**Base manifold (three dimensional)**

**Metric tensor (cartesian coordinates - norm = False)**

$$g = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

**Two dimensioanal submanifold - Unit sphere**

Basis not normalised

$(\theta, \phi) \to (r, \theta, \phi) = [1,\ \theta,\ \phi]$

$e_\theta | e_\theta = 1$

$e_\phi | e_\phi = \sin(\theta)^2$

$$g = \begin{bmatrix} 1 & 0 \\ 0 & \sin(\theta)^2 \end{bmatrix}$$

$$\text{g\_inv} = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{\sin^2(\theta)} \end{bmatrix}$$

Christoffel symbols of the first kind:

$$\Gamma_{1,\alpha,\beta} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\sin(2\theta)}{2} \end{bmatrix} \quad \Gamma_{2,\alpha,\beta} = \begin{bmatrix} 0 & \frac{\sin(2\theta)}{2} \\ \frac{\sin(2\theta)}{2} & 0 \end{bmatrix}$$

Christoffel symbols of the second kind:

$$\Gamma^1{}_{\alpha,\beta} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{\sin(2\theta)}{2} \end{bmatrix} \quad \Gamma^2{}_{\alpha,\beta} = \begin{bmatrix} 0 & \frac{1}{\tan(\theta)} \\ \frac{1}{\tan(\theta)} & 0 \end{bmatrix}$$

$$\nabla = \boldsymbol{e}_\theta \frac{\partial}{\partial \theta} + \boldsymbol{e}_\phi \frac{1}{\sin(\theta)^2} \frac{\partial}{\partial \phi}$$

$$\nabla f = \partial_\theta f \boldsymbol{e}_\theta + \frac{\partial_\phi f}{\sin(\theta)^2} \boldsymbol{e}_\phi$$

$$F = F^\theta \boldsymbol{e}_\theta + F^\phi \boldsymbol{e}_\phi$$

$$\nabla F = \left( \frac{F^\theta}{\tan(\theta)} + \partial_\phi F^\phi + \partial_\theta F^\theta \right) + \left( \frac{2F^\phi}{\tan(\theta)} + \partial_\theta F^\phi - \frac{\partial_\phi F^\theta}{\sin(\theta)^2} \right) \boldsymbol{e}_\theta \wedge \boldsymbol{e}_\phi$$

One dimensioanal submanifold

Basis not normalised

$(\phi) \to (\theta, \phi) = \left[\frac{\pi}{8},\ \phi\right]$

$e_\phi | e_\phi = \frac{1}{2} - \frac{\sqrt{2}}{4}$

$g = \left[\frac{1}{2} - \frac{\sqrt{2}}{4}\right]$

$$\nabla = \boldsymbol{e}_\phi \left(2\sqrt{2} + 4\right) \frac{\partial}{\partial \phi}$$

$$\nabla h = \left(2\sqrt{2} + 4\right) \partial_\phi h \boldsymbol{e}_\phi$$

$$H = H^\phi \boldsymbol{e}_\phi$$

$$\nabla H = \partial_\phi H^\phi$$