$$f = f$$

$$A = A^{r}\boldsymbol{e}_{r} + A^{\theta}\boldsymbol{e}_{\theta} + A^{\phi}\boldsymbol{e}_{\phi}$$

$$B = B^{r\theta}\boldsymbol{e}_{r}\wedge\boldsymbol{e}_{\theta} + B^{r\phi}\boldsymbol{e}_{r}\wedge\boldsymbol{e}_{\phi} + B^{\theta\phi}\boldsymbol{e}_{\theta}\wedge\boldsymbol{e}_{\phi}$$

$$\nabla f = \partial_{r} f\boldsymbol{e}_{r} + \frac{\partial_{\theta} f}{r^{2}}\boldsymbol{e}_{\theta} + \frac{\partial_{\phi} f}{r^{2}{\sin{\left (\theta \right )}}^{2}}\boldsymbol{e}_{\phi}$$

$$\nabla \cdot A = \frac{A^{\theta}}{\tan{\left (\theta \right )}} + \partial_{\phi} A^{\phi} + \partial_{r} A^{r} + \partial_{\theta} A^{\theta} + \frac{2 A^{r}}{r}$$

$$\nabla \times A = -I\nabla \wedge A = \left ( \frac{2 A^{\phi}}{\tan{\left (\theta \right )}} + \partial_{\theta} A^{\phi} - \frac{\partial_{\phi} A^{\theta}}{{\sin{\left (\theta \right )}}^{2}}\right ) \left|{\sin{\left (\theta \right )}}\right|\boldsymbol{e}_{r} + \frac{- r^{2}{\sin{\left (\theta \right )}}^{2}\partial_{r} A^{\phi} - 2 r A^{\phi}{\sin{\left (\theta \right )}}^{2} + \partial_{\phi} A^{r}}{r^{2} \left|{\sin{\left (\theta \right )}}\right|}\boldsymbol{e}_{\theta} + \frac{r^{2}\partial_{r} A^{\theta} + 2 r A^{\theta} - \partial_{\theta} A^{r}}{r^{2} \left|{\sin{\left (\theta \right )}}\right|}\boldsymbol{e}_{\phi}$$

$$\nabla^{2}f = \frac{r^{2}\partial^{2}_{r} f + 2 r\partial_{r} f + \partial^{2}_{\theta} f + \frac{\partial_{\theta} f}{\tan{\left (\theta \right )}} + \frac{\partial^{2}_{\phi} f}{{\sin{\left (\theta \right )}}^{2}}}{r^{2}}$$

$$\nabla \wedge B = \frac{r^{2}\partial_{r} B^{\theta\phi} + 4 r B^{\theta\phi} - \frac{2 B^{r\phi}}{\tan{\left (\theta \right )}} - \partial_{\theta} B^{r\phi} + \frac{\partial_{\phi} B^{r\theta}}{{\sin{\left (\theta \right )}}^{2}}}{r^{2}}\boldsymbol{e}_{r}\wedge\boldsymbol{e}_{\theta}\wedge\boldsymbol{e}_{\phi}$$

Derivatives in Paraboloidal Coordinates

$$f = f$$

$$A = A^{u}\boldsymbol{e}_{u} + A^{v}\boldsymbol{e}_{v} + A^{\phi}\boldsymbol{e}_{\phi}$$

$$B = B^{uv}\boldsymbol{e}_{u}\wedge\boldsymbol{e}_{v} + B^{u\phi}\boldsymbol{e}_{u}\wedge\boldsymbol{e}_{\phi} + B^{v\phi}\boldsymbol{e}_{v}\wedge\boldsymbol{e}_{\phi}$$

$$\nabla f = \frac{\partial_{u} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e}_{u} + \frac{\partial_{v} f}{\sqrt{u^{2} + v^{2}}}\boldsymbol{e}_{v} + \frac{\partial_{\phi} f}{u v}\boldsymbol{e}_{\phi}$$

$$\nabla \cdot A = \frac{u A^{u}}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{v A^{v}}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} + \frac{\partial_{u} A^{u}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{v} A^{v}}{\sqrt{u^{2} + v^{2}}} + \frac{A^{v}}{v \sqrt{u^{2} + v^{2}}} + \frac{A^{u}}{u \sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} A^{\phi}}{u v}$$

$$\begin{aligned} \nabla \times A = -I\nabla \wedge A = & \frac{u v \sqrt{u^{2} + v^{2}}\partial_{v} A^{\phi} + u \sqrt{u^{2} + v^{2}} A^{\phi} - \left(u^{2} + v^{2}\right)\partial_{\phi} A^{v}}{u v \left(u^{2} + v^{2}\right)}\boldsymbol{e}_{u} \\ & + \frac{- u v \sqrt{u^{2} + v^{2}}\partial_{u} A^{\phi} - v \sqrt{u^{2} + v^{2}} A^{\phi} + \left(u^{2} + v^{2}\right)\partial_{\phi} A^{u}}{u v \left(u^{2} + v^{2}\right)}\boldsymbol{e}_{v} \\ & + \frac{u A^{v} - v A^{u} + \left(u^{2} + v^{2}\right) \left(-\partial_{v} A^{u} + \partial_{u} A^{v}\right)}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}}\boldsymbol{e}_{\phi} \end{aligned}$$

$$\nabla \wedge B = \left ( \frac{u B^{v\phi}}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{v B^{u\phi}}{\left(u^{2} + v^{2}\right)^{\frac{3}{2}}} - \frac{\partial_{v} B^{u\phi}}{\sqrt{u^{2} + v^{2}}} + \frac{\partial_{u} B^{v\phi}}{\sqrt{u^{2} + v^{2}}} - \frac{B^{u\phi}}{v \sqrt{u^{2} + v^{2}}} + \frac{B^{v\phi}}{u \sqrt{u^{2} + v^{2}}} + \frac{\partial_{\phi} B^{uv}}{u v}\right ) \boldsymbol{e}_{u}\wedge\boldsymbol{e}_{v}\wedge\boldsymbol{e}_{\phi}$$

$$f = f$$

$$\begin{aligned} A = & A^{\xi}\boldsymbol{e}_{\xi} \\ & + A^{\eta}\boldsymbol{e}_{\eta} \\ & + A^{\phi}\boldsymbol{e}_{\phi} \end{aligned}$$

$$\begin{aligned} B = & B^{\xi\eta}\boldsymbol{e}_{\xi}\wedge\boldsymbol{e}_{\eta} \\ & + B^{\xi\phi}\boldsymbol{e}_{\xi}\wedge\boldsymbol{e}_{\phi} \\ & + B^{\eta\phi}\boldsymbol{e}_{\eta}\wedge\boldsymbol{e}_{\phi} \end{aligned}$$

$$\begin{aligned} \nabla f = & \frac{\partial_{\xi} f}{\sqrt{{\sin{\left (\eta \right )}}^{2} + {\sinh{\left (\xi \right )}}^{2}} \left|{a}\right|}\boldsymbol{e}_{\xi} \\ & + \frac{\partial_{\eta} f}{\sqrt{{\sin{\left (\eta \right )}}^{2} + {\sinh{\left (\xi \right )}}^{2}} \left|{a}\right|}\boldsymbol{e}_{\eta} \\ & + \frac{\partial_{\phi} f}{a \sin{\left (\eta \right )} \sinh{\left (\xi \right )}}\boldsymbol{e}_{\phi} \end{aligned}$$

$$\nabla\cdot A = \frac{a\left(\sin(\eta)^2+\sinh(\xi)^2\right)^3\partial_\phi A^\phi + \frac{\left(A^\eta\sin(2\eta)+A^\xi\sinh(2\xi)\right)\left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi)|a|}{2} + \left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{5}{2}}\left(\partial_\eta A^\eta+\partial_\xi A^\xi\right)\sin(\eta)\sinh(\xi)|a| + \left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{5}{2}}A^\eta\cos(\eta)\sinh(\xi)|a| + \left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{5}{2}} A}{a^2\left(\sin(\eta)^2+\sinh(\xi)^2\right)^3\sin(\eta)\sinh(\xi)}$$

$$\begin{aligned} -I\nabla\wedge A = {} & \left(-\frac{\partial_\phi A^\eta}{a\sin(\eta)\sinh(\xi)} + \frac{A^\phi|a|}{a^2\sqrt{\sin(\eta)^2+\sinh(\xi)^2}\tan(\eta)} + \frac{|a|\,\partial_\eta A^\phi}{a^2\sqrt{\sin(\eta)^2+\sinh(\xi)^2}}\right)\boldsymbol{e}_\xi \\ & + \left(\frac{\partial_\phi A^\xi}{a\sin(\eta)\sinh(\xi)} - \frac{A^\phi|a|}{a^2\sqrt{\sin(\eta)^2+\sinh(\xi)^2}\tanh(\xi)} - \frac{|a|\,\partial_\xi A^\phi}{a^2\sqrt{\sin(\eta)^2+\sinh(\xi)^2}}\right)\boldsymbol{e}_\eta \\ & + \frac{2\left(\sin(\eta)^2+\sinh(\xi)^2\right)\left(\partial_\xi A^\eta-\partial_\eta A^\xi\right)+A^\eta\sinh(2\xi)-A^\xi\sin(2\eta)}{2\left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{3}{2}}|a|}\boldsymbol{e}_\phi \end{aligned}$$

$$\nabla\wedge B = \frac{a\left(\sin(\eta)^2+\sinh(\xi)^2\right)^3\partial_\phi B^{\xi\eta} + \frac{\left(B^{\eta\phi}\sinh(2\xi)-B^{\xi\phi}\sin(2\eta)\right)\left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{3}{2}}\sin(\eta)\sinh(\xi)|a|}{2} + \left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{5}{2}}\left(\partial_\xi B^{\eta\phi}-\partial_\eta B^{\xi\phi}\right)\sin(\eta)\sinh(\xi)|a| + \left(\sin(\eta)^2+\sinh(\xi)^2\right)^{\frac{5}{2}}B^{\eta\phi}\sin(\eta)\cosh(\xi)|a| - \left(\sin(\eta)^2+\sinh(\right.}{a^2\left(\sin(\eta)^2+\sinh(\xi)^2\right)^3\sin(\eta)\sinh(\xi)}$$