$$g=\begin{bmatrix}1 & (e_E\cdot e_B) & (e_E\cdot e_k) & 0\\ (e_E\cdot e_B) & 1 & (e_B\cdot e_k) & 0\\ (e_E\cdot e_k) & (e_B\cdot e_k) & 1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$$

$$X=x_E\boldsymbol{e}_E+x_B\boldsymbol{e}_B+x_k\boldsymbol{e}_k+t\boldsymbol{e}_t$$

$$K=k\boldsymbol{e}_k+\omega\boldsymbol{e}_t$$

$$K|X=(e_B\cdot e_k)\,kx_B+(e_E\cdot e_k)\,kx_E-\omega t+kx_k$$

$$\begin{aligned}F=\ &\frac{(e_B\cdot e_k)\,Be^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\wedge\boldsymbol{e}_B\\ &-\frac{Be^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\wedge\boldsymbol{e}_k\\ &+Ee^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}\boldsymbol{e}_E\wedge\boldsymbol{e}_t\\ &+\frac{(e_E\cdot e_B)\,Be^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_B\wedge\boldsymbol{e}_k\end{aligned}$$

$$\begin{aligned}\nabla F=\ &\frac{i\left(-(e_B\cdot e_k)^2Bk+Bk+E\omega\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}\right)e^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\\ &+\frac{iBk\left((e_B\cdot e_k)\,(e_E\cdot e_k)-(e_E\cdot e_B)\right)e^{i(-\omega t+k((e_B\cdot e_k)x_B+(e_E\cdot e_k)x_E+x_k))}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_B\\ &+\frac{iBk\left((e_B\cdot e_k)\,(e_E\cdot e_B)-(e_E\cdot e_k)\right)e^{i(-\omega t+k((e_B\cdot e_k)x_B+(e_E\cdot e_k)x_E+x_k))}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_k\\ &+i\,(e_E\cdot e_k)\,Eke^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}\boldsymbol{e}_t\\ &+\frac{i\,(e_B\cdot e_k)\,Bke^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\wedge\boldsymbol{e}_B\wedge\boldsymbol{e}_k\\ &+\frac{i\,(e_B\cdot e_k)\,B\omega e^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\wedge\boldsymbol{e}_B\wedge\boldsymbol{e}_t\\ &-\frac{i\left(B\omega+Ek\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}\right)e^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_E\wedge\boldsymbol{e}_k\wedge\boldsymbol{e}_t\\ &+\frac{i\,(e_E\cdot e_B)\,B\omega e^{i((e_B\cdot e_k)kx_B+(e_E\cdot e_k)kx_E-\omega t+kx_k)}}{\sqrt{-(e_B\cdot e_k)^2+2\,(e_B\cdot e_k)\,(e_E\cdot e_B)\,(e_E\cdot e_k)-(e_E\cdot e_B)^2-(e_E\cdot e_k)^2+1}}\boldsymbol{e}_B\wedge\boldsymbol{e}_k\wedge\boldsymbol{e}_t\end{aligned}$$

Substituting $e_E\cdot e_B=e_E\cdot e_k=e_B\cdot e_k=0$

$$\begin{aligned}(\boldsymbol{\nabla}F)/\left(ie^{iK\cdot X}\right)=\ &(Bk+E\omega)\,\boldsymbol{e}_E\\ &+(-B\omega-Ek)\,\boldsymbol{e}_E\wedge\boldsymbol{e}_k\wedge\boldsymbol{e}_t\end{aligned}$$

$$g=\begin{bmatrix}1 & (e_E\cdot e_B) & (e_E\cdot e_k) & 0\\ (e_E\cdot e_B) & 1 & (e_B\cdot e_k) & 0\\ (e_E\cdot e_k) & (e_B\cdot e_k) & 1 & 0\\ 0 & 0 & 0 & -1\end{bmatrix}$$

$$X = \begin{aligned} & x_E \boldsymbol{e}_E \\ & + x_B \boldsymbol{e}_B \\ & + x_k \boldsymbol{e}_k \\ & + t \boldsymbol{e}_t \end{aligned}$$

$$K = \begin{aligned} & k \boldsymbol{e}_k \\ & + \omega \boldsymbol{e}_t \end{aligned}$$

$$K|X = (e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k$$

$$F = \begin{aligned} & \frac{(e_B \cdot e_k)\, B \sin\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_E \wedge \boldsymbol{e}_B \\ & - \frac{B \sin\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_E \wedge \boldsymbol{e}_k \\ & + E \sin\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right) \boldsymbol{e}_E \wedge \boldsymbol{e}_t \\ & + \frac{(e_E \cdot e_B)\, B \sin\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_B \wedge \boldsymbol{e}_k \end{aligned}$$

$$\nabla F = \begin{aligned} & \frac{\left(-(e_B \cdot e_k)^2 Bk + Bk + E\omega \sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}\right) \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_E \\ & + \frac{Bk\left((e_B \cdot e_k)(e_E \cdot e_k) - (e_E \cdot e_B)\right) \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_B \\ & + \frac{Bk\left((e_B \cdot e_k)(e_E \cdot e_B) - (e_E \cdot e_k)\right) \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_k \\ & + (e_E \cdot e_k)\, Ek \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right) \boldsymbol{e}_t \\ & + \frac{(e_B \cdot e_k)\, Bk \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_E \wedge \boldsymbol{e}_B \wedge \boldsymbol{e}_k \\ & + \frac{(e_B \cdot e_k)\, B\omega \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_E \wedge \boldsymbol{e}_B \wedge \boldsymbol{e}_t \\ & + \left(-\frac{B\omega \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} - Ek \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)\right) \boldsymbol{e}_E \wedge \boldsymbol{e}_k \wedge \boldsymbol{e}_t \\ & + \frac{(e_E \cdot e_B)\, B\omega \cos\left((e_B \cdot e_k)\, k x_B + (e_E \cdot e_k)\, k x_E - \omega t + k x_k\right)}{\sqrt{-(e_B \cdot e_k)^2 + 2 (e_B \cdot e_k)(e_E \cdot e_B)(e_E \cdot e_k) - (e_E \cdot e_B)^2 - (e_E \cdot e_k)^2 + 1}} \boldsymbol{e}_B \wedge \boldsymbol{e}_k \wedge \boldsymbol{e}_t \end{aligned}$$

Substituting $e_E \cdot e_B = e_E \cdot e_k = e_B \cdot e_k = 0$

$$(\boldsymbol{\nabla} F) / (\cos(K \cdot X)) = \begin{aligned} & (Bk + E\omega)\, \boldsymbol{e}_E \\ & + (-B\omega - Ek)\, \boldsymbol{e}_E \wedge \boldsymbol{e}_k \wedge \boldsymbol{e}_t \end{aligned}$$
