Problem Set #2

Tuesday, September 6, 2022 2:24 PM

1. Imagine the following situations involving two events, A and B as described below, when drawing a single card from a 52 card deck (you should assume Ace equals 1).

Event A	Event B
Draw a Facecard	Draw a Black card
Draw a Red card	Draw a card ≤ 7
Draw a Red card	Draw a One-eyed card
Draw a Facecard	Draw a card ≤ 7

   a. For each scenario above, calculate P(A), P(B), P(A or B) and P(A and B).
      - A = draw face-card, B = draw black card:
         - $P(A) = \frac{3}{13}, P(B) = \frac{1}{2}$
         - $P(A\ and\ B) = \frac{3}{13} * \frac{1}{2} = \frac{3}{26}$
         - $P(A\ or\ B) = \frac{6}{26} + \frac{13}{26} - \frac{3}{26} = \frac{16}{26} = \frac{8}{13}$
      - A = draw red card, B = draw card <= 7:
         - $P(A) = \frac{1}{2}, P(B) = \frac{7}{13}$
         - $P(A\ and\ B) = \frac{7}{26}$
         - $P(A\ or\ B) = \frac{13}{26} + \frac{14}{26} - \frac{7}{26} = \frac{20}{26} = \frac{10}{13}$
      - A = draw red card, B = draw one-eyed card
         - $P(A) = \frac{1}{2}, P(B) = \frac{3}{52}$
         - $\mathrm{P}(A\ and\ B) = \frac{3}{104}$
         - $\mathrm{P}(A\ or\ B) = \frac{52}{104} + \frac{6}{104} - \frac{3}{104} = \frac{55}{104}$
      - A = draw face-card, B = draw card <= 7:
         - $P(A) = \frac{3}{13}, P(B) = \frac{7}{13}$
         - $P(A\ and\ B) = \frac{21}{169}$
         - $P(A\ or\ B) = \frac{39}{169} + \frac{91}{169} - \frac{21}{169} = \frac{109}{169}$

   b. Can you use your results to generalize the equations for P(A or B) and P(A and B)? Hint: think about how concepts of independence and mutually exclusive come into play.
      - $P(A\ and\ B) = P(A) * P(B)$
         - If you draw a tree out, you can that the first thing has x options and the second thing has y options, so for each one in x there are y options. Therefore, the total number of possibilities is x * y. Then, the chance of both occurring would be the probability of one happening times the probability of the second one happening since that's the amount of the tree where both things occur.
      - $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$
         - You add the chance A happens and the chance B happens, then subtract the chance they both happen since it repeats

2. Imagine the following situations involving two events, A and B when rolling two rolls of a fair six-sided die.

Event A	Event B
First $= 3$	Second $= 5$
First $= 5$	Second $= 3$
First $= 3$	Sum is even
First $= 5$	Sum is ≥ 10

   a. For each scenario above, calculate P(A), P(B), P(A or B) and P(A and B).

- A = first die lands on 3, B = second die lands on 5
  - $P(A) = \frac{1}{6}, P(B) = \frac{1}{6}$
  - $P(A\ and\ B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$
  - $P(A\ or\ B) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$
- A = first die lands on 5, B = second die lands on 3
  - $P(A) = \frac{1}{6}, P(B) = \frac{1}{6}$
  - $P(A\ and\ B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$
  - $P(A\ or\ B) = \frac{6}{36} + \frac{6}{36} - \frac{1}{36} = \frac{11}{36}$
- A = first die lands on 3, B = second die lands on side that makes sum even (aka lands on odd number)
  - $P(A) = \frac{1}{6}, P(B) = \frac{1}{2}$
  - $P(A\ and\ B) = \frac{1}{6} * \frac{1}{2} = \frac{1}{12}$
  - $P(A\ or\ B) = \frac{6}{36} + \frac{18}{36} - \frac{3}{36} = \frac{21}{36} = \frac{7}{12}$
- A = first die lands on 5, B = second dice lands on side that makes sum >= 10 (aka 5 or 6)
  - $P(A) = \frac{1}{6}, P(B) = \frac{1}{3}$
  - $P(A\ and\ B) = \frac{1}{6} * \frac{1}{3} = \frac{1}{18}$
  - $P(A\ or\ B) = \frac{3}{18} + \frac{6}{18} - \frac{1}{18} = \frac{8}{18} = \frac{4}{9}$

b. Can you use your results to generalize the equations for P(A or B) and P(A and B)? Hint: think about how concepts of independence and mutually exclusive come into play.
   - $P(A\ and\ B) = P(A) * P(B)$
   - $P(A\ or\ B) = P(A) + P(B) - P(A\ and\ B)$

3. Elliot has a 60% chance of getting an A in Calculus but only a 35% chance of getting an A in Statistics. The probability that he gets an A in both is 25%.
   a. How can you visualize the relationship between these two events?

   60%=3/5, 35%=7/20
   A = getting A in calc, B = getting A in stats

A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A & B	A & B	A & B	B	B
A	A	A		
A	A	A		
A	A	A		
A	A	A		
A	A	A		
A	A	A		
A	A	A		
A	A	A		

A	A	A		
A	A	A		
A	A	A		
A	A	A		
A	A	A		

b. What is the probability of getting an A in neither? Please use variables to define your events, and then give the answer using probability notation as well as its value.
   - A = getting A in calc, B = getting A in stats
   - $P(!A \text{ and } !B) = (1 - 60\%) * (1 - 35\%) = 40\% * 65\% =$ 26%

c. What is the probability of getting an A in at least 1 of the courses? Again give the answer using probability notation as well as its value.
   - A = getting A in calc, B = getting A in stats
   - $P(A \text{ or } B) = 60\% + 35\% - 21\% =$ 74%
   - $100\% - 26\% \ (part\ b) =$ 74%

d. What is the probability of getting an A in Stats but not in Calculus? Again give the answer using probability notation as well as its value.
   - A = getting A in calc, B = getting A in stats
   - $P(!A \text{ and } B) = P(B) - P(A \text{ and } B) = 35\% - 21\% =$ 14%

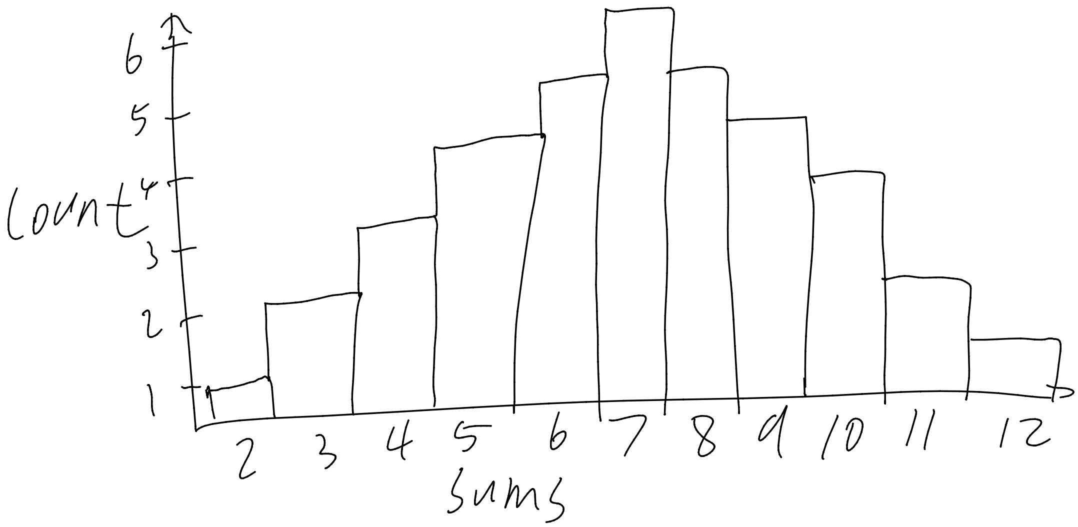
e. Do you think these events are independent? Why or why not and be quantitative in your answer.
   - In the real world maybe not but in this scenario yes since they don't affect the results of one another.

4. When rolling two fair 6-sixed die:
   a. What is the probability of rolling 2, 3, or 12?
      a. 2 has to be 1 and 1 so 1/36
      b. 3 can be 1 and 2 or 2 and 1 so 2/36 or 1/18
      c. 12 has to be 6 and 6 so 1/36
   b. Create a table or otherwise visualize the probabilities of all possible outcomes (sums).

Die 1\Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Sum	**Count**
2	1
3	2
4	3
5	4
6	5
7	6
8	5
9	4
10	3
11	2
12	1

c. Sketch by hand a histogram of the distribution of outcomes (sums).



d. Now imagine that one of the die is weighted, such that each value is no longer equally likely. Assume the following probabilities: 1 - 2/8, 2 - 1/8, 3 - 1/8, 4 - 1/8, 5 - 1/8, 6 - 2/8. In this case what is the probability of the sum of the two dice being a 7?

If the first one is 1, we need 6, if 2, then 5, if 3, then 4, … Since all these can occur, we can just average everything (see below for mathematical explanation)

$$\frac{1}{6}*\frac{2}{8}+\frac{1}{6}*\frac{1}{8}+\frac{1}{6}*\frac{1}{8}+\frac{1}{6}*\frac{1}{8}*\frac{1}{6}*\frac{1}{8}*\frac{1}{6}*\frac{2}{8}=\frac{2}{48}+\frac{1}{48}+\frac{1}{48}+\frac{1}{48}+\frac{1}{48}+\frac{2}{48}=\frac{8}{48}=\frac{1}{6}$$

It's the same as before since it doesn't matter which side it lands on because 7 can be made with any side of the die and average will end up being the same.