

$$\boldsymbol{A} = A + A^x \boldsymbol{e}_x + A^y \boldsymbol{e}_y + A^z \boldsymbol{e}_z + A^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y + A^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z + A^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z + A^{xyz} \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$\boldsymbol{A} = A^x \boldsymbol{e}_x + A^y \boldsymbol{e}_y + A^z \boldsymbol{e}_z$$

$$\boldsymbol{B} = B^{xy} \boldsymbol{e}_x \wedge \boldsymbol{e}_y + B^{xz} \boldsymbol{e}_x \wedge \boldsymbol{e}_z + B^{yz} \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$\boldsymbol{\nabla} f = \partial_x f \boldsymbol{e}_x + \partial_y f \boldsymbol{e}_y + \partial_z f \boldsymbol{e}_z$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{A} = \partial_x A^x + \partial_y A^y + \partial_z A^z$$

$$\boldsymbol{\nabla} \boldsymbol{A} = (\partial_x A^x + \partial_y A^y + \partial_z A^z) + (-\partial_y A^x + \partial_x A^y) \boldsymbol{e}_x \wedge \boldsymbol{e}_y + (-\partial_z A^x + \partial_x A^z) \boldsymbol{e}_x \wedge \boldsymbol{e}_z + (-\partial_z A^y + \partial_y A^z) \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$-I(\boldsymbol{\nabla} \wedge \boldsymbol{A}) = (-\partial_z A^y + \partial_y A^z) \boldsymbol{e}_x + (\partial_z A^x - \partial_x A^z) \boldsymbol{e}_y + (-\partial_y A^x + \partial_x A^y) \boldsymbol{e}_z$$

$$\boldsymbol{\nabla} \boldsymbol{B} = (-\partial_y B^{xy} - \partial_z B^{xz}) \boldsymbol{e}_x + (\partial_x B^{xy} - \partial_z B^{yz}) \boldsymbol{e}_y + (\partial_x B^{xz} + \partial_y B^{yz}) \boldsymbol{e}_z + (\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}) \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$\boldsymbol{\nabla} \wedge \boldsymbol{B} = (\partial_z B^{xy} - \partial_y B^{xz} + \partial_x B^{yz}) \boldsymbol{e}_x \wedge \boldsymbol{e}_y \wedge \boldsymbol{e}_z$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{B} = (-\partial_y B^{xy} - \partial_z B^{xz}) \boldsymbol{e}_x + (\partial_x B^{xy} - \partial_z B^{yz}) \boldsymbol{e}_y + (\partial_x B^{xz} + \partial_y B^{yz}) \boldsymbol{e}_z$$

$$g_{ij} = \begin{bmatrix} (a \cdot a) & (a \cdot b) & (a \cdot c) & (a \cdot d) \\ (a \cdot b) & (b \cdot b) & (b \cdot c) & (b \cdot d) \\ (a \cdot c) & (b \cdot c) & (c \cdot c) & (c \cdot d) \\ (a \cdot d) & (b \cdot d) & (c \cdot d) & (d \cdot d) \end{bmatrix}$$

$$\boldsymbol{a} \cdot (\boldsymbol{bc}) = -(a \cdot c) \boldsymbol{b} + (a \cdot b) \boldsymbol{c}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) = -(a \cdot c) \boldsymbol{b} + (a \cdot b) \boldsymbol{c}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) = (a \cdot d) \boldsymbol{b} \wedge \boldsymbol{c} - (a \cdot c) \boldsymbol{b} \wedge \boldsymbol{d} + (a \cdot b) \boldsymbol{c} \wedge \boldsymbol{d}$$

$$\boldsymbol{a} \cdot (\boldsymbol{b} \wedge \boldsymbol{c}) + \boldsymbol{c} \cdot (\boldsymbol{a} \wedge \boldsymbol{b}) + \boldsymbol{b} \cdot (\boldsymbol{c} \wedge \boldsymbol{a}) = 0$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b}) = 3\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}$$

$$\boldsymbol{a}(\boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) - \boldsymbol{b}(\boldsymbol{a} \wedge \boldsymbol{c} \wedge \boldsymbol{d}) + \boldsymbol{c}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{d}) - \boldsymbol{d}(\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c}) = 4\boldsymbol{a} \wedge \boldsymbol{b} \wedge \boldsymbol{c} \wedge \boldsymbol{d}$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \cdot (\boldsymbol{c} \wedge \boldsymbol{d}) = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$((\boldsymbol{a} \wedge \boldsymbol{b}) \cdot \boldsymbol{c}) \cdot \boldsymbol{d} = -(a \cdot c) (b \cdot d) + (a \cdot d) (b \cdot c)$$

$$(\boldsymbol{a} \wedge \boldsymbol{b}) \times (\boldsymbol{c} \wedge \boldsymbol{d}) = -(b \cdot d) \boldsymbol{a} \wedge \boldsymbol{c} + (b \cdot c) \boldsymbol{a} \wedge \boldsymbol{d} + (a \cdot d) \boldsymbol{b} \wedge \boldsymbol{c} - (a \cdot c) \boldsymbol{b} \wedge \boldsymbol{d}$$

$$E = \boldsymbol{e}_1 \wedge \boldsymbol{e}_2 \wedge \boldsymbol{e}_3$$

$$E^2 = (e_1 \cdot e_2)^2 - 2 (e_1 \cdot e_2) (e_1 \cdot e_3) (e_2 \cdot e_3) + (e_1 \cdot e_3)^2 + (e_2 \cdot e_3)^2 - 1$$

$$E1 = (e2 \wedge e3)E = \left((e_2 \cdot e_3)^2 - 1 \right) \boldsymbol{e}_1 + ((e_1 \cdot e_2) - (e_1 \cdot e_3) (e_2 \cdot e_3)) \boldsymbol{e}_2 + (- (e_1 \cdot e_2) (e_2 \cdot e_3) + (e_1 \cdot e_3)) \boldsymbol{e}_3$$

$$E2 = -(e1 \wedge e3)E = ((e_1 \cdot e_2) - (e_1 \cdot e_3) (e_2 \cdot e_3)) \boldsymbol{e}_1 + \left((e_1 \cdot e_3)^2 - 1 \right) \boldsymbol{e}_2 + (- (e_1 \cdot e_2) (e_1 \cdot e_3) + (e_2 \cdot e_3)) \boldsymbol{e}_3$$

$$E3 = (e1 \wedge e2)E = (- (e_1 \cdot e_2) (e_2 \cdot e_3) + (e_1 \cdot e_3)) \boldsymbol{e}_1 + (- (e_1 \cdot e_2) (e_1 \cdot e_3) + (e_2 \cdot e_3)) \boldsymbol{e}_2 + \left((e_1 \cdot e_2)^2 - 1 \right) \boldsymbol{e}_3$$

$$E1 \cdot e2 = 0$$

$$E1 \cdot e3 = 0$$

$$E2 \cdot e1 = 0$$

$$E2 \cdot e3 = 0$$

$$E3 \cdot e1 = 0$$

$$E3 \cdot e2 = 0$$

$$(E1 \cdot e1)/E^2 = 1$$

$$(E2 \cdot e2)/E^2 = 1$$

$$(E3 \cdot e3)/E^2 = 1$$

$$A = A^r \boldsymbol{e}_r + A^\theta \boldsymbol{e}_\theta + A^\phi \boldsymbol{e}_\phi$$

$$B = B^{r\theta} \boldsymbol{e}_r \wedge \boldsymbol{e}_\theta + B^{r\phi} \boldsymbol{e}_r \wedge \boldsymbol{e}_\phi + B^{\phi\phi} \boldsymbol{e}_\theta \wedge \boldsymbol{e}_\phi$$

$$\nabla f = \partial_r f \mathbf{e}_r + \frac{1}{r} \partial_\theta f \mathbf{e}_\theta + \frac{\partial_\phi f}{r \sin(\theta)} \mathbf{e}_\phi$$

$$\nabla \cdot A = \frac{1}{r} \left(r \partial_r A^r + 2A^r + \frac{A^\theta}{\tan(\theta)} + \partial_\theta A^\theta + \frac{\partial_\phi A^\phi}{\sin(\theta)} \right)$$

$$-I(\nabla \wedge A) = \frac{1}{r} \left(\frac{A^\phi}{\tan(\theta)} + \partial_\theta A^\phi - \frac{\partial_\phi A^\theta}{\sin(\theta)} \right) \mathbf{e}_r + \frac{1}{r} \left(-r \partial_r A^\phi - A^\phi + \frac{\partial_\phi A^r}{\sin(\theta)} \right) \mathbf{e}_\theta + \frac{1}{r} (r \partial_r A^\theta + A^\theta - \partial_\theta A^r) \mathbf{e}_\phi$$

$$\nabla \wedge B = \frac{1}{r} \left(r \partial_r B^{\phi\phi} - \frac{B^{r\phi}}{\tan(\theta)} + 2B^{\phi\phi} - \partial_\theta B^{r\phi} + \frac{\partial_\phi B^{r\theta}}{\sin(\theta)} \right) \mathbf{e}_r \wedge \mathbf{e}_\theta \wedge \mathbf{e}_\phi$$

$$B = \mathbf{B}\boldsymbol{\gamma}_t = -B^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - B^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - B^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$$

$$E = \mathbf{E}\boldsymbol{\gamma}_t = -E^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$$

$$F = E + IB = -E^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z - B^z \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y + B^y \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z - B^x \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$$

$$J = J^t \boldsymbol{\gamma}_t + J^x \boldsymbol{\gamma}_x + J^y \boldsymbol{\gamma}_y + J^z \boldsymbol{\gamma}_z$$

$$\nabla F = J$$

$$R = \cosh\left(\frac{\alpha}{2}\right) + \sinh\left(\frac{\alpha}{2}\right) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x$$

$$t\boldsymbol{\gamma}_t + x\boldsymbol{\gamma}_x = t'\boldsymbol{\gamma}'_t + x'\boldsymbol{\gamma}'_x = R(t'\boldsymbol{\gamma}_t + x'\boldsymbol{\gamma}_x)R^\dagger$$

$$t\boldsymbol{\gamma}_t + x\boldsymbol{\gamma}_x = (t' \cosh(\alpha) - x' \sinh(\alpha)) \boldsymbol{\gamma}_t + (-t' \sinh(\alpha) + x' \cosh(\alpha)) \boldsymbol{\gamma}_x$$

$$\sinh(\alpha) = \gamma\beta$$

$$\cosh(\alpha) = \gamma$$

$$t\boldsymbol{\gamma}_t + x\boldsymbol{\gamma}_x = \gamma(-\beta x' + t') \boldsymbol{\gamma}_t + \gamma(-\beta t' + x') \boldsymbol{\gamma}_x$$

$$\mathbf{A} = A^t \boldsymbol{\gamma}_t + A^x \boldsymbol{\gamma}_x + A^y \boldsymbol{\gamma}_y + A^z \boldsymbol{\gamma}_z$$

$$\boldsymbol{\psi} = \psi + \psi^{tx} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x + \psi^{ty} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y + \psi^{tz} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z + \psi^{xy} \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y + \psi^{xz} \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z + \psi^{yz} \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z + \psi^{txyz} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$$