Pseudo Scalar $I = \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

$I_{xyz} = \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

$$\text{Electromagnetic Field Bi-Vector } F = \begin{aligned} & - E^{x} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \\ & - E^{y} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \\ & - E^{z} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z \\ & - B^{z} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\ & + B^{y} e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\ & - B^{x} e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \end{aligned}$$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\nabla F = 0 = \begin{aligned} & - i \left(E^{x} k_x + E^{y} k_y + E^{z} k_z\right) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_t \\ & + i \left(B^{y} k_z - B^{z} k_y - E^{x} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_x \\ & + i \left(-B^{x} k_z + B^{z} k_x - E^{y} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_y \\ & + i \left(B^{x} k_y - B^{y} k_x - E^{z} \omega\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_z \\ & + i \left(-B^{z} \omega - E^{x} k_y + E^{y} k_x\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\ & + i \left(B^{y} \omega - E^{x} k_z + E^{z} k_x\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\ & + i \left(-B^{x} \omega - E^{y} k_z + E^{z} k_y\right) e^{i(\omega t - k_x x - k_y y - k_z z)} \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \\ & - i \left(B^{x} k_x + B^{y} k_y + B^{z} k_z\right) e^{-i(-\omega t + k_x x + k_y y + k_z z)} \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \end{aligned}$$

$$\left(\boldsymbol{\nabla} F\right) / \left(i e^{iK\cdot X}\right) = 0 = \begin{aligned} & \left(-E^{x} k_x - E^{y} k_y - E^{z} k_z\right) \boldsymbol{\gamma}_t \\ & + \left(B^{y} k_z - B^{z} k_y - E^{x} \omega\right) \boldsymbol{\gamma}_x \\ & + \left(-B^{x} k_z + B^{z} k_x - E^{y} \omega\right) \boldsymbol{\gamma}_y \\ & + \left(B^{x} k_y - B^{y} k_x - E^{z} \omega\right) \boldsymbol{\gamma}_z \\ & + \left(-B^{z} \omega - E^{x} k_y + E^{y} k_x\right) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\ & + \left(B^{y} \omega - E^{x} k_z + E^{z} k_x\right) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\ & + \left(-B^{x} \omega - E^{y} k_z + E^{z} k_y\right) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \\ & + \left(-B^{x} k_x - B^{y} k_y - B^{z} k_z\right) \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \end{aligned}$$

set $e_E \cdot e_k = e_B \cdot e_k = 0$ and $e_E \cdot e_E = e_B \cdot e_B = e_k \cdot e_k = -e_t \cdot e_t = 1$

$$g = \left[\begin{array}{cccc} -1 & (e_E \cdot e_B) & 0 & 0 \\ (e_E \cdot e_B) & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$K|X = \omega t - k x_k$

$$F = \begin{aligned} & - \frac{B e^{i(\omega t - k x_k)}}{\sqrt{1 - (e_E \cdot e_B)^2}} \boldsymbol{e}_E \wedge \boldsymbol{e}_k \\ & + E e^{i(\omega t - k x_k)} \boldsymbol{e}_E \wedge \boldsymbol{t} \\ & - \frac{(e_E \cdot e_B) B e^{i(\omega t - k x_k)}}{\sqrt{1 - (e_E \cdot e_B)^2}} \boldsymbol{e}_B \wedge \boldsymbol{e}_k \end{aligned}$$

$$
\nabla F = 0 = \begin{aligned}
&- \frac{i\left(Bk + E\omega\sqrt{1-(e_E\cdot e_B)^2}\right)e^{i(\omega t - kx_k)}}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_E \\
&- \frac{i\,(e_E\cdot e_B)\,Bke^{i(\omega t - kx_k)}}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_B \\
&- \frac{i\left(B\omega + Ek\sqrt{1-(e_E\cdot e_B)^2}\right)e^{i(\omega t - kx_k)}}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_E\wedge\boldsymbol{e}_k\wedge\boldsymbol{t} \\
&- \frac{i\,(e_E\cdot e_B)\,B\omega e^{i(\omega t - kx_k)}}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_B\wedge\boldsymbol{e}_k\wedge\boldsymbol{t}
\end{aligned}
$$

$$
(\boldsymbol{\nabla}F)/\left(ie^{iK\cdot X}\right) = 0 = \begin{aligned}
&\left(-\frac{Bk}{\sqrt{1-(e_E\cdot e_B)^2}} - E\omega\right)\boldsymbol{e}_E \\
&- \frac{(e_E\cdot e_B)\,Bk}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_B \\
&+ \left(-\frac{B\omega}{\sqrt{1-(e_E\cdot e_B)^2}} - Ek\right)\boldsymbol{e}_E\wedge\boldsymbol{e}_k\wedge\boldsymbol{t} \\
&- \frac{(e_E\cdot e_B)\,B\omega}{\sqrt{1-(e_E\cdot e_B)^2}}\boldsymbol{e}_B\wedge\boldsymbol{e}_k\wedge\boldsymbol{t}
\end{aligned}
$$

Previous equation requires that: $e_E\cdot e_B = 0$ if $B \neq 0$ and $k \neq 0$

$$
(\boldsymbol{\nabla}F)/\left(ie^{iK\cdot X}\right) = 0 = \begin{aligned}
&(-Bk - E\omega)\,\boldsymbol{e}_E \\
&+ (-B\omega - Ek)\,\boldsymbol{e}_E\wedge\boldsymbol{e}_k\wedge\boldsymbol{t}
\end{aligned}
$$

$0 = -Bk - E\omega$

$0 = -B\omega - Ek$

eq3 = eq1-eq2: $0 = -\frac{E\omega}{k} + \frac{Ek}{\omega}$

eq3 = (eq1-eq2)/E: $0 = -\frac{\omega}{k} + \frac{k}{\omega}$

$k = \begin{bmatrix} -\omega \\ \omega \end{bmatrix}$

$B = \begin{bmatrix} -E \\ E \end{bmatrix}$