Synchronisation of metronomes with feedback

Perfect timing inc.

Dear consultants,

When mechanical metronomes are placed on a platform that is free to swing from side to side, the metronomes will gradually shift in their phase and frequency until they are synchronised with each other. This is a well-known physical phenomenon that is the subject of several online videos (e.g. see here and this nice video from Veritaseum) and has also been studied mathematically (see, for example, [2]). The synchronisation occurs because of the mechanical connection between the platform and the metronomes; the oscillations of the metronomes cause the platform to move, which in turn affects the metronomes.

We in *Perfect Timing* are exploring various applications of synchronised systems of connected oscillators, both mechanical and electronic. Unfortunately, our planned experiments require systems of already-synchronised oscillators and the metronome synchronisation process is often fairly slow, as you can see from videos. We need to improve the speed at which oscillators synchronise with each other.

Lugo-Cardenas et al. [1] have investigated whether certain types of control (generally based on supplying some additional torque to the platform) can be used to speed the synchronisation of metronomes. Their system is interesting, and we would like to use mathematical modelling to understand better what they have achieved and how to modify it for more general systems of oscillators (e.g. the coupled oscillator systems described in [3], or systems of more than two oscillators) We would also like to understand the trade-off between the time it takes for a system to synchronise and the amount of energy we put into our control. If we place a given value on the additional energy supplied to the platform and a given value on the time to synchronisation, what is the best strategy available to us?

References

[1]   I. Lugo-Cérdenas et al. "Metronome synchronization using feedback control".
      In: *2012 9th International Conference on Electrical Engineering, Computing Science and Automatic Control (CCE)*. 2012, pp. 1–5.
      DOI: 10.1109/ICEEE.2012.6421188.

[2]   James Pantaleone. "Synchronization of metronomes".
      In: *American Journal of Physics* 70.10 (Oct. 2002), pp. 992–1000.
      DOI: 10.1119/1.1501118.

[3]   Mason A. Porter and James P. Gleeson.
      Dynamical Systems on Networks: A Tutorial. 2015. arXiv: 1403.7663 [nlin.AO].
      URL: https://arxiv.org/abs/1403.7663.