3d orthogonal ($A$ is vector function)

$$A = A^{x}\boldsymbol{e}_{x} + A^{y}\boldsymbol{e}_{y} + A^{z}\boldsymbol{e}_{z}$$

$$A^{2} = A^{x2} + A^{y2} + A^{z2}$$

$$\nabla \cdot A = \partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z}$$

$$\nabla A = \left ( \partial_{x} A^{x} + \partial_{y} A^{y} + \partial_{z} A^{z} \right ) + \left ( - \partial_{y} A^{x} + \partial_{x} A^{y} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left ( - \partial_{z} A^{x} + \partial_{x} A^{z} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left ( - \partial_{z} A^{y} + \partial_{y} A^{z} \right ) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$v \cdot (\nabla A) = \left ( v^{y} \partial_{y} A^{x} - v^{y} \partial_{x} A^{y} + v^{z} \partial_{z} A^{x} - v^{z} \partial_{x} A^{z} \right ) \boldsymbol{e}_{x} + \left ( - v^{x} \partial_{y} A^{x} + v^{x} \partial_{x} A^{y} + v^{z} \partial_{z} A^{y} - v^{z} \partial_{y} A^{z} \right ) \boldsymbol{e}_{y} + \left ( - v^{x} \partial_{z} A^{x} + v^{x} \partial_{x} A^{z} - v^{y} \partial_{z} A^{y} + v^{y} \partial_{y} A^{z} \right ) \boldsymbol{e}_{z}$$

2d general ($A$ is vector function)

$$A = A^{u}\boldsymbol{e}_{u} + A^{v}\boldsymbol{e}_{v}$$

$$A^{2} = \left ( e_{u}\cdot e_{u} \right ) A^{u2} + 2 \left ( e_{u}\cdot e_{v} \right ) A^{u} A^{v} + \left ( e_{v}\cdot e_{v} \right ) A^{v2}$$

$$\nabla \cdot A = \partial_{u} A^{u} + \partial_{v} A^{v}$$

$$\nabla A = \left ( \partial_{u} A^{u} + \partial_{v} A^{v} \right ) + \frac{- \left ( e_{u}\cdot e_{u} \right ) \partial_{v} A^{u} + \left ( e_{u}\cdot e_{v} \right ) \partial_{u} A^{u} - \left ( e_{u}\cdot e_{v} \right ) \partial_{v} A^{v} + \left ( e_{v}\cdot e_{v} \right ) \partial_{u} A^{v}}{\left ( e_{u}\cdot e_{u} \right ) \left ( e_{v}\cdot e_{v} \right ) - \left ( e_{u}\cdot e_{v} \right )^{2}} \boldsymbol{e}_{u}\wedge \boldsymbol{e}_{v}$$

3d orthogonal ($A,\ B$ are linear transformations)

$$A = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & A_{xx}\boldsymbol{e}_{x} + A_{yx}\boldsymbol{e}_{y} + A_{zx}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & A_{xy}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{zy}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & A_{xz}\boldsymbol{e}_{x} + A_{yz}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right \}$$

$$mat \left ( A \right ) = \left [ \begin{array}{ccc} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{array} \right ]$$

$$\det \left ( A \right ) = A_{xz} \left ( A_{yx} A_{zy} - A_{yy} A_{zx} \right ) - A_{yz} \left ( A_{xx} A_{zy} - A_{xy} A_{zx} \right ) + A_{zz} \left ( A_{xx} A_{yy} - A_{xy} A_{yx} \right )$$

$$\overline{A} = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & A_{xx}\boldsymbol{e}_{x} + A_{xy}\boldsymbol{e}_{y} + A_{xz}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & A_{yx}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{yz}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & A_{zx}\boldsymbol{e}_{x} + A_{zy}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right \}$$

$$\mathrm{Tr} \left ( A \right ) = A_{xx} + A_{yy} + A_{zz}$$

$$A \left ( e_{x}\wedge e_{y} \right ) = \left ( A_{xx} A_{yy} - A_{xy} A_{yx} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left ( A_{xx} A_{zy} - A_{xy} A_{zx} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left ( A_{yx} A_{zy} - A_{yy} A_{zx} \right ) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$A \left ( e_{x} \right ) \wedge A \left ( e_{y} \right ) = \left ( A_{xx} A_{yy} - A_{xy} A_{yx} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{y} + \left ( A_{xx} A_{zy} - A_{xy} A_{zx} \right ) \boldsymbol{e}_{x}\wedge \boldsymbol{e}_{z} + \left ( A_{yx} A_{zy} - A_{yy} A_{zx} \right ) \boldsymbol{e}_{y}\wedge \boldsymbol{e}_{z}$$

$$g = \left [ \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array} \right ]$$

$$g^{-1} = \left[\begin{matrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{matrix}\right]$$

$$A + B = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & \left ( A_{xx} + B_{xx} \right ) \boldsymbol{e}_{x} + \left ( A_{yx} + B_{yx} \right ) \boldsymbol{e}_{y} + \left ( A_{zx} + B_{zx} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & \left ( A_{xy} + B_{xy} \right ) \boldsymbol{e}_{x} + \left ( A_{yy} + B_{yy} \right ) \boldsymbol{e}_{y} + \left ( A_{zy} + B_{zy} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & \left ( A_{xz} + B_{xz} \right ) \boldsymbol{e}_{x} + \left ( A_{yz} + B_{yz} \right ) \boldsymbol{e}_{y} + \left ( A_{zz} + B_{zz} \right ) \boldsymbol{e}_{z} \end{array} \right \}$$

$$AB = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & \left ( A_{xx} B_{xx} + A_{xy} B_{yx} + A_{xz} B_{zx} \right ) \boldsymbol{e}_{x} + \left ( A_{yx} B_{xx} + A_{yy} B_{yx} + A_{yz} B_{zx} \right ) \boldsymbol{e}_{y} + \left ( A_{zx} B_{xx} + A_{zy} B_{yx} + A_{zz} B_{zx} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & \left ( A_{xx} B_{xy} + A_{xy} B_{yy} + A_{xz} B_{zy} \right ) \boldsymbol{e}_{x} + \left ( A_{yx} B_{xy} + A_{yy} B_{yy} + A_{yz} B_{zy} \right ) \boldsymbol{e}_{y} + \left ( A_{zx} B_{xy} + A_{zy} B_{yy} + A_{zz} B_{zy} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & \left ( A_{xx} B_{xz} + A_{xy} B_{yz} + A_{xz} B_{zz} \right ) \boldsymbol{e}_{x} + \left ( A_{yx} B_{xz} + A_{yy} B_{yz} + A_{yz} B_{zz} \right ) \boldsymbol{e}_{y} + \left ( A_{zx} B_{xz} + A_{zy} B_{yz} + A_{zz} B_{zz} \right ) \boldsymbol{e}_{z} \end{array} \right \}$$

$$A - B = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & \left ( A_{xx} - B_{xx} \right ) \boldsymbol{e}_{x} + \left ( A_{yx} - B_{yx} \right ) \boldsymbol{e}_{y} + \left ( A_{zx} - B_{zx} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & \left ( A_{xy} - B_{xy} \right ) \boldsymbol{e}_{x} + \left ( A_{yy} - B_{yy} \right ) \boldsymbol{e}_{y} + \left ( A_{zy} - B_{zy} \right ) \boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & \left ( A_{xz} - B_{xz} \right ) \boldsymbol{e}_{x} + \left ( A_{yz} - B_{yz} \right ) \boldsymbol{e}_{y} + \left ( A_{zz} - B_{zz} \right ) \boldsymbol{e}_{z} \end{array} \right \}$$

General Symmetric Linear Transformation

$$A = \left \{ \begin{array}{ll} L \left ( \boldsymbol{e}_{x} \right ) = & A_{xx}\boldsymbol{e}_{x} + A_{xy}\boldsymbol{e}_{y} + A_{xz}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{y} \right ) = & A_{xy}\boldsymbol{e}_{x} + A_{yy}\boldsymbol{e}_{y} + A_{yz}\boldsymbol{e}_{z} \\ L \left ( \boldsymbol{e}_{z} \right ) = & A_{xz}\boldsymbol{e}_{x} + A_{yz}\boldsymbol{e}_{y} + A_{zz}\boldsymbol{e}_{z} \end{array} \right \}$$

General Antisymmetric Linear Transformation

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_x\right) = & -A_{xy}\boldsymbol{e}_y - A_{xz}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_y\right) = & A_{xy}\boldsymbol{e}_x - A_{yz}\boldsymbol{e}_z \\ L\left(\boldsymbol{e}_z\right) = & A_{xz}\boldsymbol{e}_x + A_{yz}\boldsymbol{e}_y \end{array} \right\}$$

2d general ($A$, $B$ are linear transformations)

$$g = \left[ \begin{array}{cc} (e_u\cdot e_u) & (e_u\cdot e_v) \\ (e_u\cdot e_v) & (e_v\cdot e_v) \end{array} \right]$$

$$g^{-1} = \left[ \begin{array}{cc} \frac{(e_v\cdot e_v)}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} & -\frac{(e_u\cdot e_v)}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} \\ -\frac{(e_u\cdot e_v)}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} & \frac{(e_u\cdot e_u)}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} \end{array} \right]$$

$$gg^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A_{uv}+(e_v\cdot e_v)A_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{-(e_u\cdot e_v)A_{vv}+(e_v\cdot e_v)A_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{(e_u\cdot e_u)A_{uv}-(e_u\cdot e_v)A_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A_{vv}-(e_u\cdot e_v)A_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$mat\left(A\right) = \left[ \begin{array}{cc} A_{uu} & A_{uv} \\ A_{vu} & A_{vv} \end{array} \right]$$

$$\det\left(A\right) = \frac{-\left(\frac{(e_u\cdot e_v)A_{vv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} - \frac{(e_v\cdot e_v)A_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\right)\left(-\frac{(e_u\cdot e_u)A_{uv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} + \frac{(e_u\cdot e_v)A_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\right)\boldsymbol{e}_u\wedge\boldsymbol{e}_v + \left(\frac{(e_u\cdot e_u)A_{vv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} - \frac{(e_u\cdot e_v)A_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\right)\left(-\frac{(e_u\cdot e_v)A_{uv}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2} + \frac{(e_v\cdot e_v)A_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\right)\boldsymbol{e}_u\wedge\boldsymbol{e}_v}{\sqrt{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}}$$

$$\overline{A} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_u)^2(e_u\cdot e_v)A_{uv}+(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{uv}+(e_u\cdot e_u)(e_v\cdot e_v)^2A_{uu}+(e_u\cdot e_v)^3A_{vu}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vv}+(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_u + \frac{A_{uv}(e_u\cdot e_u)^3-(e_u\cdot e_u)^2(e_u\cdot e_v)A_{uu}+(e_u\cdot e_u)^2(e_u\cdot e_v)A_{vv}+(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{vu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^3A_{vv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{vv}+(e_u\cdot e_v)^3A_{uu}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uv}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vu}+(e_u\cdot e_v)(e_v\cdot e_v)^2A_{uu}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vv}+A_{vu}(e_v\cdot e_v)^3}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)^2(e_u\cdot e_v)A_{uv}+(e_u\cdot e_u)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}+(e_u\cdot e_v)^3A_{uv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_v \end{array} \right\}$$

$$mat\left(\overline{A}\right) = \left[ \begin{array}{cc} -\frac{(e_u\cdot e_u)\left((e_u\cdot e_u)^2(e_u\cdot e_v)A_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uu}+(e_u\cdot e_u)(e_u\cdot e_v)^2A_{vv}+(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{uv}-(e_u\cdot e_u)(e_v\cdot e_v)^2A_{uu}-(e_u\cdot e_v)^3A_{vu}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vu}\right)+(e_u\cdot e_v)\left((e_u\cdot e_u)(e_u\cdot e_v)^2A_{uv}+(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)^3A_{uu}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vu}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{uu}+(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vv}-A_{vu}(e_v\cdot e_v)^3\right)}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4} & \frac{-(e_u\cdot e_u)^2(e_u\cdot e_v)^2A_{uv}+(e_u\cdot e_u)(e_u\cdot e_v)^3A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)^3A_{vv}-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uv}+(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)^2A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vv}+A_{vu}(e_u\cdot e_v)^4+(e_u\cdot e_v)^3(e_v\cdot e_v)A_{uu}-(e_u\cdot e_v)^3(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)^2(e_v\cdot e_v)^2A_{uv}+2(e_u\cdot e_v)^2(e_v\cdot e_v)^2A_{vu}+(e_u\cdot e_v)A_{uu}(e_v\cdot e_v)^3-(e_u\cdot e_v)A_{vv}(e_v\cdot e_v)^3+A_{vu}(e_v\cdot e_v)^4}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4} \\ \frac{(e_u\cdot e_u)\left(A_{uv}(e_u\cdot e_u)^3-(e_u\cdot e_u)^2(e_u\cdot e_v)A_{uu}+(e_u\cdot e_u)^2(e_u\cdot e_v)A_{vv}+(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{vu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^3A_{vv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vu}\right)+(e_u\cdot e_v)\left((e_u\cdot e_u)^2(e_u\cdot e_v)A_{uv}+(e_u\cdot e_u)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}+(e_u\cdot e_v)^3A_{uv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vu}\right)}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4} & \frac{(e_u\cdot e_v)\left(A_{uv}(e_u\cdot e_u)^3-(e_u\cdot e_u)^2(e_u\cdot e_v)A_{uu}+(e_u\cdot e_u)^2(e_u\cdot e_v)A_{vv}+(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{vu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^3A_{vv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vu}\right)+(e_v\cdot e_v)\left((e_u\cdot e_u)^2(e_u\cdot e_v)A_{uv}+(e_u\cdot e_u)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)^2A_{uu}-(e_u\cdot e_u)(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}+(e_u\cdot e_v)^3A_{uv}-(e_u\cdot e_v)^2(e_v\cdot e_v)A_{uu}+(e_u\cdot e_v)^2(e_v\cdot e_v)A_{vv}-(e_u\cdot e_v)(e_v\cdot e_v)^2A_{vu}\right)}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4} \end{array} \right]$$

$$\mathrm{Tr}\left(A\right) = -\frac{\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)A_{vv}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} + \frac{\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2A_{vv}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} + \frac{\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A_{uv}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} + \frac{\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)\left(e_v\cdot e_v\right)A_{vu}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} - \frac{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right)^2A_{uu}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} - \frac{\left(e_u\cdot e_v\right)^3A_{uv}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} - \frac{\left(e_u\cdot e_v\right)^3A_{vu}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4} + \frac{\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)A_{uu}}{-\left(e_u\cdot e_u\right)^2\left(e_v\cdot e_v\right)^2+2\left(e_u\cdot e_u\right)\left(e_u\cdot e_v\right)^2\left(e_v\cdot e_v\right)-\left(e_u\cdot e_v\right)^4}$$

$$A\left(e_u\wedge e_v\right) = \frac{A_{uu}A_{vv} - A_{uv}A_{vu}}{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}\boldsymbol{e}_u\wedge\boldsymbol{e}_v$$

$$A\left(e_u\right)\wedge A\left(e_v\right) = \frac{A_{uu}A_{vv} - A_{uv}A_{vu}}{\left(e_u\cdot e_u\right)\left(e_v\cdot e_v\right) - \left(e_u\cdot e_v\right)^2}\boldsymbol{e}_u\wedge\boldsymbol{e}_v$$

$$B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)B_{uv}+(e_v\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{-(e_u\cdot e_v)B_{vv}+(e_v\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{(e_u\cdot e_u)B_{uv}-(e_u\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)B_{vv}-(e_u\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$A + B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A_{uv}-(e_u\cdot e_v)B_{uv}+(e_v\cdot e_v)A_{uu}+(e_v\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{-(e_u\cdot e_v)A_{vv}-(e_u\cdot e_v)B_{vv}+(e_v\cdot e_v)A_{vu}+(e_v\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{(e_u\cdot e_u)A_{uv}+(e_u\cdot e_u)B_{uv}-(e_u\cdot e_v)A_{uu}-(e_u\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A_{vv}+(e_u\cdot e_u)B_{vv}-(e_u\cdot e_v)A_{vu}-(e_u\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$AB = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_u)(e_u\cdot e_v)A_{uv}B_{vv}+(e_u\cdot e_u)(e_v\cdot e_v)A_{uv}B_{vu}+(e_u\cdot e_v)^2A_{uu}B_{vv}+(e_u\cdot e_v)^2A_{uv}B_{uv}-(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}B_{uv}-(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}B_{vu}-(e_u\cdot e_v)(e_v\cdot e_v)A_{uv}B_{uu}+(e_v\cdot e_v)^2A_{uu}B_{uu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_u + \frac{-(e_u\cdot e_u)(e_u\cdot e_v)A_{vv}B_{vv}+(e_u\cdot e_u)(e_v\cdot e_v)A_{vv}B_{vu}+(e_u\cdot e_v)^2A_{vu}B_{vv}+(e_u\cdot e_v)^2A_{vv}B_{uv}-(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}B_{uv}-(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}B_{vu}-(e_u\cdot e_v)(e_v\cdot e_v)A_{vv}B_{uu}+(e_v\cdot e_v)^2A_{vu}B_{uu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{(e_u\cdot e_u)^2A_{uv}B_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{uu}B_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{uv}B_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{uv}B_{vu}+(e_u\cdot e_u)(e_v\cdot e_v)A_{uu}B_{uv}+(e_u\cdot e_v)^2A_{uu}B_{vu}+(e_u\cdot e_v)^2A_{uv}B_{uu}-(e_u\cdot e_v)(e_v\cdot e_v)A_{uu}B_{uu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)^2A_{vv}B_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{vu}B_{vv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{vv}B_{uv}-(e_u\cdot e_u)(e_u\cdot e_v)A_{vv}B_{vu}+(e_u\cdot e_u)(e_v\cdot e_v)A_{vu}B_{uv}+(e_u\cdot e_v)^2A_{vu}B_{vu}+(e_u\cdot e_v)^2A_{vv}B_{uu}-(e_u\cdot e_v)(e_v\cdot e_v)A_{vu}B_{uu}}{(e_u\cdot e_u)^2(e_v\cdot e_v)^2-2(e_u\cdot e_u)(e_u\cdot e_v)^2(e_v\cdot e_v)+(e_u\cdot e_v)^4}\boldsymbol{e}_v \end{array} \right\}$$

$$A - B = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_u\right) = & \frac{-(e_u\cdot e_v)A_{uv}+(e_u\cdot e_v)B_{uv}+(e_v\cdot e_v)A_{uu}-(e_v\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{-(e_u\cdot e_v)A_{vv}+(e_u\cdot e_v)B_{vv}+(e_v\cdot e_v)A_{vu}-(e_v\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \\ L\left(\boldsymbol{e}_v\right) = & \frac{(e_u\cdot e_u)A_{uv}-(e_u\cdot e_u)B_{uv}-(e_u\cdot e_v)A_{uu}+(e_u\cdot e_v)B_{uu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_u + \frac{(e_u\cdot e_u)A_{vv}-(e_u\cdot e_u)B_{vv}-(e_u\cdot e_v)A_{vu}+(e_u\cdot e_v)B_{vu}}{(e_u\cdot e_u)(e_v\cdot e_v)-(e_u\cdot e_v)^2}\boldsymbol{e}_v \end{array} \right\}$$

$$a\cdot\overline{A}\left(b\right) - b\cdot\underline{A}\left(a\right) = 0$$

$$g = \left[ \begin{array}{cccc} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{array} \right]$$

$$\underline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{t}\right) = & T_{tt}\boldsymbol{e}_{t} + T_{xt}\boldsymbol{e}_{x} + T_{yt}\boldsymbol{e}_{y} + T_{zt}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{x}\right) = & -T_{tx}\boldsymbol{e}_{t} - T_{xx}\boldsymbol{e}_{x} - T_{yx}\boldsymbol{e}_{y} - T_{zx}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & -T_{ty}\boldsymbol{e}_{t} - T_{xy}\boldsymbol{e}_{x} - T_{yy}\boldsymbol{e}_{y} - T_{zy}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & -T_{tz}\boldsymbol{e}_{t} - T_{xz}\boldsymbol{e}_{x} - T_{yz}\boldsymbol{e}_{y} - T_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\overline{T} = \left\{ \begin{array}{ll} L\left(\boldsymbol{e}_{t}\right) = & T_{tt}\boldsymbol{e}_{t} + T_{tx}\boldsymbol{e}_{x} + T_{ty}\boldsymbol{e}_{y} + T_{tz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{x}\right) = & -T_{xt}\boldsymbol{e}_{t} - T_{xx}\boldsymbol{e}_{x} - T_{xy}\boldsymbol{e}_{y} - T_{xz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{y}\right) = & -T_{yt}\boldsymbol{e}_{t} - T_{yx}\boldsymbol{e}_{x} - T_{yy}\boldsymbol{e}_{y} - T_{yz}\boldsymbol{e}_{z} \\ L\left(\boldsymbol{e}_{z}\right) = & -T_{zt}\boldsymbol{e}_{t} - T_{zx}\boldsymbol{e}_{x} - T_{zy}\boldsymbol{e}_{y} - T_{zz}\boldsymbol{e}_{z} \end{array} \right\}$$

$$\det\left(\underline{T}\right) = T_{tz}\left(T_{xt}T_{yx}T_{zy} - T_{xt}T_{yy}T_{zx} - T_{xx}T_{yt}T_{zy} + T_{xx}T_{yy}T_{zt} + T_{xy}T_{yt}T_{zx} - T_{xy}T_{yx}T_{zt}\right) - T_{xz}\left(T_{tt}T_{yx}T_{zy} - T_{tt}T_{yy}T_{zx} - T_{tx}T_{yt}T_{zy} + T_{tx}T_{yy}T_{zt} + T_{ty}T_{yt}T_{zx} - T_{ty}T_{yx}T_{zt}\right) + T_{yz}\left(T_{tt}T_{xx}T_{zy} - T_{tt}T_{xy}T_{zx} - T_{tx}T_{xt}T_{zy} + T_{tx}T_{xy}T_{zt} + T_{ty}T_{xt}T_{zx} - T_{ty}T_{xx}T_{zt}\right) - T_{zz}\left(T_{tt}T_{xx}T_{yy} - T_{tt}T_{xy}T_{yx} - T_{tx}T_{xt}T_{yy} + T_{tx}T_{xy}T_{yt} + T_{ty}T_{xt}T_{yx} - T_{ty}T_{xx}T_{yt}\right)$$

$$\operatorname{tr}\left(\underline{T}\right) = T_{tt} - T_{xx} - T_{yy} - T_{zz}$$

$$a \cdot \overline{T}\left(b\right) - b \cdot \underline{T}\left(a\right) = 0$$

$$f = f$$

$$\nabla f = \partial_{u} f \boldsymbol{e}_{u} + \frac{\partial_{v} f}{\sin\left(u\right)}\boldsymbol{e}_{v}$$

$$F = F^{u}\boldsymbol{e}_{u} + F^{v}\boldsymbol{e}_{v}$$

$$\nabla F = \left(\frac{F^{u}}{\tan\left(u\right)} + \partial_{u} F^{u} + \frac{\partial_{v} F^{v}}{\sin\left(u\right)}\right) + \left(\frac{F^{v}}{\tan\left(u\right)} + \partial_{u} F^{v} - \frac{\partial_{v} F^{u}}{\sin\left(u\right)}\right)\boldsymbol{e}_{u}\wedge \boldsymbol{e}_{v}$$

$$f = f$$

$$\nabla f = \partial_{\theta} f \boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f}{\sin\left(\theta\right)}\boldsymbol{e}_{\phi}$$

$$F = F^{\theta}\boldsymbol{e}_{\theta} + F^{\phi}\boldsymbol{e}_{\phi}$$

$$\nabla F = \left(\frac{F^{\theta}}{\tan\left(\theta\right)} + \partial_{\theta} F^{\theta} + \frac{\partial_{\phi} F^{\phi}}{\sin\left(\theta\right)}\right) + \left(\frac{F^{\phi}}{\tan\left(\theta\right)} + \partial_{\theta} F^{\phi} - \frac{\partial_{\phi} F^{\theta}}{\sin\left(\theta\right)}\right)\boldsymbol{e}_{\theta}\wedge \boldsymbol{e}_{\phi}$$