Problem Set #7

Wednesday, September 28, 2022 9:08 AM

1. Unfair coin with P(H) = 0.55, 4 flips
   a.

Heads	Combs	Individual Probability
0	!H, !H, !H, !H	(.45)^4=0.041
1	H, !H, !H, !H !H, H, !H, !H !H, !H, H, !H !H, !H, !H, H	(.45)^3*.55=0.0501
2	H, H, !H, !H H, !H, H, !H H, !H, !H, H !H, H, H, !H !H, H, !H, H !H, !H, H, H	(.45)^2*(.55)^2=0.0613
3	!H, H, H, H H, !H, H, H H, H, !H, H H, H, H, !H	(.45)*(.55)^3=0.0749
4	H, H, H, H	(.55)^4=0.0915

   b. 5: 0, 1, 2, 3, and 4
   c. HHTH and THHH are the same but TTHT is different as the number of tails and heads differs. If this was a fair coin, they would have all been the same
      i. HHTH: 0.55*0.55*0.45*0.55=0.0749
      ii. THHH: 0.45*0.55*0.55*0.55=0.0749
      iii. TTHT: 0.45*0.45*0.55*0.45=0.0501
   d. 4
   e. $\frac{0.0749 * 3}{1} = 0.2247$
   f. Because the coin is not fair so the probabilities differ across the different number of heads
2. Unfair coin with P(H) = 0.55, 4 flips
   a.

Heads	Combs	Individual Prob	Total Prob
0	!H, !H, !H, !H	(.45)^4=0.041	(.45)^4=0.041
1	H, !H, !H, !H !H, H, !H, !H !H, !H, H, !H !H, !H, !H, H	(.45)^3*.55=0.0501	(.45)^3*.55*4=0.2005
2	H, H, !H, !H H, !H, H, !H H, !H, !H, H !H, H, H, !H !H, H, !H, H !H, !H, H, H	(.45)^2*(.55)^2=0.0613	(.45)^2*(.55)^2*6=0.3675
3	!H, H, H, H H, !H, H, H H, H, !H, H H, H, H, !H	(.45)*(.55)^3=0.0749	(.45)*(.55)^3*4=0.2995

4	H, H, H, H	(.55)^4=0.0915	(.55)^4=0.0915

i. 0.041+0.2005+0.3675+0.2995+0.0915=1

b. It's a normal distribution that's slightly skewed due to the coin not being fair

   i. 

c. It would still be a normal distribution but would be exactly symmetrical

d. What are the chances that the first 2 are heads and how does that align with the table?

   i. 0.55*0.55=0.3025

   ii. In the table, there are 4 options or 0.25, which would be 0.5 * 0.5. Makes sense since table does not include the chances of the coin. If we add the individual probabilities, we get 0.0915+0.0749+0.0749+0.0613=0.3026, which is basically the same (rounding error probably)

3. $\frac{n!}{k!\,(n-k)!}$

   a. $\frac{4!}{3!\,(4-3)!} = \frac{4}{1} = 4, \text{they match}$

   b. They all match

      i. $\frac{4!}{0!\,(4-0)!} = 1$

      ii. $\frac{4!}{1!\,(4-1)!} = 4$

      iii. $\frac{4!}{2!\,(4-2)!} = \frac{4*3}{2!} = 6$

      iv. $\frac{4!}{4!\,(4-4)!} = 1$

   c. With tails, the function would not change at all since it's just checking number of combinations

      i. $n\ choose\ n-k\colon \frac{n!}{(n-k)!\,(n-n+k)!} = \frac{n!}{(n-k)!\,(k)!} = \frac{n!}{k!\,(n-k)!}$

   d. The rows reflect how many coin flips occur and the numbers in the middle tell you how many times each outcome occurs

4. At least 16 with 3 dice

   a. 4, 6, 6; 6, 4, 6; 6, 6, 4

   b. 5, 5, 6; 5, 6, 5; 5, 5, 6

   c. 5, 6, 6; 6, 5, 6; 6, 6, 5;

   d. 6, 6, 6

   e. 10/216=5/108