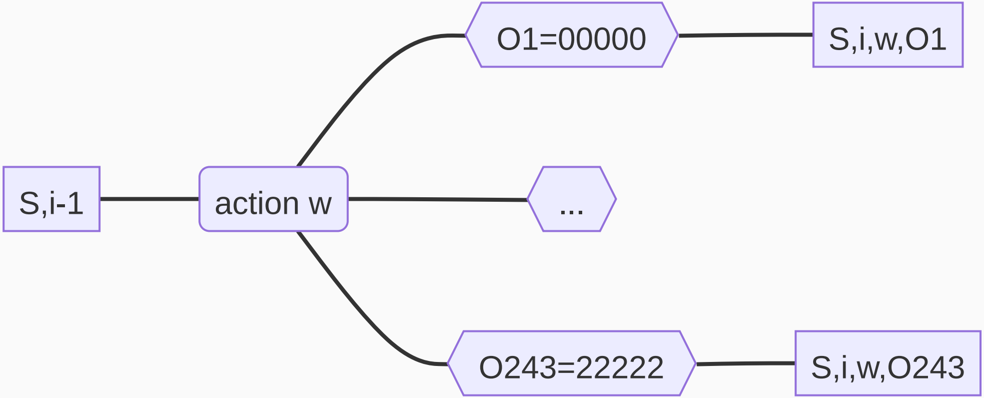
The idea here is to calculate the probability of drawing the letters in word w from letters in C_{i-1} with replacement.

Answer 6.2c.1 [Using Transition, Known Agent]

State h_2, explain how h_2 can be computed and prove dominance.

Idea: We want to reduce C_i as much as possible; $(0, 1, 2) = (\text{Green}, \text{Yellow}, \text{Grey})$.

Since there are 5 letters each with 3 possible observations - $3^5 = 243$ observations.



For every word w,

- We have each $C_i^{(j)}$ from every observation $j \in [1, 3^5]$.
- Now, we can calculate statistics using each $C_i^{(j)}$.

One way is to take the expectation over all observations:

$$h_2(s_i) = \mathbb{E}\left[\frac{|C_i|}{|D|}\right] = \mathbb{E}\left[\forall_j \frac{|C_i^{(j)}|}{|D|}\right]$$

Or using information entropy; idea is to choose the state that gives us the most info:

$$h_2(s_i) = \sum_{j \in [1, 3^5]} \left[\frac{|C_i^{(j)}|}{|C_{i-1}|} \log_2 \frac{|C_i^{(j)}|}{|C_{i-1}|}\right]$$

To generalize, consider $l > 1$ lookahead; note that we are currently looking ahead 1 step.

h_2 dominates h_1:

- $h_1 = 0$ for all states.
- We note that the smallest h_2 is when each observation $C_i^{(j)}$ is very imbalanced.
- But in that case, it is non-zero > 0.

$$\min[h_2(s_i)] > 0 = h_1$$

Answer 6.2c.2

Discuss how h_2 is useful to a local search algorithm.

- We take a 1 step lookahead to see the effect of taking the action w
- We compute over the possible observations to calculate the information gain
- We choose the state with the highest expected information gain

Answer 6.2c.3

Proof inadmissibility and that there cannot be any other admissible heuristics.

Let s_i to be the goal state, then $h^*(s_i) = 0$, but $h_2(s_i)$ is non zero. Assume an admissible heuristic h':

- At the current state, the target word can be the action.
- If the agent takes the action, the goal state will be reached.
- The $h^*(g) = 0$, and consequently $h'(g) = 0$ for admissible heuristic.
- For $h'(g) = 0$, h' must have known g is the goal state or $h' = 0$.
- But h' does not have access to the exact next state but only an estimate.
- So, h' cannot be admissible unless $h' = 0$. There are no other admissible heuristics.

Inadmissible. $h_2(g) \neq 0$

Note: Usually we proof inadmissiblity by giving example; But this is an exception.