Assignment #1

Monday, September 12, 2022 1:54 AM

Apprentice

1. Let A be catching mending book, let B be a zombie spawning nearby
   a. $P(A) = 0.025 = \frac{1}{40}, P(B) = 0.16 = \frac{4}{25}$
      i. Outcomes: A, !A and B, !B, as in catching book, not catching book and zombie spawning and zombie not spawning
   b. Yes, catching a mending book and a zombie spawning nearby are completely separate events and do not impact the chances of the other happening
   c. $P(A\ and\ !B) = \frac{1}{40} * \left(1 - \frac{4}{25}\right) = \frac{1}{40} * \frac{21}{25} = \frac{21}{1000} = 2.1\%$
   d.

	A	!A	
B	0.004	0.156	0.16
!B	0.021	0.819	0.84
	0.025	0.975	1

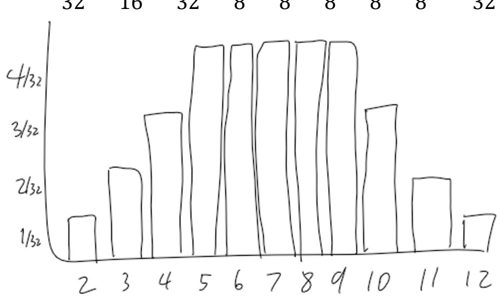
      i. $P(A\ and\ B) = \frac{1}{40} * \frac{4}{25} = \frac{1}{250} = 0.004$
      ii. $P(!A\ and\ B) = \left(1 - \frac{1}{40}\right) * \frac{4}{25} = \frac{39}{250} = 0.156$
   e. A or B happening is the sum of A and !B, A and B, and !A and B, basically every option except for both not occurring, which is !A and !B. Therefore, P(A or B) + P(!A and !B) = 1 because they add up to all the options.
      i. $P(A\ or\ B) = 0.004 + 0.021 + 0.156 = 1 - 0.819 = 0.181 = \frac{181}{1000}$
      ii. $P(!A\ and\ !B) = \frac{39}{40} * \frac{21}{25} = \frac{819}{1000} = 0.819$
2. Let B be zombie spawning nearby, let R be raining
   a.

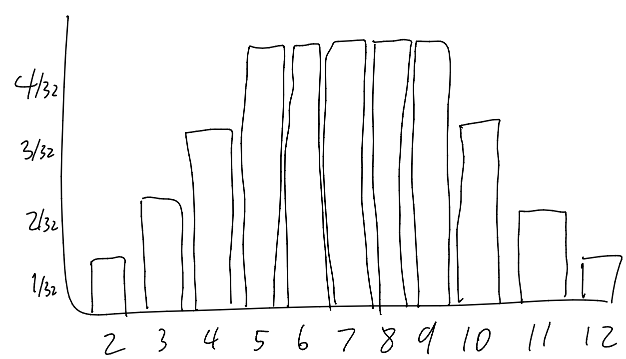
	R	!R	
B	0.06	0.1	0.16
!B	0.26	0.58	0.84
	0.32	0.68	1

   b. $P(B\ or\ R) = 0.06 + 0.1 + 0.26 = 1 - 0.58 = 0.42$
   c. No because normally, P(R and B) would be 0.0512 but it is 0.06 because the events are dependent and affect the chances of one another
3. Sum of 4-sided and 8-sided dice
   a. Define stuff
      i. Random variable: Outcome of 4-sided die and outcome of 8-sided die
      ii. Event: Sum of the 4-sided die and 8-sided die being a certain number
      iii. Possible outcomes: 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12
   b.

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	2	3	4	5	6	7	8	9
2	3	4	5	6	7	8	9	10
3	4	5	6	7	8	9	10	11
4	5	6	7	8	9	10	11	12

      i. $2{:}\frac{1}{32}, 3{:}\frac{1}{16}, 4{:}\frac{3}{32}, 5{:}\frac{1}{8}, 6{:}\frac{1}{8}, 7{:}\frac{1}{8}, 8{:}\frac{1}{8}, 9{:}\frac{1}{8}, 10{:}\frac{3}{32}, 11{:}\frac{1}{16}, 12{:}\frac{1}{32}$
   c.





Master

4.

Name	2020 Acceptance Rate
Stanford	5.2%
UCLA	14.3%
USC	16.1%
Berkeley	17.5%
University of Washington	55.9%
Washington State University	79.9%
Oregon State University	82.2%
University of Oregon	83.8%

a. Stanford, UCLA, Berkeley, UW: Chance of getting at least 1 is 1 - chance of not getting any
   i. Let S be getting into Stanford, L be getting into UCLA, B be getting into Berkeley, W be getting into UW
   ii. $P(!S \text{ and } !L \text{ and } !B \text{ and } !W) = (1-0.052)*(1-0.143)*(1-0.175)*(1-0.559)$ $= 0.948*0.857*0.825*0.441 = 0.2956 = 29.56\%$
   iii. $1 - 0.2956 = 0.7044 =$ 70.44%

b. I think that logically they are independent events since getting in or getting rejected by one university won't affect the chance another one's decision. However, if you are a very exceptional student, then it is more likely for you to get into more schools. Acceptances are not completely random and are based on the student's profile, so that does impact how independent the events are.

c. No because rather than being randomly accepted or rejected based on probability, they view your profile relative to other students. If a student has 4.0 GPA and does a ton of volunteering and extracurriculars, they are more likely to get in than someone who has a 2.5 GPA and only plays video games. These percentages are determined by looking at the total number accepted divided by the total number of applicants. Therefore, depending on your place in the applicant pool and other factors, your chances could be higher or lower than the given acceptance rate.

d. Situations with >0.95 chance of at least 1 acceptance. Commonality is they include universities with really high acceptance rates so that the chance of not getting into either is minimized.
   i. Let O be getting into Oregon State University, U be getting into University of Oregon
      1) $P(!O \text{ and } !U) = (1-0.822)*(1-0.838) = 0.178*0.162 = 0.0288$
      2) $1 - 0.0288 = 0.9721 = 97.21\%$
      3) $P(O \text{ or } U) = 0.9721 > 0.95$
   ii. Let U be getting into University of Oregon, A be getting into Washington State University
      1) $P(!U \text{ and } !A) = (1-0.838)*(1-0.799) = 0.162*0.201 = 0.0326$
      2) $1 - 0.0326 = 0.9674 = 96.74\%$
      3) $P(U \text{ or } A) = 0.9674 > 0.95$

e. That is not possible since even if the percentage gets increasingly close to 100%, the chance of not getting into any will always be nonzero since multiplication by a nonzero number cannot be 0. You would need a university with an 100% acceptance rate.

Wizard

5. Risk™, let D be defender's roll, let A_1 be attacker's first roll and A_2 be attacker's second roll, let A be the larger value between A_1 and A_2
   a. Assuming it's a fair die, each side [1 - 6] would be 1/6
   b. $1: \frac{1}{36}, 2: \frac{3}{36}, 3: \frac{5}{36}, 4: \frac{7}{36}, 5: \frac{9}{36}, 6: \frac{11}{36}$
      i.

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

      ii. Should sum up to 1, 1/36+3/36+5/36+7/36+9/36+11/36=1

   c.

D\A	1	2	3	4	5	6	
1	1/216	3/216	5/216	7/216	9/216	11/216	1/6
2	1/216	3/216	5/216	7/216	9/216	11/216	1/6
3	1/216	3/216	5/216	7/216	9/216	11/216	1/6
4	1/216	3/216	5/216	7/216	9/216	11/216	1/6
5	1/216	3/216	5/216	7/216	9/216	11/216	1/6
6	1/216	3/216	5/216	7/216	9/216	11/216	1/6

	1/36	3/36	5/36	7/36	9/36	11/36	

d. $P(A > D) = \frac{3}{216} + \frac{10}{216} + \frac{21}{216} + \frac{36}{216} + \frac{55}{216} = \frac{125}{216}$

   i. $P(D \geq A) = 1 - \frac{125}{216} = \frac{6}{216} + \frac{15}{216} + \frac{20}{216} + \frac{21}{216} + \frac{18}{216} + \frac{11}{216} = \frac{91}{216}$

e. Everything would be the exact same except for the chances in the second table between defenders and attackers. We would just need to adjust all the fractions there for the chances of each matchup happening and that would be it. For the 3 dice maximums, we can first look at what we did for 2 dice. The table above has , we have these patterns for maximums that look like the bottom and right edges of a square and keeps expanding.

   i.

	1	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	2	3	4	5	6
3	3	3	3	4	5	6
4	4	4	4	4	5	6
5	5	5	5	5	5	6
6	6	6	6	6	6	6

Assignment 1 3 Dice...

   ii. If we were to move on to 3 dice, that pattern would continue, but with the 3 outer faces of a cube. For example, if we looked at 3 fair 3-sided dice (table below), we see that the chance of getting n as the maximum where n is a nonzero positive integer is $n^3 - (n-1)^3$, because the inner cubes $(n-1)^3$ are where the maximums only include values below n.

      1)

Die #1	Die #2	Die #3	Max		Num	Count	num^3-(num-1)^3
1	1	1	1		1	1	1^3-0^3=1
1	2	1	2		2	7	2^3-1^3=7
1	3	1	3		3	19	3^3-2^3=19
1	1	2	2				
1	2	2	2				
1	3	2	3				
1	1	3	3				
1	2	3	3				
1	3	3	3				
2	1	1	2				
2	2	1	2				
2	3	1	3				
2	1	2	2				
2	2	2	2				
2	3	2	3				
2	1	3	3				
2	2	3	3				
2	3	3	3				
3	1	1	3				
3	2	1	3				
3	3	1	3				
3	1	2	3				
3	2	2	3				
3	3	2	3				
3	1	3	3				
3	2	3	3				
3	3	3	3				

   iii. Based on the reasoning above, we can get the chances for each maximum to be:
      1) 1: 1^3-0^3=1, 1/216
      2) 2: 2^3-1^3=7, 7/216
      3) 3: 3^3-2^3=19, 19/216
      4) 4: 4^3-3^3=37, 37/216
      5) 5: 5^3-4^3=61, 61/216
      6) 6: 6^3-5^3=91, 91/216
      7) Check: 1/216+7/216+19/216+37/216+61/216+91/216=1, Good

   iv. Then, you just apply these chances with the 2 dice chances from before for the defender to figure out who wins