COMP2119 Introduction to Data Structures and Algorithms Assignment 1 - Recursion, Mathematical Induction and Algorithm Analysis

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1 Asymptotic Bounds

$n^\pi, \pi^n, n^n, \log n, \pi^{\log n}, n^{\log \pi}, \frac{n}{\log \pi}, \frac{n}{\log n}, \frac{n}{\log \log n}, \log \frac{n}{\log n},$
$\pi^{\log(2n)}, n^{\log 2\pi}, \sqrt{\sum_{i=1}^{n}(i+1)}, 1910n! + 316n^n$

2 Recurrence Relations

(a) $T(n) = T(n-1) + 3 \text{for } n > 0$ $\quad\therefore T(n) = \Theta(n)$

$= T(n-2) + 3 + 3 \text{for } n > 1$

$= \dots$

$= T(0) + 3n \text{for } n > 1$

$= 3n \text{for } n > 1$

(b) Assume that n is a power of 3, i.e. $n = 3^k$ for $k \in \mathbb{N}$, and $\log_3 n = k$, $\quad\therefore T(n) = \Theta(n \log n)$

$$\begin{aligned}
\because T(n) &= 3T(\frac{n}{3}) + n \text{for } n \neq 1 \\
&= 3(3T(\frac{n}{9}) + \frac{n}{3}) + n \text{for } n >= 9 \\
&= 9T(\frac{n}{9}) + n + n \text{for } n >= 9 \\
&= ... \\
\therefore T(n) &= 3^k T(\frac{n}{3^k}) + k * n \\
&= 3^k T(1) + k * n \\
&= 0 + k * n \\
&= kn \\
&= n * \log_3 n
\end{aligned}$$

(c) Assume that n is a power of 3, i.e. $n = 3^k$ for $k \in \mathbb{N}$, and $\log_3 n = k$, $\quad\therefore T(n) = \Theta()$

$\because T(n) = 4T(\frac{n}{3}) + 1$

(d) Assume that n is a power of 2, i.e. $n = 2^k$ for $k \in \mathbb{N}$, and $\log_2 n = k$, $\quad\therefore T(n) = \Theta()$

$\because T(n) = nT(\frac{n}{2}) + n - 1$

3 Mathematical Induction

(a) Let $f(n)$ be the predicate "$1 * 2^1 + 2 * 2^2 + 3 * 2^3 + ... + n * 2^n = (n-1)2^{n+1} + 2$" for $\forall n \in \mathbb{Z}^+$.

For $n = 1$,
$$\begin{aligned} \text{L.H.S.} &= 1 * 2^1 \\ &= 2 \\ \text{R.H.S.} &= (1-1)2^{1+1} + 2 \\ &= 2 \end{aligned}$$

$\because$ L.H.S. = R.H.S.

$\therefore f(1)$ is true.

4 Algorithm Design