CS3243 Tutorial 9

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Annoucements

1. Assignment 7 scores are now on Luminus.
2. Highly recommended to send in your teaching feedback - I appreciate it greatly!
3. This tutorial is the last 'content' tutorial; next week is a recap tutorial.

Student Feedback on Teaching (SFT)

Feedback is *optional* but *highly encouraged*, access here: **https://es.nus.edu.sg/blue/**

- **[Tutorial Feedback]** Your feedback is important to me, and will be used to improve my teaching.
  - If I have helped your learning in any way, your positive feedback will be an encouragement to me.
  - If you find your learning can be enhanced by some action on my part, that feedback will be used to improve my teaching.
- **[Module Feedback]** Your feedback will be used to improve the module.
- Feedback is confidential to the university and anonymous to us.
- Avoid mixing the feedback; ie. project feedback to tutorial feedback.

Past student feedback had been used to improve teaching; ie. Telegram access to provide faster feedback.

Previously from T08, Q1

Consider below, a Vertex Cover where it is a set of vertices that covers all edges.

i. Write down the constraints as logical
   statements for a vertex cover of size 1.
ii. Apply the resolution algorithm in order
    to prove that the vertex 1 must be part
    of the vertex cover.

Recap

- What is a Vertex Cover of size k?
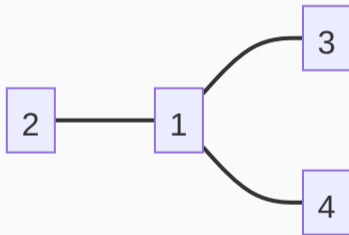- How to formulate a KB problem?

Figure 1: Graph for Vertex Cover CSP

Answer T8.Q1.i - Formulating KB

Variables:

- x_i represents i node on the graph that is in the vertex cover.
- $\neg x_i$ represents not in the vertex cover.

Constraints:

- Edge cover constraints:
    - $x_1 \vee x_2$;
    - $x_1 \vee x_3$;
    - $x_1 \vee x_4$
- Size $k = 1$ constraints: (We ignore contrapositive)
    - x_1 set then... $x_1 \implies \neg x_2$; $x_1 \implies \neg x_3$; $x_1 \implies \neg x_4$
    - x_2 set then... $x_2 \implies \neg x_3$; $x_2 \implies \neg x_4$;
    - x_3 set then... $x_3 \implies \neg x_4$

Then convert to CNF!

Answer T8.Q1.ii

Show $KB \models \alpha = x_1$; we resolve $KB \wedge \neg x_1$

1. $\neg x_1 \oplus x_1 \vee x_2 \implies x_2$
2. $x_2 \oplus \neg x_2 \vee \neg x_3 \implies \neg x_3$
3. $\neg x_1 \oplus x_1 \vee x_3 \implies x_3$
4. $x_3 \oplus \neg x_3 \implies \square$

Question 1

Having both good grades (G) and good communication skills (C) will increase your chances of performing well in your interview (I).

Table 1: Probability of I; $Pr[G = 1] = 0.7, Pr[C = 1] = 0.2$

Figure 2: Belief Network

G	C	$Pr[I = 1 \| G, C]$
1	1	0.9
1	0	0.5
0	1	0.5
0	0	0.1

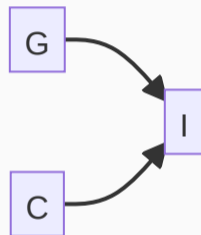**Recap**

- How to read a Bayes Network?
- What is Marginalisation?

Answer

What is the probability that

a. Alice, who has poor grades and communication skills, performs well on her interview? - $Pr[I = 1|G = 0, C = 0] = 0.1$
b. Bob is a student with great communication skills, assuming we do not know anything about him? - $Pr[C = 1] = 0.2$
c. A student has good communication skills, given that he or she has performed well in an interview? - $Pr[C = 1|I = 1] = \frac{Pr[C=1,I=1]}{Pr[I=1]} = \frac{\sum_g Pr[G=g,C=1,I=1]}{\sum_{g,c} Pr[G=g,C=c,I=1]} = 0.339$
   a. Are good communication skills independent of good performance in an interview? - Remember independence; $Pr[C = 1|I = 1] \neq Pr[C = 1] = 0.2$.

Question 2

Assume that 2% of the population in a country carry a particular virus (Y is a carrier).
A test kit developed to detect the presence (X is positive test).

Y	$Pr[X = 1 \| Y]$
1	0.998
0	$1 - 0.996 = 0.004$

Recap
- What is Conditional Probability?
- What is Conditional Independence?

Question 2

Assume that 2% of the population in a country carry a particular virus (Y is a carrier).
A test kit developed to detect the presence (X is positive test).

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0	$1 - 0.996 = 0.004$

Recap

- What is Conditional Probability?
- What is Conditional Independence?

Conditional independence is a situation when an observation is redundant,
i.e. $Pr[A \mid B, C] = Pr[A \mid C]$. Careful with the conditional independence here. . .

Answer 2a

Given that a patient is tested to be positive using this kit, what is the posterior belief that he is not a carrier?

$$Pr[Y=0|X=1] = \frac{Pr[Y=0,X=1]}{Pr[X=1]} = \frac{Pr[Y=0,X=1]}{\sum_y Pr[Y=y,X=1]} = \frac{Pr[X=1|Y=0] \times Pr[Y=0]}{\sum_y Pr[Y=y,X=1]} = 0.164$$

Answer 2b

Patient tested positive again using the second kit (X_2 is the second test, X_1 is the first test). Assume conditional independence between results of different test kits given the patient's state of virus contraction.

$$Pr[Y=0|X_1=1, X_2=1] = 0.0008$$

- $= \frac{Pr[Y=0,X_2=1,X_1=1]}{Pr[X_1=1,X_2=1]} = \frac{Pr[X_2=1,X_1=1|Y=0]Pr[Y=0]}{Pr[X_1=1,X_2=1]}$ (Conditional Probability)
- $= \frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{Pr[X_1=1,X_2=1]}$ (Conditional Independence)
- $= \frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{\sum_y Pr[X_1=1,X_2=1,Y=y]}$ (Marginalisation)
- $= \frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{\sum_y Pr[X_1=1|Y=y]Pr[X_2=1|Y=y]Pr[Y=y]}$ (Conditional Probability, Independence)

10

Very interesting, read more...

- Can you solve the false positive riddle?: https://youtu.be/1csFTDXXULY
- False-positive paradox: https://en.wikipedia.org/wiki/Base_rate_fallacy
- Prior, Posterior...

$$\underset{\text{Posterior}}{P(A|B)} = \frac{\overset{\text{Likelihood}}{P(B|A)} * \overset{\text{Prior}}{P(A)}}{\underset{\text{Evidence}}{P(B)}}$$

Figure 3: Prior vs Posterior

Construct a Bayesian network and determine the probability

$$Pr[WG = 1, RS = 1, R = 0, S = 0]$$

Recap
- Any useless variables from the table?

Answer

Lemma 1. Given two random boolean variables A and B, if $Pr[A|B] = 0$ and $Pr[A|\neg B] = 1$ then $Pr[A] = 1 - Pr[B]$; in fact, $A \equiv \neg B$.

Not difficult to proof (*Note the notation change.*):

$$Pr[A|B] = 0 \land Pr[A|\neg B] = 1 \implies Pr[A] = 1 - Pr[B]$$

Pr[A]

- $= Pr[A \land B] + Pr[A \land \neg B]$ (Marginalisation)
- $= Pr[A|B] \times Pr[B] + Pr[A|\neg B] \times Pr[\neg B]$ (Conditional Probability)
- $= Pr[\neg B] = 1 - Pr[B]$ (Subt given)

So, $Pr[A \land \neg B \land \cdots] = Pr[\cdots | A \land \neg B] \times Pr[A \land \neg B] = Pr[\cdots | A] \times Pr[A]$,

- Since, $Pr[A \land \neg B] = Pr[A|\neg B] \times Pr[\neg B] = Pr[A]$;
- And, $Pr[\cdots | A \land \neg B] = Pr[\cdots \land \neg B | A] / Pr[\neg B | A] = Pr[\cdots | A]$
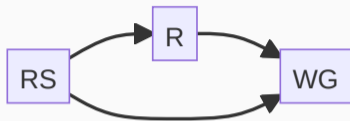
Since $S \equiv \neg RS$, so:

$Pr[RS = 1] = 0.7$

RS	$Pr[R = 1|RS]$
1	0.9
0	0.1

Figure 4: RS, WG, R network.

RS	R	$Pr[WG = 1|RS, R]$
1	1	0.8
1	0	0.1
0	1	0.95
0	0	0.9

$Pr[WG = 1, RS = 1, R = 0, S = 0] = Pr[WG = 1, RS = 1, R = 0]$

- $= Pr[WG = 1, R = 0|RS = 1] \times Pr[RS = 1]$ (Conditional Probability)
- $= Pr[WG = 1|RS = 1, R = 0] \times Pr[R = 0|RS = 1] \times Pr[RS = 1]$ (Cond. Prob.)
- $= 0.1 \times (1 - 0.9) \times 0.7 = 0.007$