R11922138 蕭[illegible]

7.1 Assume $p_0 = \frac{p}{2}$, $\psi(n,p) < \psi(n,p_0)$

$\Rightarrow \sigma(n) + \varphi(n)/p + c\log p > \sigma(n) + \varphi(n)/p_0 + c\log p_0$

$\Rightarrow \varphi(n)/p + c\log p > 2\varphi(n)/p + c\log p - c$

$\Rightarrow c > \frac{\varphi(n)}{p}$

For any p, we can find a $p_0 = \frac{p}{2}$ with $c > \frac{\varphi(n)}{p}$ such that

$\psi(n,p) < \psi(n,p_0)$ #

7.2 $\varepsilon(n,p') \le \varepsilon(n,p)$

$\Rightarrow p'\sigma(n) + \varphi(n) + p'\kappa(n,p') \ge p\sigma(n) + \varphi(n) + p\kappa(n,p)$

$\Rightarrow (p'-p)\sigma(n) + p'\kappa(n,p') - p\kappa(n,p) > 0$

$\because p' > p$, $\therefore (p'-p)\sigma(n) > 0$

$\kappa(n,p)$ is an increasing function, meaning $p'\kappa(n,p')) - p\kappa(n,p) > 0$ #

7.3 Sequential time : $(n-1)\chi$, parrallel time : $(\lceil n/p \rceil - 1)\chi + \lceil \log p \rceil(\lambda + \chi)$

So the speedup will be $\frac{\text{Sequential time}}{\text{parrellel time}} = \frac{(n-1)\chi}{(\lceil n/p \rceil - 1)\chi + \lceil \log p \rceil(\lambda + \chi)}$

P	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
speedup	1	1.67	1.87	2.22	2	2.14	2.26	2.35	1.96	2	2.04	2.07	2.1	2.12	2.14	2.16

7.4 By Amdahl's law, maximum speedup will be $\frac{1}{0.05 + \frac{0.95}{10}} \approx 6.9$ #

7.5 Equivalent to solving $\frac{1}{0.06 + \frac{0.94}{p}} = 10$, $p = 23.5$, which means we need at least 24 processors #

7.6 $\lim_{p \to \infty} \frac{1}{x + \frac{(1-x)}{p}} = 50 \Rightarrow x = 0.02$ #

7.7 $9 = \frac{1}{x + \frac{(1-x)}{10}} \Rightarrow x = \frac{1}{81}$ #

7.8 $s = \frac{9}{242}$, by gustatson Law, $p + (1-p)s = 16 + (1-16) \times \frac{9}{242} = 15.44$ #

7.9 $S = 0.01$, by gustafson law, $P + (1-P)S = 40 + (1-40) \times 0.01 = 39.61$ #

7.10 I II III IV V VI

B C A A C A

7.11 Amdahl's Law assume the problem size is fixed, so the speedup will be bounded by sequence time. On the other hand, Gustafson's Law assume fixed time, speedup will will increase when the problem size increase.

7.12 No, if the problem itself has large sequential portion, we can only use one processor to run it. So, if the sequental portion need a huge amount of time to solve, we can't guarantee to solve this problem within a specified time limit.

7.13 The scalability function $M(f(P))/P$ is at below

a. $(cp)^2/p = c^2 p$

b. $(C\sqrt{P}\log p)^2/p = c^2 \log^2 p$

c. $(c\sqrt{p})^2/p = c$

d. $(cp\log p)^2/p = cp\log^2 p$

e. $cp/p = c$

f. $p^c/p = p^{c-1}$, $1 < c < 2$

g. $p^c/p = p^{c-1}$, $c > 2$

Ranking : $c = e > b > f > a > d > g$

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