COMP2119 Introduction to Data Structures and Algorithms
Assignment 1 - Recursion, Mathematical Induction and Algorithm Analysis

Due Date: Sept 27, 2024 7:00pm

Question 1 - Asymptotic Bounds [20%]

Rank the following functions by order of growth in increasing order and partition your list into equivalence classes such that two functions $f(n)$ and $g(n)$ are in the same class if and only if $f(n) = \Theta(g(n))$. You do not need to prove your answer. [In this question, we assume $\log n$ stands for $\log_2 n$.]

(a) n^{π},

(b) π^n,

(c) n^n,

(d) $\log n$,

(e) $\pi^{\log n}$,

(f) $n^{\log \pi}$,

(g) $\frac{n}{\log \pi}$,

(h) $\frac{n}{\log n}$,

(i) $\frac{n}{\log \log n}$,

(j) $\log \frac{n}{\log n}$,

(k) $\pi^{\log(2n)}$,

(l) $n^{\log(2\pi)}$,

(m) $\sqrt{\sum_{i=1}^{n}(i+1)}$,

(n) $1910n! + 316n^n$

Question 2 - Recurrence Relations [20%]

For each of the following recurrence relations, solve the recurrence and state the Θ bound.

(a) $T(n) = T(n-1) + 3$; $T(0) = 0$.

(b) $T(n) = 3T(\frac{n}{3}) + n$; $T(1) = 0$, you may assume that n is a power of 3.

(c) $T(n) = 4T(\frac{n}{3}) + 1$; $T(1) = 0$, you may assume that n is a power of 3.

(d) $T(n) = nT(\frac{n}{2}) + n - 1$; $T(1) = 1$, you may assume that n is a power of 2.

Question 3 - Mathematical Induction [30%]

Prove the following equations with mathematical induction.

(a) $1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n-1) \cdot 2^{n+1} + 2$ for all positive integers n.

(b) $\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{n}+\sqrt{n+1}} = \sqrt{n+1} - 1$ for all positive integers n.

Question 4 - Algorithm Design [30%]

Given a positive integer n, please design an algorithm that calculates 2^n faster than $O(n)$ time.

You should clearly explain the algorithm, demonstrate the correctness of the algorithm with a complete proof and show the running time of the algorithm.

For example, when $n = 13$, you may calculate 2^{13} in the following steps:

$$\begin{aligned} 2^3 &= 2 \cdot 2 \cdot 2 = 8; \\ 2^6 &= 2^3 \cdot 2^3 = 8 \cdot 8 = 64; \\ 2^{13} &= 2^6 \cdot 2^6 \cdot 2 = 64 \cdot 64 \cdot 2 = 8192. \end{aligned}$$

Submission

Please submit your assignment (one **PDF** file) to moodle by the deadline. Make sure the content is readable. Feel free to contact the TAs if you encounter any difficulty in this assignment. We are happy to help you!