

Noise observation in CKKS

R11922138 蕭彧

My first number $A=4$, and the second number $B=2$. I add A with B 10 times and multiply A with B 10 times, respectively.

Addition	multiplication
Noise[1]: 7.844922933486487e-09	Noise[1]: 1.1458763880867195e-05
Noise[2]: 1.1539359690004858e-08	Noise[2]: 8.01704225956712e-05
Noise[3]: 1.523379822288007e-08	Noise[3]: 0.0003091811240025777
Noise[4]: 1.89282332030416e-08	Noise[4]: 0.0010304747621034949
Noise[5]: 2.262266995955997e-08	Noise[5]: 0.0029614802734556633
Noise[6]: 2.6317099610650985e-08	Noise[6]: 0.007846096143055092
Noise[7]: 3.0011534590812516e-08	Noise[7]: 0.019843690851075735
Noise[8]: 3.3705976676401406e-08	Noise[8]: 0.048173566176728855
Noise[9]: 3.740041165656294e-08	Noise[9]: 0.11478455097130791
Noise[10]: 4.109484308401079e-08	Noise[10]: 0.2698625292759971
Adding multiple times: [24.000000041094843]	Timing multiple times: [4096.269862529276]

Apparently, multiplication has bigger error than addition. The error of addition grows up **linearly**, whether the error of multiplication grows up **exponentially**.

Reason:

CKKS encode plain text m to $(a, b = \langle s, a \rangle + e + m)$, where s is secret key and a is random sampled array.

Assume we have m_1 encode to $(a_1, b_1 = \langle s, a_1 \rangle + e_1 + m_1)$ and m_2 encode to $(a_2, b_2 = \langle s, a_2 \rangle + e_2 + m_2)$

Addition:

$(a_1 + a_2, \langle s, a_1 \rangle + \langle s, a_2 \rangle + e_1 + e_2 + m_1 + m_2)$

The error increases linearly after one addition.

Multiplication:

$(a_1 * a_2, \dots + e_1 * e_2 + \dots)$

The error increases exponentially after one multiplication.

In conclusion, multiplication usually have bigger noise than addition. It's mainly because of the encoding method in HE. Error will multiply up when doing multiplication, while error will only add up when doing addition.