COMP2119C
Data Structures & Algorithms

2024-25

Tutorial 1 – Graph Terminology (Appendix)

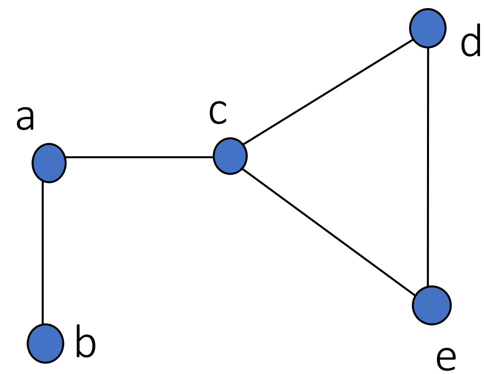
Jeff Siu

Definition of a Graph

A graph $G = (V, E)$ consists of

- V, the set of vertices (nodes, points), and
- E, the set of edges (lines)

Undirected graphs are graphs where edges have no direction (E: unordered pairs of V).

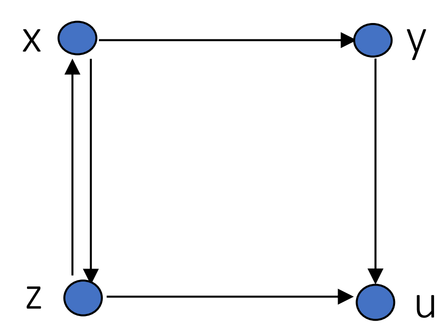


Example:

$V = \{a, b, c, d, e\}$

$E = \{\{a,b\}, \{a, c\}, \{c, d\}, \{c, e\}, \{d, e\}\}$

Directed graphs are graphs where edges have directions (E: ordered pairs of V).



Example:

$V = \{u, x, y, z\}$

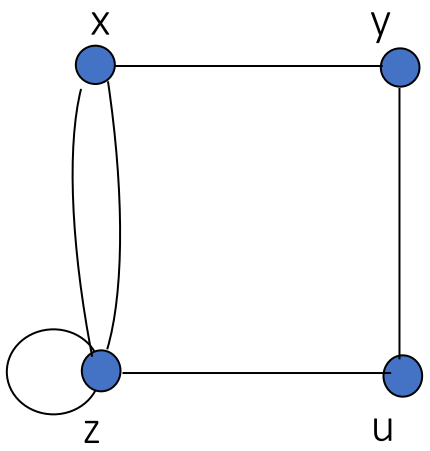
$E = \{(x, y), (x, z), (y, u), (z, x), (z, u)\}$

Simple Graphs and Multigraphs

Simple graphs - at most one edge between any pair of vertices, no self-loop.

Unless otherwise stated, a "graph" means a simple graph.

Multi-graphs - having multiple edges between the same (ordered) pair of vertices, self-loops allowed



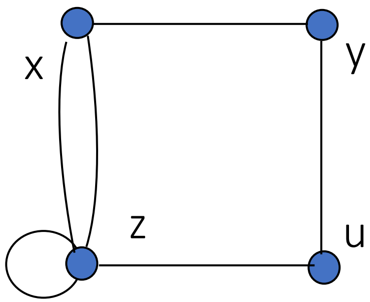
If x, y are two vertices, and e = {x, y} is in E, then

- x and y are adjacent
- e is incident with x and y
- x and y are the endpoints of e

Graph Degree

The degree of a vertex of a graph is the number of edges that are incident to the vertex.

Example: deg(y)=2, deg(u)=2, deg(x)=3, deg(z)=5

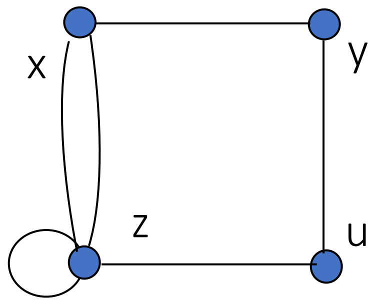


Graph Degree

For an undirected graph G = (V, E):

$$\sum_{v \in V} \deg(v) = ? \quad 2|E|$$

Proof: Every edge is counted twice in counting degrees.



e.g. sum of degrees = 3+2+5+2 = 12, |E| = 6

For a directed graph G=(V, E),

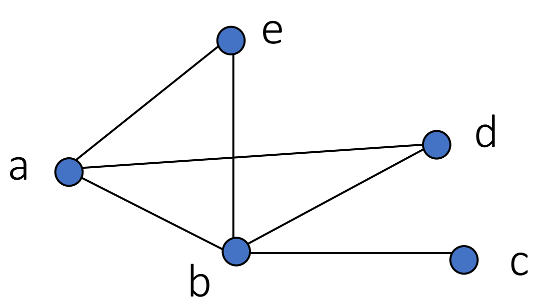
$$\sum_{v \in V} \deg^{-}(v) = \sum_{v \in V} \deg^{+}(v) = |E|$$

in-degree

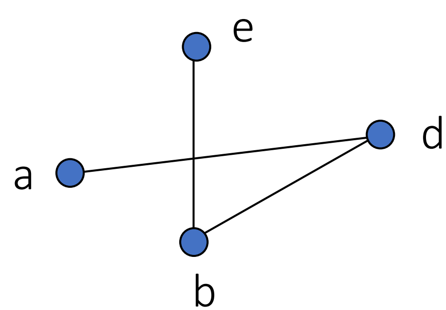
out-degree

Graph Operations

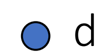
A graph $H = (V', E')$ is a **subgraph** of $G = (V, E)$ if $V' \subseteq V$, $E' \subseteq E$.



G



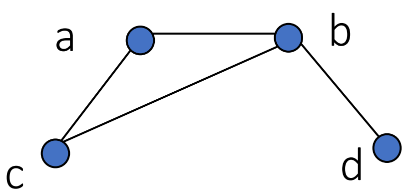
H



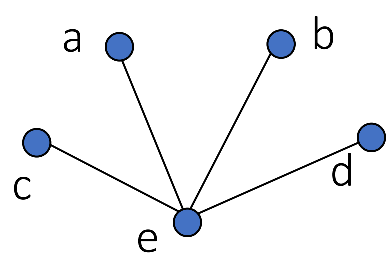
Is this a subgraph of G?

The **union** of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is

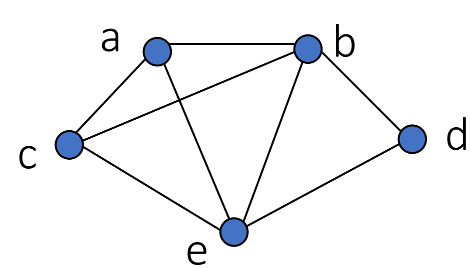
$G_1 \cup G_2 = (V_1 \cup V_2, E_1 \cup E_2)$



G_1



G_2



$G_1 \cup G_2$

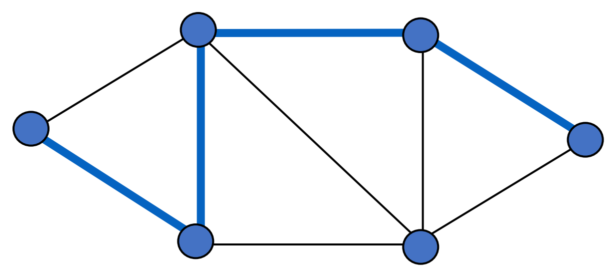
Paths

A path of length n from u to v in a graph G is a sequence of edges e_1, e_2, …, e_n, such that $e_1 = (u, x_1)$, $e_2 = (x_1, x_2)$, … , $e_n = (x_{n-1}, v)$.

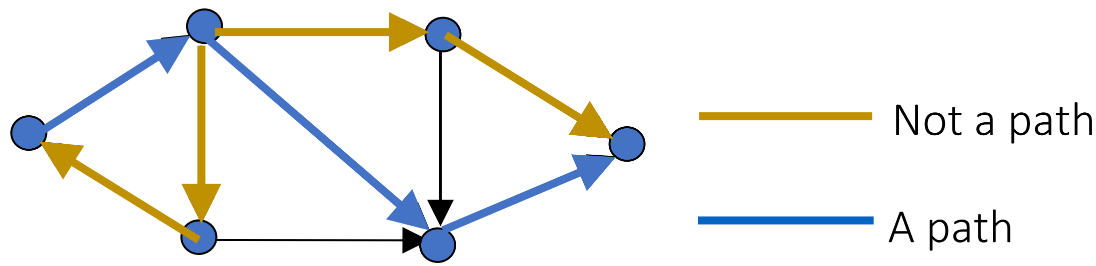
A path is simple if it doesn't contain the same vertex more than once.

Example:

- Undirected graph:



- Directed graph:

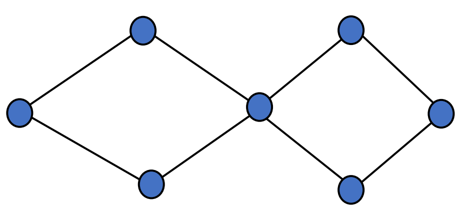


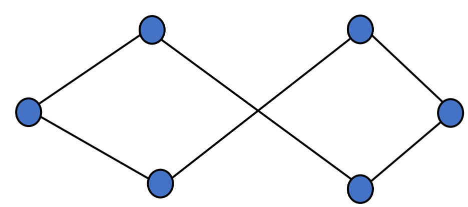
Circuits

A circuit of length n in a graph G is a sequence of edges e_1, e_2, …, e_n, such that $e_1 = (x_0, x_1)$, $e_2 = (x_1, x_2)$, … , $e_n = (x_{n-1}, x_0)$.

i.e., a circuit is a path that starts and ends at the same vertex.

A cycle is a circuit that does not contain the same vertex more than once.





Simple Paths

If there is a path between *u* and *v*, then there is a simple path between *u* and *v*.

Example:

u → a → b → c → d → e → f → a → b → v is a path

u → a → b → v is a simple path

a → b → c → d → e → f → a is a circuit