

```
def Maxwells_Equations_in_Geom_Calculus():
    Print_Function()
    X = symbols('t x y z',real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1, 1, 1, 1],coords=X)

    I = st4d.i

    B = st4d.mv('B','vector',f=True)
    E = st4d.mv('E','vector',f=True)
    B.set_coef(1,0,0)
    E.set_coef(1,0,0)
    B *= g0
    E *= g0
    J = st4d.mv('J','vector',f=True)
    F = E+I*B

    print(r'\T{Pseudo Scalar \;\;} I =',I)
    print(r'\T{Magnetic Field Bi Vector \;\;} B = \bs{B\gamma_{t}} =',B)
    print(r'\T{Electric Field Bi Vector \;\;} E = \bs{E\gamma_{t}} =',E)
    print(r'\T{Electromagnetic Field Bi Vector \;\;} F = E+IB =',F)
    print(r'\T{Four Current Density \;\;} J =',J)
    gradF = st4d.grad*F
    print(r'\T{Geom Derivative of Electomagnetic Field Bi Vector}')
    print(r'\nabla F =',gradF.Fmt(3))

    print(r'\T{Maxwell Equations}')
    print(r'\nabla F = J')
    print(r'\T{Div $E$ and Curl $H$ Equations}')
    print(r'\grade{\nabla F}{1} J = 0 =',(gradF.get_grade(1)J).Fmt(3))
    print(r'\T{Curl $E$ and Div $B$ equations}')
    print(r'\grade{\nabla F}{3} = 0 =',(gradF.get_grade(3)).Fmt(3))
    return
```

Code Output:

Pseudo Scalar $I = \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

Magnetic Field Bi-Vector $B = \boldsymbol{B\gamma_t} = -B^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - B^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - B^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$

Electric Field Bi-Vector $E = \boldsymbol{E\gamma_t} = -E^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z$

Electromagnetic Field Bi-Vector $F = E + IB = -E^x \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x - E^y \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y - E^z \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_z - B^z \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y + B^y \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z - B^x \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z$

Four Current Density $J = J^t \boldsymbol{\gamma}_t + J^x \boldsymbol{\gamma}_x + J^y \boldsymbol{\gamma}_y + J^z \boldsymbol{\gamma}_z$

Geom Derivative of Electomagnetic Field Bi-Vector

$$\begin{aligned} & (\partial_x E^x + \partial_y E^y + \partial_z E^z) \boldsymbol{\gamma}_t \\ & + (-\partial_z B^y + \partial_y B^z - \partial_t E^x) \boldsymbol{\gamma}_x \\ & + (\partial_z B^x - \partial_x B^z - \partial_t E^y) \boldsymbol{\gamma}_y \\ & + (-\partial_y B^x + \partial_x B^y - \partial_t E^z) \boldsymbol{\gamma}_z \\ \nabla F = & + (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \\ & + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_z \\ & + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \boldsymbol{\gamma}_t \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \\ & + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \boldsymbol{\gamma}_x \wedge \boldsymbol{\gamma}_y \wedge \boldsymbol{\gamma}_z \end{aligned}$$

Maxwell Equations

$$\nabla F = J$$

Div E and Curl H Equations

$$\begin{aligned} \langle \nabla F \rangle_1 - J = 0 = & (-J^t + \partial_x E^x + \partial_y E^y + \partial_z E^z) \gamma_t \\ & + (-J^x - \partial_z B^y + \partial_y B^z - \partial_t E^x) \gamma_x \\ & + (-J^y + \partial_z B^x - \partial_x B^z - \partial_t E^y) \gamma_y \\ & + (-J^z - \partial_y B^x + \partial_x B^y - \partial_t E^z) \gamma_z \end{aligned}$$

Curl E and Div B equations

$$\begin{aligned} \langle \nabla F \rangle_3 = 0 = & (-\partial_t B^z + \partial_y E^x - \partial_x E^y) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + (\partial_t B^y + \partial_z E^x - \partial_x E^z) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + (-\partial_t B^x + \partial_z E^y - \partial_y E^z) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + (\partial_x B^x + \partial_y B^y + \partial_z B^z) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

```
def Dirac_Equation_in_Geom_Calculus():
    Print_Function()
    coords = symbols('t x y z',real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1, 1, 1, 1],coords=coords)
    I = st4d.i

    (m,e) = symbols('m e')

    psi = st4d.mv('psi','spinor',f=True)
    A = st4d.mv('A','vector',f=True)
    sig_z = g3*g0

    print(r'\T{4 Vector Potential\;\;}\bs{A} =',A)
    print(r'\T{8 component real spinor\;\;}\bs{\psi} =',psi)

    dirac_eq = (st4d.grad*psi)*I*sig_z - e*A*psi - m*psi*g0
    dirac_eq = dirac_eq.simplify()

    print(r'\T{Dirac Equation\;\;}\nabla \bs{\psi} - I \sigma_{\{z\}} e\bs{A}\bs{\psi} - m\bs{\psi}\gamma_{\{t\}} = 0 =',dirac_eq.Fmt(3))

    return
```

Code Output:

4-Vector Potential

$$bsA = \begin{aligned} & A^t \gamma_t \\ & + A^x \gamma_x \\ & + A^y \gamma_y \\ & + A^z \gamma_z \end{aligned}$$

8-component real spinor $\psi =$

$$\begin{aligned} \psi = & \psi \\ & + \psi^{tx} \gamma_t \wedge \gamma_x \\ & + \psi^{ty} \gamma_t \wedge \gamma_y \\ & + \psi^{tz} \gamma_t \wedge \gamma_z \\ & + \psi^{xy} \gamma_x \wedge \gamma_y \\ & + \psi^{xz} \gamma_x \wedge \gamma_z \\ & + \psi^{yz} \gamma_y \wedge \gamma_z \\ & + \psi^{txyz} \gamma_t \wedge \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

Dirac Equation $\nabla \psi I \sigma_z - e \mathbf{A} \psi - m \psi \gamma_t = 0 =$

$$\begin{aligned} & (-eA^t \psi - eA^x \psi^{tx} - eA^y \psi^{ty} - eA^z \psi^{tz} - m\psi - \partial_y \psi^{tx} - \partial_z \psi^{txyz} + \partial_x \psi^{ty} + \partial_t \psi^{xy}) \gamma_t \\ & + (-eA^t \psi^{tx} - eA^x \psi - eA^y \psi^{xy} - eA^z \psi^{xz} + m\psi^{tx} + \partial_y \psi - \partial_t \psi^{ty} - \partial_x \psi^{xy} + \partial_z \psi^{yz}) \gamma_x \\ & + (-eA^t \psi^{ty} + eA^x \psi^{xy} - eA^y \psi - eA^z \psi^{yz} + m\psi^{ty} - \partial_x \psi + \partial_t \psi^{tx} - \partial_y \psi^{xy} - \partial_z \psi^{xz}) \gamma_y \\ & + (-eA^t \psi^{tz} + eA^x \psi^{xz} + eA^y \psi^{yz} - eA^z \psi + m\psi^{tz} + \partial_t \psi^{txyz} - \partial_z \psi^{xy} + \partial_y \psi^{xz} - \partial_x \psi^{yz}) \gamma_z \\ & + (-eA^t \psi^{xy} + eA^x \psi^{ty} - eA^y \psi^{tx} - eA^z \psi^{txyz} - m\psi^{xy} - \partial_t \psi + \partial_x \psi^{tx} + \partial_y \psi^{ty} + \partial_z \psi^{tz}) \gamma_t \wedge \gamma_x \wedge \gamma_y \\ & + (-eA^t \psi^{xz} + eA^x \psi^{tz} + eA^y \psi^{txyz} - eA^z \psi^{tx} - m\psi^{xz} + \partial_x \psi^{txyz} + \partial_z \psi^{ty} - \partial_y \psi^{tz} - \partial_t \psi^{yz}) \gamma_t \wedge \gamma_x \wedge \gamma_z \\ & + (-eA^t \psi^{yz} - eA^x \psi^{txyz} + eA^y \psi^{tz} - eA^z \psi^{ty} - m\psi^{yz} - \partial_z \psi^{tx} + \partial_y \psi^{txyz} + \partial_x \psi^{tz} + \partial_t \psi^{xz}) \gamma_t \wedge \gamma_y \wedge \gamma_z \\ & + (-eA^t \psi^{txyz} - eA^x \psi^{yz} + eA^y \psi^{xz} - eA^z \psi^{xy} + m\psi^{txyz} + \partial_z \psi - \partial_t \psi^{tz} - \partial_x \psi^{xz} - \partial_y \psi^{yz}) \gamma_x \wedge \gamma_y \wedge \gamma_z \end{aligned}$$

```
def Lorentz_Tranformation_in_Geog_Algebra():
    Print_Function()
    (alpha,beta,gamma) = symbols('alpha beta gamma')
    (x,t,xp,tp) = symbols("x t x' t'",real=True)
    (st2d,g0,g1) = Ga.build('gamma*t|x',g=[1, 1])

    from sympy import sinh,cosh

    R = cosh(alpha/2)+sinh(alpha/2)*(g0^g1)
    X = t*g0+x*g1
    Xp = tp*g0+xp*g1
    print('R =',R)

    print(r"\t\bm{\gamma_{t}}+x\bm{\gamma_{x}} = t'\bm{\gamma_{-t}}+x'\bm{\gamma_{-x}} = R\l p \t'\bm{\gamma_{t}}+x'\bm{\gamma_{x}}\rp R^{\dagger}")

    Xpp = R*Xp*R.rev()
    Xpp = Xpp.collect()
    Xpp = Xpp.trigsimp()
    print(r'\t\bm{\gamma_{t}}+x\bm{\gamma_{x}} =',Xpp)
    Xpp = Xpp.subs({sinh(alpha):gamma*beta,cosh(alpha):gamma})

    print(r'\f{\sinh}{\alpha} = \gamma\beta')
    print(r'\f{\cosh}{\alpha} = \gamma')

    print(r'\t\bm{\gamma_{t}}+x\bm{\gamma_{x}} =',Xpp.collect())
    return
```

Code Output:

$$R = \cosh\left(\frac{\alpha}{2}\right) + \sinh\left(\frac{\alpha}{2}\right)\gamma_t \wedge \gamma_x$$
$$t\gamma_{\boldsymbol{t}} + x\gamma_{\boldsymbol{x}} = t'\gamma'_{\boldsymbol{t}} + x'\gamma'_{\boldsymbol{x}} = R(t'\gamma_{\boldsymbol{t}} + x'\gamma_{\boldsymbol{x}})R^{\dagger}$$
$$t\gamma_{\boldsymbol{t}} + x\gamma_{\boldsymbol{x}} = \frac{(t'\cosh(\alpha) - x'\sinh(\alpha))}{+(-t'\sinh(\alpha) + x'\cosh(\alpha))}\gamma_t$$
$$\sinh(\alpha) = \gamma\beta$$
$$\cosh(\alpha) = \gamma$$
$$t\gamma_{\boldsymbol{t}} + x\gamma_{\boldsymbol{x}} = \frac{\gamma(-\beta x' + t')}{+\gamma(-\beta t' + x')}\gamma_t$$

```
def General_Lorentz_Tranformation():
    Print_Function()
    (alpha,beta,gamma) = symbols('alpha beta gamma')
    (x,y,z,t) = symbols("x y z t",real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1, 1, 1, 1])

    B = (x*g1+y*g2+z*g3)^(t*g0)
    print(B)
    print(B.exp(hint='+'))
    print(B.exp(hint=' '))
```

Code Output:

$$-tx\gamma_t \wedge \gamma_x$$
$$-ty\gamma_t \wedge \gamma_y$$
$$-tz\gamma_t \wedge \gamma_z$$

$$\begin{aligned} & \cosh\left(\sqrt{x^2+y^2+z^2}\,|t|\right) \\ & - \frac{tx\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_x \\ & - \frac{ty\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_y \\ & - \frac{tz\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_z \\ & \cosh\left(\sqrt{x^2+y^2+z^2}\,|t|\right) \\ & - \frac{tx\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_x \\ & - \frac{ty\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_y \\ & - \frac{tz\sinh\left(\sqrt{x^2+y^2+z^2}\,|t|\right)}{\sqrt{x^2+y^2+z^2}\,|t|}\,\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_z \end{aligned}$$

```
def Lie_Group():
    Print_Function()
    coords = symbols('t x y z',real=True)
    (st4d,g0,g1,g2,g3) = Ga.build('gamma*t|x|y|z',g=[1, 1, 1, 1],coords=coords)
    I = st4d.i

    a = st4d.mv('a','vector')
    B = st4d.mv('B','bivector')
    print('a =',a)
    print('B =',B)
    print(r'a\cdot B =', a|B)
    print(r' (a\cdot B)\cdot B =',((a|B)|B).simplify()).Fmt(3))
    print(r' ((a\cdot B)\cdot B)\cdot B =',(((a|B)|B)|B).simplify()).Fmt(3))

    return
```

Code Output:

$$\begin{aligned} & a^t\boldsymbol{\gamma}_t \\ a = & + a^x\boldsymbol{\gamma}_x \\ & + a^y\boldsymbol{\gamma}_y \\ & + a^z\boldsymbol{\gamma}_z \\ & B^{tx}\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_x \\ & + B^{ty}\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_y \\ B = & + B^{tz}\boldsymbol{\gamma}_t\wedge\boldsymbol{\gamma}_z \\ & + B^{xy}\boldsymbol{\gamma}_x\wedge\boldsymbol{\gamma}_y \\ & + B^{xz}\boldsymbol{\gamma}_x\wedge\boldsymbol{\gamma}_z \\ & + B^{yz}\boldsymbol{\gamma}_y\wedge\boldsymbol{\gamma}_z \\ a\cdot B = & \left(B^{tx}a^x+B^{ty}a^y+B^{tz}a^z\right)\boldsymbol{\gamma}_t \\ & + \left(B^{tx}a^t+B^{xy}a^y+B^{xz}a^z\right)\boldsymbol{\gamma}_x \\ & + \left(B^{ty}a^t-B^{xy}a^x+B^{yz}a^z\right)\boldsymbol{\gamma}_y \\ & + \left(B^{tz}a^t-B^{xz}a^x-B^{yz}a^y\right)\boldsymbol{\gamma}_z \end{aligned}$$

$$\begin{aligned}
& \left((B^{tx})^2 a^t + B^{tx} B^{xy} a^y + B^{tx} B^{xz} a^z + (B^{ty})^2 a^t - B^{ty} B^{xy} a^x + B^{ty} B^{yz} a^z + (B^{tz})^2 a^t - B^{tz} B^{xz} a^x - B^{tz} B^{yz} a^y \right) \gamma_t \\
& + \left((B^{tx})^2 a^x + B^{tx} B^{ty} a^y + B^{tx} B^{tz} a^z + B^{ty} B^{xy} a^t + B^{tz} B^{xz} a^t - (B^{xy})^2 a^x + B^{xy} B^{yz} a^z - (B^{xz})^2 a^x - B^{xz} B^{yz} a^y \right) \gamma_x \\
(a \cdot B) \cdot B = & + \left(B^{tx} B^{ty} a^x - B^{tx} B^{xy} a^t + (B^{ty})^2 a^y + B^{ty} B^{tz} a^z + B^{tz} B^{yz} a^t - (B^{xy})^2 a^y - B^{xy} B^{xz} a^z - B^{xz} B^{yz} a^x - (B^{yz})^2 a^y \right) \gamma_y \\
& + \left(B^{tx} B^{tz} a^x - B^{tx} B^{xz} a^t + B^{ty} B^{tz} a^y - B^{ty} B^{yz} a^t + (B^{tz})^2 a^z - B^{xy} B^{xz} a^y + B^{xy} B^{yz} a^x - (B^{xz})^2 a^z - (B^{yz})^2 a^z \right) \gamma_z \\
& \left((B^{tx})^3 a^x + (B^{tx})^2 B^{ty} a^y + (B^{tx})^2 B^{tz} a^z + B^{tx} (B^{ty})^2 a^x + B^{tx} (B^{tz})^2 a^x - B^{tx} (B^{xy})^2 a^x + B^{tx} B^{xy} B^{yz} a^z - B^{tx} (B^{xz})^2 a^x - B^{tx} B^{xz} B^{yz} a^y + (B^{ty})^3 a^y + (B^{ty})^2 B^{tz} a^z + B^{ty} (B^{tz})^2 a^y - B^{ty} (B^{xy})^2 a^y - B^{ty} B^{xy} B^{yz} a^z - B^{ty} (B^{xz})^2 a^y - B^{ty} B^{xz} B^{yz} a^x \right) \\
((a \cdot B) \cdot B) \cdot B = & + \left((B^{tx})^3 a^t + (B^{tx})^2 B^{xy} a^y + (B^{tx})^2 B^{xz} a^z + B^{tx} (B^{ty})^2 a^t + B^{tx} B^{ty} B^{yz} a^z + B^{tx} (B^{tz})^2 a^t - B^{tx} B^{tz} B^{yz} a^y - B^{tx} (B^{xy})^2 a^t - B^{tx} (B^{xz})^2 a^t + (B^{ty})^2 B^{xy} a^y + B^{ty} B^{tz} B^{xy} a^z + B^{ty} B^{tz} B^{xz} a^y - B^{ty} B^{xz} B^{yz} a^t + (B^{tz})^2 B^{xy} a^x + (B^{tz})^2 B^{yz} a^y - B^{tz} B^{xz} a^x - B^{tz} B^{yz} a^y \right) \\
& + \left((B^{tx})^2 B^{ty} a^t - (B^{tx})^2 B^{xy} a^x + B^{tx} B^{ty} B^{xz} a^z - B^{tx} B^{tz} B^{xy} a^z + B^{tx} B^{tz} B^{yz} a^x - B^{tx} B^{xz} B^{yz} a^t + (B^{ty})^3 a^t - (B^{ty})^2 B^{xy} a^x + (B^{ty})^2 B^{yz} a^z + B^{ty} (B^{tz})^2 a^t - B^{ty} B^{tz} B^{xz} a^x - B^{ty} (B^{xy})^2 a^t - B^{ty} (B^{yz})^2 a^t + (B^{tz})^2 B^{xy} a^x + (B^{tz})^2 B^{yz} a^y - B^{tz} B^{xz} a^x - B^{tz} B^{yz} a^y \right) \\
& + \left((B^{tx})^2 B^{tz} a^t - (B^{tx})^2 B^{xz} a^x - B^{tx} B^{ty} B^{xz} a^y - B^{tx} B^{ty} B^{yz} a^x + B^{tx} B^{tz} B^{xy} a^y + B^{tx} B^{xy} B^{yz} a^t + (B^{ty})^2 B^{tz} a^t - (B^{ty})^2 B^{yz} a^y - B^{ty} B^{tz} B^{xy} a^x - B^{ty} B^{xy} B^{xz} a^t + (B^{tz})^3 a^t - (B^{tz})^2 B^{xz} a^x - (B^{tz})^2 B^{yz} a^y - B^{tz} B^{xz} a^x - B^{tz} B^{yz} a^y \right)
\end{aligned}$$