

$$(u, v) \rightarrow (r, \theta, \phi) = [1, u, v]$$

Unit Sphere Manifold:

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin^2(u) \end{bmatrix}$$

$$a = a^u \mathbf{e}_u + a^v \mathbf{e}_v$$

$$f = f^u \mathbf{e}_u + f^v \mathbf{e}_v$$

$$\nabla = \mathbf{e}_u \frac{\partial}{\partial u} + \mathbf{e}_v \frac{1}{\sin^2(u)} \frac{\partial}{\partial v}$$

$$a \cdot \nabla = a^u \frac{\partial}{\partial u} + a^v \frac{\partial}{\partial v}$$

$$(a \cdot \nabla) \mathbf{e}_u = \frac{a^v}{\tan(u)} \mathbf{e}_v$$

$$(a \cdot \nabla) \mathbf{e}_v = -\frac{a^v}{2} \sin(2u) \mathbf{e}_u + \frac{a^u}{\tan(u)} \mathbf{e}_v$$

$$(a \cdot \nabla) f = \left(a^u \partial_u f^u - \frac{a^v f^v}{2} \sin(2u) + a^v \partial_v f^u \right) \mathbf{e}_u + \left(\frac{a^u f^v}{\tan(u)} + a^u \partial_u f^v + \frac{a^v f^u}{\tan(u)} + a^v \partial_v f^v \right) \mathbf{e}_v$$

$$\nabla f = \left(\frac{f^u}{\tan(u)} + \partial_u f^u + \partial_v f^v \right) + \left(\frac{2f^v}{\tan(u)} + \partial_u f^v - \frac{\partial_v f^u}{\sin^2(u)} \right) \mathbf{e}_u \wedge \mathbf{e}_v$$

1-D Manifold On Unit Sphere:

$$\nabla = \mathbf{e}_s \frac{1}{\sin^2(u^s) (\partial_s v^s)^2 + (\partial_s u^s)^2} \frac{\partial}{\partial s}$$

$$\nabla g = \frac{\partial_s g}{\sin^2(u^s) (\partial_s v^s)^2 + (\partial_s u^s)^2} \mathbf{e}_s$$

$$\nabla \cdot \mathbf{h} = \frac{1}{\sin^2(u^s) (\partial_s v^s)^2 + (\partial_s u^s)^2} \left(\left(\sin^2(u^s) (\partial_s v^s)^2 + (\partial_s u^s)^2 \right) \partial_s h^s + \left(\sin^2(u^s) \partial_s v^s \partial_s^2 v^s + \frac{\partial_s u^s}{2} \sin(2u^s) (\partial_s v^s)^2 + \partial_s u^s \partial_s^2 u^s \right) h^s \right)$$