chap11slow

October 14, 2019

[13]:
```
from functools import reduce
from sympy import simplify, sqrt, Rational, Symbol
from galgebra.ga import Ga
from galgebra.mv import Mv
from galgebra.printer import Format
Format()
def hline():
    print('\n')
    return
```

[14]:
```
Ga.dual_mode('Iinv+')

GA_list = [
    Ga("e*0|1|2", g='-1 0 0, 0 1 0, 0 0 1'),
    Ga("e*0|1|2|3", g='-1 0 0 0, 0 1 0 0, 0 0 1 0, 0 0 0 1'),
    Ga("e*0|1|2|3|5", g='-1 0 0 0 0, 0 1 0 0 0, 0 0 1 0 0, 0 0 0 1 0, 0 0 0 0␣
 ↪1'),
    ]

for GA in GA_list:
    e_0 = GA.mv_basis[0]
    e_0_inv = e_0.inv()

    Ip = GA.I()
    Ip_inv = Ip.inv()
    print('I =',Ip)
    print('I^{-1} =',Ip_inv)
    Ir = e_0_inv < Ip
    Ir_inv = Ir.inv()
    #self.assertEquals(Ip, e_0 ^ Ir)
    #self.assertEquals(Ip, e_0 * Ir)

    p = GA.mv([1] + [Symbol('p%d' % i) for i in range(1, GA.n)], 'vector')

    print('p =',p)

    v = [
```

```
    GA.mv([0] + [Symbol('q%d' % i) for i in range(1, GA.n)], 'vector'),
    GA.mv([0] + [Symbol('r%d' % i) for i in range(1, GA.n)], 'vector'),
    GA.mv([0] + [Symbol('s%d' % i) for i in range(1, GA.n)], 'vector'),
    GA.mv([0] + [Symbol('t%d' % i) for i in range(1, GA.n)], 'vector'),
    ]
print('v =',v)
```

$$I = \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2$$

$$I^{-1} = \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2$$

$$p = \begin{aligned} & \boldsymbol{e}_0 \\ & + p_1 \boldsymbol{e}_1 \\ & + p_2 \boldsymbol{e}_2 \end{aligned}$$

$$v = [\begin{aligned} & q_1 \boldsymbol{e}_1 \\ & + q_2 \boldsymbol{e}_2 \end{aligned}, \begin{aligned} & r_1 \boldsymbol{e}_1 \\ & + r_2 \boldsymbol{e}_2 \end{aligned}, \begin{aligned} & s_1 \boldsymbol{e}_1 \\ & + s_2 \boldsymbol{e}_2 \end{aligned}, \begin{aligned} & t_1 \boldsymbol{e}_1 \\ & + t_2 \boldsymbol{e}_2 \end{aligned}]$$

$$I = \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3$$

$$I^{-1} = -\boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3$$

$$p = \begin{aligned} & \boldsymbol{e}_0 \\ & + p_1 \boldsymbol{e}_1 \\ & + p_2 \boldsymbol{e}_2 \\ & + p_3 \boldsymbol{e}_3 \end{aligned}$$

$$v = [\begin{aligned} & q_1 \boldsymbol{e}_1 \\ & + q_2 \boldsymbol{e}_2 \\ & + q_3 \boldsymbol{e}_3 \end{aligned}, \begin{aligned} & r_1 \boldsymbol{e}_1 \\ & + r_2 \boldsymbol{e}_2 \\ & + r_3 \boldsymbol{e}_3 \end{aligned}, \begin{aligned} & s_1 \boldsymbol{e}_1 \\ & + s_2 \boldsymbol{e}_2 \\ & + s_3 \boldsymbol{e}_3 \end{aligned}, \begin{aligned} & t_1 \boldsymbol{e}_1 \\ & + t_2 \boldsymbol{e}_2 \\ & + t_3 \boldsymbol{e}_3 \end{aligned}]$$

$$I = \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5$$

$$I^{-1} = -\boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5$$

$$p = \begin{aligned} & \boldsymbol{e}_0 \\ & + p_1 \boldsymbol{e}_1 \\ & + p_2 \boldsymbol{e}_2 \\ & + p_3 \boldsymbol{e}_3 \\ & + p_4 \boldsymbol{e}_5 \end{aligned}$$

$$v = \left[\begin{array}{c} q_{1}\boldsymbol{e}_{1} \\ + q_{2}\boldsymbol{e}_{2} \\ + q_{3}\boldsymbol{e}_{3} \\ + q_{4}\boldsymbol{e}_{5} \end{array}, \begin{array}{c} r_{1}\boldsymbol{e}_{1} \\ + r_{2}\boldsymbol{e}_{2} \\ + r_{3}\boldsymbol{e}_{3} \\ + r_{4}\boldsymbol{e}_{5} \end{array}, \begin{array}{c} s_{1}\boldsymbol{e}_{1} \\ + s_{2}\boldsymbol{e}_{2} \\ + s_{3}\boldsymbol{e}_{3} \\ + s_{4}\boldsymbol{e}_{5} \end{array}, \begin{array}{c} t_{1}\boldsymbol{e}_{1} \\ + t_{2}\boldsymbol{e}_{2} \\ + t_{3}\boldsymbol{e}_{3} \\ + t_{4}\boldsymbol{e}_{5} \end{array} \right]$$

[15]:

```
# We test available finite k-flats
for k in range(1, GA.n):
    A = reduce(Mv.__xor__, v[:k])
    X = (p ^ A)
    hline()
    print('X =',X.Fmt(3))
    #self.assertNotEquals(X, 0)
    M = e_0_inv < (e_0 ^ X)
    hline()
    print('M =',M.Fmt(3))
    # Very slow
    d = (e_0_inv < (e_0 ^ X)) / (e_0_inv < X)
    hline()
    print('d =',d.Fmt(3))
    #d_inv = d.inv()
```

$$X = \begin{array}{l} q_{1}\boldsymbol{e}_{0}\wedge \boldsymbol{e}_{1} \\ + q_{2}\boldsymbol{e}_{0}\wedge \boldsymbol{e}_{2} \\ + q_{3}\boldsymbol{e}_{0}\wedge \boldsymbol{e}_{3} \\ + q_{4}\boldsymbol{e}_{0}\wedge \boldsymbol{e}_{5} \\ + \left (p_{1} q_{2} - p_{2} q_{1}\right) \boldsymbol{e}_{1}\wedge \boldsymbol{e}_{2} \\ + \left (p_{1} q_{3} - p_{3} q_{1}\right) \boldsymbol{e}_{1}\wedge \boldsymbol{e}_{3} \\ + \left (p_{1} q_{4} - p_{4} q_{1}\right) \boldsymbol{e}_{1}\wedge \boldsymbol{e}_{5} \\ + \left (p_{2} q_{3} - p_{3} q_{2}\right) \boldsymbol{e}_{2}\wedge \boldsymbol{e}_{3} \\ + \left (p_{2} q_{4} - p_{4} q_{2}\right) \boldsymbol{e}_{2}\wedge \boldsymbol{e}_{5} \\ + \left (p_{3} q_{4} - p_{4} q_{3}\right) \boldsymbol{e}_{3}\wedge \boldsymbol{e}_{5} \end{array}$$

$$M = \begin{aligned}&(p_1q_2 - p_2q_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_2\\&+ (p_1q_3 - p_3q_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_3\\&+ (p_1q_4 - p_4q_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_5\\&+ (p_2q_3 - p_3q_2)\,\boldsymbol{e}_2 \wedge \boldsymbol{e}_3\\&+ (p_2q_4 - p_4q_2)\,\boldsymbol{e}_2 \wedge \boldsymbol{e}_5\\&+ (p_3q_4 - p_4q_3)\,\boldsymbol{e}_3 \wedge \boldsymbol{e}_5\end{aligned}$$

$$d = \begin{aligned}&\frac{p_1(q_2)^2 + p_1(q_3)^2 + p_1(q_4)^2 - p_2q_1q_2 - p_3q_1q_3 - p_4q_1q_4}{(q_1)^2 + (q_2)^2 + (q_3)^2 + (q_4)^2}\boldsymbol{e}_1\\&+ \frac{-p_1q_1q_2 + p_2(q_1)^2 + p_2(q_3)^2 + p_2(q_4)^2 - p_3q_2q_3 - p_4q_2q_4}{(q_1)^2 + (q_2)^2 + (q_3)^2 + (q_4)^2}\boldsymbol{e}_2\\&+ \frac{-p_1q_1q_3 - p_2q_2q_3 + p_3(q_1)^2 + p_3(q_2)^2 + p_3(q_4)^2 - p_4q_3q_4}{(q_1)^2 + (q_2)^2 + (q_3)^2 + (q_4)^2}\boldsymbol{e}_3\\&+ \frac{-p_1q_1q_4 - p_2q_2q_4 - p_3q_3q_4 + p_4(q_1)^2 + p_4(q_2)^2 + p_4(q_3)^2}{(q_1)^2 + (q_2)^2 + (q_3)^2 + (q_4)^2}\boldsymbol{e}_5\end{aligned}$$

$$X = \begin{aligned}&(q_1r_2 - q_2r_1)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2\\&+ (q_1r_3 - q_3r_1)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_3\\&+ (q_1r_4 - q_4r_1)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_5\\&+ (q_2r_3 - q_3r_2)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3\\&+ (q_2r_4 - q_4r_2)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_5\\&+ (q_3r_4 - q_4r_3)\,\boldsymbol{e}_0 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5\\&+ (p_1q_2r_3 - p_1q_3r_2 - p_2q_1r_3 + p_2q_3r_1 + p_3q_1r_2 - p_3q_2r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3\\&+ (p_1q_2r_4 - p_1q_4r_2 - p_2q_1r_4 + p_2q_4r_1 + p_4q_1r_2 - p_4q_2r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_5\\&+ (p_1q_3r_4 - p_1q_4r_3 - p_3q_1r_4 + p_3q_4r_1 + p_4q_1r_3 - p_4q_3r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5\\&+ (p_2q_3r_4 - p_2q_4r_3 - p_3q_2r_4 + p_3q_4r_2 + p_4q_2r_3 - p_4q_3r_2)\,\boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5\end{aligned}$$

$$M = \begin{aligned}&(p_1q_2r_3 - p_1q_3r_2 - p_2q_1r_3 + p_2q_3r_1 + p_3q_1r_2 - p_3q_2r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3\\&+ (p_1q_2r_4 - p_1q_4r_2 - p_2q_1r_4 + p_2q_4r_1 + p_4q_1r_2 - p_4q_2r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_5\\&+ (p_1q_3r_4 - p_1q_4r_3 - p_3q_1r_4 + p_3q_4r_1 + p_4q_1r_3 - p_4q_3r_1)\,\boldsymbol{e}_1 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5\\&+ (p_2q_3r_4 - p_2q_4r_3 - p_3q_2r_4 + p_3q_4r_2 + p_4q_2r_3 - p_4q_3r_2)\,\boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5\end{aligned}$$

$$
d = \frac{p_1(q_2)^2(r_3)^2 + p_1(q_2)^2(r_4)^2 - 2p_1q_2q_3r_2r_3 - 2p_1q_2q_4r_2r_4 + p_1(q_3)^2(r_2)^2 + p_1(q_3)^2(r_4)^2 - 2p_1q_3q_4r_3r_4 + p_1(q_4)^2(r}{}
$$

$$
+ \frac{-p_1q_1q_2(r_3)^2 - p_1q_1q_2(r_4)^2 + p_1q_1q_3r_2r_3 + p_1q_1q_4r_2r_4 + p_1q_2q_3r_1r_3 + p_1q_2q_4r_1r_4 - p_1(q_3)^2r_1r_2 - p_1(q_4)^2r_1r_2}{}
$$

$$
+ \frac{p_1q_1q_2r_2r_3 - p_1q_1q_3(r_2)^2 - p_1q_1q_3(r_4)^2 + p_1q_1q_4r_3r_4 - p_1(q_2)^2r_1r_3 + p_1q_2q_3r_1r_2 + p_1q_3q_4r_1r_4 - p_1(q_4)^2r_1r_3 -}{}
$$

$$
+ \frac{p_1q_1q_2r_2r_4 + p_1q_1q_3r_3r_4 - p_1q_1q_4(r_2)^2 - p_1q_1q_4(r_3)^2 - p_1(q_2)^2r_1r_4 + p_1q_2q_4r_1r_2 - p_1(q_3)^2r_1r_4 + p_1q_3q_4r_1r_3 -}{}
$$

$$
\begin{aligned}
X = {} & (q_1r_2s_3 - q_1r_3s_2 - q_2r_1s_3 + q_2r_3s_1 + q_3r_1s_2 - q_3r_2s_1)\, \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \\
& + (q_1r_2s_4 - q_1r_4s_2 - q_2r_1s_4 + q_2r_4s_1 + q_4r_1s_2 - q_4r_2s_1)\, \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_5 \\
& + (q_1r_3s_4 - q_1r_4s_3 - q_3r_1s_4 + q_3r_4s_1 + q_4r_1s_3 - q_4r_3s_1)\, \boldsymbol{e}_0 \wedge \boldsymbol{e}_1 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5 \\
& + (q_2r_3s_4 - q_2r_4s_3 - q_3r_2s_4 + q_3r_4s_2 + q_4r_2s_3 - q_4r_3s_2)\, \boldsymbol{e}_0 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3 \wedge \boldsymbol{e}_5 \\
& + (p_1q_2r_3s_4 - p_1q_2r_4s_3 - p_1q_3r_2s_4 + p_1q_3r_4s_2 + p_1q_4r_2s_3 - p_1q_4r_3s_2 - p_2q_1r_3s_4 + p_2q_1r_4s_3 + p_2q_3r_1s_4 - p_2q_3r
\end{aligned}
$$

$$
M = (p_1q_2r_3s_4 - p_1q_2r_4s_3 - p_1q_3r_2s_4 + p_1q_3r_4s_2 + p_1q_4r_2s_3 - p_1q_4r_3s_2 - p_2q_1r_3s_4 + p_2q_1r_4s_3 + p_2q_3r_1s_4 - p_2q_3r_4s
$$

$$
d = \frac{p_1(q_2)^2(r_3)^2(s_4)^2 - 2p_1(q_2)^2r_3r_4s_3s_4 + p_1(q_2)^2(r_4)^2(s_3)^2 - 2p_1q_2q_3r_2r_3(s_4)^2 + 2p_1q_2q_3r_2r_4s_3s_4 + 2p_1q_2q_3r_3r_4s_2}{}
$$

$$
+ \frac{-p_1q_1q_2(r_3)^2(s_4)^2 + 2p_1q_1q_2r_3r_4s_3s_4 - p_1q_1q_2(r_4)^2(s_3)^2 + p_1q_1q_3r_2r_3(s_4)^2 - p_1q_1q_3r_2r_4s_3s_4 - p_1q_1q_3r_3r_4s_2s}{}
$$

$$
+ \frac{p_1q_1q_2r_2r_3(s_4)^2 - p_1q_1q_2r_2r_4s_3s_4 - p_1q_1q_2r_3r_4s_2s_4 + p_1q_1q_2(r_4)^2s_2s_3 - p_1q_1q_3(r_2)^2(s_4)^2 + 2p_1q_1q_3r_2r_4s_2s_4 -}{}
$$

$$
+ \frac{-p_1q_1q_2r_2r_3s_3s_4 + p_1q_1q_2r_2r_4(s_3)^2 + p_1q_1q_2(r_3)^2s_2s_4 - p_1q_1q_2r_3r_4s_2s_3 + p_1q_1q_3(r_2)^2s_3s_4 - p_1q_1q_3r_2r_3s_2s_4 -}{}
$$

$$X = (q_1r_2s_3t_4 - q_1r_2s_4t_3 - q_1r_3s_2t_4 + q_1r_3s_4t_2 + q_1r_4s_2t_3 - q_1r_4s_3t_2 - q_2r_1s_3t_4 + q_2r_1s_4t_3 + q_2r_3s_1t_4 - q_2r_3s_4t_1 -$$

$$M = 0$$

$$d = 0$$

```
[ ]:
```