Base manifold (three dimensional)

Metric tensor (cartesian coordinates - norm = False)

$$g = \left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right]$$

Two dimensioanal submanifold - Unit sphere

Basis not normalised

$(\theta,\phi) \rightarrow (r,\theta,\phi) = [1,\ \theta,\ \phi]$

$e_\theta | e_\theta = 1$

$e_\phi | e_\phi = \sin{(\theta)}^{2}$

$$g = \left[\begin{array}{cc} 1 & 0 \\ 0 & \sin{(\theta)}^{2} \end{array}\right]$$

$$\text{g\_inv} = \begin{bmatrix}1 & 0\\0 & \frac{1}{\sin^{2}{(\theta)}}\end{bmatrix}$$

Christoffel symbols of the first kind:

$$\Gamma_{1,\alpha,\beta} = \begin{bmatrix}0 & 0\\0 & - \frac{\sin{(2\theta)}}{2}\end{bmatrix} \quad \Gamma_{2,\alpha,\beta} = \begin{bmatrix}0 & \frac{\sin{(2\theta)}}{2}\\\frac{\sin{(2\theta)}}{2} & 0\end{bmatrix}$$

Christoffel symbols of the second kind:

$$\Gamma^{1}{}_{\alpha,\beta} = \begin{bmatrix}0 & 0\\0 & - \frac{\sin{(2\theta)}}{2}\end{bmatrix} \quad \Gamma^{2}{}_{\alpha,\beta} = \begin{bmatrix}0 & \frac{1}{\tan{(\theta)}}\\\frac{1}{\tan{(\theta)}} & 0\end{bmatrix}$$

$$\nabla = \boldsymbol{e}_{\theta} \frac{\partial}{\partial \theta} + \boldsymbol{e}_{\phi} \frac{1}{\sin{(\theta)}^{2}} \frac{\partial}{\partial \phi}$$

$$\nabla f = \partial_{\theta} f \boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f}{\sin{(\theta)}^{2}} \boldsymbol{e}_{\phi}$$

$$F = F^{\theta} \boldsymbol{e}_{\theta} + F^{\phi} \boldsymbol{e}_{\phi}$$

$$\nabla F = \left(\frac{F^{\theta}}{\tan{(\theta)}} + \partial_{\phi} F^{\phi} + \partial_{\theta} F^{\theta} \right) + \left(\frac{2 F^{\phi}}{\tan{(\theta)}} + \partial_{\theta} F^{\phi} - \frac{\partial_{\phi} F^{\theta}}{\sin{(\theta)}^{2}} \right) \boldsymbol{e}_{\theta}\wedge \boldsymbol{e}_{\phi}$$

One dimensioanal submanifold

Basis not normalised

$(\phi) \rightarrow (\theta,\phi) = \left[\frac{\pi}{8},\ \phi\right]$

$$e_\phi | e_\phi = \frac{1}{2} - \frac{\sqrt{2}}{4}$$

$$g = \left[\begin{array}{c} \frac{1}{2} - \frac{\sqrt{2}}{4} \end{array}\right]$$

$$\nabla = \boldsymbol{e}_{\phi} \left(2 \sqrt{2} + 4\right) \frac{\partial}{\partial \phi}$$

$$\nabla h = \left(2 \sqrt{2} + 4\right) \partial_{\phi} h \boldsymbol{e}_{\phi}$$

$H = H^{\phi} \boldsymbol{e}_{\phi}$

$\nabla H = \partial_{\phi} H^{\phi}$