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def Product_of_Rotors():
    Print_Function()
    (na,nb,nm,alpha,th,th_a,th_b) = symbols('n_a n_b n_m alpha theta theta_a theta_b',\
                                    real = True)
    g = [[na, 0, alpha],[0, nm, 0],[alpha, 0, nb]] #metric tensor
    """
    Values of metric tensor components
    [na,nm,nb] = [+1/ 1,+1/ 1,+1/ 1]  alpha = ea|eb
    """
    (g3d, ea, em, eb) = Ga.build('e_a e_m e_b', g=g)
    print('g =',g3d.g)
    print(r'n_{a} = \bm{e}_{a}^{2}\;\;n_{b} = \bm{e}_{b}^{2}\;\;n_{m} = \bm{e}_{m}^{2}'+\
        r'\;\;\alpha = \bm{e}_{a}\cdot\bm{e}_{b}')
    (ca,cb,sa,sb) = symbols('c_a c_b s_a s_b',real=True)
    Ra = ca + sa*ea*em  # Rotor for ea^em plane
    Rb = cb + sb*em*eb  # Rotor for em^eb plane
    print(r'\mbox{Rotor in }\bm{e}_{a}\bm{e}_{m}\mbox{ plane } R_{a} =',Ra)
    print(r'\mbox{Rotor in }\bm{e}_{m}\bm{e}_{b}\mbox{ plane } R_{b} =',Rb)
    Rab = Ra*Rb  # Compound Rotor
    """
    Show that compound rotor is scalar plus bivector
    """
    print(r'R_{a}R_{b} = S+\bm{B} =', Rab)
    Rab2 = Rab.get_grade(2)
    print(r'\bm{B} =',Rab2)
    Rab2sq = Rab2*Rab2  # Square of compound rotor bivector part
    Ssq = (Rab.scalar())**2  # Square of compound rotor scalar part
    Bsq =  Rab2sq.scalar()
    print(r'S^{2} =',Ssq)
    print(r'\bm{B}^{2} =',Bsq)
    Dsq = (Ssq Bsq).expand().simplify()
    print('S^{2} B^{2} =', Dsq)
    Dsq = Dsq.subs(nm**2,S(1))  # (e_m)**4 = 1
    print('S^{2} B^{2} =', Dsq)
    Cases = [S( 1),S(1)]  # 1/+1 squares for each basis vector
    print(r'\T{Consider all combinations of} \bm{e}_{a}^{2}, \bm{e}_{b}^{2}'+\
          r' \T{and} \bm{e}_{m}^2:')
    for Na in Cases:
        for Nb in Cases:
            for Nm in Cases:
                Ba_sq =  Na*Nm
                Bb_sq =  Nb*Nm
                if Ba_sq < 0:
                    Ca_th = cos(th_a)
                    Sa_th = sin(th_a)
                else:
                    Ca_th = cosh(th_a)
                    Sa_th = sinh(th_a)
                if Bb_sq < 0:
                    Cb_th = cos(th_b)
                    Sb_th = sin(th_b)
                else:
                    Cb_th = cosh(th_b)
                    Sb_th = sinh(th_b)
                print(r'\left [ \bm{e}_{a}^{2},\bm{e}_{b}^{2},\bm{e}_{m}^2\right ] =',\
                    [Na,Nb,Nm])
                Dsq_tmp = Dsq.subs({ca:Ca_th,sa:Sa_th,cb:Cb_th,sb:Sb_th,na:Na,nb:Nb,nm:Nm})
                print(r'S^{2}\bm{B}^{2} =',Dsq_tmp,' =',trigsimp(Dsq_tmp))
    print(r'\T{Thus we have shown that }R_{a}R_{b} = S+\bm{D} = e^{\bm{C}} \T{where} \bm{C}'+\
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        r' \T{is a bivector blade.}')
    return
```

Code Output:

$$g = \left[\begin{array}{ccc} n_a & 0 & \alpha \\ 0 & n_m & 0 \\ \alpha & 0 & n_b \end{array} \right]$$

$n_a = \boldsymbol{e}_a^2 \;\; n_b = \boldsymbol{e}_b^2 \;\; n_m = \boldsymbol{e}_m^2 \;\; \alpha = \boldsymbol{e}_a \cdot \boldsymbol{e}_b$

Rotor in $\boldsymbol{e}_a\boldsymbol{e}_m$ plane $R_a = c_a + s_a \boldsymbol{e}_a \wedge \boldsymbol{e}_m$

Rotor in $\boldsymbol{e}_m\boldsymbol{e}_b$ plane $R_b = c_b + s_b \boldsymbol{e}_m \wedge \boldsymbol{e}_b$

$R_a R_b = S + \boldsymbol{B} = (\alpha n_m s_a s_b + c_a c_b) + c_b s_a \boldsymbol{e}_a \wedge \boldsymbol{e}_m + n_m s_a s_b \boldsymbol{e}_a \wedge \boldsymbol{e}_b + c_a s_b \boldsymbol{e}_m \wedge \boldsymbol{e}_b$

$\boldsymbol{B} = c_b s_a \boldsymbol{e}_a \wedge \boldsymbol{e}_m + n_m s_a s_b \boldsymbol{e}_a \wedge \boldsymbol{e}_b + c_a s_b \boldsymbol{e}_m \wedge \boldsymbol{e}_b$

$S^2 = (\alpha n_m s_a s_b + c_a c_b)^2$

$\boldsymbol{B}^2 = \alpha^2 (n_m)^2 (s_a)^2 (s_b)^2 + 2\alpha c_a c_b n_m s_a s_b - (c_a)^2 n_b n_m (s_b)^2 - (c_b)^2 n_a n_m (s_a)^2 - n_a n_b (n_m)^2 (s_a)^2 (s_b)^2$

$S^2 - B^2 = (c_a)^2 (c_b)^2 + (c_a)^2 n_b n_m (s_b)^2 + (c_b)^2 n_a n_m (s_a)^2 + n_a n_b (n_m)^2 (s_a)^2 (s_b)^2$

$S^2 - B^2 = (c_a)^2 (c_b)^2 + (c_a)^2 n_b n_m (s_b)^2 + (c_b)^2 n_a n_m (s_a)^2 + n_a n_b (s_a)^2 (s_b)^2$

Consider all combinations of$\boldsymbol{e}_a^2, \boldsymbol{e}_b^2and\boldsymbol{e}_m^2$:

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, -1, -1]$

$S^2 - \boldsymbol{B}^2 = \sin(\theta_a)^2 \sin(\theta_b)^2 + \sin(\theta_a)^2 \cos(\theta_b)^2 + \sin(\theta_b)^2 \cos(\theta_a)^2 + \cos(\theta_a)^2 \cos(\theta_b)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, -1, 1]$

$S^2 - \boldsymbol{B}^2 = \sinh(\theta_a)^2 \sinh(\theta_b)^2 - \sinh(\theta_a)^2 \cosh(\theta_b)^2 - \sinh(\theta_b)^2 \cosh(\theta_a)^2 + \cosh(\theta_a)^2 \cosh(\theta_b)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, 1, -1]$

$S^2 - \boldsymbol{B}^2 = -\sin(\theta_a)^2 \sinh(\theta_b)^2 + \sin(\theta_a)^2 \cosh(\theta_b)^2 - \cos(\theta_a)^2 \sinh(\theta_b)^2 + \cos(\theta_a)^2 \cosh(\theta_b)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [-1, 1, 1]$

$S^2 - \boldsymbol{B}^2 = -\sin(\theta_b)^2 \sinh(\theta_a)^2 + \sin(\theta_b)^2 \cosh(\theta_a)^2 - \cos(\theta_b)^2 \sinh(\theta_a)^2 + \cos(\theta_b)^2 \cosh(\theta_a)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, -1, -1]$

$S^2 - \boldsymbol{B}^2 = -\sin(\theta_b)^2 \sinh(\theta_a)^2 + \sin(\theta_b)^2 \cosh(\theta_a)^2 - \cos(\theta_b)^2 \sinh(\theta_a)^2 + \cos(\theta_b)^2 \cosh(\theta_a)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, -1, 1]$

$S^2 - \boldsymbol{B}^2 = -\sin(\theta_a)^2 \sinh(\theta_b)^2 + \sin(\theta_a)^2 \cosh(\theta_b)^2 - \cos(\theta_a)^2 \sinh(\theta_b)^2 + \cos(\theta_a)^2 \cosh(\theta_b)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, 1, -1]$

$S^2 - \boldsymbol{B}^2 = \sinh(\theta_a)^2 \sinh(\theta_b)^2 - \sinh(\theta_a)^2 \cosh(\theta_b)^2 - \sinh(\theta_b)^2 \cosh(\theta_a)^2 + \cosh(\theta_a)^2 \cosh(\theta_b)^2 = 1$

$\left[\boldsymbol{e}_a^2, \boldsymbol{e}_b^2, \boldsymbol{e}_m^2\right] = [1, 1, 1]$

$S^2 - \boldsymbol{B}^2 = \sin(\theta_a)^2 \sin(\theta_b)^2 + \sin(\theta_a)^2 \cos(\theta_b)^2 + \sin(\theta_b)^2 \cos(\theta_a)^2 + \cos(\theta_a)^2 \cos(\theta_b)^2 = 1$

Thus we have shown that $R_a R_b = S + \boldsymbol{D} = e^{\boldsymbol{C}}$where$\boldsymbol{C}$is a bivector blade.