Example

One can easily prove (by M.I.) that

$$f(n) = \frac{n(n+1)(2n+1)}{6}, \forall n >= 0 \tag{1}$$

Proof

Let $f(n)$ be "$0^2 + 1^2 + 2^2 + 3^2 + ... + n^2 = \frac{n(n+1)(2n+1)}{6}$". For $\forall n \geq 0$,

For $n = 0$,

$$\text{L.H.S.} = 0$$
$$\text{R.H.S.} = 0$$

$\because$ L.H.S. = R.H.S.
$\therefore f(0)$ is true.

Assume $S(n)$ is true for some $n = k$ where $k \geq 0$, i.e.
$0^2 + 1^2 + 2^2 + 3^2 + ... + k^2 = \frac{k(k+1)(2k+1)}{6}$

For $n = k + 1$,

$$\begin{aligned}
\text{L.H.S.} &= 0^2 + 1^2 + 2^2 + 3^2 + ... + k^2 + (k+1)^2 \\
&= \frac{k(k+1)(2k+1)}{6} + (k+1)^2 \text{(By induction assumption)} \\
&= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6} \\
&= \frac{(k+1)(k(2k+1) + 6(k+1))}{6} \\
&= \frac{(k+1)(2k^2 + 7k + 6)}{6} \\
&= \frac{(k+1)(k+2)(2k+3)}{6} \\
&= \frac{(k+1)((k+1)+1)(2(k+1)+1)}{6} \\
&= \text{R.H.S.}
\end{aligned}$$

$\because$ L.H.S. = R.H.S.
$\therefore$ By the principle of mathematical induction, $f(n)$ is true for all $n \geq 0$.