ENGINEERING TRIPOS PART IA

Wednesday 8 June 2011 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (**short**) A two-dimensional hydraulic jump occurs in a channel as shown in Fig. 1. Well upstream and downstream of the jump, at sections 1 and 2, the flow is uniform and steady with velocity V_1 and V_2, and depth h_1 and h_2.

(a) Explain why the pressure distribution in the fluid at sections 1 and 2 may be treated as hydrostatic. [2]

(b) Using the Force-Momentum Equation, or otherwise, show that:

$$h_1 V_1^2 + \frac{1}{2} g h_1^2 = h_2 V_2^2 + \frac{1}{2} g h_2^2 .$$

State carefully any assumptions you have made in addition to those in part (a). [4]

(c) The above expression can be rearranged to give the relationship between two non-dimensional groups, the depth ratio h_2/h_1, and the upstream Froude Number $Fr = V_1 / \sqrt{g h_1}$:

$$Fr^2 = \frac{1}{2} \frac{h_2}{h_1} \left(1 + \frac{h_2}{h_1} \right).$$

Determine the upstream velocity of a ¼ scale laboratory model that has dynamic similarity to a real hydraulic jump in which $V_1 = 10.0 \text{ ms}^{-1}$. [4]

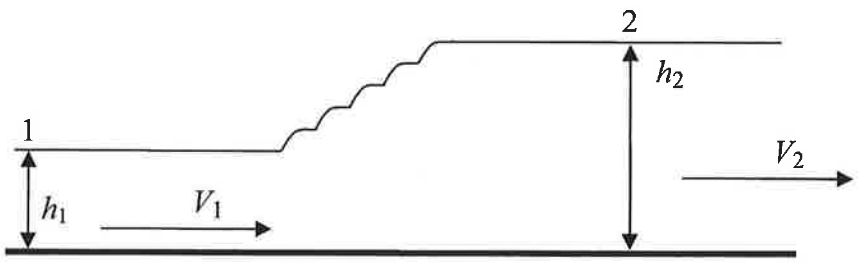

Fig. 1

2 (**short**) A two-dimensional air jet of height h_1 = 20 mm and speed V_1 = 40 ms^{-1} impinges on a stationary curved plate and is deflected by 30° as shown in Fig. 2. The flow is steady and can be treated as inviscid.

(a) Calculate the height, h_2, and speed, V_2, of the air jet leaving the plate at location 2, which is sufficiently far away from the impinging point. State the assumptions you have made in deriving this. [4]

(b) Calculate the aerodynamic force per unit width acting on the plate. [6]

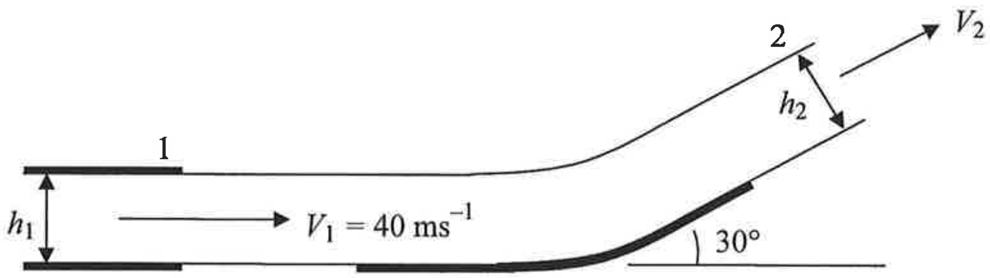

Fig. 2

3 (**short**) In a polytropic process, the pressure, p, and specific volume, v, of a gas are related by

$$pv^n = k \ ,$$

where k is a constant.

(a) State the value of n for a perfect gas during
- (i) an isentropic process (constant entropy);
- (ii) an isothermal process (constant temperature);
- (iii) an isobaric process (constant pressure);
- (iv) an isochoric process (constant volume). [4]

(b) A gas in a cylinder does displacement work as it expands against a piston and satisfies the polytropic equation above. Derive an expression for the work done by the gas in terms of its initial state (p_1,v_1) and its final state (p_2,v_2) when n is not equal to 1. [4]

(c) Explain why the expression derived in (b) breaks down when $n = 1$. [2]

4 (**short**) An industrial air filter consists of a porous mesh inside a thermally-insulated tube with constant area 0.05 m^2. Air enters the filter at a pressure of 1 MPa, a temperature of 850 °C and a mass flowrate of 30 kgs^{-1}. In this question, assume that air is a perfect gas.

(a) Calculate the velocity of the air upstream of the filter. [2]

(b) Far downstream of the filter, the temperature is 846 °C. Calculate the velocity and pressure there. [4]

(c) Calculate the change in specific entropy between the upstream and downstream conditions and explain the reason for this change. [4]

5 (**long**) A small reservoir has a submerged underwater gate to control its water level. The underwater gate is a two-dimensional rectangular flat plate, inclined at 45° as shown in Fig. 3, and may be considered to be weightless. The width of the gate is w and the length is L. The gate is hinged at B and opened by applying a vertical force, T, upwards at A. The vertical distance between the bottom of the gate and the floor of the reservoir is h_1, where $h_1 << h_0$. Far downstream, the exiting stream of water is uniform and parallel and has height h_2.

(a) Calculate the total hydrostatic force acting on the gate. [6]

(b) The reservoir is fed water from upstream at a volumetric flowrate, $\dot{Q}$. Assume that the discharge coefficient $h_2 / h_1 = 0.8$, and that the kinetic energy of the upstream flow can be neglected. Calculate the opening height of the gate, h_1, required to maintain the water level at $h_0 = \left(\frac{1}{4} + \frac{\sqrt{2}}{2}\right)L$. [9]

(c) Calculate the minimum force, T, required to lift the gate. [9]

(d) Without detailed calculations, compare the flat plate gate in Fig. 3 with the quarter-circular cylindrical gate in Fig. 4. Comment on the force required to lift the cylindrical gate. [6]

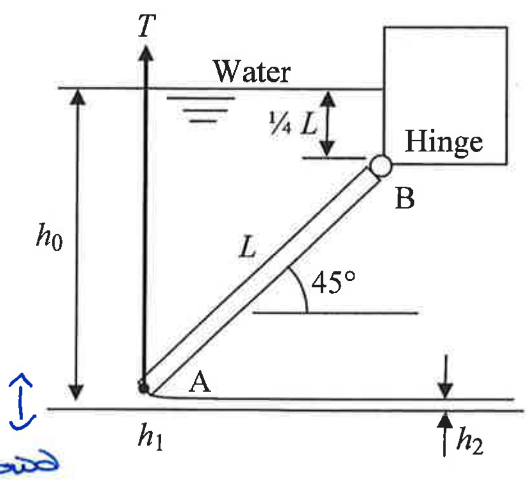

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Fig. 3

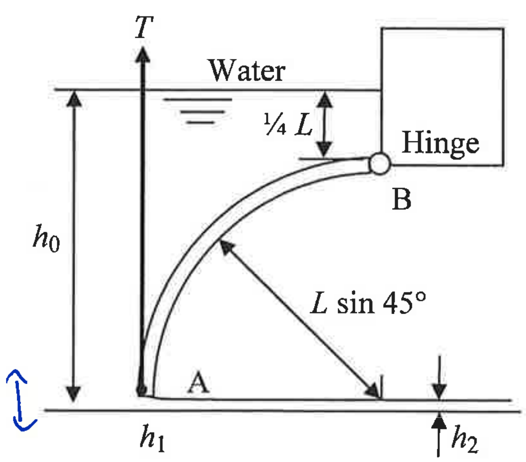

Fig. 4

(TURN OVER

6 (**long**) A stationary jet engine is shown schematically in Fig. 5. It is in an environment at 25 °C and 1 bar. The mass flow of air through the engine is 50 kgs^{-1}. You may assume that the working fluid is a perfect gas with the properties of air at normal atmospheric conditions, that the processes in the compressor, the turbine and the nozzle are reversible and adiabatic, that the temperatures and static pressures at locations 1 and 2 are equal, and that there is no pressure drop in the combustion chamber.

(a) The pressure ratio across the compressor is 20:1. Assuming that kinetic energy terms may be neglected, determine the temperature at location 3, and the power input to the compressor. [5]

(b) Fuel is burnt between locations 3 and 4 and results in an average temperature at location 4 of 950 °C. Assuming that the mass flow of fuel is very small relative to that of the air, and that kinetic energy terms may be neglected, calculate the apparent rate at which heat is added in the combustion chamber. [2]

(c) Determine the temperature and pressure at location 5, the turbine exit. [5]

(d) The gas accelerates in the nozzle until the pressure at location 6 drops to 2.60 bar. Calculate the temperature and the speed of the gas at location 6. Calculate the speed of sound at location 6. Comment on these two values. [8]

(e) On a *T-s* diagram, sketch the thermodynamic path of the working fluid between points 2 and 6. [5]

(f) The flow area at location 2 is 0.75 m^2. Is it reasonable to assume that the pressure and temperature at locations 1 and 2 are equal? [5]

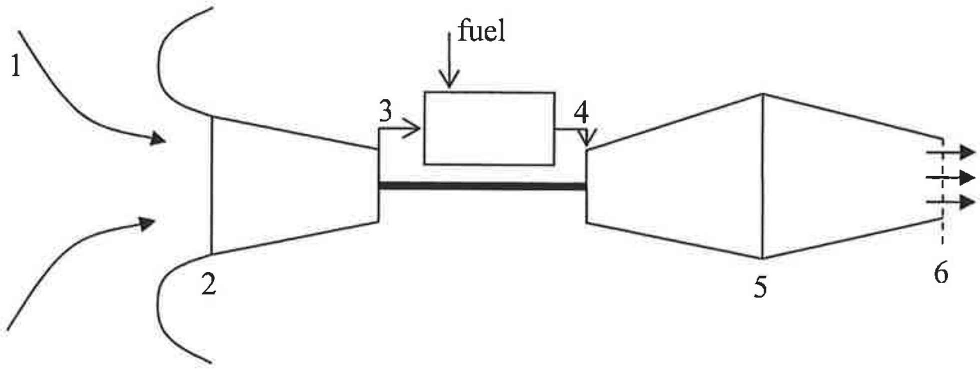

Fig. 5.

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SECTION B

7 (**short**) An elliptical lamina of mass ρ per unit area is held in a vertical plane and hinged at the fixed point O, as shown in Fig. 6. The centre of the lamina is at point G, and its major and minor axes are of length h and v, respectively.

(a) What is the moment of inertia of the lamina about an axis that is oriented perpendicular to the plane of the lamina and passes through point G? [4]

(b) What is the moment of inertia of the lamina about an axis that is oriented perpendicular to the plane of the lamina and passes through point O? [2]

(c) If the lamina is held in the position shown and then released, what will be its initial angular acceleration? [4]

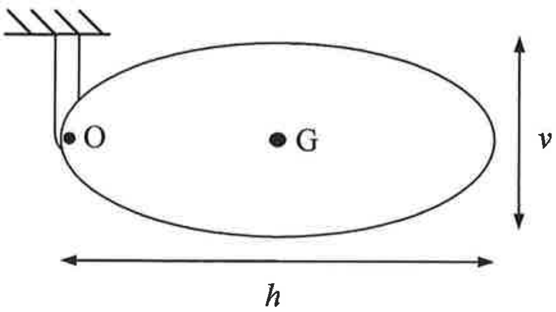

Fig. 6

8 (short) A planar mechanism oriented in the vertical plane is shown in Fig. 7. It is composed of a heavy, uniform rigid bar AB of length x attached to two light rigid links AC and BD. AC is of length l and forms an angle of 30° to the vertical. BD is of length $2l$ and forms an angle of 60° to the vertical. At the instant shown, link AC is rotating freely counter-clockwise with angular velocity ω.

(a) Using instantaneous centres or otherwise, express the angular velocity of the link BD in terms of ω. [4]

(b) The angular velocity of bar AB is found to be $\omega/2$ at this instant. What is length x? [3]

(c) At the instant shown, is the mechanism moving towards, or away from, its equilibrium position? Give a reason for your answer. [3]

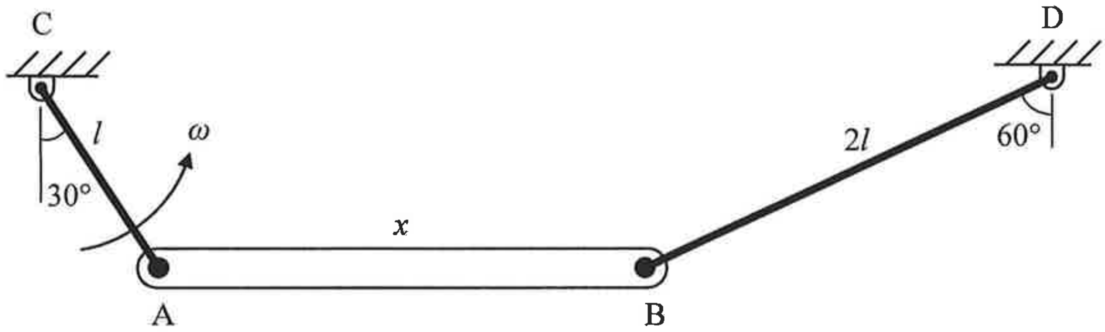

Fig. 7

9 (**short**) The planar mechanism in Fig. 8 consists of a rigid uniform rod AB of length l with a pivoting slider attached at B, through which runs a rigid uniform rod CD of length $3l$. At the instant shown, rod AB is vertical and is being driven counter-clockwise with constant angular velocity ω, rod CD makes an angle of 30° to the horizontal, and the slider at B is a distance of $2l$ from C.

(a) Sketch a velocity diagram for the mechanism, and use it to find the angular velocity of CD, and the velocity of D. [5]

(b) There is a frictional torque of M in the hinge of the slider at B, and a force F is applied vertically downwards at D. Find the torque T needed to drive the rod AB at this instant. You may neglect the mass of the mechanism and all other potential sources of friction. [5]

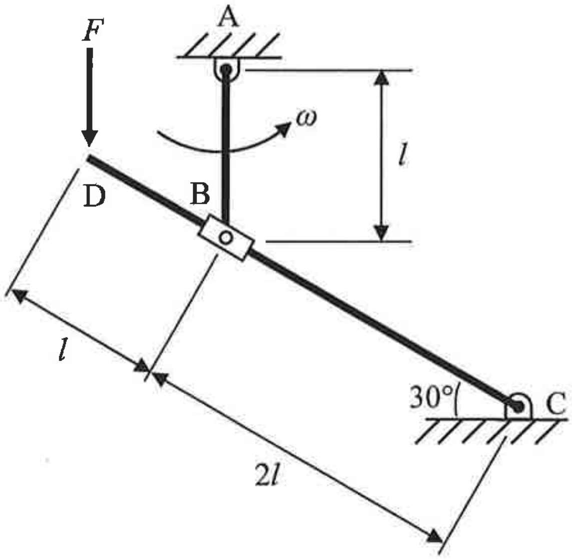

Fig. 8

10 **(short)** A machine of mass m is mounted to a vertical wall with four springs, each of stiffness k, and a damper of rate λ (see Fig. 9). The machine operates with an out-of-balance force of magnitude F and frequency ω.

(a) By writing the equation of horizontal motion for the machine in standard form, show that the natural frequency ω_n and damping ratio ζ are given by

$$\omega_n = \sqrt{\frac{4k}{m}} \quad \text{and} \quad \zeta = \frac{\lambda}{4\sqrt{km}} \ .$$

[4]

It is known that: $m = 250$ kg ; $k = 250$ Nm^{-1} ; $\lambda = 50$ Nsm^{-1} ; $F = 50$ N ; $\omega = 1$ rad s^{-1}.

(b) Estimate the amplitude of vibration of the machine and the phase angle relative to the force. [4]

(c) Describe (qualitatively) the motion of the machine if only one of the springs is connected. [2]

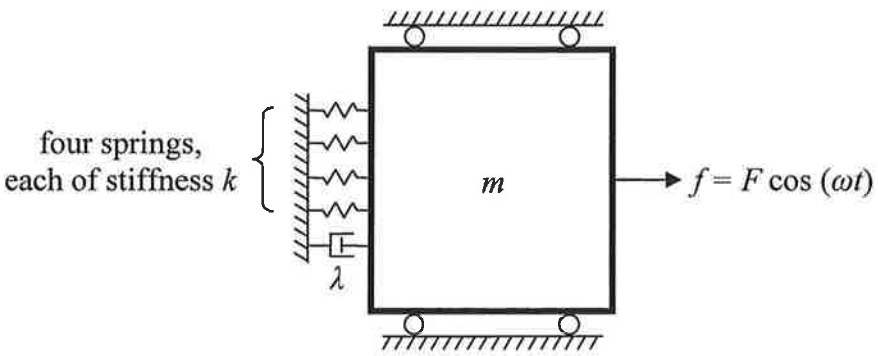

Fig. 9

11 **(long)** A particle of mass 0.5 kg lies on a frictionless horizontal table and is attached to a fixed point O by means of a spring of unstretched length 0.5 m and stiffness 50 Nm^{-1}. A second particle, of mass 1 kg, slides freely across the table with a velocity of 5 ms^{-1} in a direction that is perpendicular to the length of the spring (see Fig. 10). The two particles collide and fuse together on impact.

(a) What is the velocity of the new combined particle immediately after the collision? [3]

(b) During subsequent motion, the length of the spring increases to a maximum at point B. Sketch in plan view a curve to represent the path of the combined particle from A to B. Show on your sketch vectors to represent the velocities at A and B. Describe the motion of the particle after it has passed through point B. [6]

(c) Show that at point B the length of the spring is 1 m. [12]

(d) What are the maximum and minimum speeds of the combined particle, during the subsequent motion? [4]

(e) What is the acceleration of the combined particle, when its speed is at a minimum, and what is the radius of curvature of its path? [5]

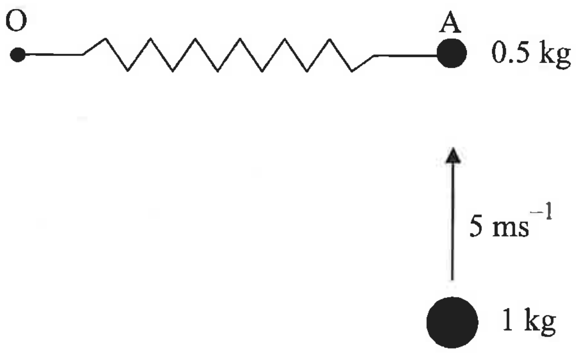

Fig. 10

12 (**long**) A machine can be modelled as a casing that contains two components of mass m_1 and m_2. The lower mass m_1 is held off the base of the casing by a spring of stiffness k_1 and a damper of rate λ. The upper mass m_2 is suspended from the roof of the casing by a spring of stiffness k_2. The gap between the top of m_1 and the bottom of m_2 is 10 mm when the machine is at rest (see Fig. 11). Note that m_1 and m_2 are constrained so that they cannot rotate relative to each other (i.e. they only go up and down).

It is known that: $m_1 = 1$ kg ; $m_2 = 9$ kg ; $k_1 = 4$ Nm^{-1} ; $k_2 = 25$ Nm^{-1} ; $\lambda = 2$ Nsm^{-1}.

The casing is vibrated with a vertical position input of frequency 2 rad/s and amplitude X. y_1 represents the distance between the base of the casing and the top of m_1. y_2 represents the distance between the base of the casing and the bottom of m_2.

(a) Express in terms of X, the amplitude of vibration of m_1 relative to the casing, and state the phase angle. [6]

(b) Sketch a plot of y_1 and y_2 against time. [14]

(c) Find the maximum amplitude at which the casing can be vibrated before m_1 and m_2 collide. [10]

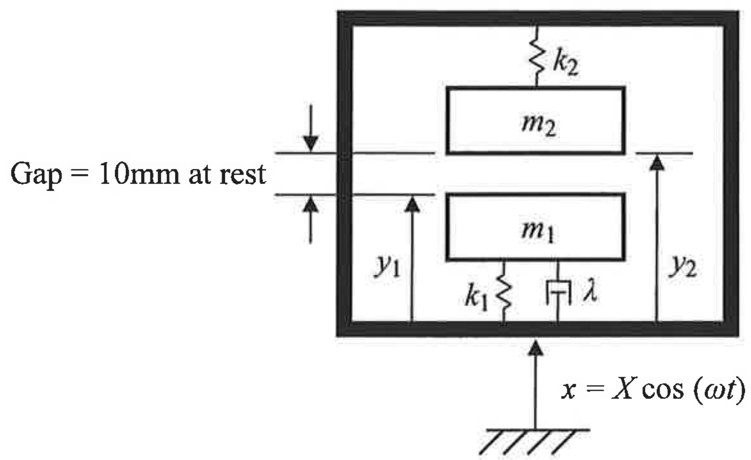

Fig. 11

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