COMP3278C Introduction to Database Management Systems
Assignment 3: Functional Dependencies and BCNF Decomposition

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Question 1

Given relation $R(A, B, C, D, E)$ and functional dependencies, $F = \{A \to B, B \to C, CA \to D, C \to A, D \to E\}$. Let $\alpha = \{A, C\}$. Compute the closure α^+ under F.

Answer:
$\alpha^+ = \{A, B, C, D, E\}$

Question 2

Given a relation $R(A, B, C)$ and the following functional dependencies:

$$F = \{A \to B, \quad C \to B\},$$

compute F^+, the closure of all functional dependencies in F. Your final answer should present a comprehensive list of all functional dependencies that can be derived from F.

Answer:

	{A}	**{B}**	**{C}**	**{AB}**	**{AC}**	**{BC}**	**{ABC}**
Attribute set closure	$\{A, B\}$	$\{B\}$	$\{B, C\}$	$\{A, B\}$	$\{A, B, C\}$	$\{B, C\}$	$\{A, B, C\}$
FD	$A \to A$ $A \to B$	$B \to B$	$C \to B$ $C \to C$	$AB \to A$ $AB \to B$ $AB \to AB$	$AC \to A$ $AC \to B$ $AC \to C$ $AC \to AB$ $AC \to AC$ $AC \to BC$ $AC \to ABC$	$BC \to B$ $BC \to C$ $BC \to BC$	$ABC \to A$ $ABC \to B$ $ABC \to C$ $ABC \to AB$ $ABC \to AC$ $ABC \to BC$ $ABC \to ABC$

Question 3

Consider the relation schema:

$$R = (A, B, C, D, E)$$

with the following functional dependencies:

$$F = \{A \to B, \quad C \to D, \quad AB \to E, \quad C \to E, \quad AB \to C\}.$$

(1) Identify all candidate keys for R.

Steps:

1. Is A a superkey?
   - $\{A\}^+ = \{A, B, C, D, E\} = R$
   - A is a superkey.
2. Is B a superkey?
   - $\{B\}^+ = \{B\} \neq R$
   - B is **NOT** a superkey.
3. Is C a superkey?
   - $\{C\}^+ = \{C, D, E\} \neq R$
   - C is **NOT** a superkey.
4. Is D a superkey?
   - $\{D\}^+ = \{D\} \neq R$
   - D is **NOT** a superkey.
5. Is E a superkey?
   - $\{E\}^+ = \{E\} \neq R$
   - E is **NOT** a superkey.
6. (Skipped AB, AC, AD, AE since A is a superkey)
7. Is BC a superkey?
   - $\{BC\}^+ = \{B, C, D, E\} \neq R$
   - BC is **NOT** a superkey.
8. Is BD a superkey?
   - $\{BD\}^+ = \{B, D\} \neq R$
   - BD is **NOT** a superkey.
9. Is BE a superkey?
   - $\{BE\}^+ = \{B, E\} \neq R$
   - BE is **NOT** a superkey.
10. Is CD a superkey?
    - $\{CD\}^+ = \{C, D, E\} \neq R$
    - CD is **NOT** a superkey.
11. Is CE a superkey?

- $\{CE\}^+ = \{C, D, E\} \neq R$
- CE is **NOT** a superkey.

12. Is DE a superkey?
    - $\{DE\}^+ = \{D, E\} \neq R$
    - DE is **NOT** a superkey.
13. (Skipped ABC, ABD, ABE, ACD, ACE, ADE since A is a superkey)
14. Is BCD a superkey?
    - $\{BCD\}^+ = \{B, C, D, E\} \neq R$
    - BCD is **NOT** a superkey.
15. Is BCE a superkey?
    - $\{BCE\}^+ = \{B, C, D, E\} \neq R$
    - BCE is **NOT** a superkey.
16. Is BDE a superkey?
    - $\{BDE\}^+ = \{B, D, E\} \neq R$
    - BDE is **NOT** a superkey.
17. Is CDE a superkey?
    - $\{CDE\}^+ = \{C, D, E\} \neq R$
    - CDE is **NOT** a superkey.
18. (Skipped ABCD, ABCE, ABDE, ACDE since A is a superkey)
19. Is BCDE a superkey?
    - $\{BCDE\}^+ = \{B, C, D, E\} \neq R$
    - $BCDE$ is **NOT** a superkey.
20. (Skipped ABCDE since A is a superkey)

Candidate Keys: $\{A\}$

(2) Determine whether R is in Boyce-Codd Normal Form (BCNF).

R is **NOT** in BCNF.

> BCNF requires that for every non-trivial FD $\alpha \to \beta$, α must be a superkey.
> Candidate key found: $\{A\}$
> To check for violations of BCNF, we only have to check the FDs in F.

- $A \to B$: $\{A\}$ is a superkey. (OK)
- $C \to D$: $\{C\}$ is not a superkey. (Violation)
- $AB \to E$: $\{AB\}$ is a superkey since A is a superkey. (OK)
- $C \to E$: $\{C\}$ is not a superkey. (Violation)
- $AB \to C$: $\{AB\}$ is a superkey since A is a superkey. (OK)

(3) BCNF Decomposition.

Decomposition Process:

Iteration 1:

- Start with relation: $R(A, B, C, D, E)$ with $F = \{A \rightarrow B, C \rightarrow D, AB \rightarrow E, C \rightarrow E, AB \rightarrow C\}$.
- Choose a violating FD: $C \rightarrow D$.
- Decompose R into:
  - $R_1(C, D)$
  - $R_2(A, B, C, E)$
- Check for BCNF:
  - F_1: $\{C \rightarrow D, trivials\}$, since $\{C\}^+ = \{C, D\} = R_1$, R_1 is in BCNF.
  - F_2: $\{A \rightarrow B, C \rightarrow E, AB \rightarrow C, AB \rightarrow E, trivials\}$, check each FD:
    * $A \rightarrow B$: $\{A\}^+ = \{A, B, C, E\} = R_2$. (OK)
    * $C \rightarrow E$: $\{C\}^+ = \{C, E\} \neq R_2$. (Violation)
    * $AB \rightarrow C$: $\{AB\}^+ = \{A, B, C, E\} = R_2$. (OK)
    * $AB \rightarrow E$: $\{AB\}^+ = \{A, B, C, E\} = R_2$. (OK)
    * R_2 is **NOT** in BCNF.
- Check for dependency preserving:
  - Since $(F_1 \cup F_2)^+ = F^+$, this decomposition is dependency preserving.
- Check for loseless join:
  - $(schema(R_1) \cap schema(R_2) \rightarrow schema(R_1)) \Leftrightarrow (C \rightarrow CD)$, where $C \rightarrow CD$ is true.
  - This decomposition is loseless join.

Iteration 2:

- Start with relation: $R_2(A, B, C, E)$ with $F_2 = \{A \rightarrow B, C \rightarrow E, AB \rightarrow C, AB \rightarrow E, trivials\}$.
- Choose a violating FD: $C \rightarrow E$.
- Decompose R_2 into:
  - $R_3(C, E)$
  - $R_4(A, B, C)$
- Check for BCNF:
  - F_3: $\{C \rightarrow E, trivials\}$, since $\{C\}^+ = \{C, E\} = R_3$, R_3 is in BCNF.
  - F_4: $\{A \rightarrow B, AB \rightarrow C, trivials\}$, check each FD:
    * $A \rightarrow B$: $\{A\}^+ = \{A, B, C\} = R_4$. (OK)
    * $AB \rightarrow C$: $\{AB\}^+ = \{A, B, C\} = R_4$. (OK)
    * R_4 is in BCNF.
- Check for dependency preserving:
  - $(F_1 \cup F_3 \cup F_4)^+ = F^+$, this decomposition is dependency preserving.
- Check for loseless join:
  - $(schema(R_3) \cap schema(R_4) \rightarrow schema(R_3)) \Leftrightarrow (C \rightarrow CE)$, where $C \rightarrow CE$ is true.
  - This decomposition is loseless join.

Result of a BCNF decomposition on R:

- $R_1(C, D)$

- $R_3(C, E)$
- $R_4(A, B, C)$
- $F_1 = \{C \to D, trivials\}$
- $F_3 = \{C \to E, trivials\}$
- $F_4 = \{A \to B, AB \to C, trivials\}$