Neural Networks from scratch (IT GM)

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Neural Networks (NN)

Neural Network is a term that is so popular that it has become mainstream.

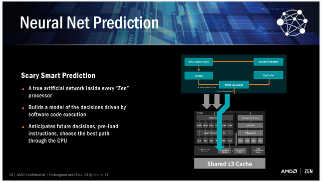

Figure 1: AMD Ryzen Marketing Material.

Defintion (Oxford)

Neural Network is a system with a structure that is similar to the human brain and nervous system.

History of NN - The downfall

The promise: If it works in nature, it must be able to work in computers.

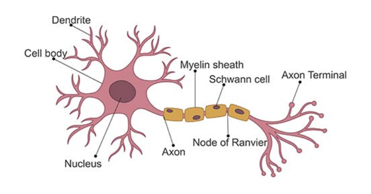

Figure 2: Neurons in the brain.

1949 - Started as a curosity of the human brain

In 'A Logical Calculus Of The Ideas Immanent In Nervous Activity', electrical circuits model a simple neural network. Promise of computer intelligence.

1950s - Computers became more advanced

John von Neumann actually worked on NN; while all the hype was on Systems and the von Neumann architecture.

1970s - Many problems

Issues arising from Single layed neural network ro Multi-layered; Unfulfilled promises of NN -weak results; Very little interest due to the lack of funding.

History of NN - The rise

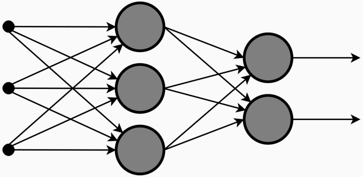

Figure 3: Multi-layered NN.

1980s - Multi-layer NNs

Researchers work on the forumulation of multi-layered neural networks.

1986 - Learning via Back-propagation

David Rumelhart came up with the idea now called back propagation networks because it distributes pattern recognition errors throughout the network; but its slow.

2010s - GPU Computation

With GPU compute, we are able to get around the problem of slow compute. Massively crunch a lot of data with many iterations.

Today

Neural networks is a very ‘hot’ research topic now due to its possibilites.

Neural Network Applications

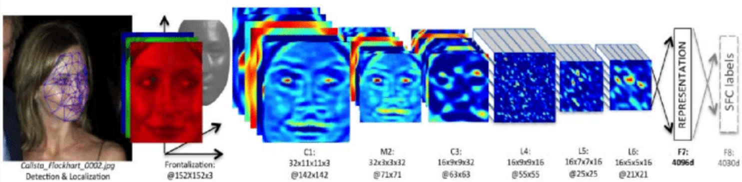

Figure 4: Facial Recognition: FaceNet:99.63%, DeepFace:97%, Human:97.53%, FBI:85%

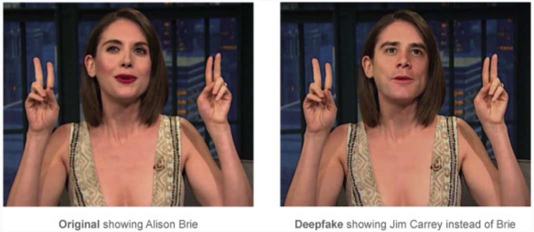

Figure 5: Adversarial Neural Networks - Deepfake: Trick or treat?

Impressive, most impressive... But...

The key mechanics for neural networks are the same; haven't really changed:

1. Neurons, input/output
2. Backpropagation

Goal for today

1. Understand a neuron
2. Understand fully connected layer
3. Understand backpropagation

Neuron

Function

A neuron takes an input (x) and produces an output $f(x)$:

1. A very simple neuron is to just simply repeat the output: $f(x) = x$
2. Another is to negate the output: $f(x) = -x$

With this, you can build things but it is not very interesting.

Tweakable function

Here we add new variables so that we can tweak them; we want some form of linear function and the ability to tweak them. Remembering secondary school $y = mx + c$.

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We are inspired to define with weights w and bias b:

$$f(x) = wx + b$$

Learning Neuron

We have some parameters w, b which we want to discover:

1. Let's start with some random values of w_1, b_1.
2. We have an input/output that we want the neuron to learn (x_1, y_1).
3. We can simply try x_1 with the neuron is, $f(x_1) = w_1 x_1 + b_1 = y_1'$.
4. Now we know how bad the neuron is, $\delta_y = y_1 - y_1'$
5. We can use differentiation to find how bad w is, $\delta_y / \delta_w = x_1 \implies \delta_w = x_1 / \delta_y$!
6. We update $w_2 = w_1 - \delta_w$, same for b_2

Keep running the procedure for many different (x, y) points and its learning!

Neural Network Layers

A layer is a set of neurons that take in a vector X and output Y:

$$\Theta_{k,k+1}(x_k) = x_k W_{k,k+1} + b_{k+1}$$

We see here, its essentially the same as just now but with matrices.

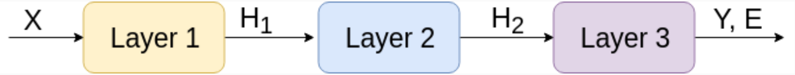


Figure 6: A stack of layers.

Activation function

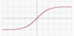
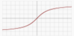
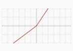
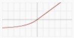
Name	Plot	Equation	Derivative
Identity		$f(x) = x$	$f'(x) = 1$
Binary step		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x \neq 0 \\ ? & \text{for } x = 0 \end{cases}$
Logistic (a.k.a Soft step)		$f(x) = \frac{1}{1+e^{-x}}$	$f'(x) = f(x)(1 - f(x))$
TanH		$f(x) = \tanh(x) = \frac{2}{1+e^{-2x}} - 1$	$f'(x) = 1 - f(x)^2$
ArcTan		$f(x) = \tan^{-1}(x)$	$f'(x) = \frac{1}{x^2+1}$
Rectified Linear Unit (ReLU)		$f(x) = \begin{cases} 0 & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} 0 & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Parameteric Rectified Linear Unit (PReLU)[2]		$f(x) = \begin{cases} \alpha x & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
Exponential Linear Unit (ELU)[3]		$f(x) = \begin{cases} \alpha(e^x - 1) & \text{for } x < 0 \\ x & \text{for } x \geq 0 \end{cases}$	$f'(x) = \begin{cases} f(x) + \alpha & \text{for } x < 0 \\ 1 & \text{for } x \geq 0 \end{cases}$
SoftPlus		$f(x) = \log_e(1 + e^x)$	$f'(x) = \frac{1}{1+e^{-x}}$

Figure 7: We want the neuron to 'fire' on some conditions; don't always fire: σ

Now our layer becomes: $\Theta_{k,k+1}(x_k) = \sigma(x_k W_{k,k+1} + b_{k+1})$

Fully-Connected Neural Network

A neural network Θ is a function that transform x to y, ie. $\Theta(x) = y$. We will also let x to be the row vector that corresponds to values of layer 0, which is the input layer. We will also let y to be the row vector that corresponds to the values of the final output layer. Using the defintion of weights W and biases b in the appendix and assuming the activation function is $\sigma(z)$, we are able to formulate the network mathematically for any layer$(k, k+1)$ transiting to the next:

$$\Theta_{k,k+1}(x_k) = \sigma(x_k W_{k,k+1} + b_{k+1})$$

Then, the neural network Θ with n hidden layers would be the composition of these fuctions(Note that $x = x_0$):

$$\Theta(x) = \Theta_{n,n+1}\Big(\cdots \Theta_{1,2}(\Theta_{0,1}(x_0))\Big)$$

Back Propagation

Essentially the same as the one neuron case, but with layers (ie. matrices).

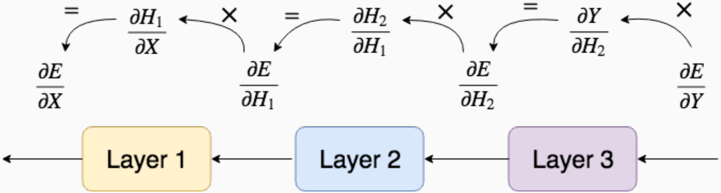


Figure 8: The errors from each layer propagate upwards.

Example - ‘Compressing’ Neural Networks

Some nice or strange properties: We can ‘compress’ a 3 layer neural network into a neural network with one layer:

$$\Theta(x) = \Theta_{2,3}\Big(\Theta_{1,2}(\Theta_{0,1}(x))\Big)$$

So, expanding them:

$$\begin{aligned}
\Theta(x) &= \big[(xW_{0,1} + b_1)W_{1,2} + b_2\big]W_{2,3} + b_3 \\
&= \big[xW_{0,1}W_{1,2} + b_1W_{1,2} + b_2\big]W_{2,3} + b_3 \\
&= xW_{0,1}W_{1,2}W_{2,3} + b_1W_{1,2}W_{2,3} + b_2W_{2,3} + b_3 \\
&= x(W_{0,1}W_{1,2}W_{2,3}) + (b_1W_{1,2}W_{2,3} + b_2W_{2,3} + b_3)
\end{aligned}$$

Code Example - ‘Compressing’ Neural Networks

Access here: https://github.com/eric-vader/simple-nn

```
i = lambda x: x # Identity function
i_ = lambda x: 1 # Identity function derivative

large_nn = NNBuilder().load(
  [i,i,i],
  [i_,i_,i_],
  nn_data_path,
  [FullyConnectedLayer,FullyConnectedLayer,FullyConnectedLayer],
  5)
small_nn = NNBuilder().load(
  [i], [i_], nn_result_path, [FullyConnectedLayer], 5)
```