CS3243 Tutorial 3

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Annoucements

Important admin:

1. Attendance Marking via CAS:
   - 2120/CS3243/T14 10-02-2022 13:00-14:00
   - 2120/CS3243/T15 10-02-2022 15:00-16:00
   - 2120/CS3243/T16 10-02-2022 16:00-17:00
2. Assignment 2 scores and comments are out on turnitin, do check!

Question 1

In the SameGame puzzle, a player is given a two-dimensional, rectangular, $n \times m$ grid of coloured squares. An example of such a grid is depicted in the figure below.

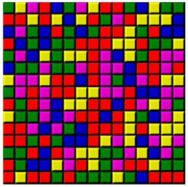

Figure 1: Q1 eg.

- **State Space**: $n \times m$ where each cell value is color, $[0, c]$.
- **Initial State**: Random matrix where each cell $[1, c]$.
- **Final State**: Zero matrix.
- **Action**: Delete a group of the same color.
- **Transition Model**: Replace cell values with 0, any cell which has 0 below will move down until zeros are ontop, columns move left if zero column.

Design an admissible heuristic for this puzzle game. Your heuristic may not be $h(s) = 0$ for all states s, the optimal heuristic, or a linear combination/simple function thereof. You may assume that the tile layout in the initial grid is solvable i.e. there is some path to a goal state. **You must prove that your heuristic is admissible.**

Recap

- What is an admissible heuristic?
- How is a heuristic *useful*?
- What is the h^* heuristic?
- What is a potential downfall of choosing an optimal heuristic?

Recap

- What is an admissible heuristic?
- How is a heuristic *useful*?
- What is the h^* heuristic?
- What is a potential downfall of choosing an optimal heuristic?

$$h(N) \leq h^*(N)$$

Question 1 - Answer crowd-sourced from T14

$h(n) =$

1. Number of different colors [Admissible]
   - One action will get rid of at most one color
2. Number of groups [Not Admissible]
   - B,G,B chooose G and left 1 step
3. Minimum of rows, columns [Not Admissible]
   - 2×3 grid of 1 color

Question 1 - Answer crowd-sourced from T15

$h(n) =$

1. Number of remaining colors [Admissible]
   - Everytime you select an action, it will take at most the number of colors.
2. Number of groups that is sized more than 1 [Not Admissible]
   - RRR, GGG, RRR $h(n) = 3$ but optimal is 1.

Question 1 - Answer crowd-sourced from T16

$h(n) =$

1. Number of remaining colors [Admissible]
   1.1 Improvement on 1 to treat singletons ad infinite cost.
      - Everytime you select an action, it will take at most the number of colors.
2. The number of groups on the board [Not Admissible]
   - Delete a group, can combine another 2 groups
3. Number of Singletons [Not Admissible]
   - Single color row at the bottom, with the top interspaced with different colors

Question 1 - Answer

$h(s) =$ number of colors remaining.

Proof of Admissibility

$h^*(s)$ is the number of optimal moves from s to goal.

Each group contains exactly 1 colour. So for each remaining colour, there can be 1 or more groups. For every color that still exist, there is at least 1 group and therefore at least requires 1 move.

Hence, $h(s)$ is less than or equals to at least the minimum moves on the board $\implies h(s) \leq h^*(s)$.

In summary, whenever every action is taken, $h(s) \leq h^*(s)$ is maintained:

- Reduce the number of colors - Number of groups (maximally) reduced by 1.
- Do not reduce the number of colors.

Question 2

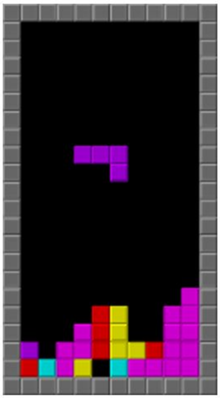

Figure 2: Q2 eg.

Each turn, when a new piece appears to be placed, the player must select the location and orientation before it falls.

- **State Space**: Matrix where each cell is 0 empty or 1 filled.
- **Initial State**: Empty matrix.
- **Final State**: Filled matrix, with all 1s.
- **Action**: Orientation and column position.
- **Transition Model**: Transition cost - 1, add 1 to the position of the dropped tetriminos.

Recap

- What is an admissible heuristic? $h(N) \leq h^*(N)$
- What are some properties of admissiblity?

Question 2a

Admissible, or inadmissible?

- $h_1(n)$ = number of unfielded tetriminos
- $h_2(n)$ = number of gaps
- $h_3(n)$ = number of incomplete rows
- $h_4(n)$ = number of blocked gaps
- None of the options are admissible.

Question 2a

Admissible, or inadmissible?

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- None of the options are admissible.

Answer

- $h_1(n)$ - Admissible, optimal steps must be $\geq$ than the number of teriminos.
- $h_2(n)$ - Inadmissible, yellow square block fills a gap of 4 but optimal cost is 1.
- $h_3(n)$ - Inadmissible, cyan vertical block fills a gap of 4 rows but optimal cost is 1.
- $h_4(n)$ - Admissible (Assuming that blocked gaps cannot be filled):
  - Whenever there is a blocked gap, h^* is infinite.
  - Whenever there is none, $h_4(.) = 0 \leq h^*(.)$

Question 2b

h	Admissiblity
h_1	Admissible
h_2	Inadmissible
h_3	Inadmissible
h_4	Admissible

Select all of the following that are True:

- $\max(h_1, h_2)$ is admissible
- $\min(h_2, h_3)$ is admissible
- $\max(h_3, h_4)$ is inadmissible
- $\min(h_1, h_4)$ is admissible

Question 2b - Answer

h	Admissiblity
h_1	Admissible
h_2	Inadmissible
h_3	Inadmissible
h_4	Admissible

Select all of the following that are True:

- **[False]** $\max(h_1, h_2)$ is admissible
- **[False]** $\min(h_2, h_3)$ is admissible
  - L shaped piece gap, $h_2 = 4, h_3 = 2$ but $h^* = 1$
- **[True]** $\max(h_3, h_4)$ is inadmissible
- **[True]** $\min(h_1, h_4)$ is admissible

Question 2b - Answer

h	Admissiblity
h_1	Admissible
h_2	Inadmissible
h_3	Inadmissible
h_4	Admissible

Select all of the following that are True:

- **[False]** $\max(h_1, h_2)$ is admissible
- **[False]** $\min(h_2, h_3)$ is admissible
  - L shaped piece gap, $h_2 = 4, h_3 = 2$ but $h^* = 1$
- **[True]** $\max(h_3, h_4)$ is inadmissible
- **[True]** $\min(h_1, h_4)$ is admissible

Analysis, cases, which are always true for general case?

- max(Admissible, Admissible)
- max(Admissible, Inadmissible) - Inadmissible ?
- max(Inadmissible, Inadmissible) - Inadmissible ?
- min(Admissible, Admissible) - Admissible ?
- min(Admissible, Inadmissible)
- min(Inadmissible, Inadmissible) - Inadmissible ?

Question 2c

Select all of the following that are True:

- h_1 dominates h_2
- h_2 dominates h_4
- h_3 does not dominate h_2
- h_4 does not dominate $h_2/2$

Recap

- What is dominates? $h_2(n) \geq h_1(n)$ for every state n, then h_2 dominates h_1.

Question 2c - Answer

h	Admissiblity
h_1	Admissible
h_2	Inadmissible
h_3	Inadmissible
h_4	Admissible

Recall the hueristics:

- $h_1(n) =$ number of unfielded tetriminos
- $h_2(n) =$ number of gaps
- $h_3(n) =$ number of incomplete rows
- $h_4(n) =$ number of blocked gaps

Select all of the following that are True:

- **[False]** h_1 dominates h_2 - Admissible cannot dominate inadmissible.
- **[True]** h_2 dominates h_4 - Blocked gaps $\leq$ Gaps.
- **[True]** h_3 does not dominate h_2 - Consider inital state: $0 = h_3(s_0) < h_2(s_0) \neq 0$
- **[True]** h_4 does not dominate $h_2/2$ - Consider inital state: $0 = h_4(s_0) < h_2(s_0)/2 \neq 0$

Question 3

Assignment Question; we will go through this question next week.

For next question, we will assume all hueristics are admissible.

Question 4

- h_1 : Number of pellets left at any point in time.
- h_2 : Number of pellets left + the minimum among all Manhattan distances from each remaining pellet to the current position of Pac-Man.
- h_3 : The Maximum among all Manhattan distances from each remaining pellet to the current position of Pac-Man.
- h_4 : The average over all Euclidean distances from each remaining pellet to the current position of Pac-Man.

Answer

- We pick h_1 to analyse dominate relationship:
  - h_2 - **False**; Trival to see $h_2(.) \geq h_1(.)$.
  - h_3 - **False**, h_4 - **False**; see example
    - Case where 1 pellet left, pacman is 10 units away: $h_1 = 1 < h_3 = h_4 = 10$
    - Case where 4 pellets left, pacman is 1 units away: $h_1 = 4 > h_3 = h_4 = 1$
- We pick h_2 with h_3, h_4 to analyse dominate relationship.
  - Similar to h_1 above.
- We pick h_3 with h_4 to analyse dominate relationship.
  - h_3 dominates h_4; We consider h_3' which is average of all manhattan distance. Then, h_3 dominates h_3' dominates h_4.