ENGINEERING TRIPOS PART IA

Wednesday 6 June 2012 9 to 12

Paper 1

MECHANICAL ENGINEERING

*Answer **all** questions.*

*The **approximate** number of marks allocated to each part of a question is indicated in the right margin.*

Answers to questions in each section should be tied together and handed in separately.

There are no attachments

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS
Engineering Data Book
CUED approved calculator allowed

You may not start to read the questions printed on the subsequent pages of this question paper until instructed that you may do so by the Invigilator

SECTION A

1 (**short**) A pitot-static pressure probe is used to measure the flow velocity in a wind tunnel. The probe is placed in the centre of the tunnel, aligned facing the air flow. It is attached to a vertical U-tube that contains liquid of density 850.0 kg m^{-3}, as shown in Fig. 1. Assume that the air density is uniform at 1.20 kg m^{-3}. The U-tube reading is 10.0 mm.

(a) Find the air flow velocity at the point of measurement. [4]

(b) The U-tube can be read accurately up to ± 0.5 mm. Estimate the measurement uncertainty in the velocity. [4]

(c) Discuss whether or not the effect of the air column Δh can be neglected in the calculation. [2]

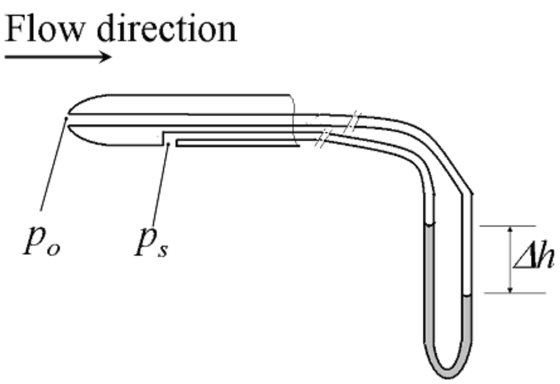

Fig. 1

2 (**short**) Figure 2 shows water flowing over a weir. The flow is steady and is uniform in the direction into the page. The flow is measured at four points along section $Y\!-\!Y$, at which the total height of water is 400 mm. For each point, Table 2 shows the vertical coordinate, y, above the corner, the flow speed, v, and the angle of the velocity vector from the horizontal, α.

Estimate the volumetric flow rate per unit width at section $Y\!-\!Y$. [10]

y (mm)	50.0	150.0	250.0	350.0
v (m s^{-1})	3.0	5.0	7.0	7.5
α (deg)	30°	25°	21°	17°

Table 2

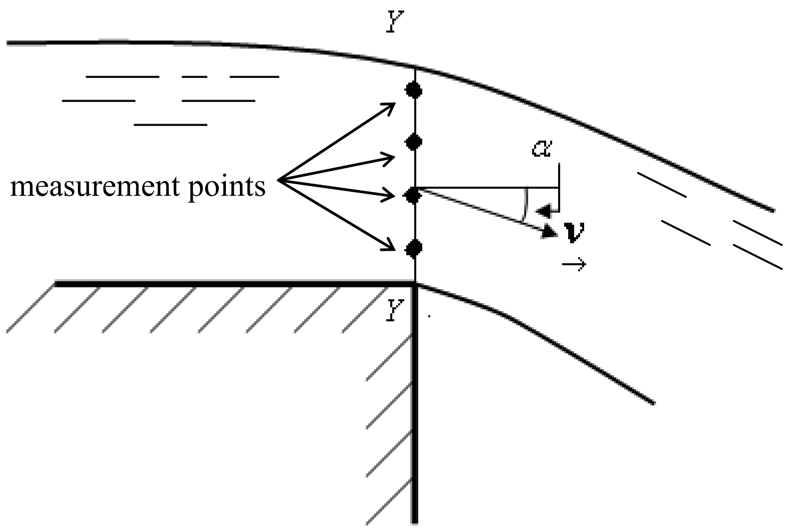

Fig. 2

3 (**long**) A pump supplies water at speed V_1 to a $90°$ convergent nozzle, as shown in Figure 3. The water jet exits the nozzle vertically, at uniform velocity V_e, and reaches a maximum height of 20.0 m above the nozzle exit. Assume that there is no loss of stagnation pressure in the pipe, the bend and the nozzle.

(a) Calculate the upstream stagnation pressure, $p_{o,1}$, and the velocity, V_e, of the water jet as it leaves the nozzle. [8]

(b) The exit area of the nozzle is $0.16\ \mathrm{m}^2$. Calculate the power required to drive the jet, assuming that the water upstream of the pump is at zero velocity and at atmospheric pressure. [7]

(c) The area ratio of the nozzle, A_1/A_e, is 10.

(i) Calculate the static pressure, p_1, at the entry of the bend. [5]

(ii) Calculate the horizontal and vertical force components required to hold the nozzle, in terms of non-dimensional force coefficients $C_{f,x} = \dfrac{F_x}{\rho V_e^2 A_e}$ and $C_{f,y} = \dfrac{F_y}{\rho V_e^2 A_e}$. [10]

The gravitational acceleration is $9.81\ \mathrm{m\ s^{-2}}$, the atmospheric pressure is $10^5\ \mathrm{N\ m^{-2}}$, and the density of water is $1000\ \mathrm{kg\ m^{-3}}$.

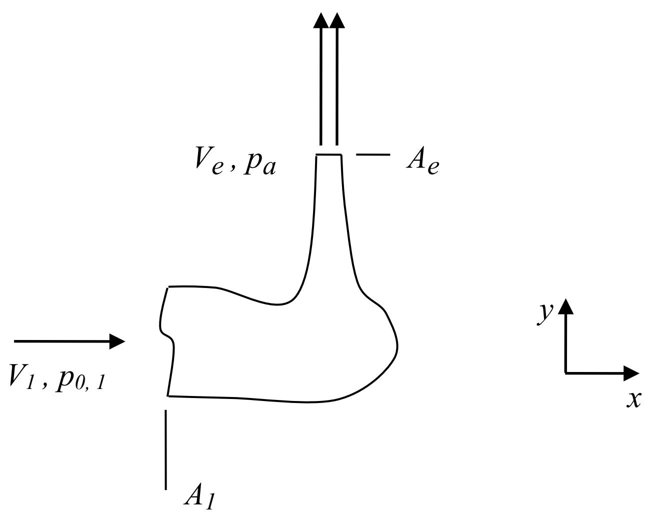

Fig. 3

4 (**short**) A cyclic refrigerator removes heat from a cold reservoir at the rate of 1.25 kW. The reservoir is insulated from the environment, which is at 27 °C.

(a) What is the minimum power that must be supplied to the refrigerator when the cold reservoir is at −23 °C ? What is the corresponding Coefficient of Performance, COP_R , of the refrigerator? [6]

(b) If the actual power supplied is 0.50 kW when the cold reservoir is at −23 °C , calculate the overall rate of entropy generation due to irreversibility. [4]

5 (**short**) A perfect gas with gas constant $R = 287\ \mathrm{J\,kg^{-1}\,K^{-1}}$ and ratio of specific heats $\gamma = 1.4$ is initially at 270 K and 1 bar. It is contained within a cylinder and compressed to 10 bar by a piston, reversibly and adiabatically.

(a) What is the final temperature of the gas? [3]

(b) How much work is done on the gas, per unit mass of gas? [4]

(c) Sketch this process on a *T-s* diagram, indicating the lines of constant pressure that pass through the initial and final states. [2]

(d) On the *T-s* diagram, mark clearly the final state for an irreversible adiabatic compression starting from the same initial state and having the same final pressure. [1]

6 (**long**) An evacuated vessel with an internal volume V is connected by a valve to a large compressed air main. The air in the main has pressure p_0 and temperature T_0 and can be treated as a perfect gas. The temperature outside the vessel is also T_0 .

(a) The valve is opened and then closed again as soon as the pressure in the vessel reaches p_0 . This process takes place quickly. Show that the temperature in the vessel rises to $T_1 = \gamma T_0$ and derive an expression for the mass of air that enters the vessel, m_1 , in terms of p_0, V, T_0, and the physical properties of air. [6]

(b) The vessel does not have any leaks but, after the valve has been closed, the pressure inside is found to fall gradually with time. Explain why this happens. Calculate T_2/T_0 when the pressure reaches $p_2/p_0 = 0.8$. [4]

(c) When the pressure in the vessel reaches $0.8\,p_0$, the valve is re-opened and then closed again as soon as the pressure in the vessel reaches p_0 . This process takes place quickly and, immediately afterwards, the vessel contains a mass of air m_3 at temperature T_3 . Stating your assumptions, calculate T_3/T_0 and m_3/m_1 . [20]

SECTION B

7 (**short**)

(a) Under what circumstances is Moment of Momentum conserved? [2]

(b) A planar mechanism lies in the horizontal plane and is shown viewed from above in Figure 7. A light rigid rod OA of length $2l$ pivots freely about O. A small heavy particle, P, of mass m has a hole through its centre and is held by a thin wire at a distance l from O. Another small heavy particle, Q, of mass m is fixed at the end of the rod at point A.

At the instant shown, when the rod is rotating about O with angular velocity ω_1, the wire breaks and P is free to slide without friction along the rod.

(i) Express, in terms of ω_1, the angular velocity of the rod at the instant that P strikes Q. [3]

(ii) Express, in terms of ω_1 and l, the velocity of P along the rod at the instant that it strikes Q. [5]

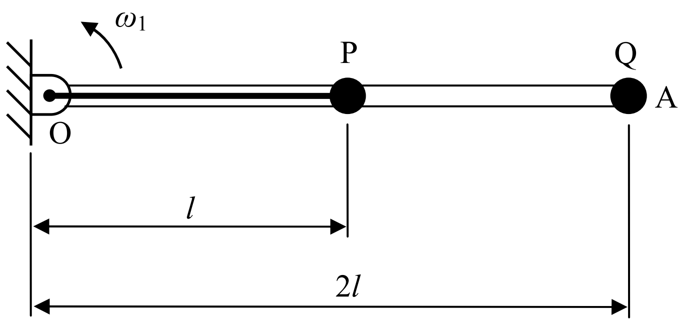

Fig. 7

8 (**short**) A child is flying a kite, as shown in Figure 8. The child is running into the wind at a steady speed of 3 m s^{-1}, with the kite flying directly downwind of the child. At the instant shown, the length of the string between the child and the kite is 20 m, and is increasing at a steady rate of 2 m s^{-1}. The angle θ that the string makes with the vertical is $\pi/4$ radians, and is decreasing at a steady rate of 0.2 radians per second.

(a) How fast is the kite travelling, and what angle does its path make with the horizontal? [4]

(b) What is the magnitude of the kite's vertical acceleration? [4]

(c) Is the kite's path curving upwards or downwards? Give a brief reason for your answer. [2]

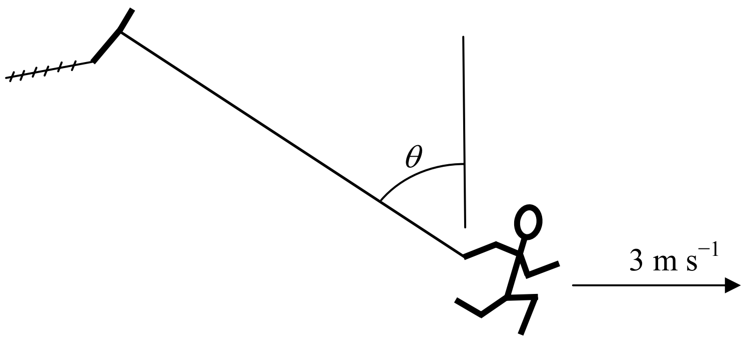

Fig. 8

9 (**short**) Figure 9 shows a schematic elevation of a toggle switch, which is being operated by a vertical force F acting at point A. In the position shown, the spring is under compression, exerting a horizontal force of 20 N on the slider at D, and the link AC is rotating clockwise with an angular velocity of 10 rad s^{-1} .

(a) What is the angular velocity of link CD? [3]

(b) What is the linear velocity of the slider at D? [3]

(c) How big must F be to cause this motion if there is no friction and all the moving parts have negligible mass? [4]

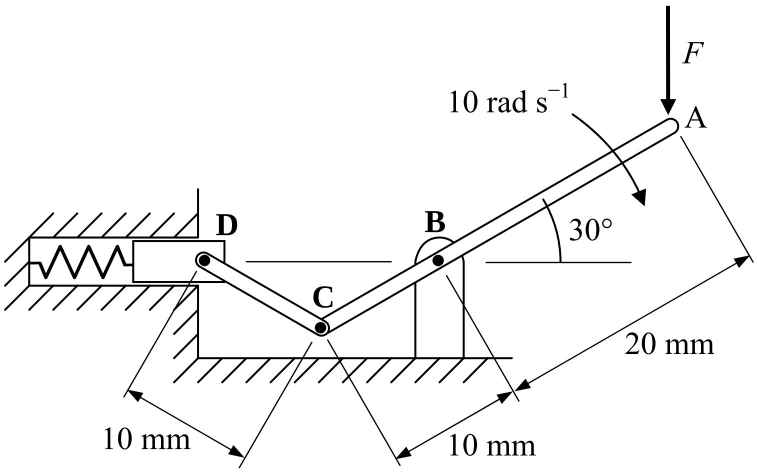

Fig. 9

10 (**short**) The left-hand side of a light rigid frame is attached to a wall by a spring of stiffness k , and a viscous dashpot of rate λ_1 . The right-hand side of the frame is connected to a viscous dashpot with a rate of λ_2 . The system has an input displacement of x , and an output displacement of y , as shown in Figure 10. Initially, the system is at rest with $x = y = 0$, and the spring is uncompressed.

(a) Show that x and y are related by the following differential equation

$$T\dot{y} + y = b\dot{x}$$

and express T and b in terms of k , λ_1 and λ_2 . [3]

(b) If x increases at a constant rate, α , derive an expression for y . [4]

(c) It is required that $y = x/2$ when $t = T$. Determine the value of λ_2 (expressed in terms of λ_1) that will satisfy this requirement. [3]

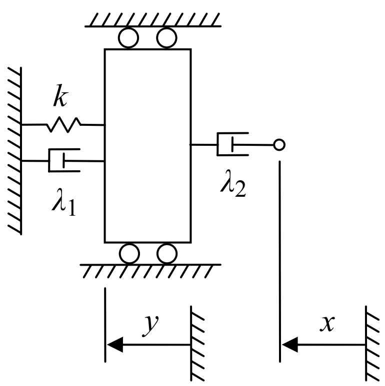

Fig. 10

11 (**long**) Two masses are suspended by three springs and constrained to move vertically, as shown in Figure 11. The mass and stiffness matrices for the system are

$$\mathbf{M} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{kg} \quad \text{and} \quad \mathbf{K} = \begin{bmatrix} 300 & -200 \\ -200 & 400 \end{bmatrix} \text{N m}^{-1}.$$

(a) Find the values of m_1 and m_2, and of k_1, k_2 and k_3. [8]

(b) Find the resonant frequencies of the system. [6]

(c) The system is subjected to a harmonic excitation $f = F \sin \omega t$, causing the two masses to move sinusoidally with $y_1 = Y_1 \sin \omega t$ and $y_2 = Y_2 \sin \omega t$.

Find any values of ω for which one or other of the masses does not move. [6]

(d) If the system is excited as in (c) above with $F = 10$ N , sketch a graph showing how Y_1 and Y_2 vary with frequency, ω , between 0 and 40 radians per second. Your sketch need only be approximate, but it should show Y_1 and Y_2 to be negative when they are out of phase with f . It should also show the magnitudes of Y_1 and Y_2 at $\omega = 0$, and at any frequency at which one or other of the masses remains stationary. [10]

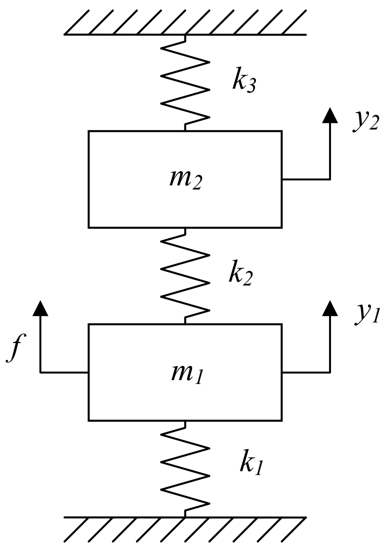

Fig. 11

12 (**long**) The gearbox shown in Figure 12(a), has a central sun gear which rotates about a fixed point at its centre, O. The system includes a ring gear that also has its centre at O. The sun gear and the ring gear are separated by three equally spaced planet gears with centres at A, B and C. The planet gears are held by a fixed Y-shaped arm that carries the pivots (A, B and C) about which the planet gears rotate. The planet gears are each of radius r ; the sun gear is of radius $2r$. The mass of the system may be neglected, and there is no slipping at the gear interfaces.

The sun gear is driven clockwise with an angular velocity of ω_1 .

(a) Express the angular velocity of the ring gear in terms of ω_1 . [8]

(b) A second gearbox is shown in Figure 12(b). All the gears have the same dimensions as those in the first gearbox but the arm is free to rotate and the ring gear is fixed. The sun gear of the second gearbox is also driven clockwise with an angular velocity of ω_1 .

(i) For the second gearbox, sketch a velocity diagram showing the velocities for pivots A, B and C. [6]

(ii) For the second gearbox, express the angular velocity of the arm that carries the planet gears in terms of ω_1 . [4]

(c) For both gearboxes, the torque driving the sun gear has a value of $12T$.

(i) If there is no friction in the gearboxes, express the output torque of the arm in the second gearbox in terms of the output torque of the ring gear in the first gearbox. [4]

(ii) If there is a frictional torque T acting at each of the pivots A, B and C, express the output torque of the arm in the second gearbox in terms of the output torque of the ring gear in the first gearbox. [8]

(cont.

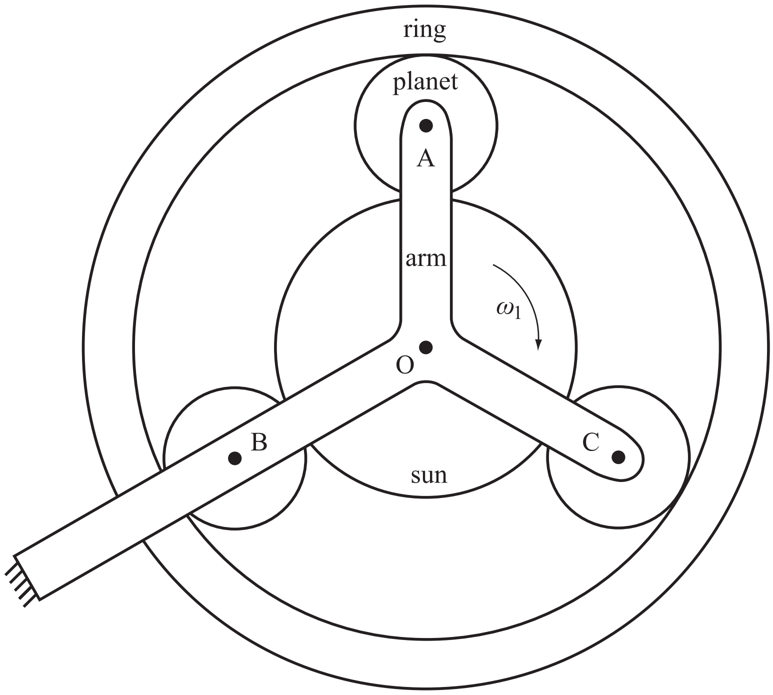

Fig. 12(a)

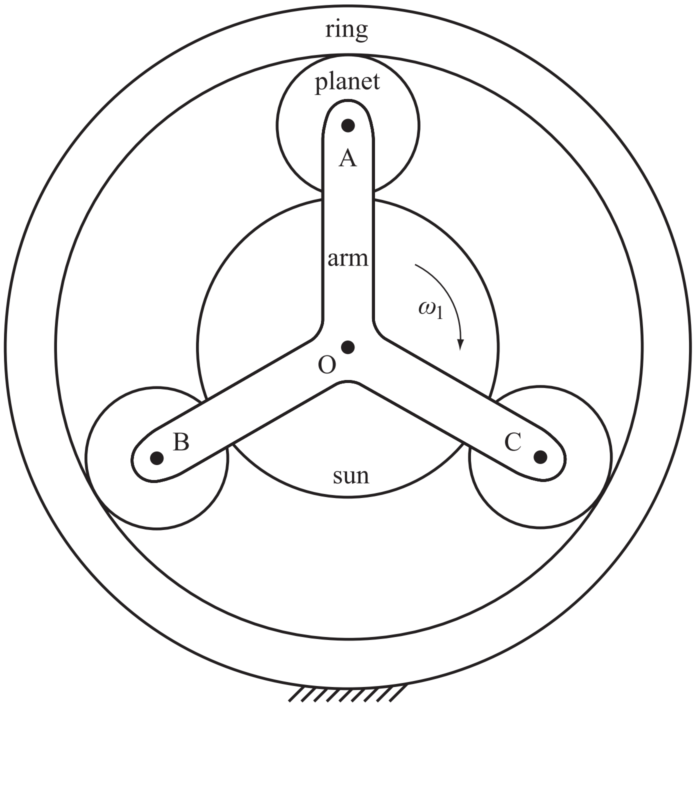

Fig. 12(b)

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