Tutorial 5

Database Design

COMP3278C

Introduction to Database Management Systems

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Acknowledgement: Dr. Chui Chun Kit, Dr. Reynold Cheng, Dr. Ping Luo

Question 0

- $R = (A, B, C, D, E)$
- $F = \{A \to E, BCE \to D, CD \to BE, C \to AB, BD \to AC\}$
- Please calculate C^+

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
2. Transitivity - if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$
3. Augmentation - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
4. Union - if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$
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result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
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$$result = \{C\}$$

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$result = \alpha$

while (changes to $result$){

  for each $\beta \rightarrow \gamma$ in F{

    if ($\beta \subseteq result$){

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$$C^+ = \{A, B, C, D, E\}$$

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- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

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$$AB \dashrightarrow ? \dashrightarrow D$$

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$$\begin{tikzcd}
AB \arrow[r, dotted] \arrow[d] & ? \arrow[r, dotted] & D \\
C \arrow[urr, dotted] & &
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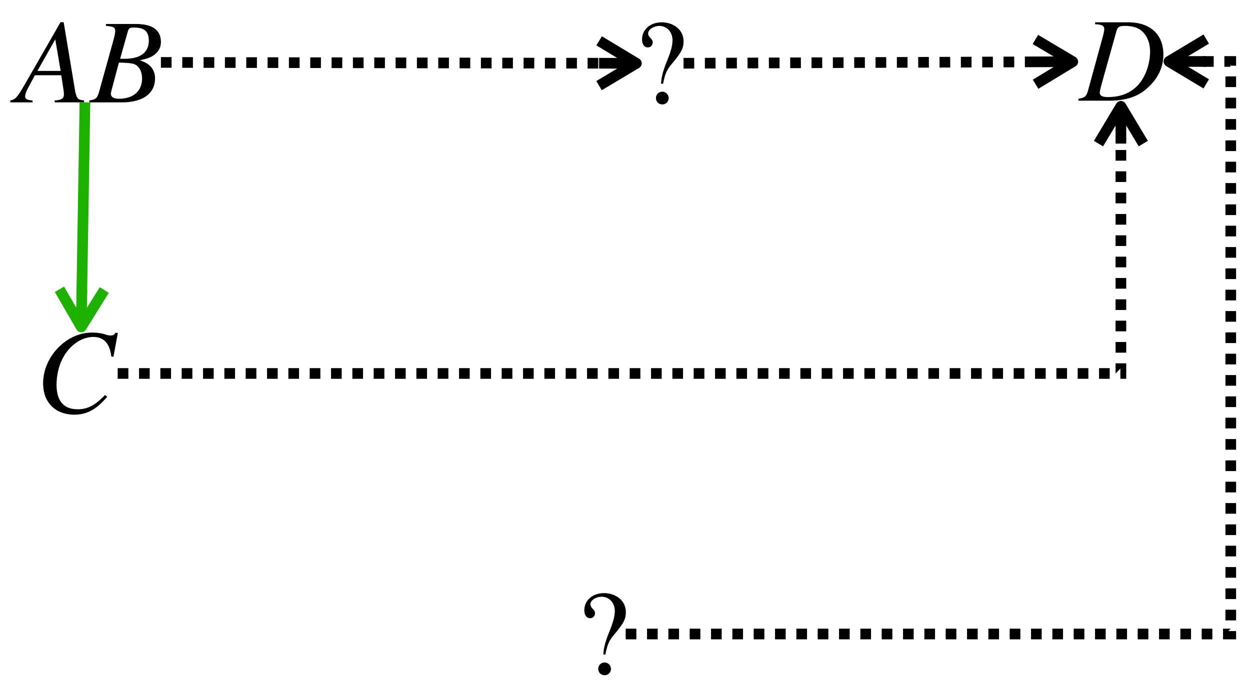

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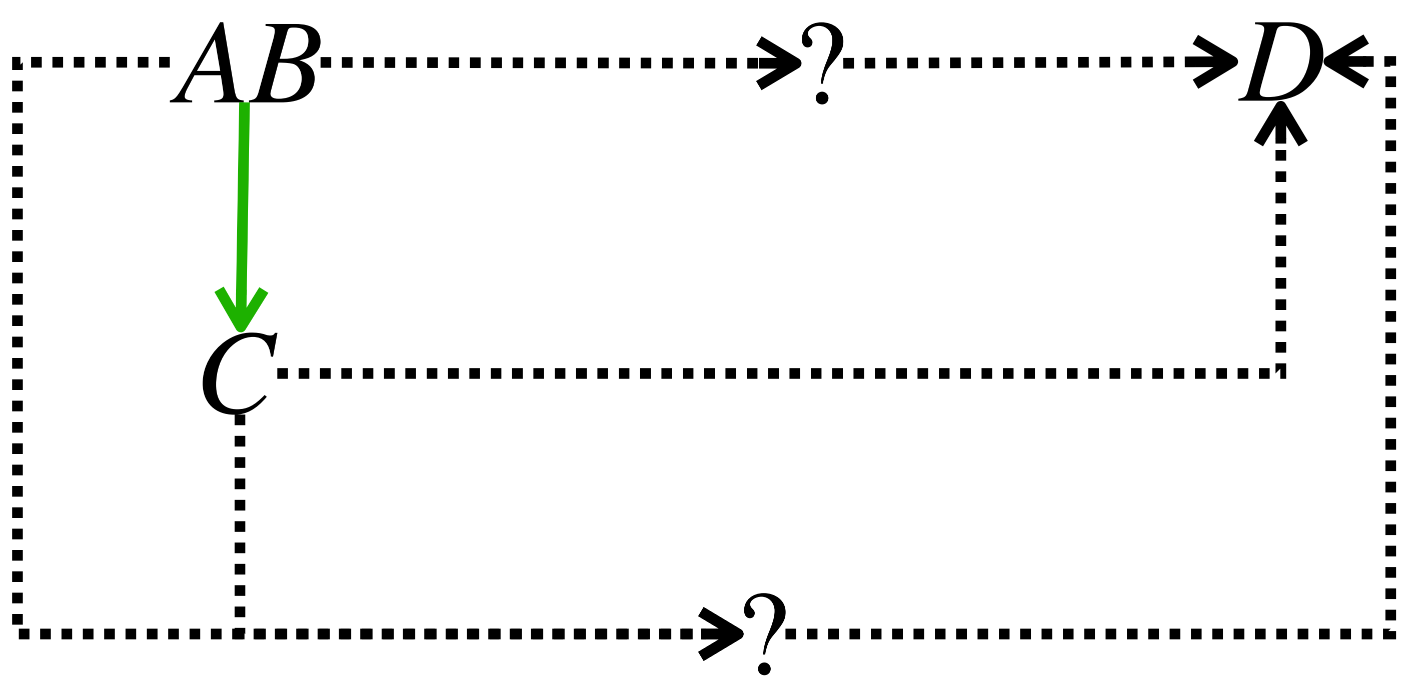

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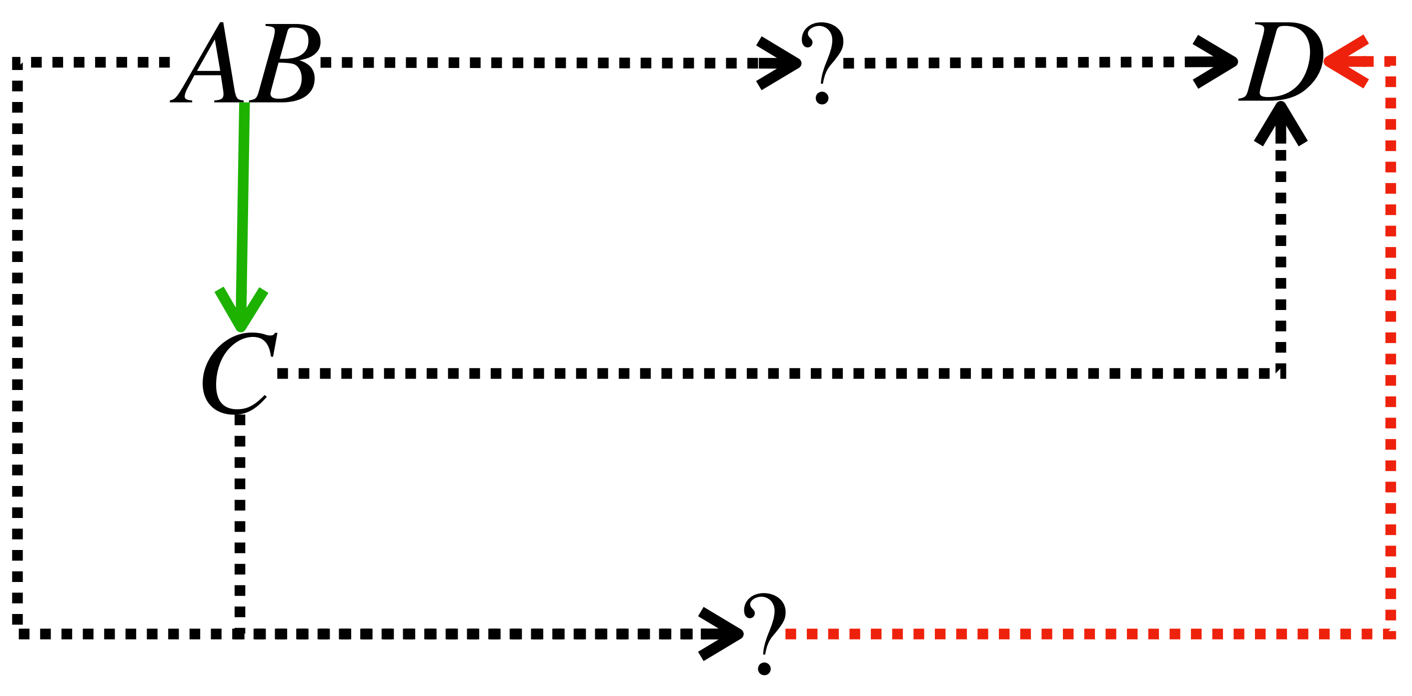


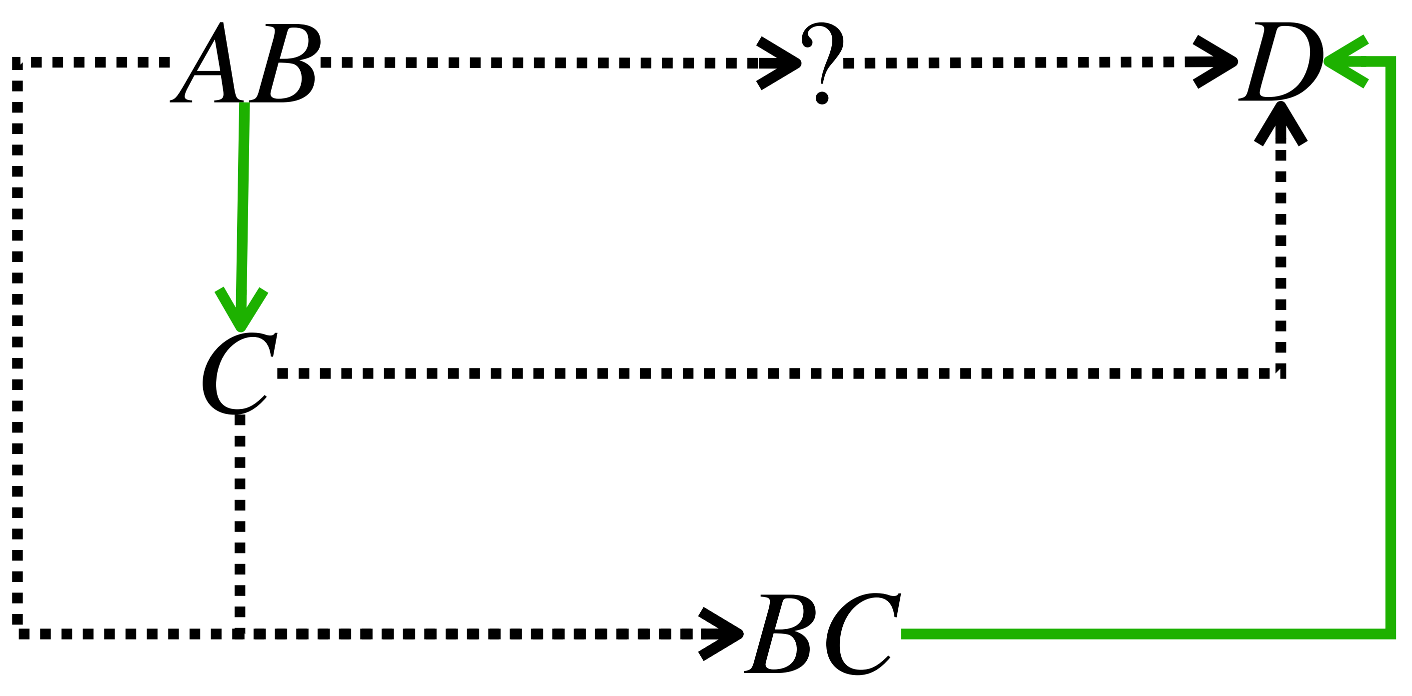
Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
2. Transitivity - if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$
3. Augmentation - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
4. Union - if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$
5. Decomposition - if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$
6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \to C, BC \to AD, D \to E, CF \to B\}$
- Does $AB \to D$ holds?

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
2. Transitivity - if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$
3. Augmentation - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
4. Union - if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$
5. Decomposition - if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$
6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$



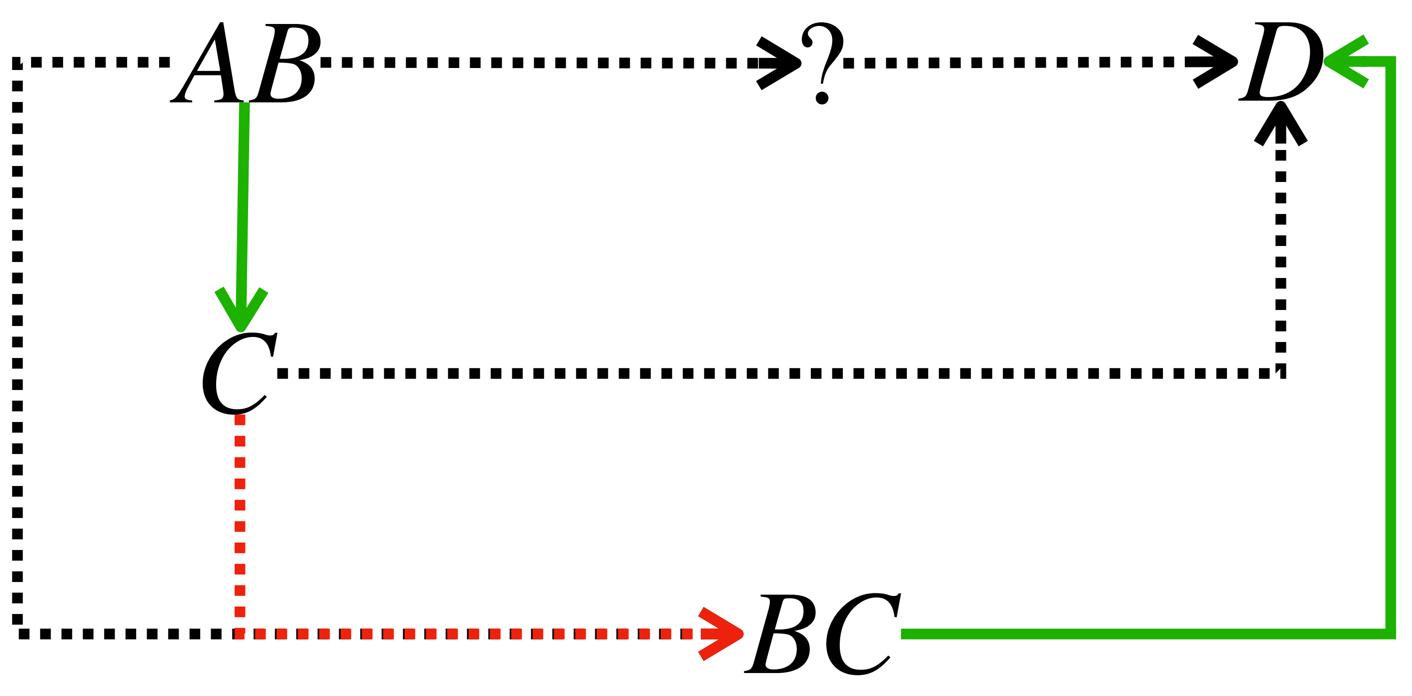
- Since $BC \to AD$, $BC \to D$ by Decomposition

Question 1

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$
4. Union - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$
5. Decomposition - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$
6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?



- Since $BC \rightarrow AD$, $BC \rightarrow D$ by Decomposition

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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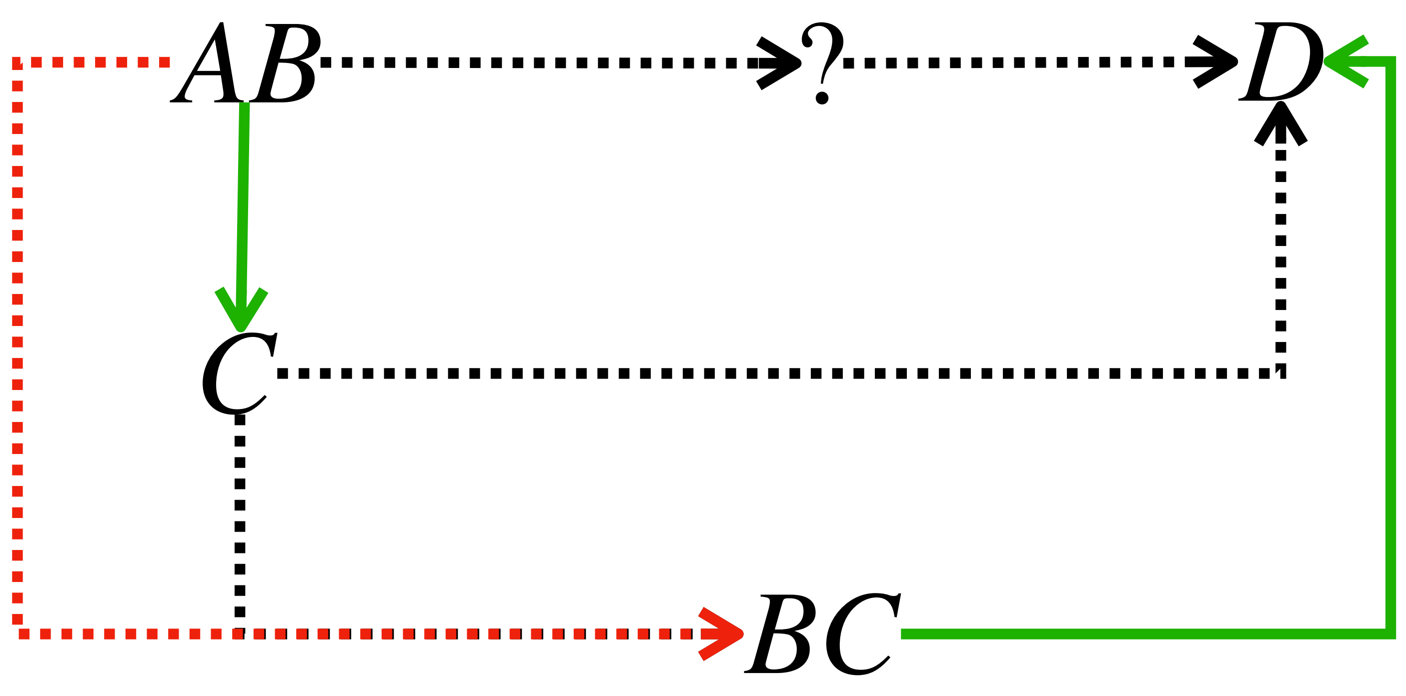


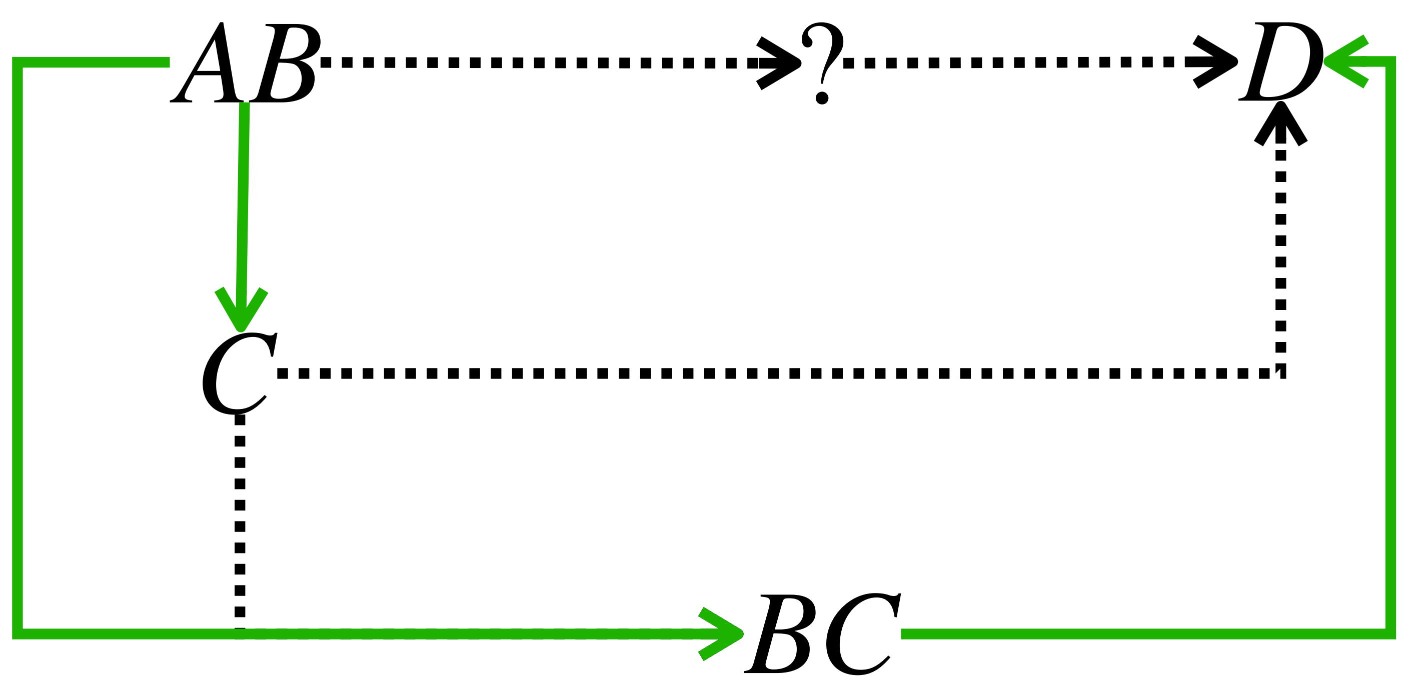
- Since $BC \rightarrow AD, BC \rightarrow D$ by Decomposition

Question 1

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
2. Transitivity - if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$
3. Augmentation - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
4. Union - if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$
5. Decomposition - if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$
6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$

- $R = (A, B, C, D, E, F), F = \{AB \to C, BC \to AD, D \to E, CF \to B\}$
- Does $AB \to D$ holds?



- Since $BC \to AD, BC \to D$ by Decomposition
- Since $AB \to C, AB \to BC$ by Augmentation

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
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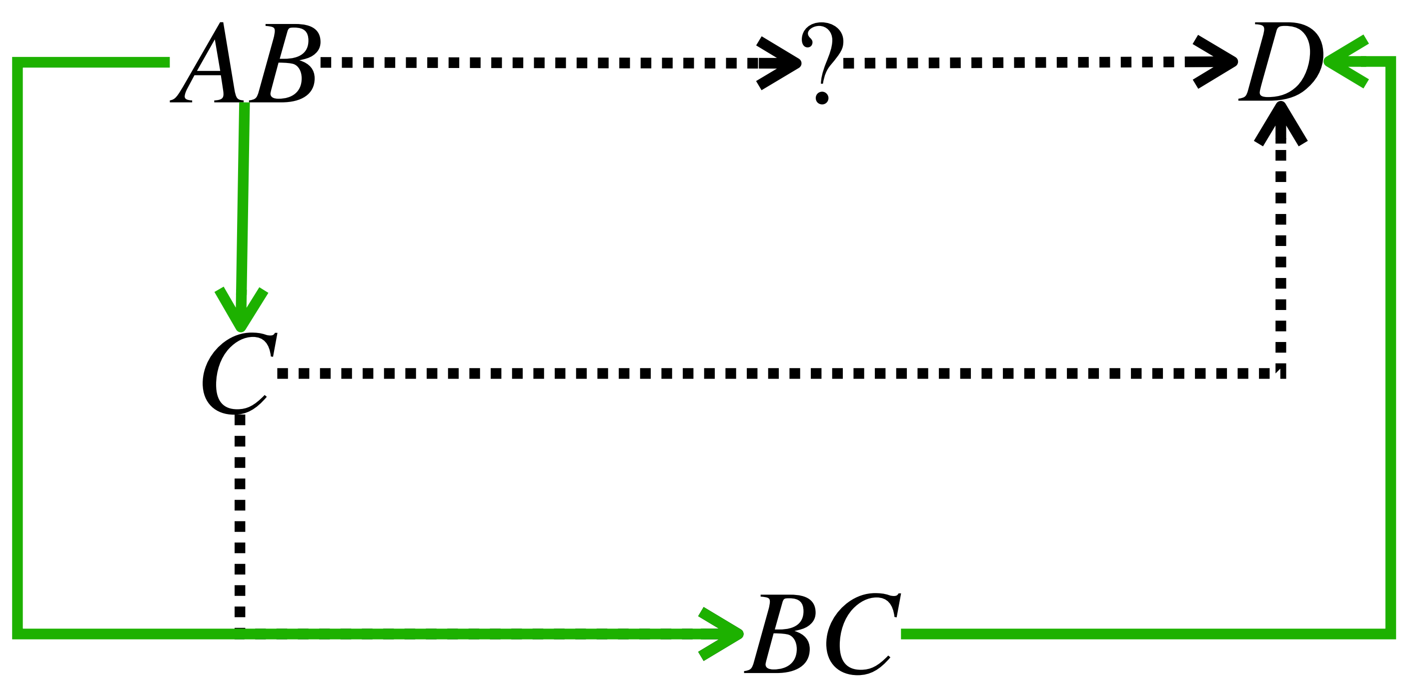

- Since $BC \rightarrow AD, BC \rightarrow D$ by Decomposition
- Since $AB \rightarrow C, AB \rightarrow BC$ by Augmentation

- Prove
  - Since $AB \rightarrow C, AB \rightarrow BC$ by Augmentation
  - Since $BC \rightarrow AD, BC \rightarrow D$ by Decomposition
  - Since $AB \rightarrow BC, BC \rightarrow D, AB \rightarrow D$ by Transitivity

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$
4. Union - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$
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Question 1

Armstrong's axioms

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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B\}$$

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B\}$$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B\}$$

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{AB \rightarrow C, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$result = \{A, B\}$

Armstrong's axioms

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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \rightarrow C}, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}\}$$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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4. Union - if $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$, then $\alpha \rightarrow \beta\gamma$
5. Decomposition - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$
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- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \rightarrow C}, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}\}$$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \to C}, BC \to AD, D \to E, CF \to B\}$
- Does $AB \to D$ holds?

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
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- If $D \subseteq \{AB\}^+, AB \to D$

$$result = \{A, B, \overset{1}{C}\}$$

```
result = α
while (changes to result){
   for each β → γ in F{
      if (β ⊆ result){
         result = result ∪ γ
      }
   }
}
```

Question 1

Armstrong's axioms

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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \rightarrow C}, BC \rightarrow AD, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}\}$$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{\overbrace{AB \rightarrow C}^{1}, \overbrace{BC \rightarrow AD}^{2}, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
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```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \rightarrow C}, \overset{2}{BC \rightarrow AD}, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

- Use Armstrong's axioms to prove

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \to C}, \overset{2}{BC \to AD}, D \to E, CF \to B\}$
- Does $AB \to D$ holds?

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
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6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$

- If $D \subseteq \{AB\}^+, AB \to D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

- Use Armstrong's axioms to prove

$$AB \longrightarrow D$$

Question 1

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \to C}, \overset{2}{BC \to AD}, D \to E, CF \to B\}$
- Does $AB \to D$ holds?

- If $D \subseteq \{AB\}^+, AB \to D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

- Use Armstrong's axioms to prove

$$\begin{tikzcd} AB \arrow[rr] \arrow[dr] & & D \\ & BC \arrow[ur, "2"'] & \end{tikzcd}$$

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
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6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 1

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
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5. Decomposition - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$
6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \rightarrow C}, \overset{2}{BC \rightarrow AD}, D \rightarrow E, CF \rightarrow B\}$
- Does $AB \rightarrow D$ holds?

- If $D \subseteq \{AB\}^+, AB \rightarrow D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

$result = \alpha$
while (changes to $result$){
    for each $\beta \rightarrow \gamma$ in F{
        if ($\beta \subseteq result$){
            $result = result \cup \gamma$
        }
    }
}

- Use Armstrong's axioms to prove

$$\begin{tikzcd}
AB \arrow[rr] \arrow[dr, "1"'] & & D \\
& BC \arrow[ur, "2"'] &
\end{tikzcd}$$

Question 1

- $R = (A, B, C, D, E, F), F = \{\overset{1}{AB \to C}, \overset{2}{BC \to AD}, D \to E, CF \to B\}$
- Does $AB \to D$ holds?

- If $D \subseteq \{AB\}^+, AB \to D$

$$result = \{A, B, \overset{1}{C}, \overset{2}{D}\}$$

- Use Armstrong's axioms to prove

$$\begin{tikzcd} AB \arrow[rr] \arrow[dr, "1"'] & & D \\ & BC \arrow[ur, "2"'] & \end{tikzcd}$$

- Prove
  - Since $AB \to C, AB \to BC$ by Augmentation
  - Since $BC \to AD, BC \to D$ by Decomposition
  - Since $AB \to BC, BC \to D, AB \to D$ by Transitivity

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \to \beta$
2. Transitivity - if $\alpha \to \beta$ and $\beta \to \gamma$, then $\alpha \to \gamma$
3. Augmentation - if $\alpha \to \beta$, then $\gamma\alpha \to \gamma\beta$
4. Union - if $\alpha \to \beta$ and $\alpha \to \gamma$, then $\alpha \to \beta\gamma$
5. Decomposition - if $\alpha \to \beta\gamma$, then $\alpha \to \beta$ and $\alpha \to \gamma$
6. Pseudo-transitivity - if $\alpha \to \beta$ and $\gamma\beta \to \delta$, then $\alpha\gamma \to \delta$

```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

Armstrong's axioms

1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
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- Make $ACF \rightarrow ?$

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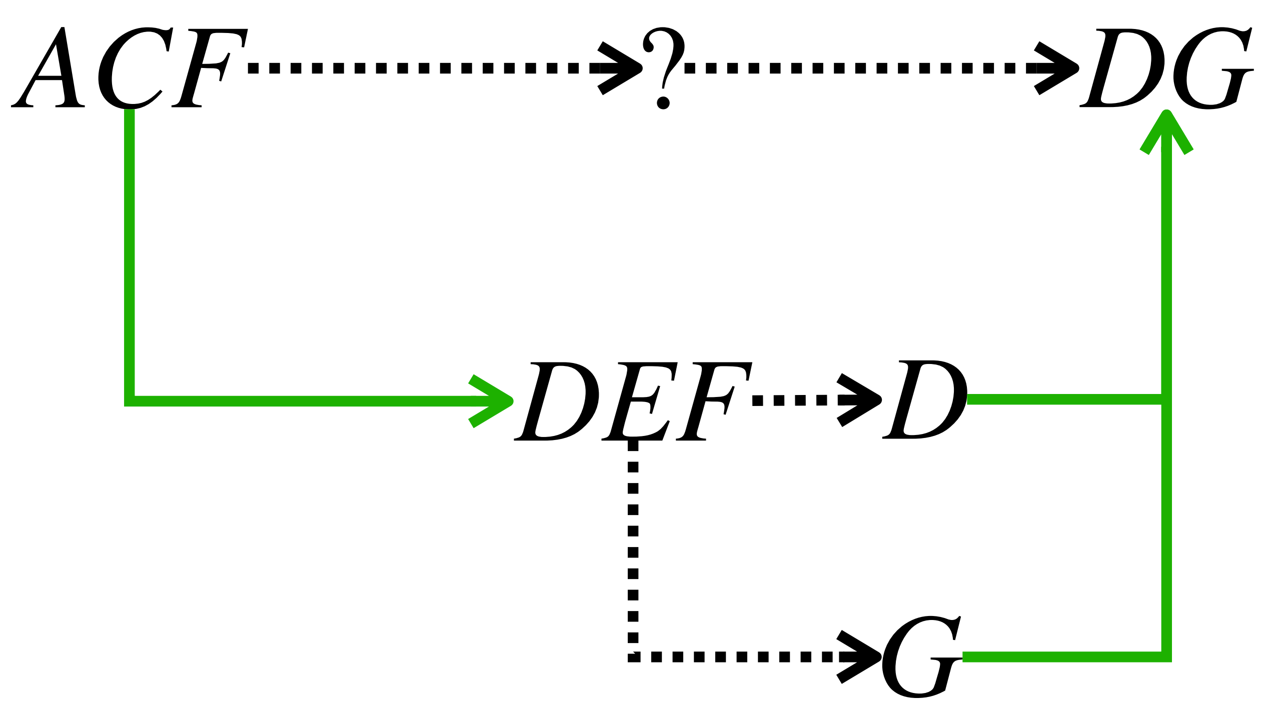

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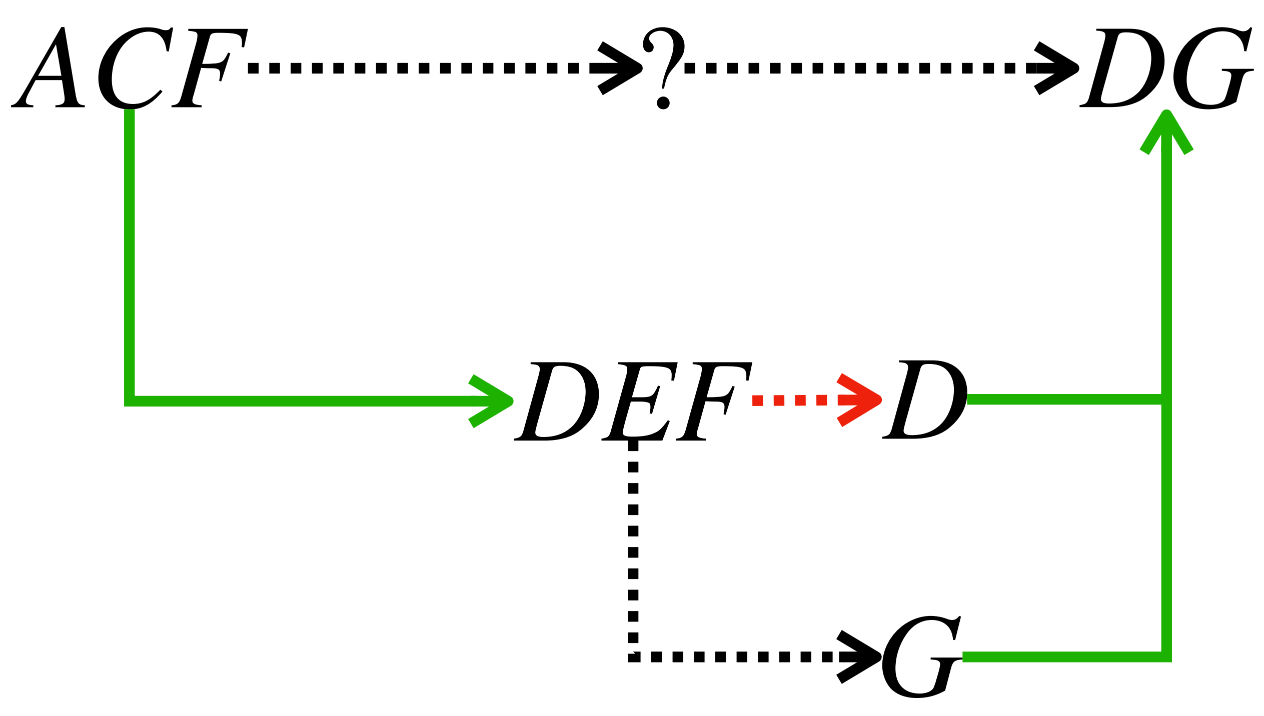

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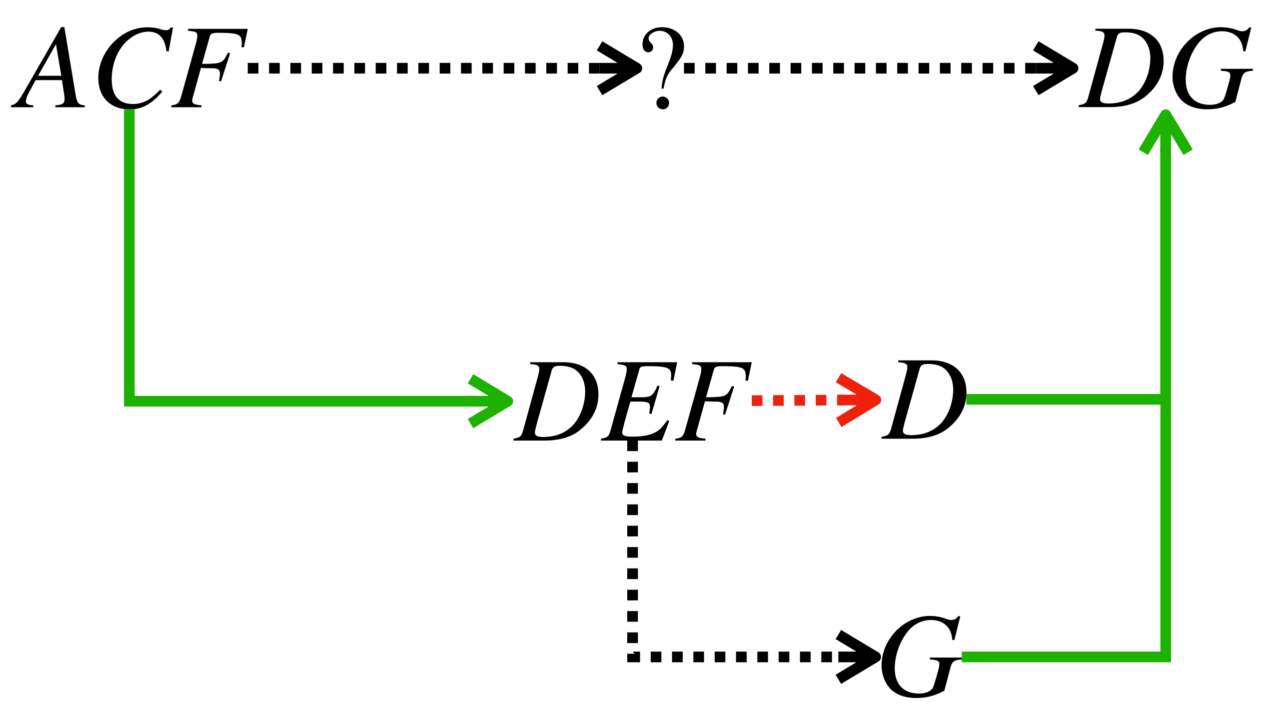

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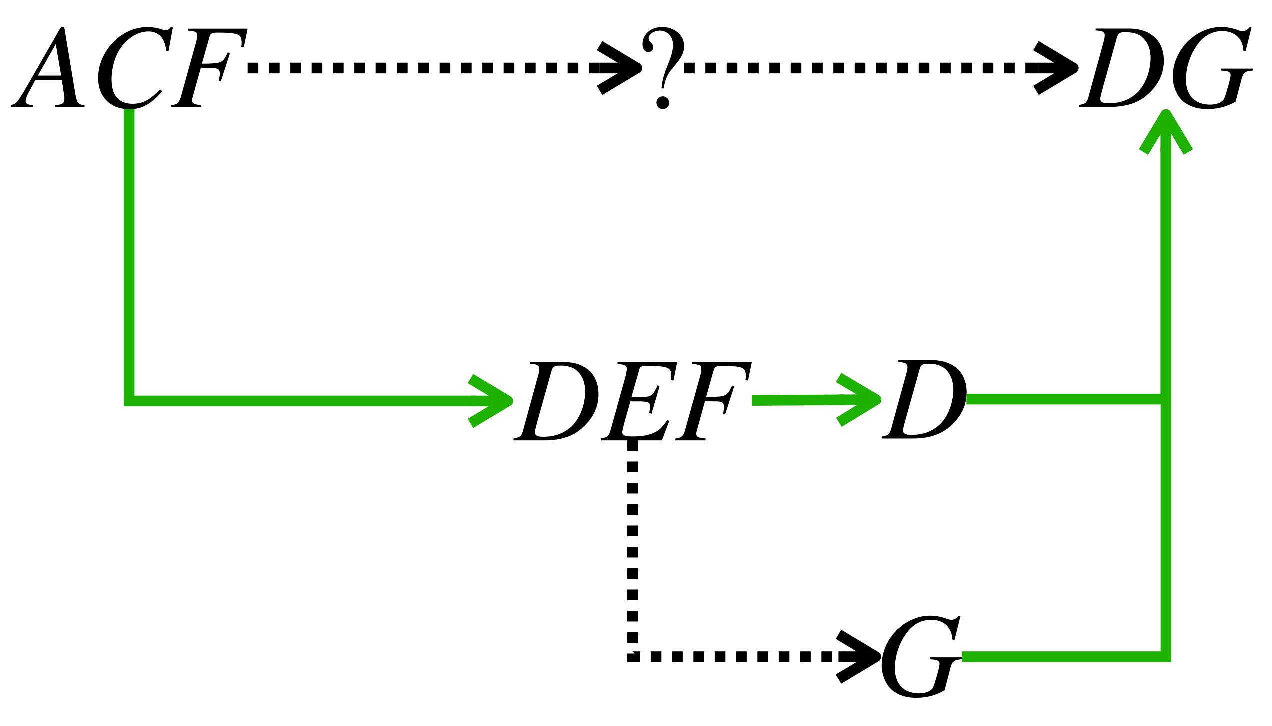

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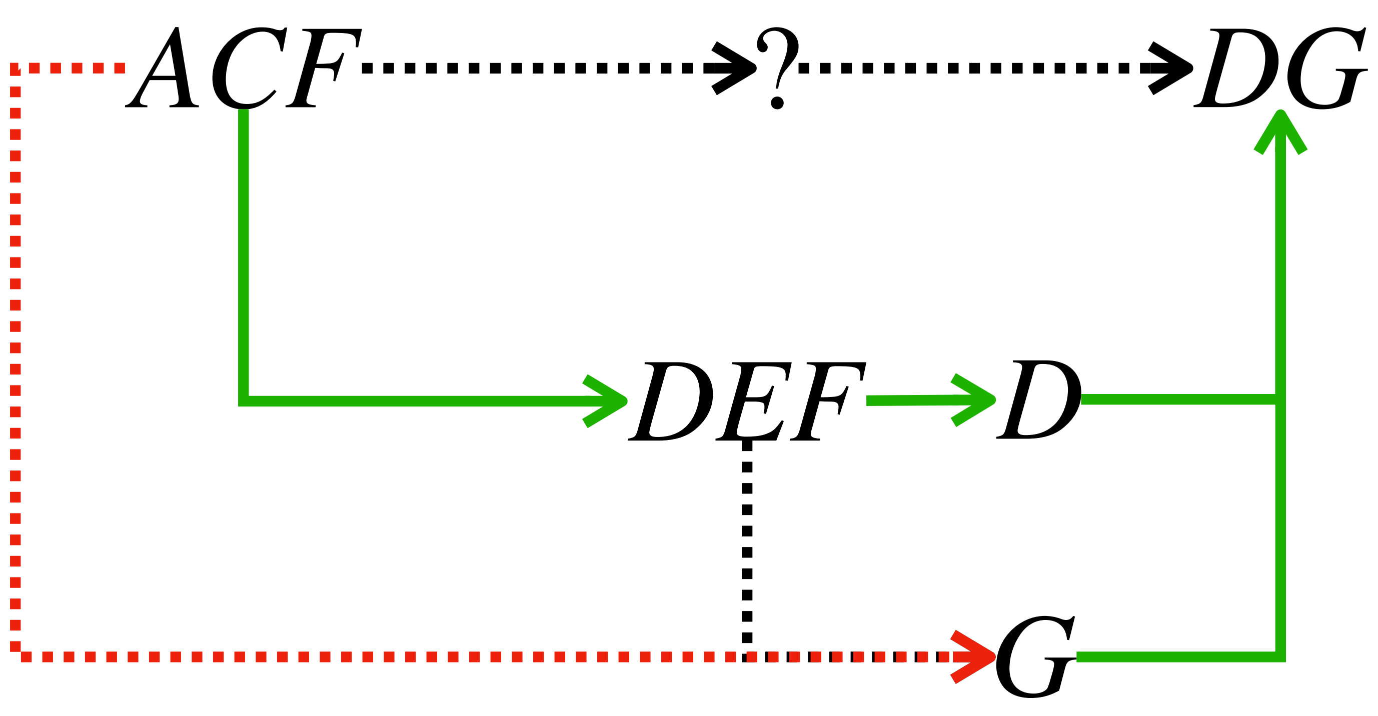

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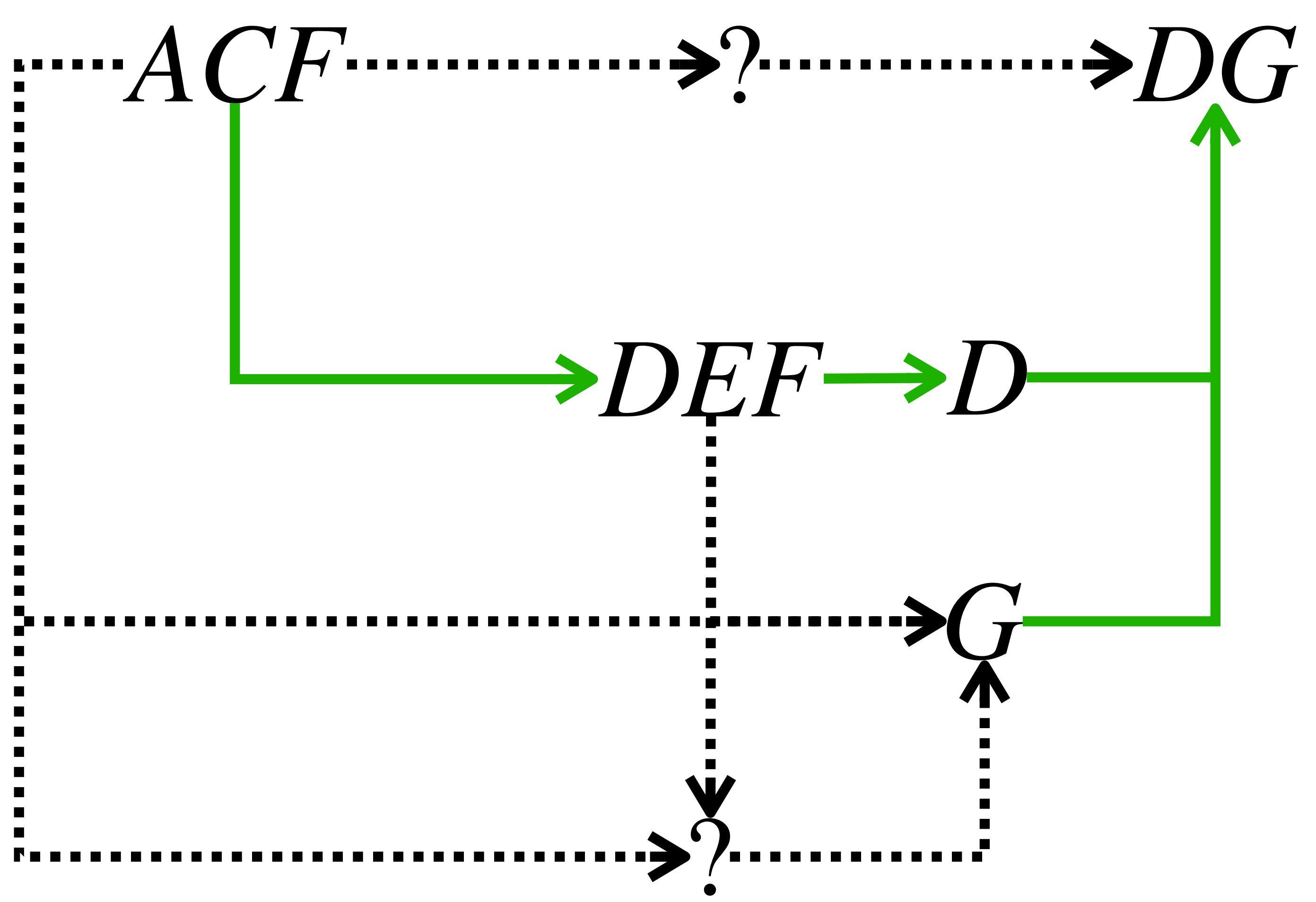

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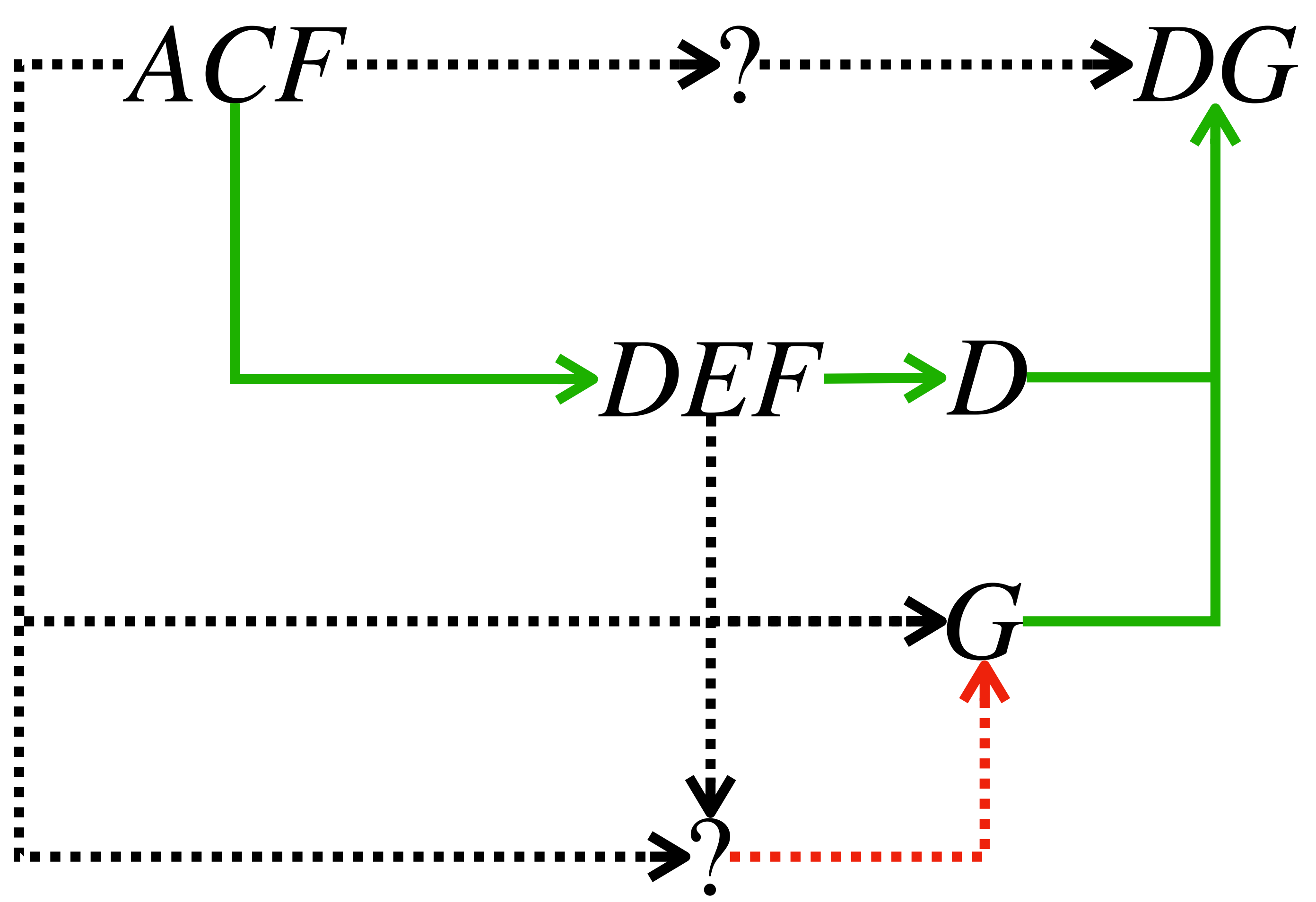

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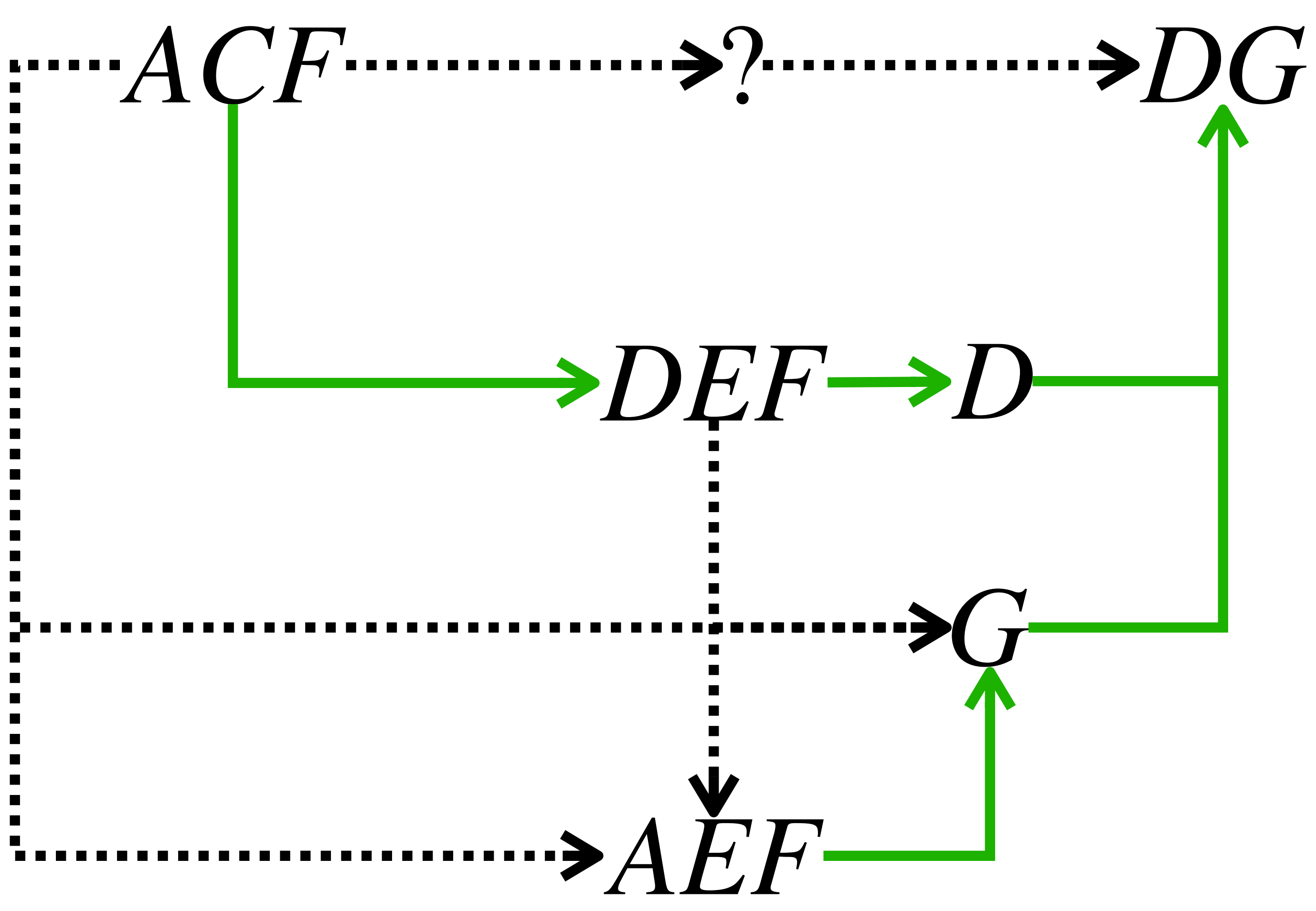

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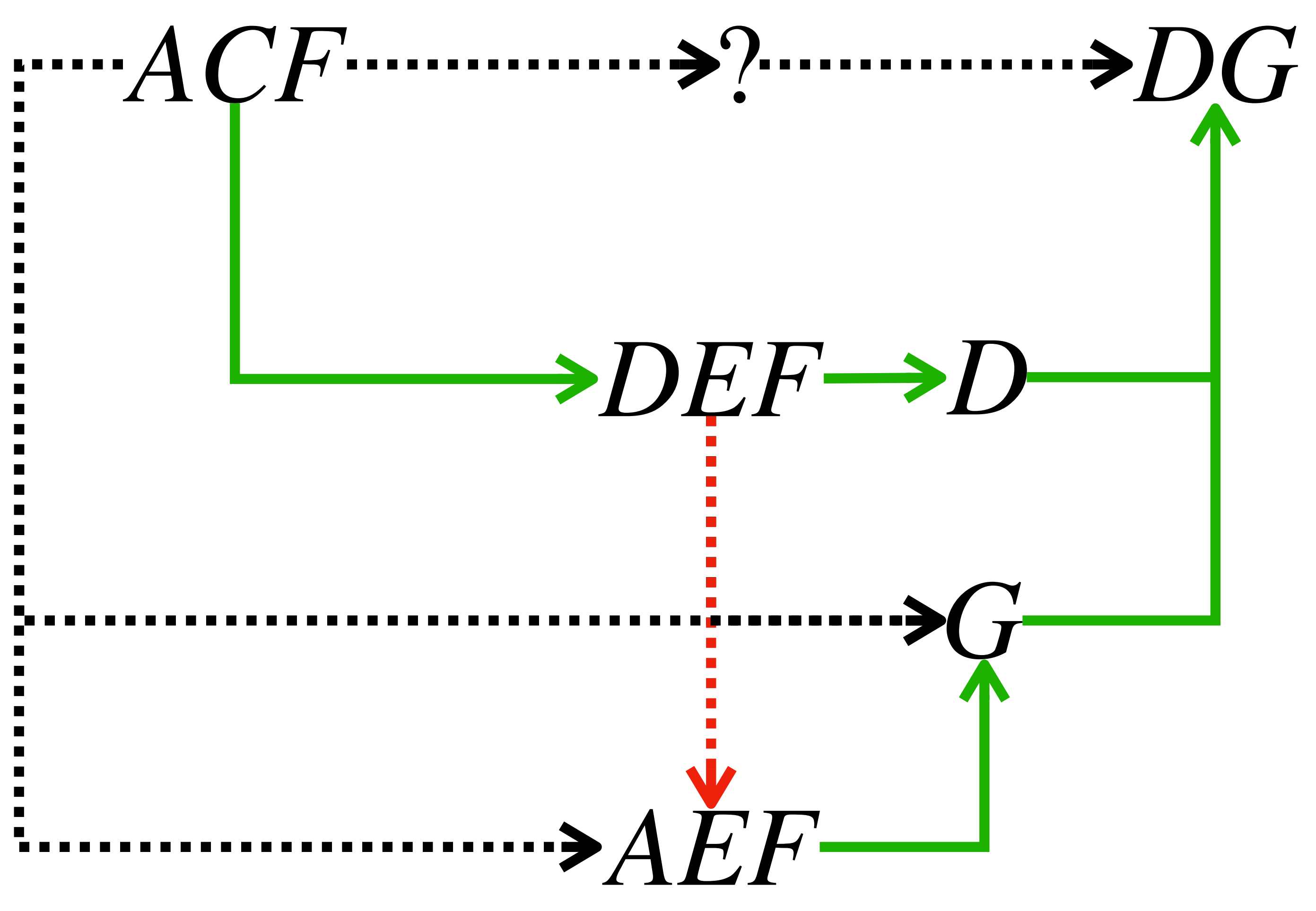

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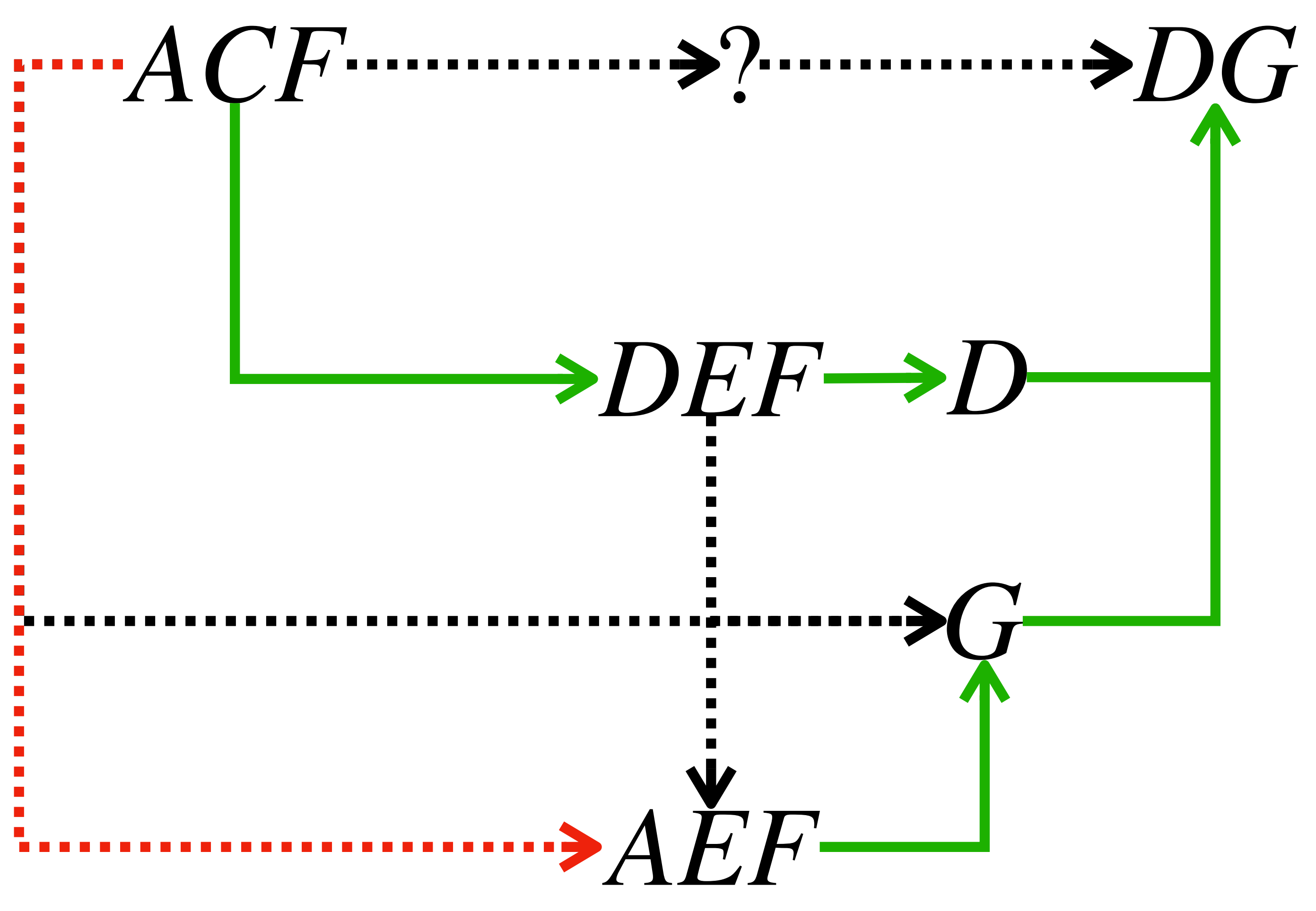

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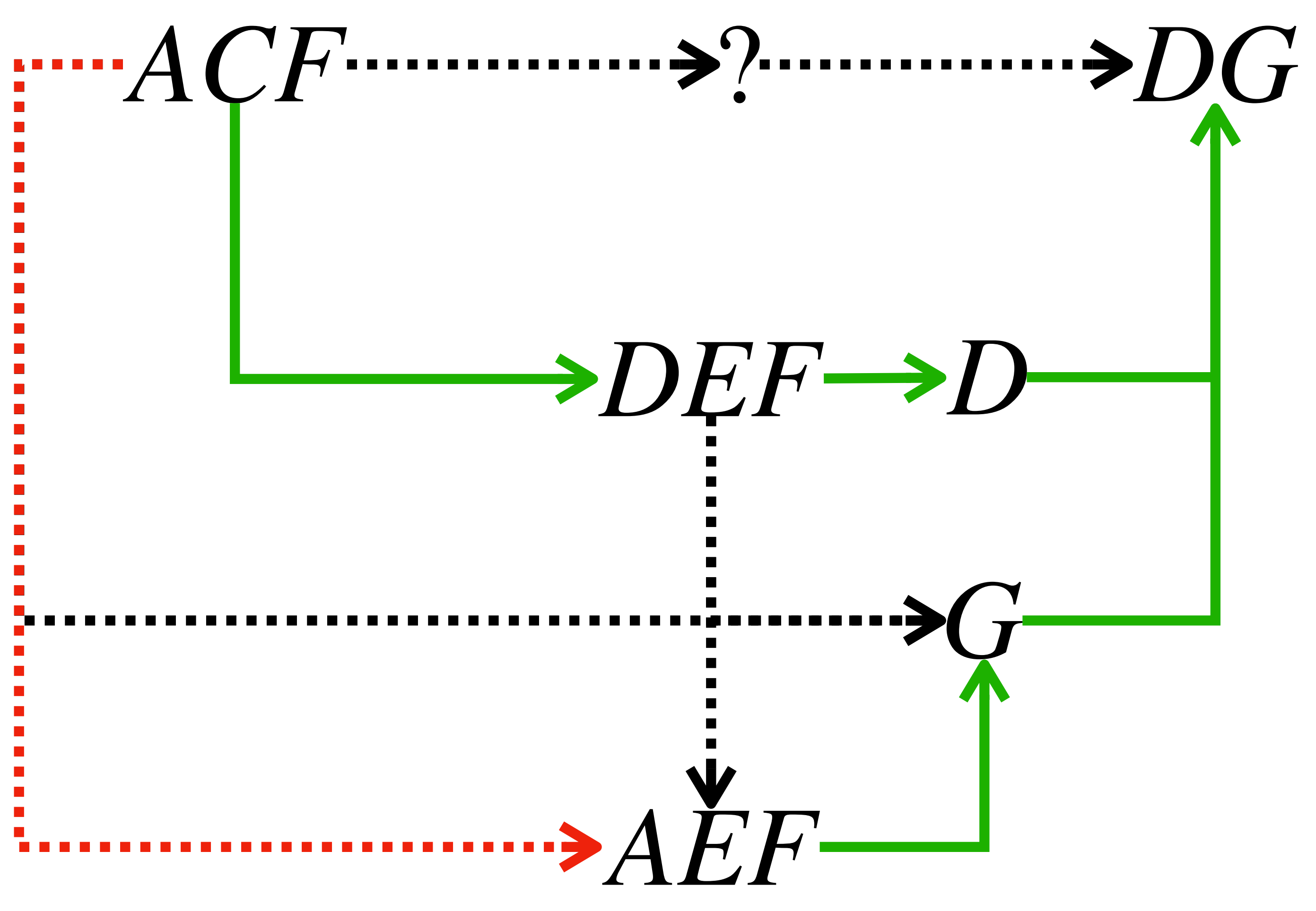

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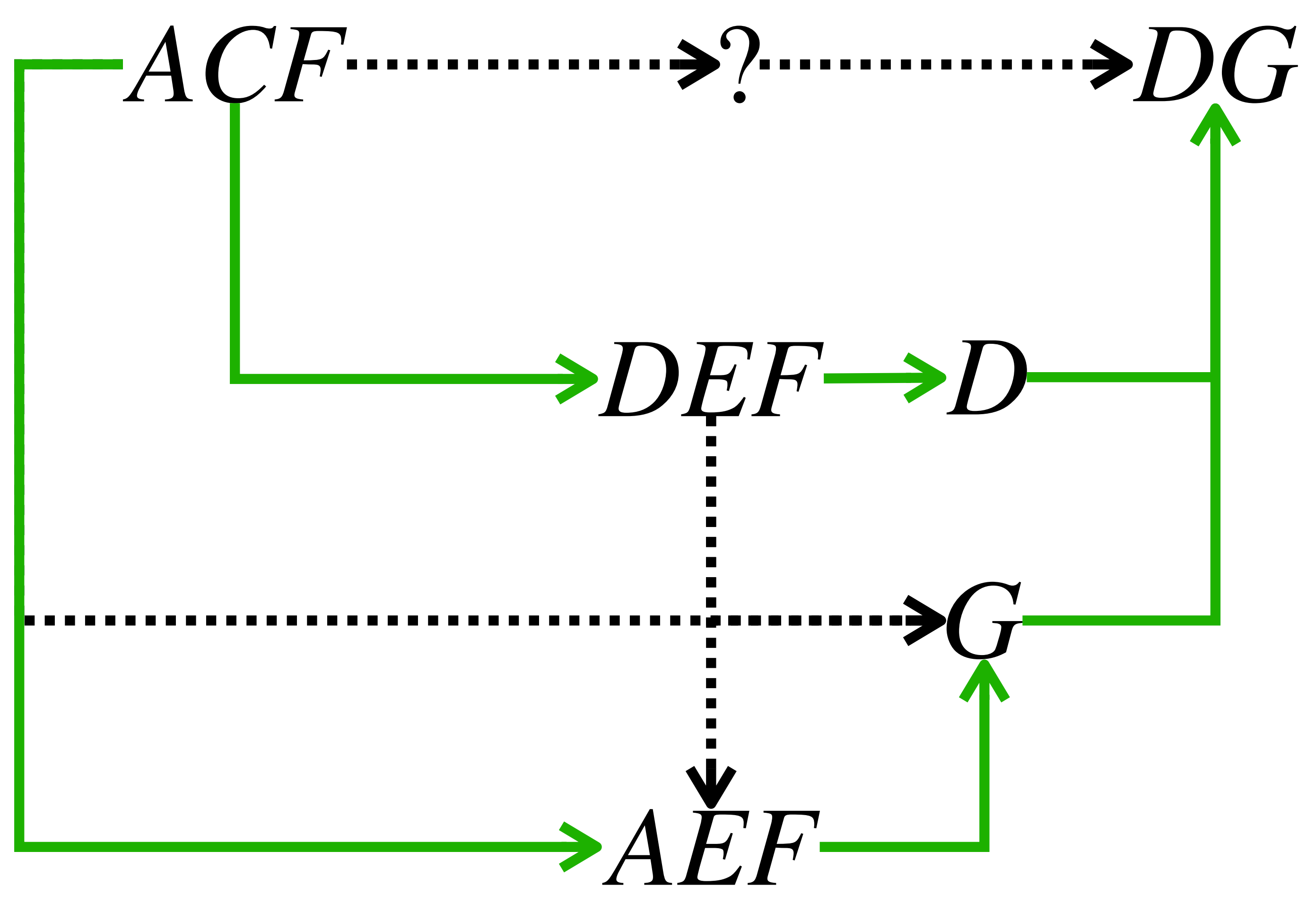

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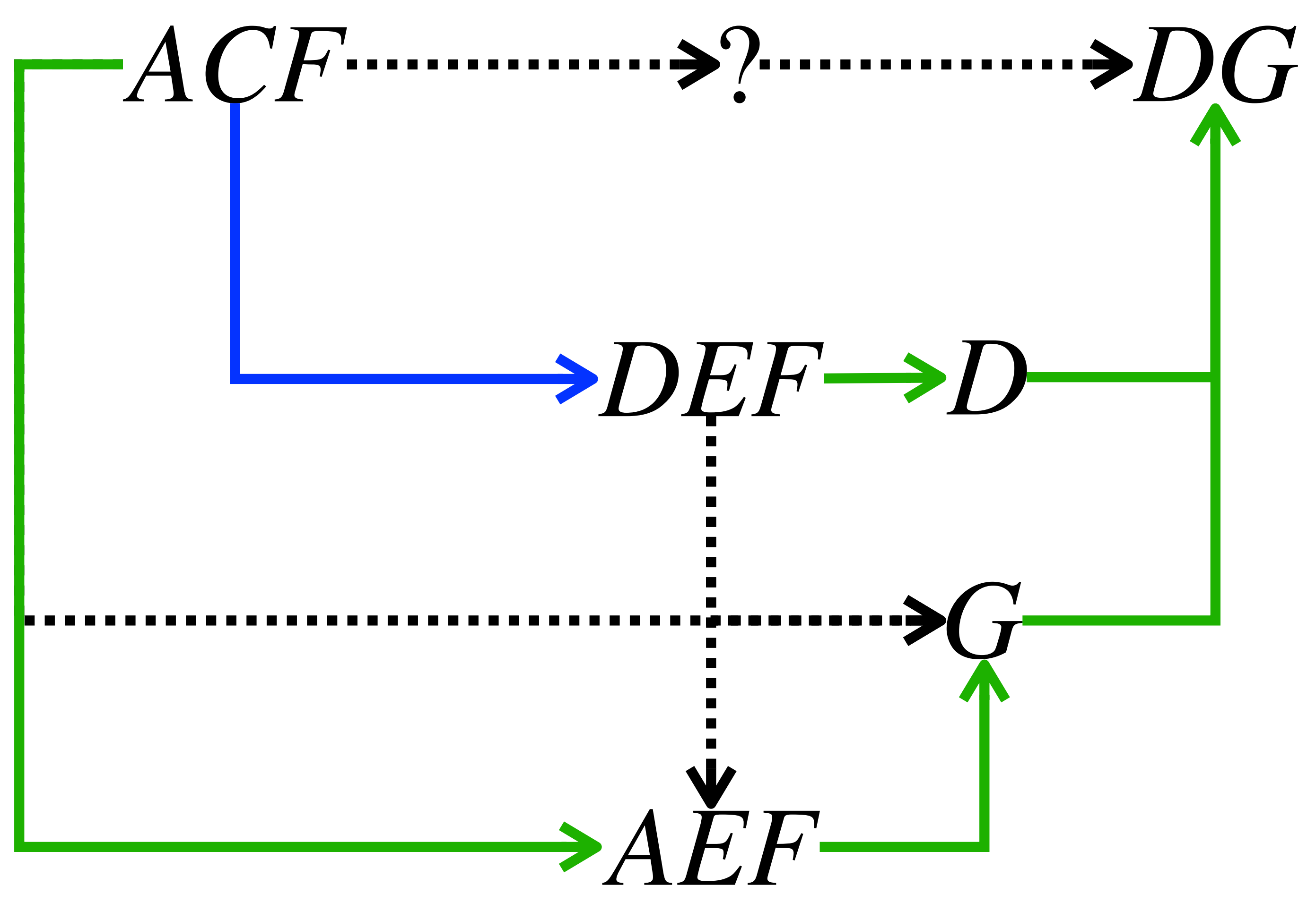

- Prove
  - Since $A \rightarrow B, BC \rightarrow DE, AC \rightarrow DE$ by Pseudo-transitivity
  - Since $AC \rightarrow DE, ACF \rightarrow DEF$ by Augmentation

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
3. Augmentation - if $\alpha \rightarrow \beta$, then $\gamma\alpha \rightarrow \gamma\beta$
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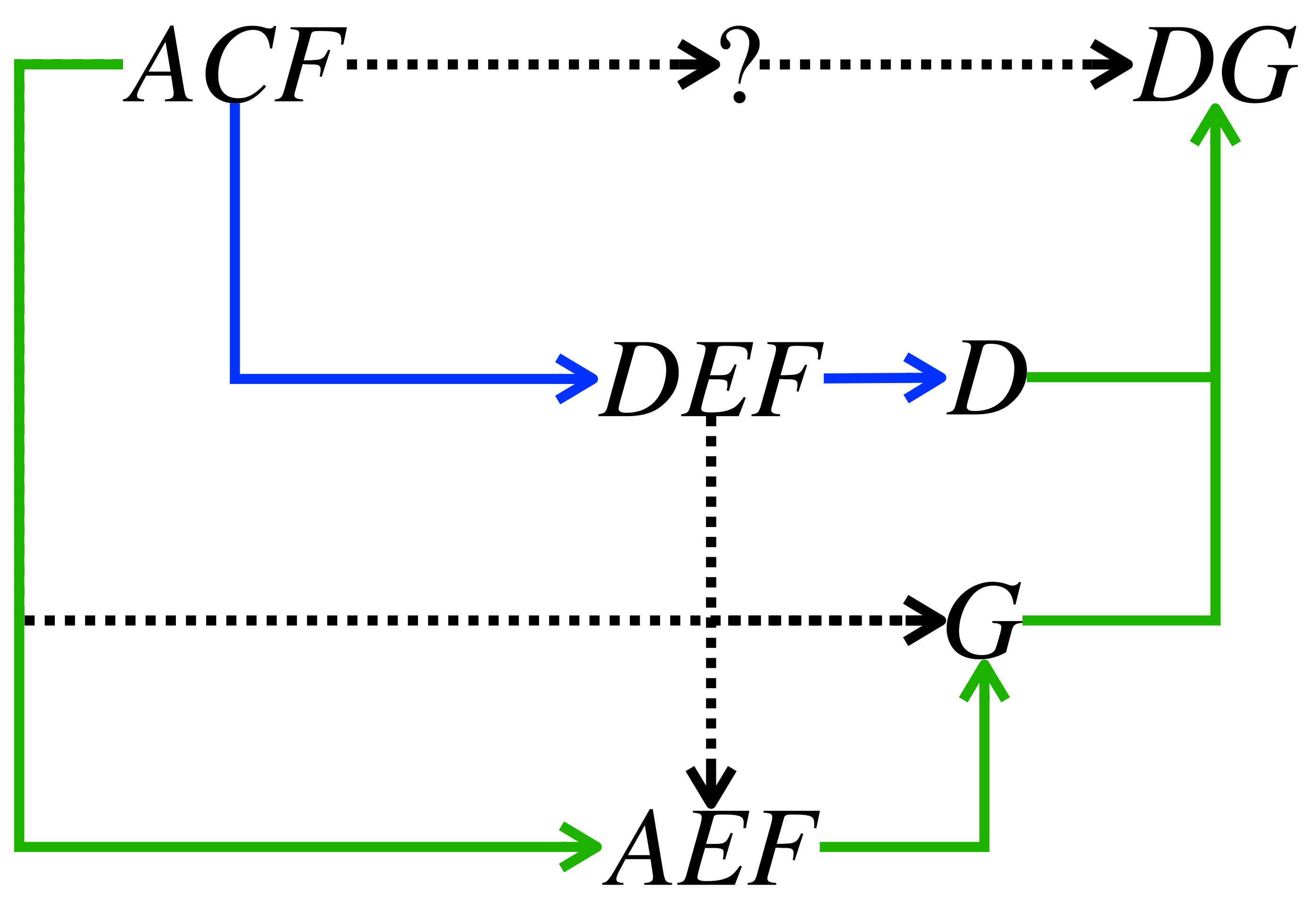
5. Decomposition - if $\alpha \rightarrow \beta\gamma$, then $\alpha \rightarrow \beta$ and $\alpha \rightarrow \gamma$
6. Pseudo-transitivity - if $\alpha \rightarrow \beta$ and $\gamma\beta \rightarrow \delta$, then $\alpha\gamma \rightarrow \delta$



- Prove
  - Since $A \rightarrow B, BC \rightarrow DE, AC \rightarrow DE$ by Pseudo-transitivity
  - Since $AC \rightarrow DE, ACF \rightarrow DEF$ by Augmentation
  - Since $D \subseteq DEF, DEF \rightarrow D$ by Reflexivity

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \to B, BC \to DE, AEF \to G\}$
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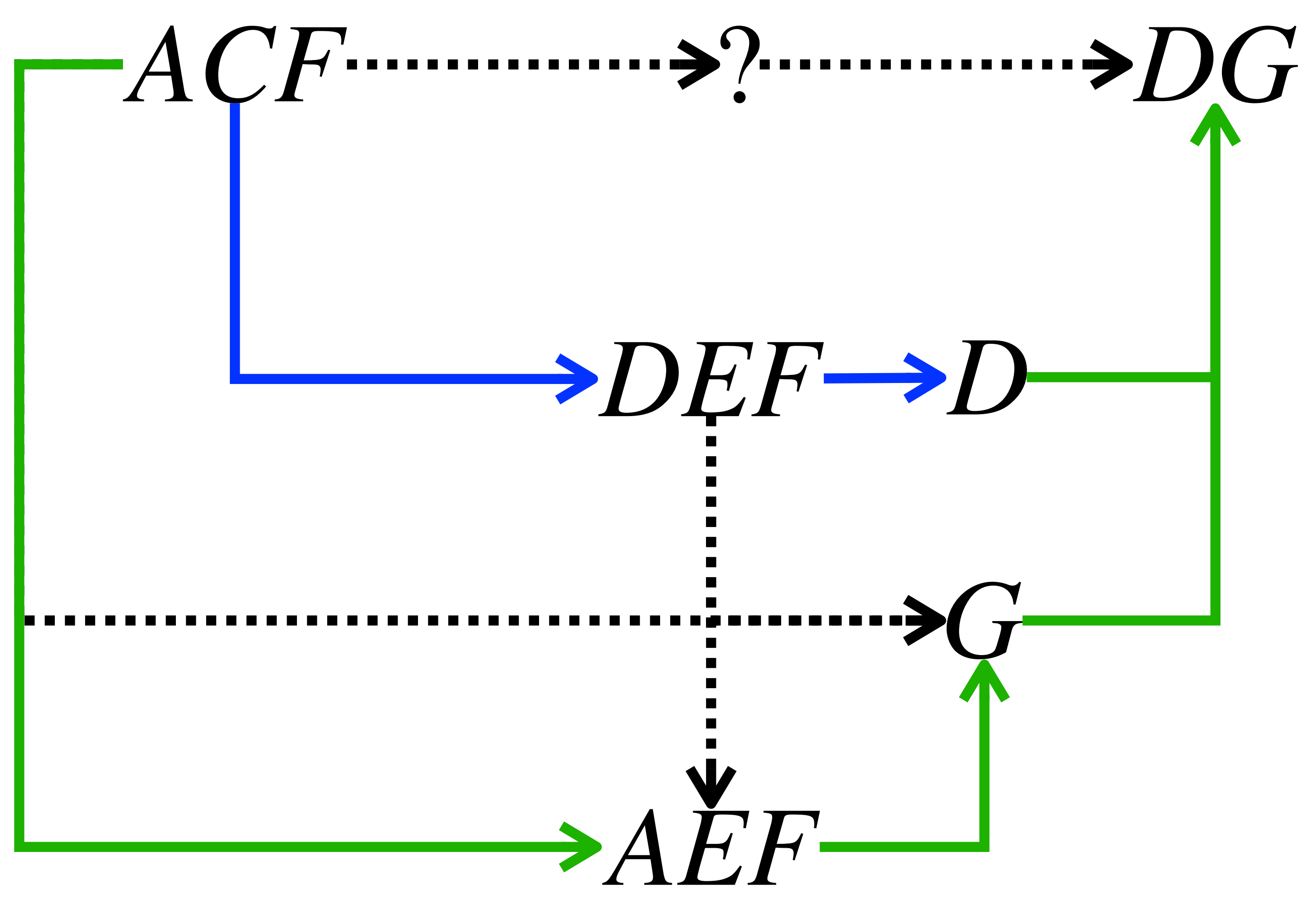

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  - Since $AC \to DE, ACF \to DEF$ by Augmentation
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  - Since $ACF \to DEF, DEF \to D, ACF \to D$ by Transitivity

Question 2

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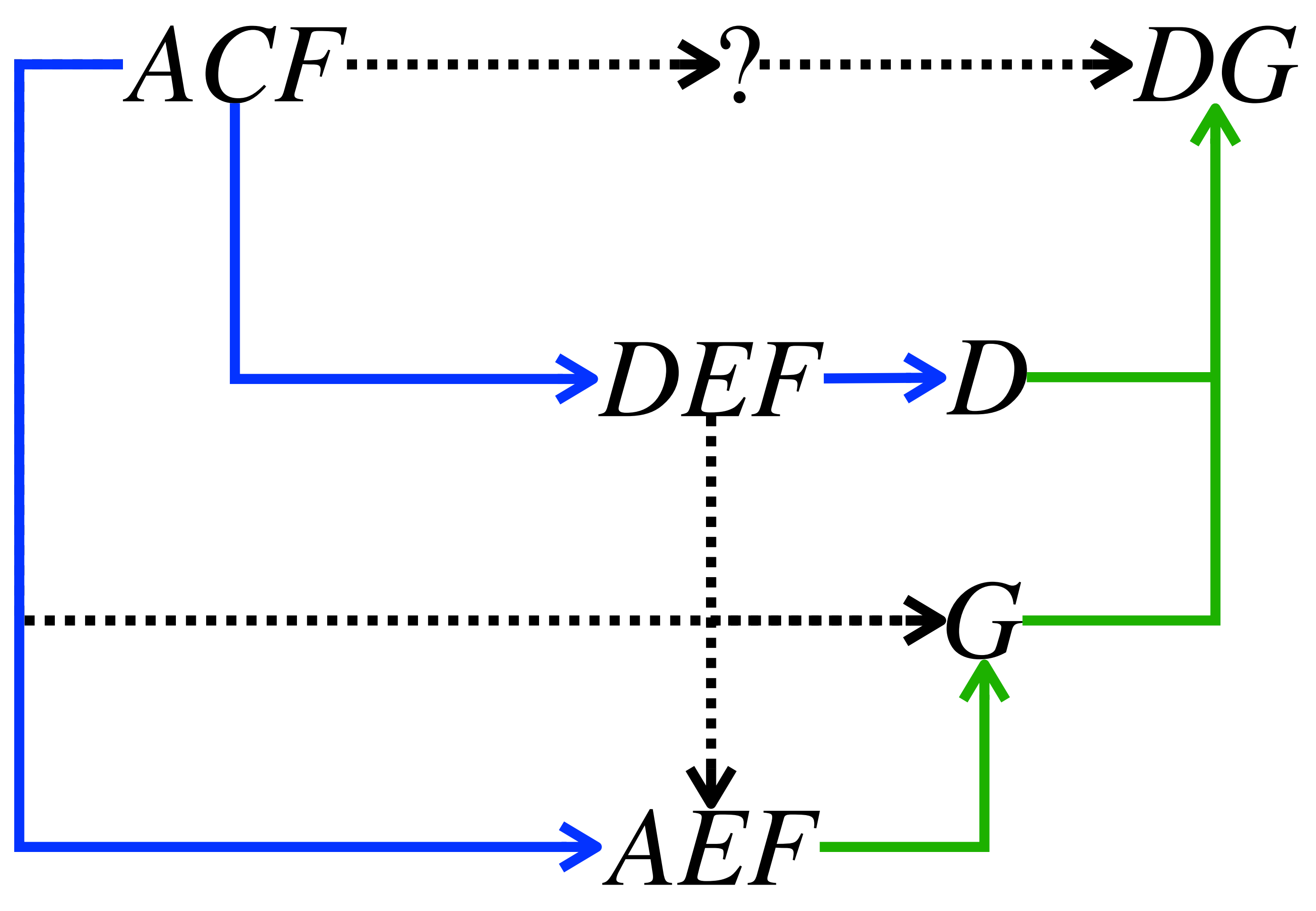

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  - Since $ACF \rightarrow DEF, DEF \rightarrow D, ACF \rightarrow D$ by Transitivity
  - Since $ACF \rightarrow DEF, ACF \rightarrow ACDEF$ by Augmentation
  - Since $ACF \rightarrow ACDEF, ACF \rightarrow AEF$ by Decomposition

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \to B, BC \to DE, AEF \to G\}$
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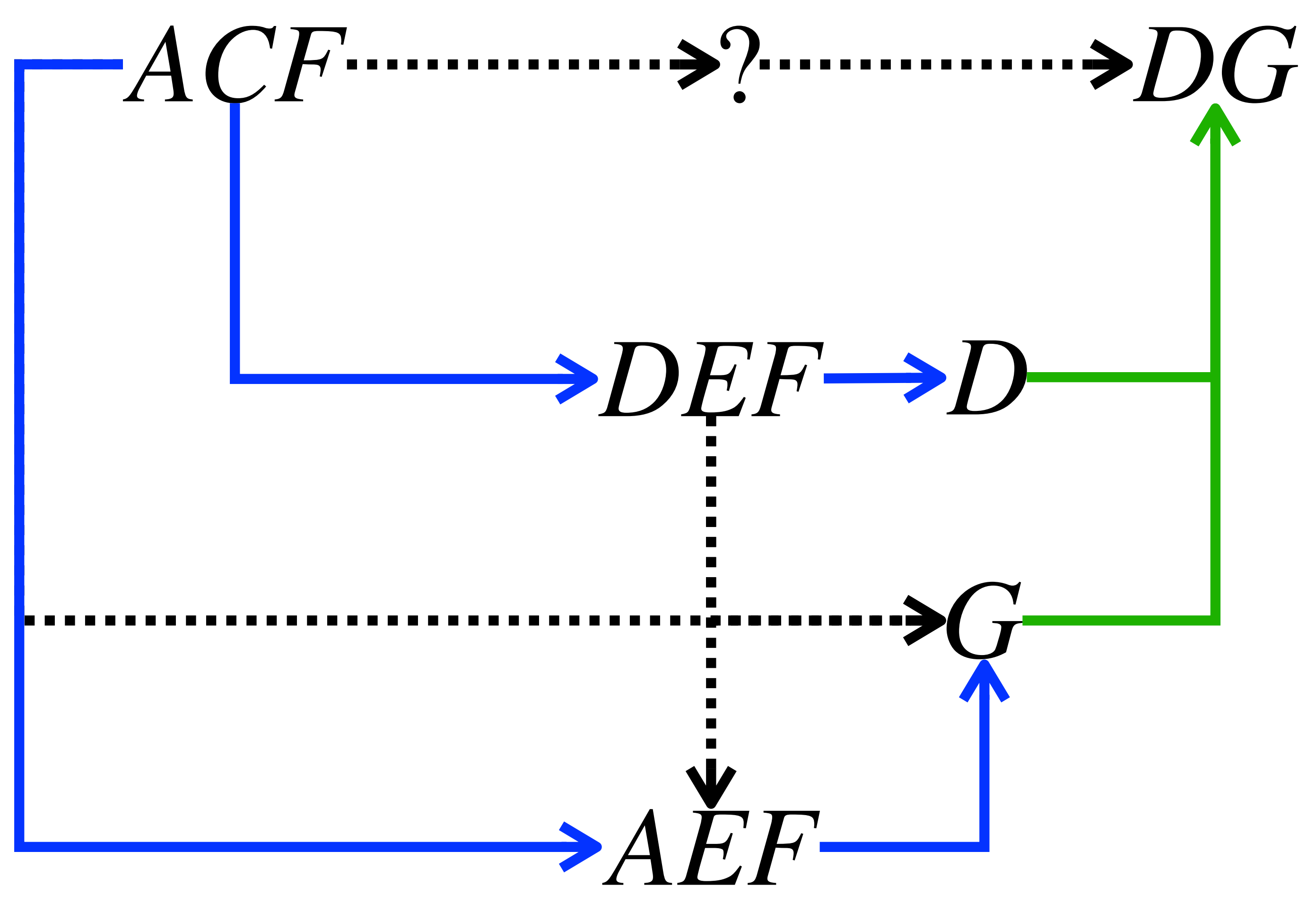

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  - Since $D \subseteq DEF, DEF \to D$ by Reflexivity
  - Since $ACF \to DEF, DEF \to D, ACF \to D$ by Transitivity
  - Since $ACF \to DEF, ACF \to ACDEF$ by Augmentation
  - Since $ACF \to ACDEF, ACF \to AEF$ by Decomposition

Question 2

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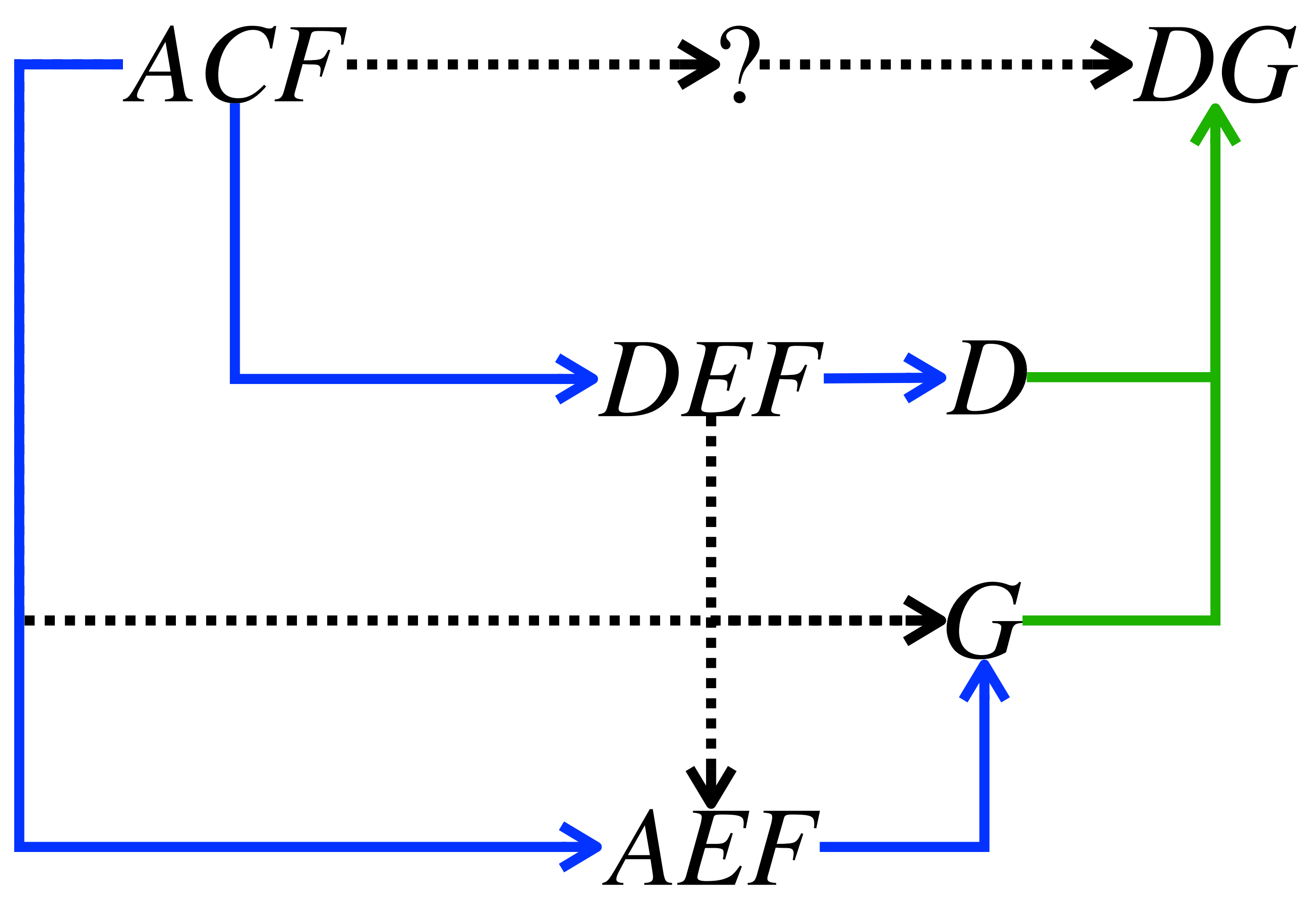

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  - Since $ACF \rightarrow DEF, DEF \rightarrow D, ACF \rightarrow D$ by Transitivity
  - Since $ACF \rightarrow DEF, ACF \rightarrow ACDEF$ by Augmentation
  - Since $ACF \rightarrow ACDEF, ACF \rightarrow AEF$ by Decomposition
  - Since $ACF \rightarrow AEF, AEF \rightarrow G, ACF \rightarrow G$ by Transitivity

Question 2

Armstrong's axioms

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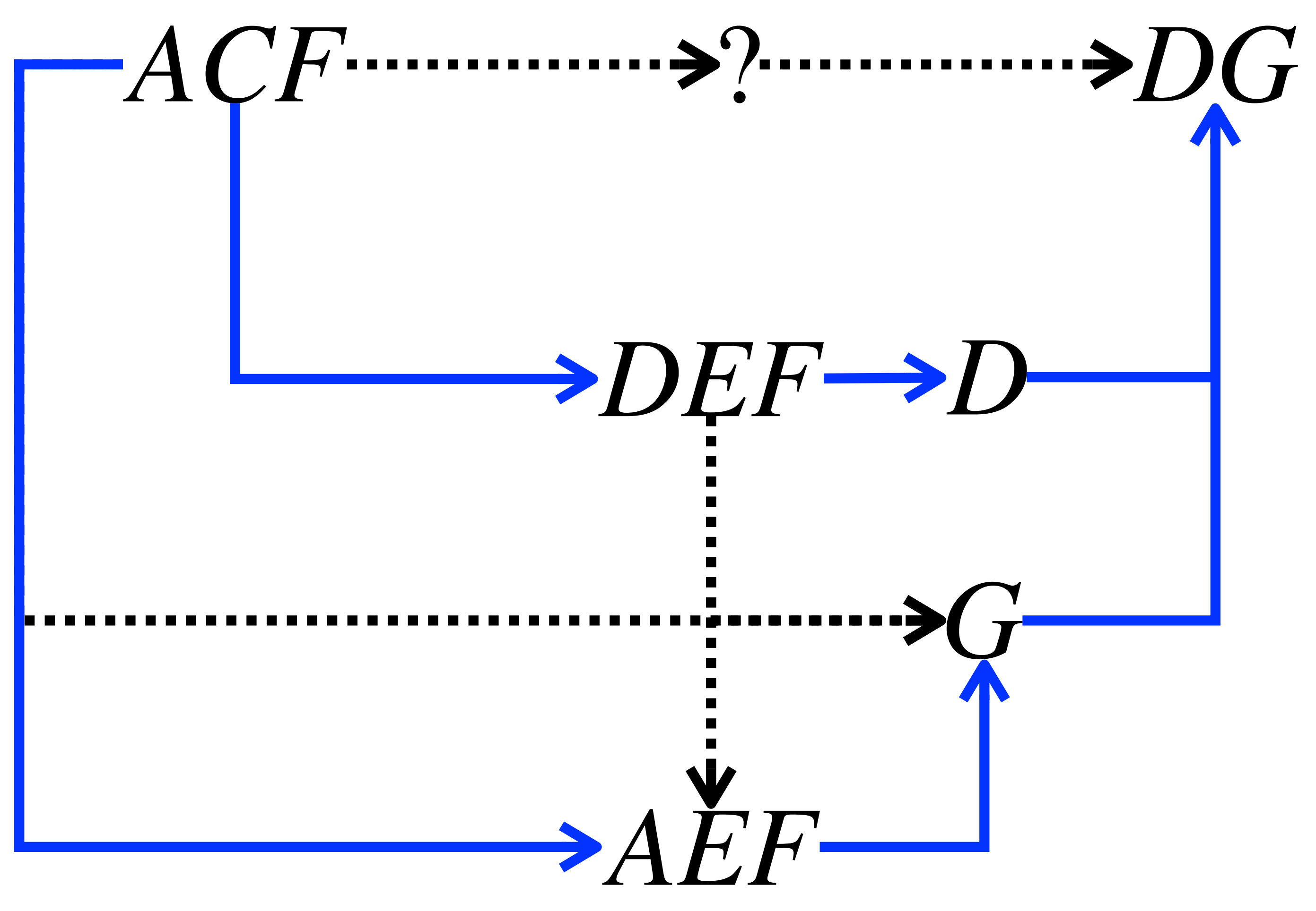
- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?



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  - Since $AC \rightarrow DE, ACF \rightarrow DEF$ by Augmentation
  - Since $D \subseteq DEF, DEF \rightarrow D$ by Reflexivity
  - Since $ACF \rightarrow DEF, DEF \rightarrow D, ACF \rightarrow D$ by Transitivity
  - Since $ACF \rightarrow DEF, ACF \rightarrow ACDEF$ by Augmentation
  - Since $ACF \rightarrow ACDEF, ACF \rightarrow AEF$ by Decomposition
  - Since $ACF \rightarrow AEF, AEF \rightarrow G, ACF \rightarrow G$ by Transitivity
  - Since $ACF \rightarrow D, ACF \rightarrow G, ACF \rightarrow DG$ by Union

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

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```
result = α
while (changes to result){
    for each β → γ in F{
        if (β ⊆ result){
            result = result ∪ γ
        }
    }
}
```

Question 2

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

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Question 2

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- $R = (A, B, C, D, E, F, G), F = \{\overset{1}{A \rightarrow B}, BC \rightarrow DE, AEF \rightarrow G\}$
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- $R = (A, B, C, D, E, F, G), F = \{\overset{1}{A \rightarrow B}, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

- $result = \{A, C, F\}$

$result = \alpha$
while (changes to $result$){
  for each $\beta \rightarrow \gamma$ in F{
    if ($\beta \subseteq result$){
      $result = result \cup \gamma$
    }
  }
}

Question 2

1

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

1

- $result = \{A, B, C, F\}$

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        }
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Question 2

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- $R = (A, B, C, D, E, F, G)$, $F = \{\overset{1}{A \rightarrow B}, \overset{2}{BC \rightarrow DE}, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

- $result = \{A, \overset{1}{B}, C, F\}$

$result = \alpha$

while (changes to $result$){

  for each $\beta \rightarrow \gamma$ in F{

    if ($\beta \subseteq result$){

      $result = result \cup \gamma$

    }

  }

}

Question 2

- $R = (A, B, C, D, E, F, G), F = \{\overset{1}{A \rightarrow B}, \overset{2}{BC \rightarrow DE}, AEF \rightarrow G\}$
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1 2 2

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- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

- $result = \{A, \overset{1}{B}, C, \overset{2}{D}, \overset{2}{E}, F\}$

$result = \alpha$
while (changes to $result$){
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    if ($\beta \subseteq result$){
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- Does $ACF \rightarrow DG$ holds?

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- $result = \{A, \overset{1}{B}, C, \overset{2}{D}, \overset{2}{E}, F\}$

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    }
  }
}

Question 2

1 2 3

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

1 2 2 3

- $result = \{A, B, C, D, E, F, G\}$

Armstrong's axioms

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Question 2

1 2 3

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1 2 2 3

- $result = \{A, B, C, D, E, F, G\}$

- Use Armstrong's axioms to prove

Armstrong's axioms
1. Reflexivity - if $\beta \subseteq \alpha$, then $\alpha \rightarrow \beta$
2. Transitivity - if $\alpha \rightarrow \beta$ and $\beta \rightarrow \gamma$, then $\alpha \rightarrow \gamma$
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        if (β ⊆ result){
            result = result ∪ γ
        }
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}
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Question 2

1 2 3

- $R = (A, B, C, D, E, F, G), F = \{A \rightarrow B, BC \rightarrow DE, AEF \rightarrow G\}$
- Does $ACF \rightarrow DG$ holds?

- If $\{DG\} \subseteq \{ACF\}^+, ACF \rightarrow DG$

1 2 2 3

- $result = \{A, B, C, D, E, F, G\}$

- Use Armstrong's axioms to prove

$ACF \longrightarrow D$ $\qquad$ $ACF \longrightarrow G$

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Question 2

Armstrong's axioms

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- Use Armstrong's axioms to prove

$$\begin{tikzcd} ACF \arrow[rr] \arrow[dr] & & D \\ & BC \arrow[ur, "2"'] & \end{tikzcd}$$

$$\begin{tikzcd} ACF \arrow[r] & G \end{tikzcd}$$

$result = \alpha$
while (changes to $result$){
  for each $\beta \to \gamma$ in F{
    if ($\beta \subseteq result$){
      $result = result \cup \gamma$
    }
  }
}

Question 2

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$$\begin{tikzcd} ACF \arrow[rr] \arrow[d, "1"'] & & D \\ & BC \arrow[ur, "2"'] & \end{tikzcd} \qquad \begin{tikzcd} ACF \arrow[r] & G \end{tikzcd}$$

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- Since $A \rightarrow B, AC \rightarrow BC$ by Augmentation
- Since $BC \rightarrow DE, BC \rightarrow D$ by Decomposition
- Since $AC \rightarrow BC, BC \rightarrow D, AC \rightarrow D$ by Transitivity
- Since $AC \rightarrow D, ACF \rightarrow DF$ by Augmentation
- Since $ACF \rightarrow DF, ACF \rightarrow D$ by Decomposition

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- Use Armstrong's axioms to prove

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& BC \arrow[ur, "2"'] &
\end{tikzcd}
\qquad
\begin{tikzcd}
ACF \arrow[rr] \arrow[dr] \arrow[dd] & & G \\
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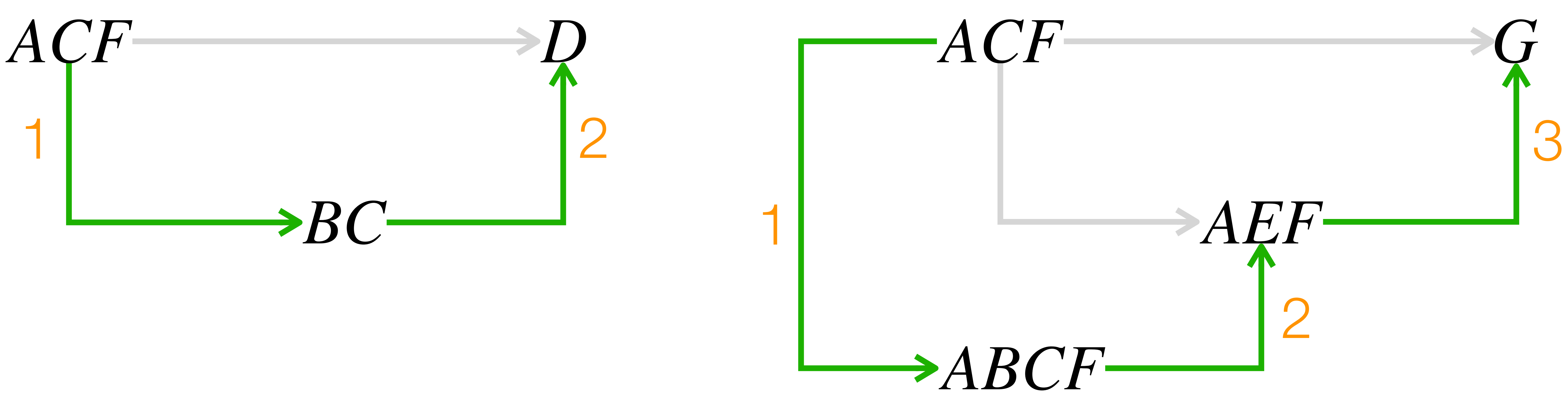

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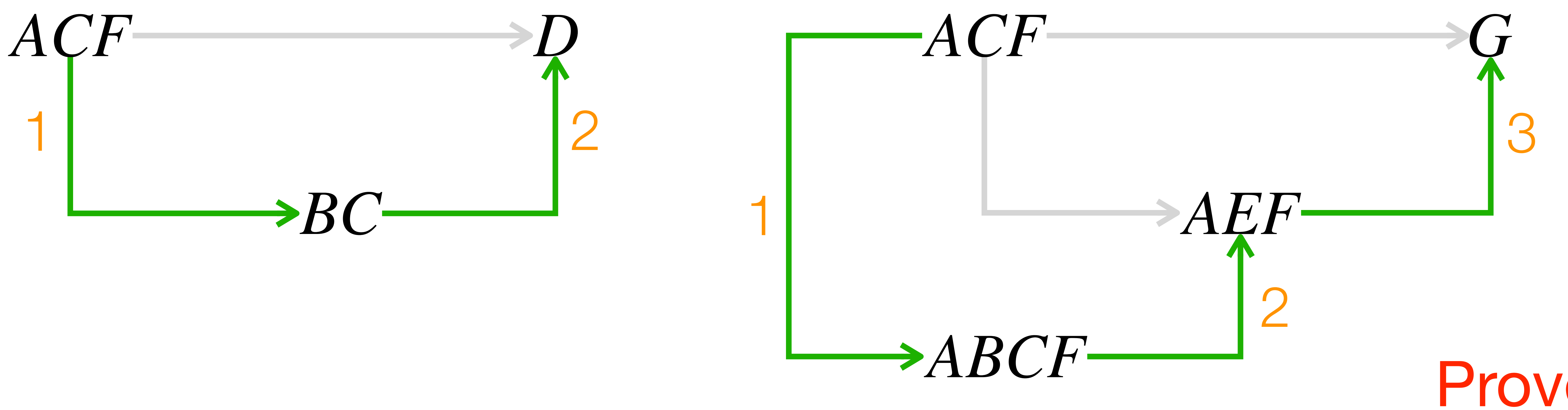
- $result = \{A, \underset{1}{B}, C, \underset{2}{D}, \underset{2}{E}, F, \underset{3}{G}\}$

- Use Armstrong's axioms to prove



$result = \alpha$
while (changes to $result$){
  for each $\beta \rightarrow \gamma$ in F{
    if ($\beta \subseteq result$){
      $result = result \cup \gamma$
    }
  }
}

Prove by your self

Question 3

- $R = (A, B, C, D), F = \{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$
- List all candidate keys of R.

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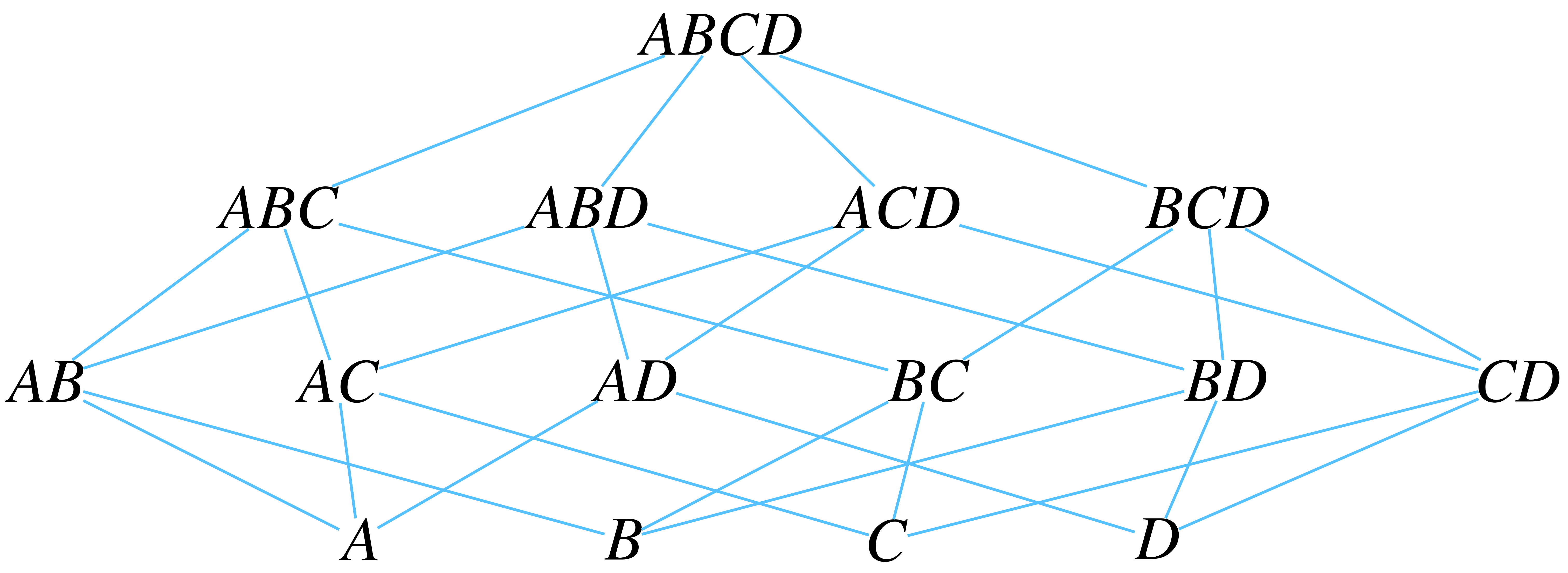

Question 3

- $R = (A, B, C, D), F = \{AB \to C, C \to D, D \to A\}$
- List all candidate keys of R.
- Is A a superkey?
  - To test whether $\{A\}^+ = R$

Armstrong's axioms

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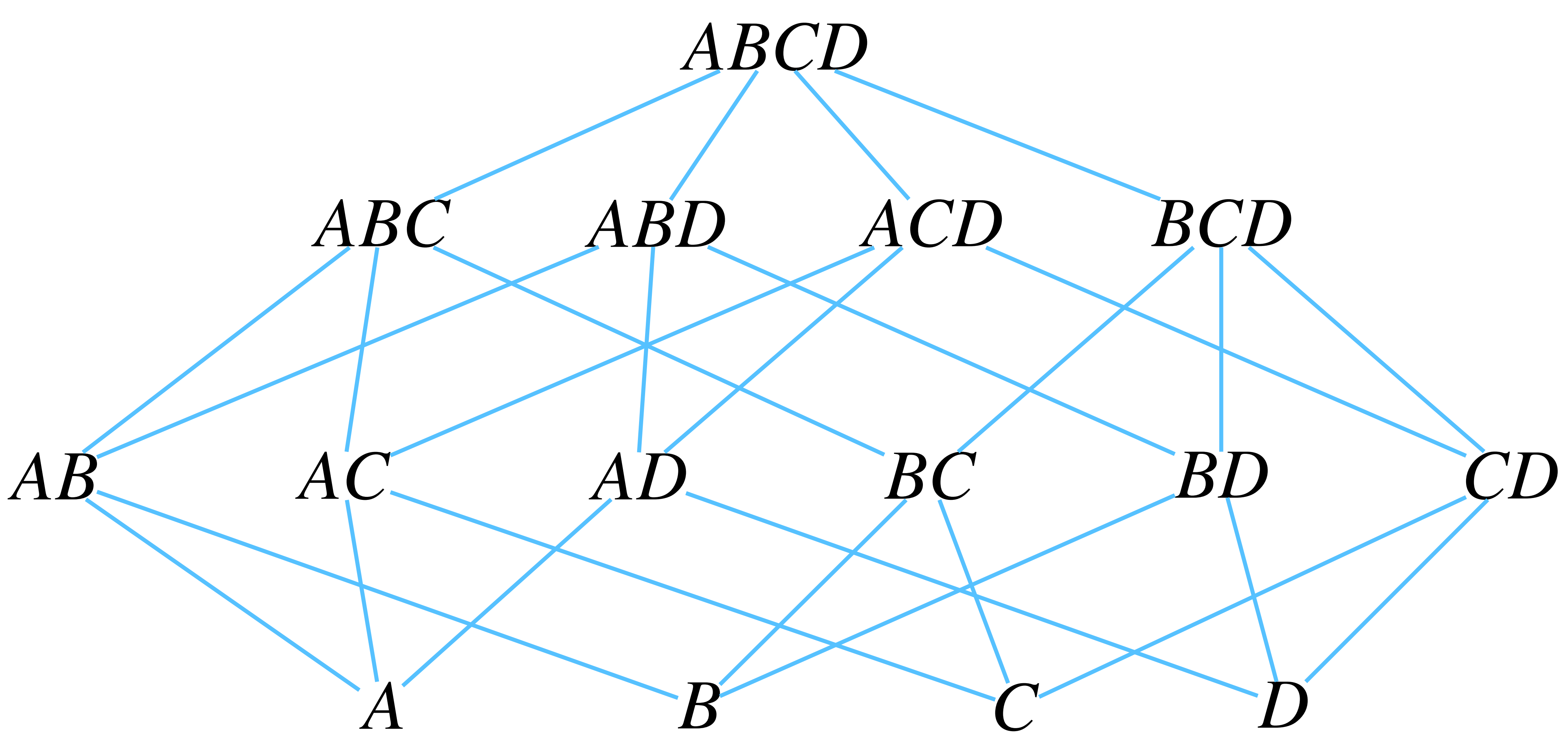

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  - $\{A\}^+ = ?$

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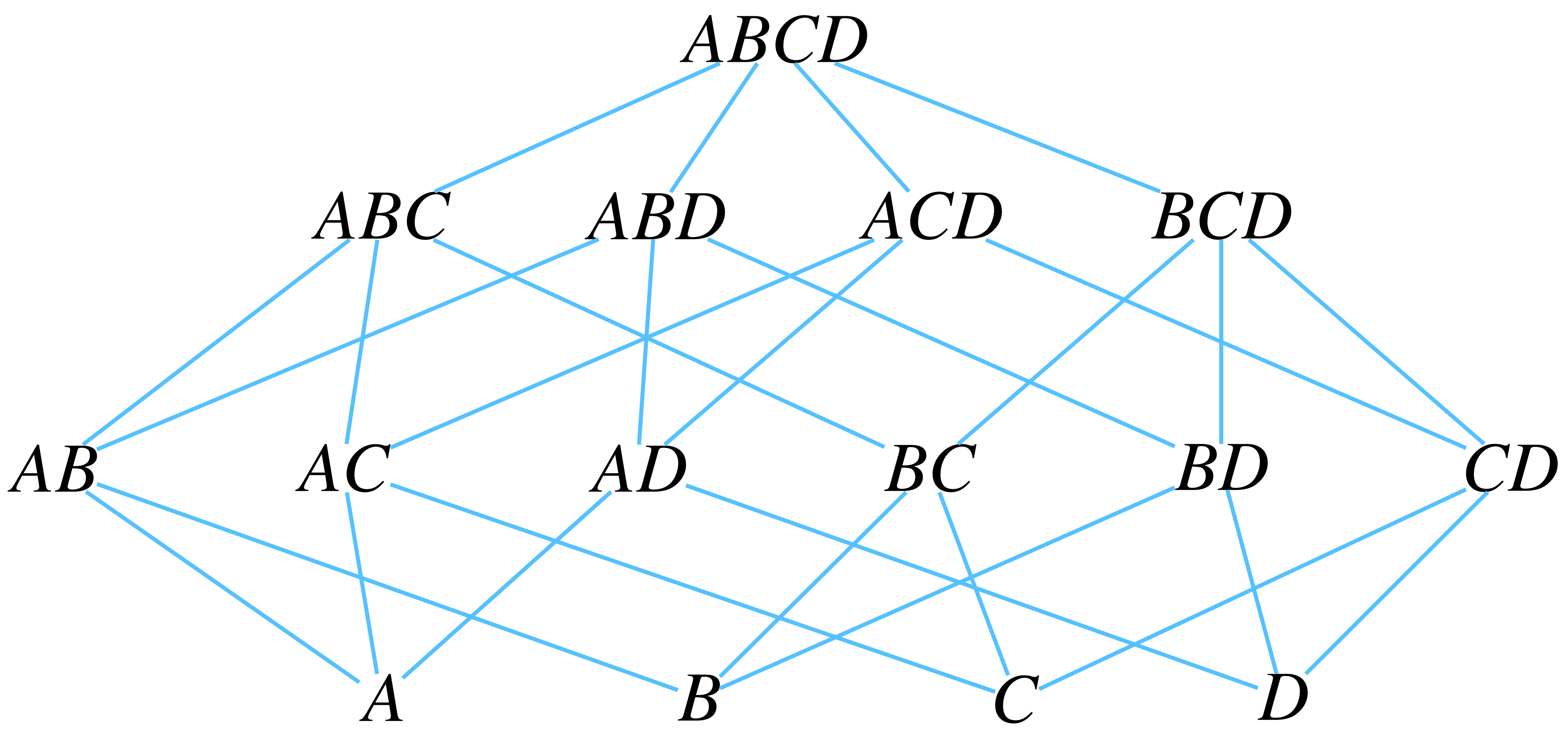

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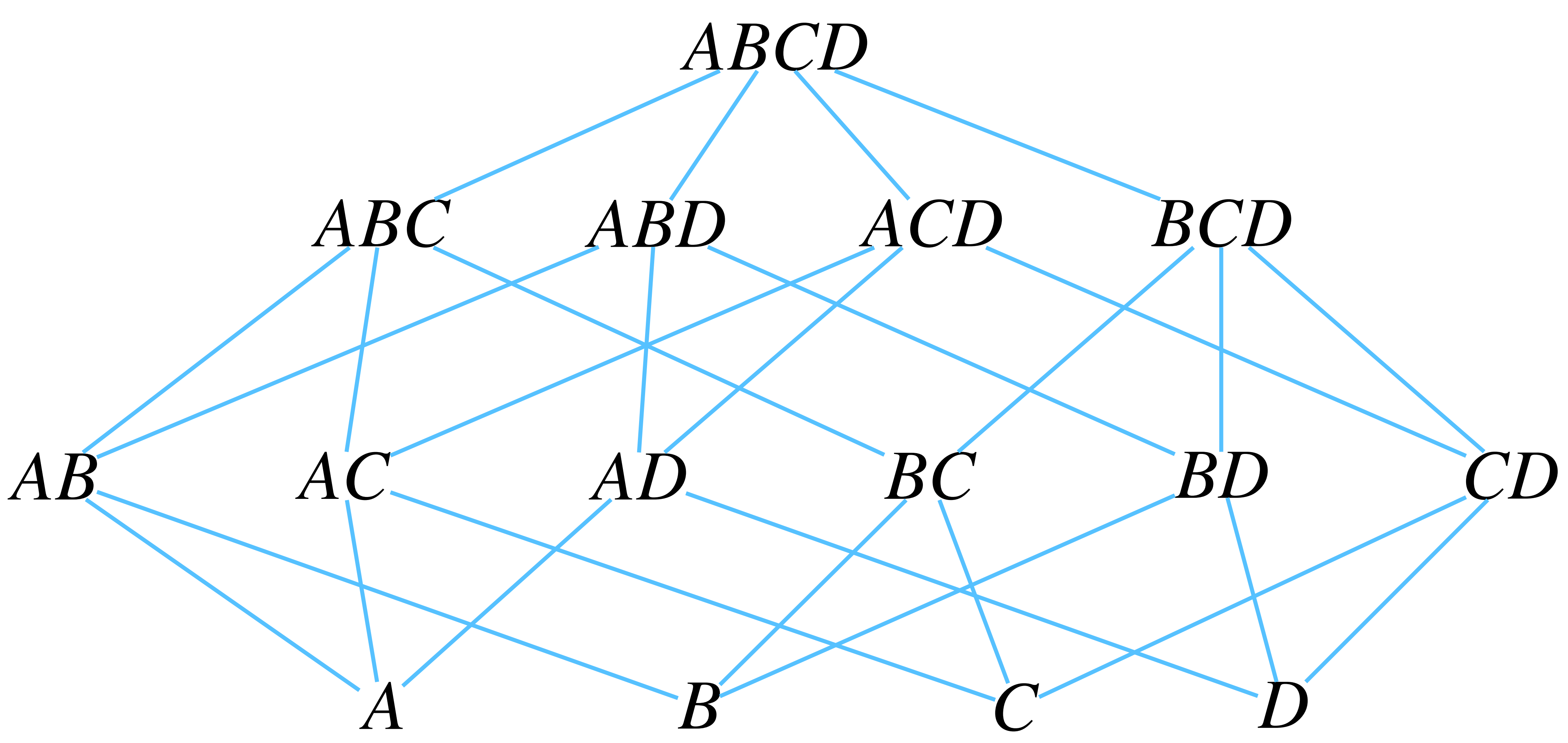
- Is A a superkey?
  - To test whether $\{A\}^+ = R$
  - $\{A\}^+ = \{A\} \neq R$
  - A is NOT a super key



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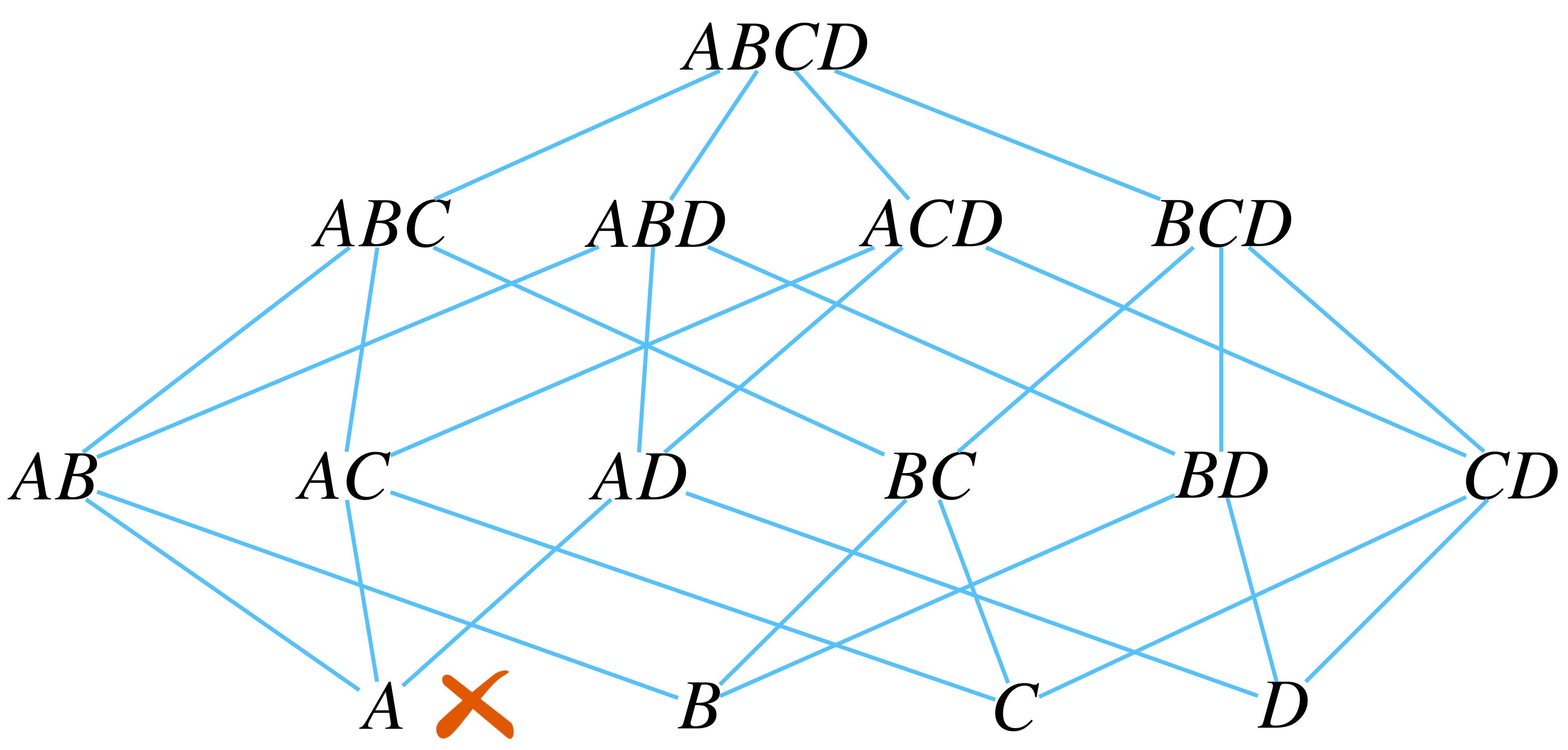

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- Is B a superkey?
  - $\{B\}^+ = ?$

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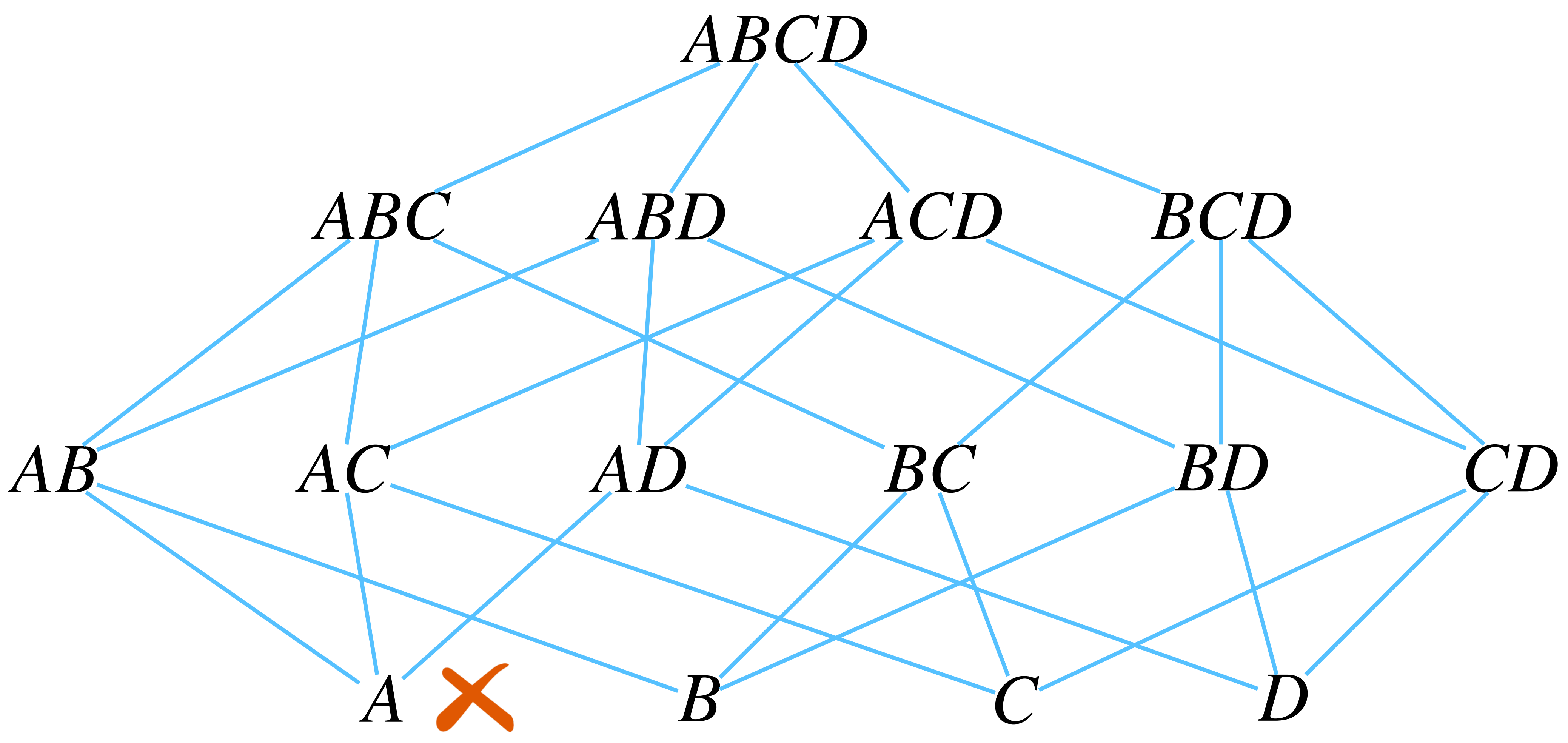

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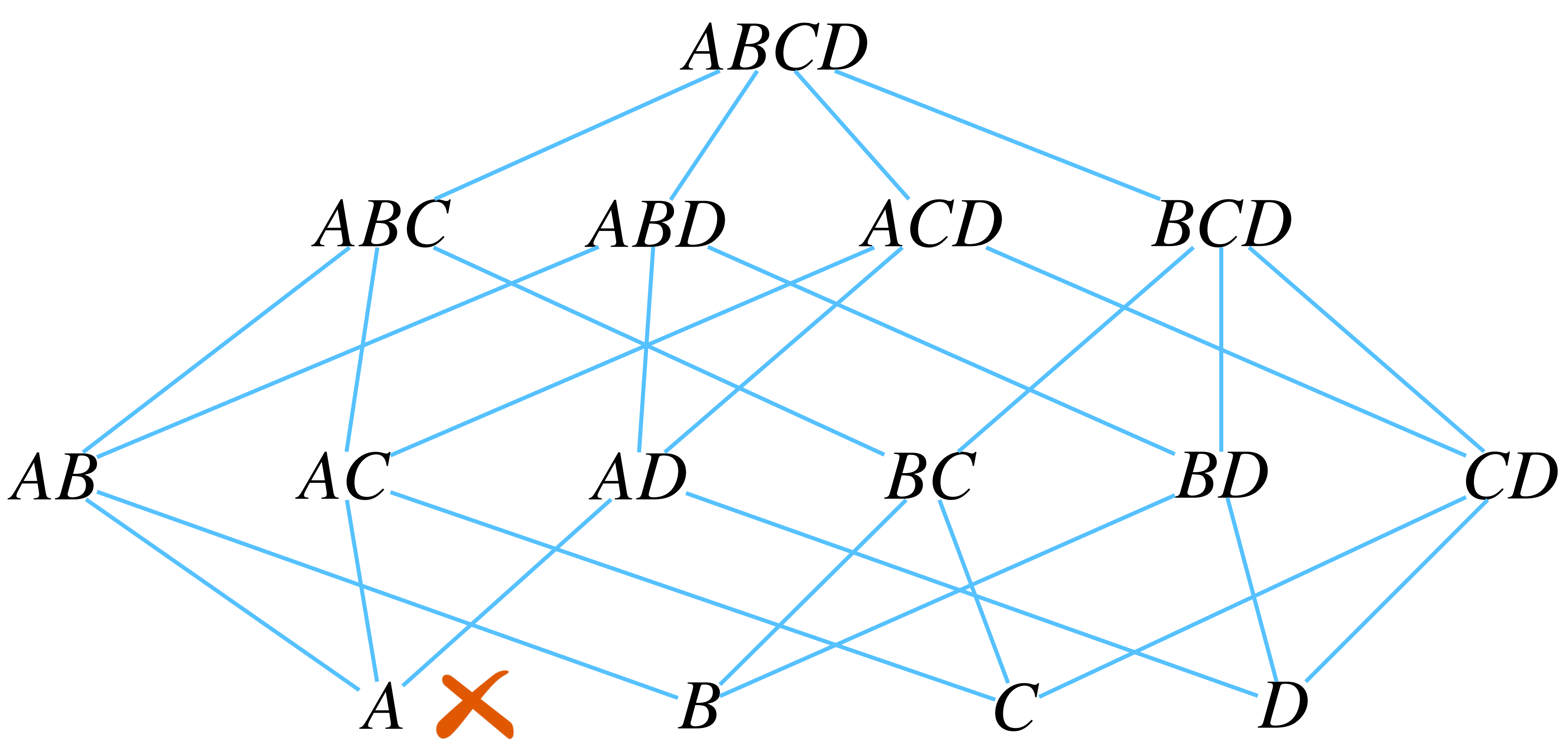

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- Is B a superkey?
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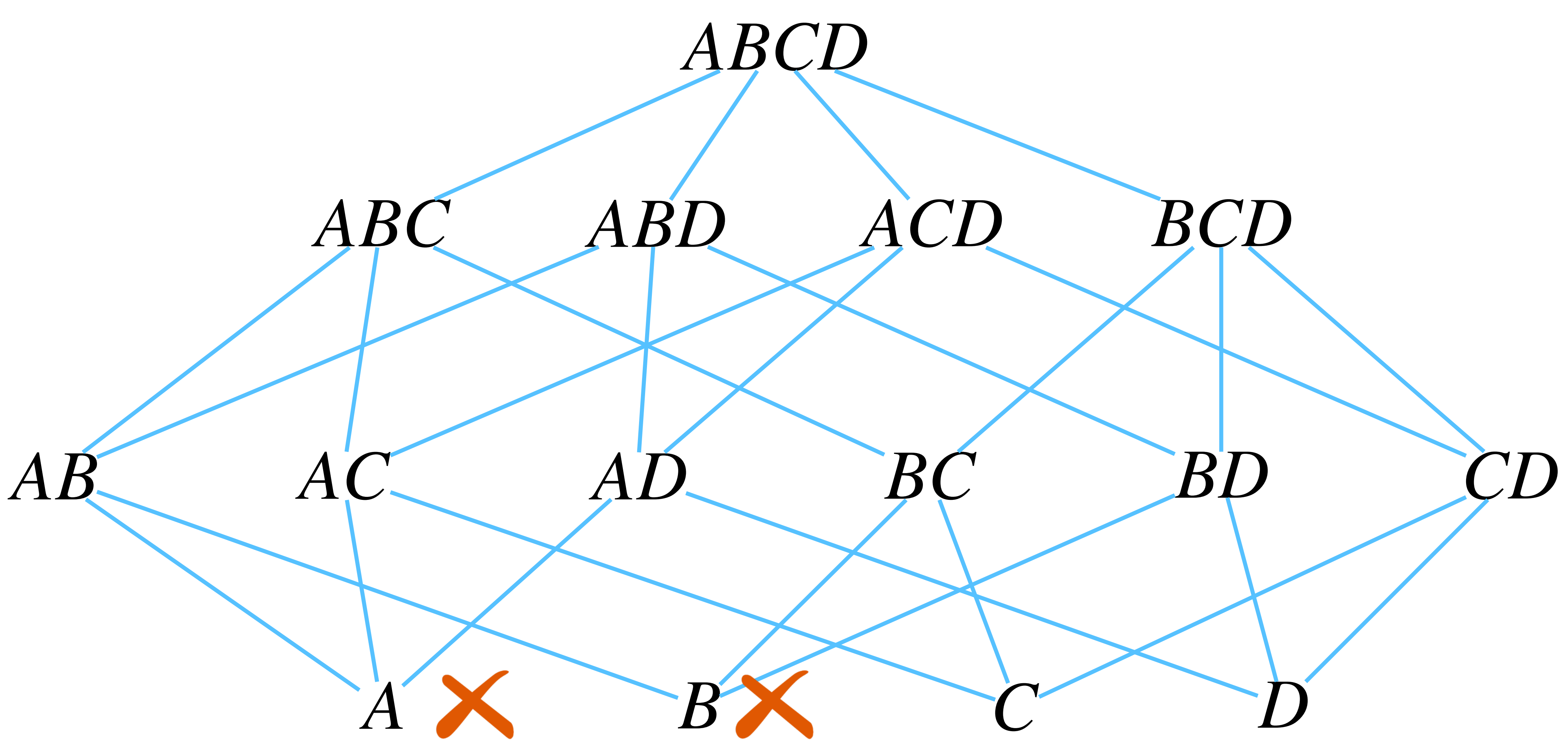

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- List all candidate keys of R.

- Is A a superkey?
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  - $\{A\}^+ = \{A\} \neq R$
  - A is NOT a super key
- Is B a superkey?
  - $\{B\}^+ = \{B\} \neq R$
  - B is NOT a super key
- Is C a superkey?
  - $\{C\}^+ = ?$

Armstrong's axioms

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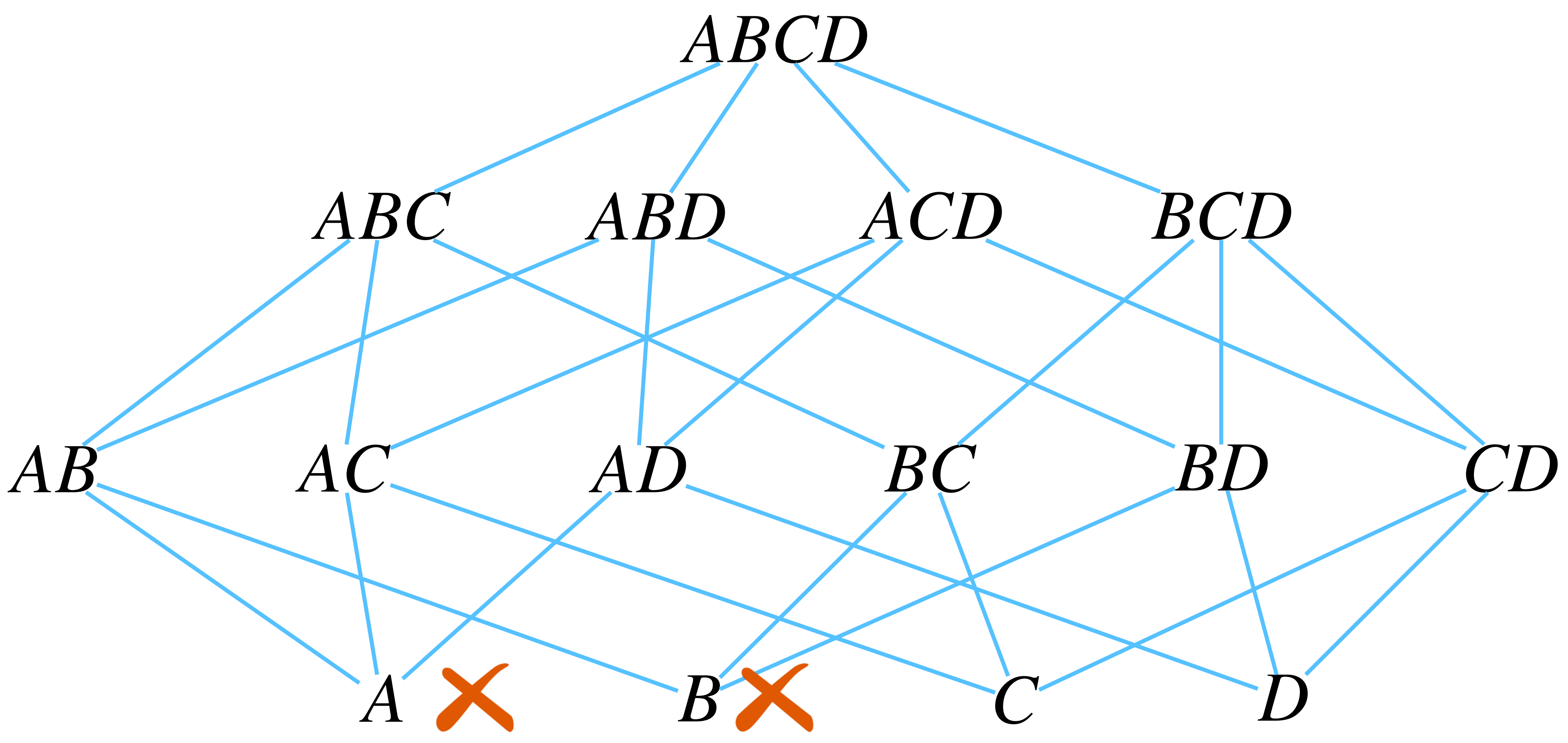

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  - $\{A\}^+ = \{A\} \neq R$
  - A is NOT a super key
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  - B is NOT a super key
- Is C a superkey?
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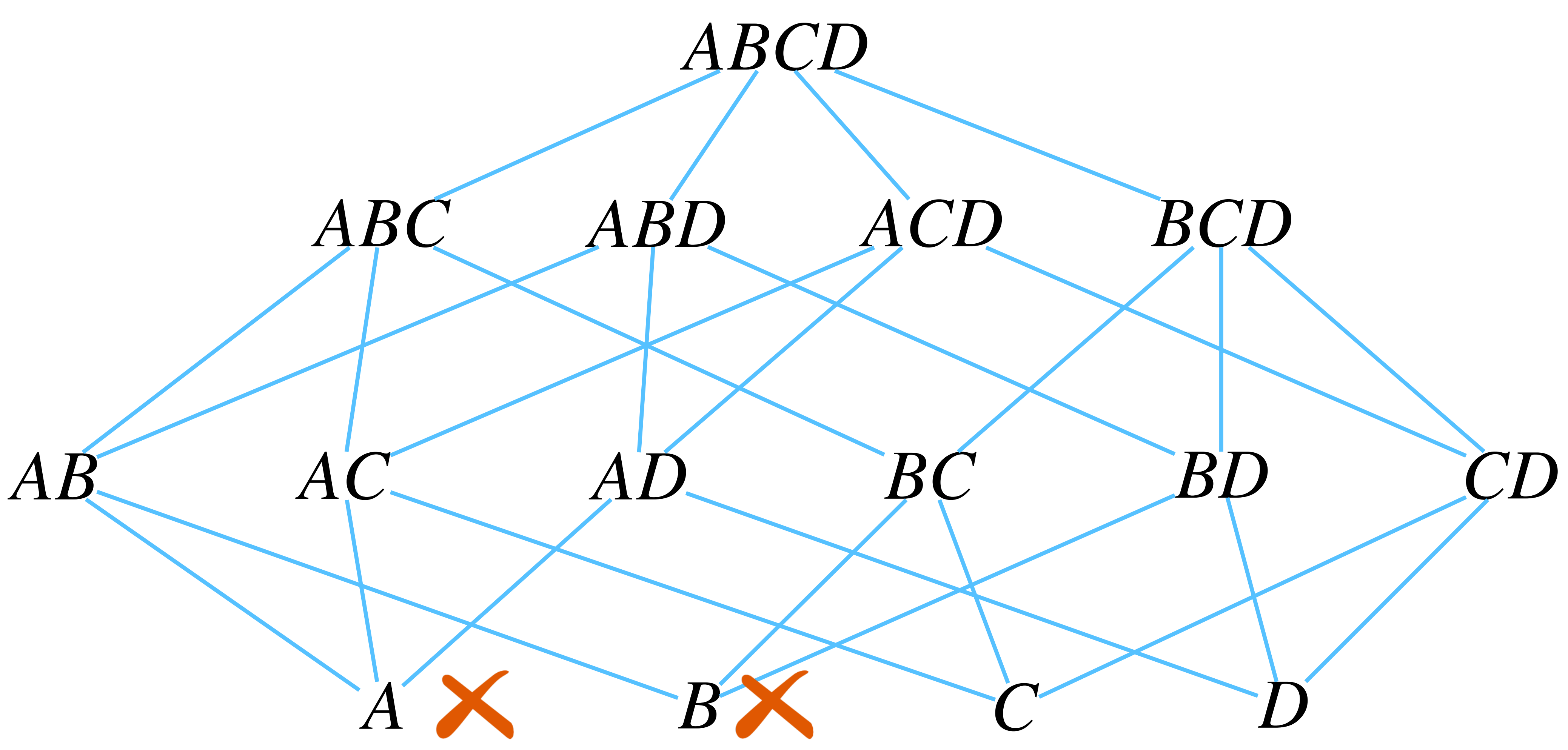

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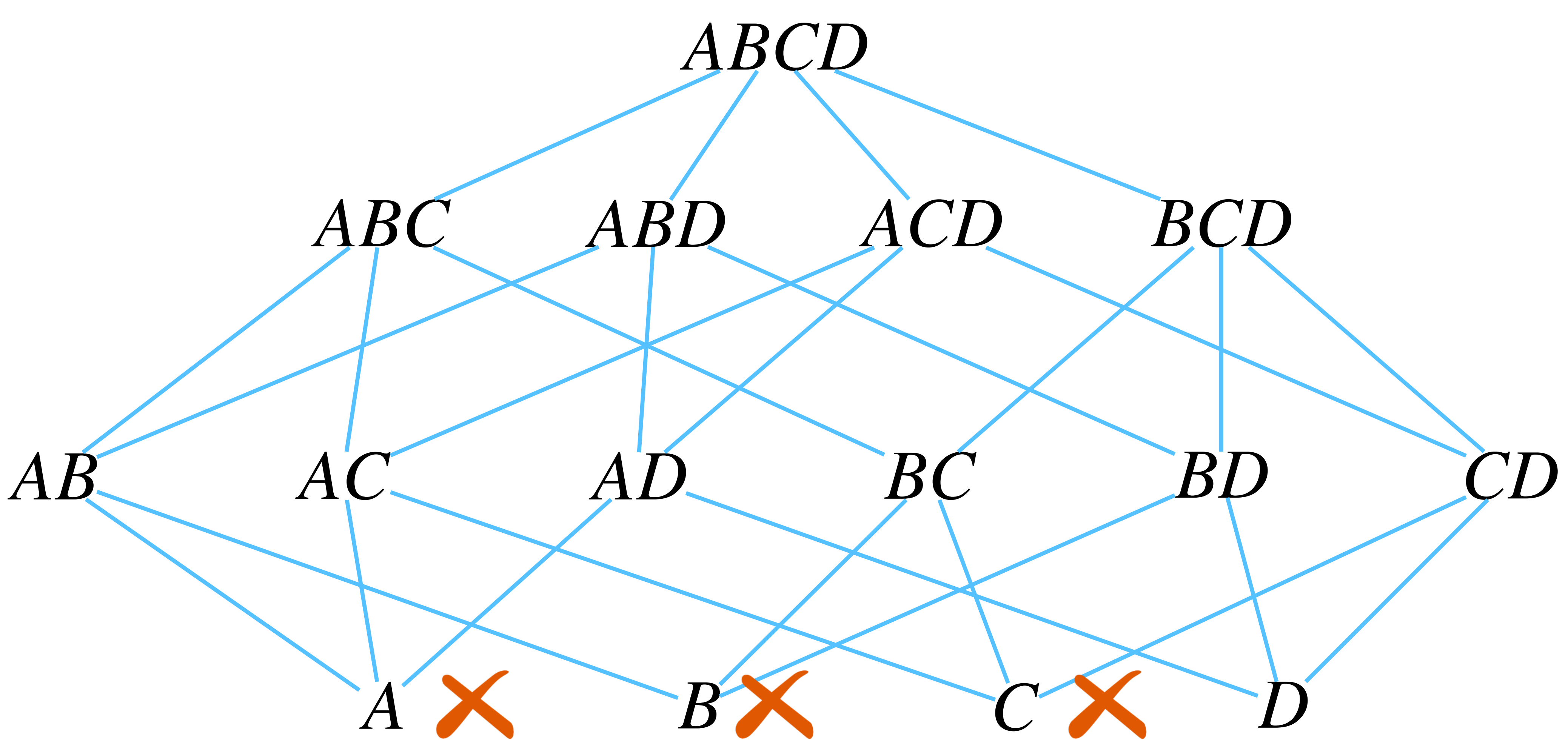

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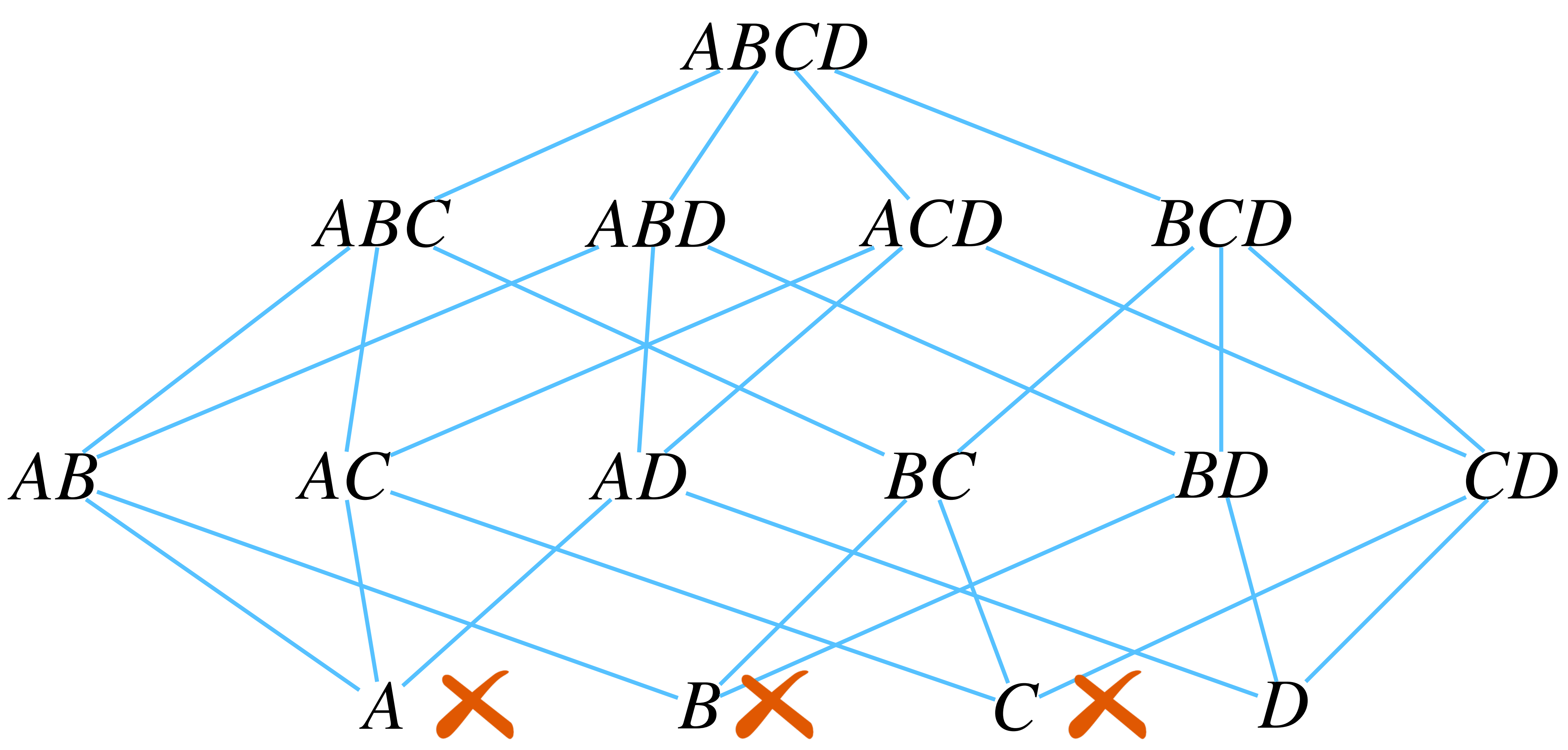
  - $\{B\}^+ = \{B\} \neq R$
  - B is NOT a super key
- Is C a superkey?
  - $\{C\}^+ = \{A, C, D\} \neq R$
  - B is NOT a super key
- Is D a superkey?
  - $\{D\}^+ = ?$



Question 3

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- List all candidate keys of R.
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  - B is NOT a super key
- Is C a superkey?
  - $\{C\}^+ = \{A, C, D\} \neq R$
  - B is NOT a super key
- Is D a superkey?
  - $\{D\}^+ = \{A, D\} \neq R$
  - D is NOT a super key

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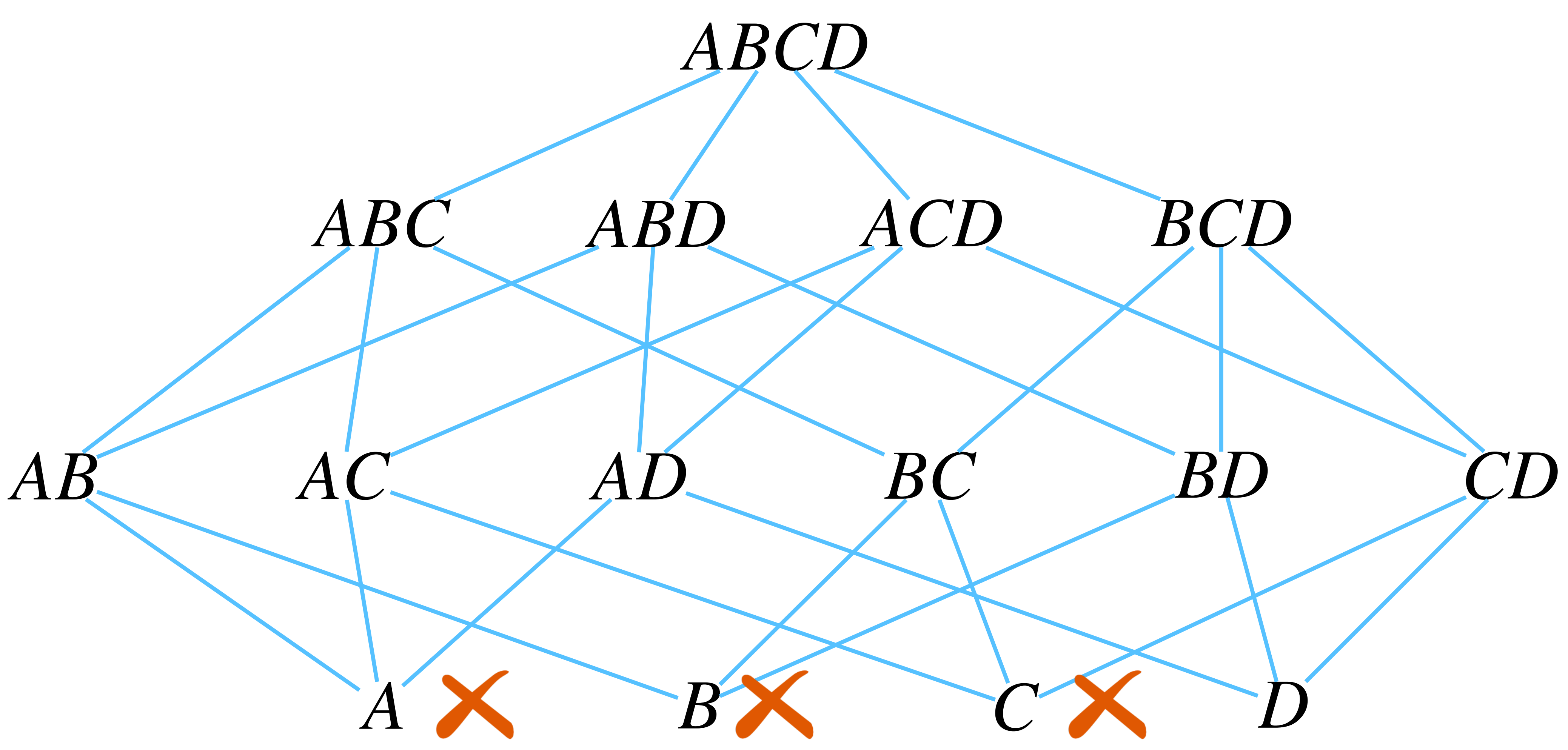

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- Is B a superkey?
  - $\{B\}^+ = \{B\} \neq R$
  - B is NOT a super key
- Is C a superkey?
  - $\{C\}^+ = \{A, C, D\} \neq R$
  - B is NOT a super key
- Is D a superkey?
  - $\{D\}^+ = \{A, D\} \neq R$
  - D is NOT a super key

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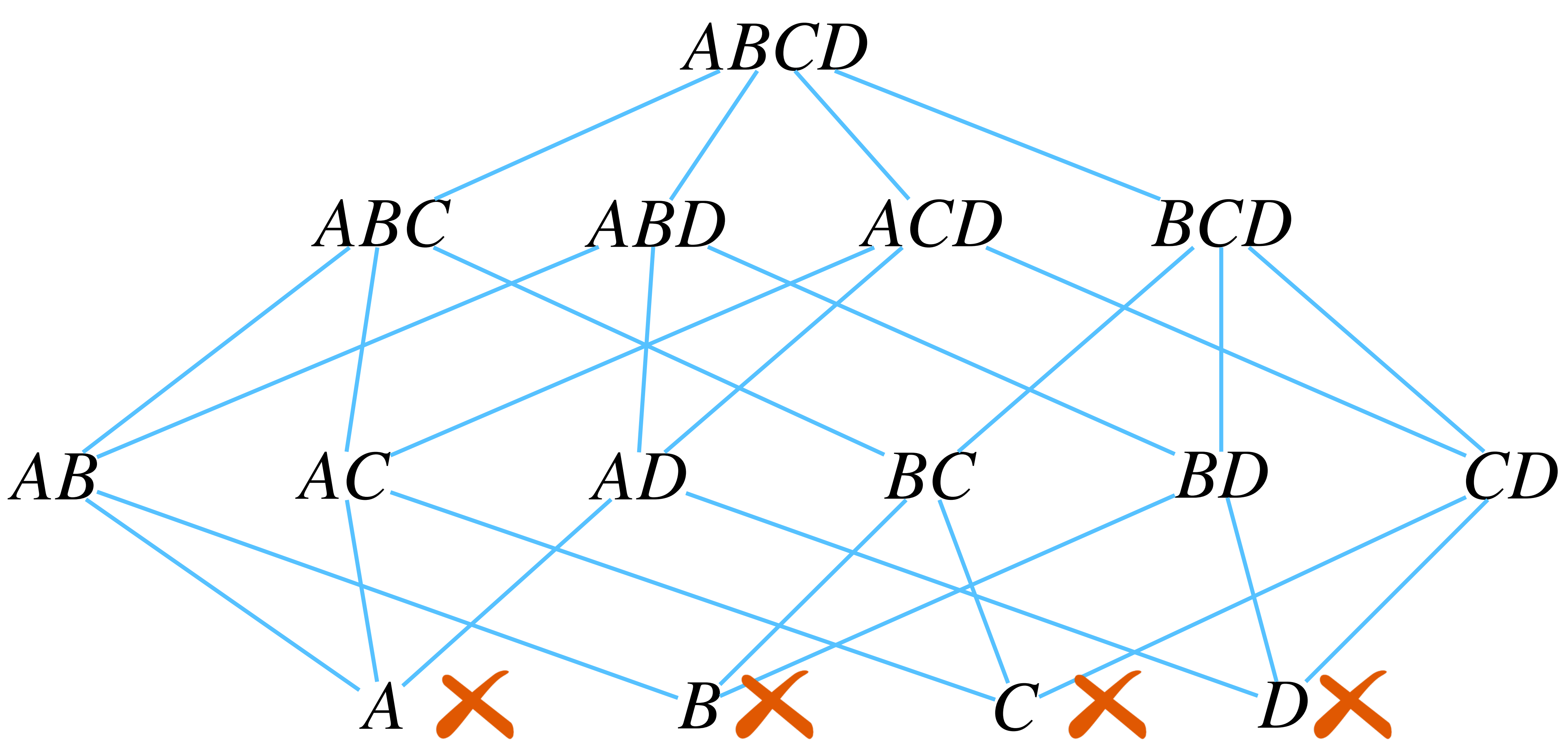

Question 3

- $R = (A, B, C, D), F = \{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$
- List all candidate keys of R.
- Is AB a superkey?
  - $\{AB\}^+ = ?$

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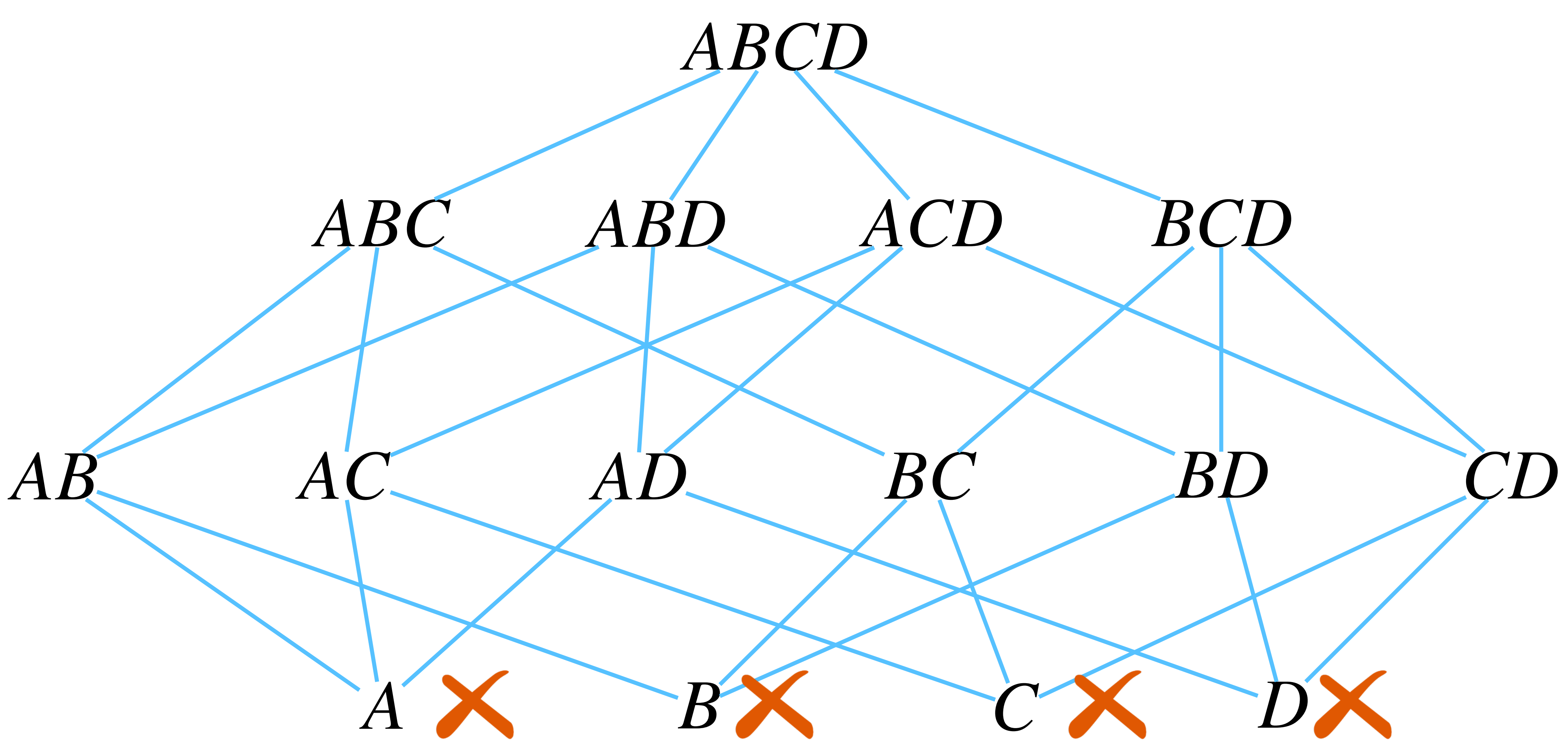

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- Is AB a superkey?
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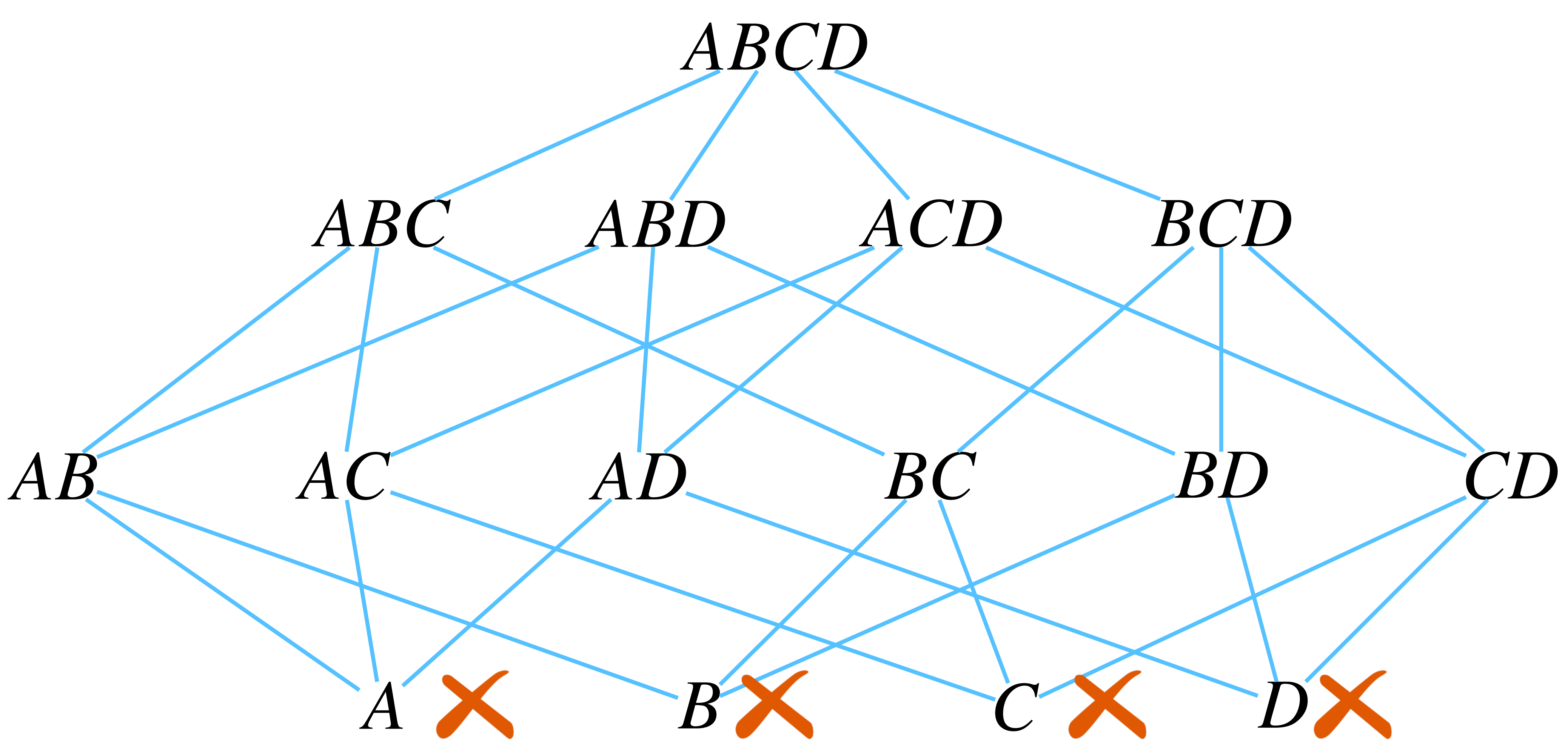

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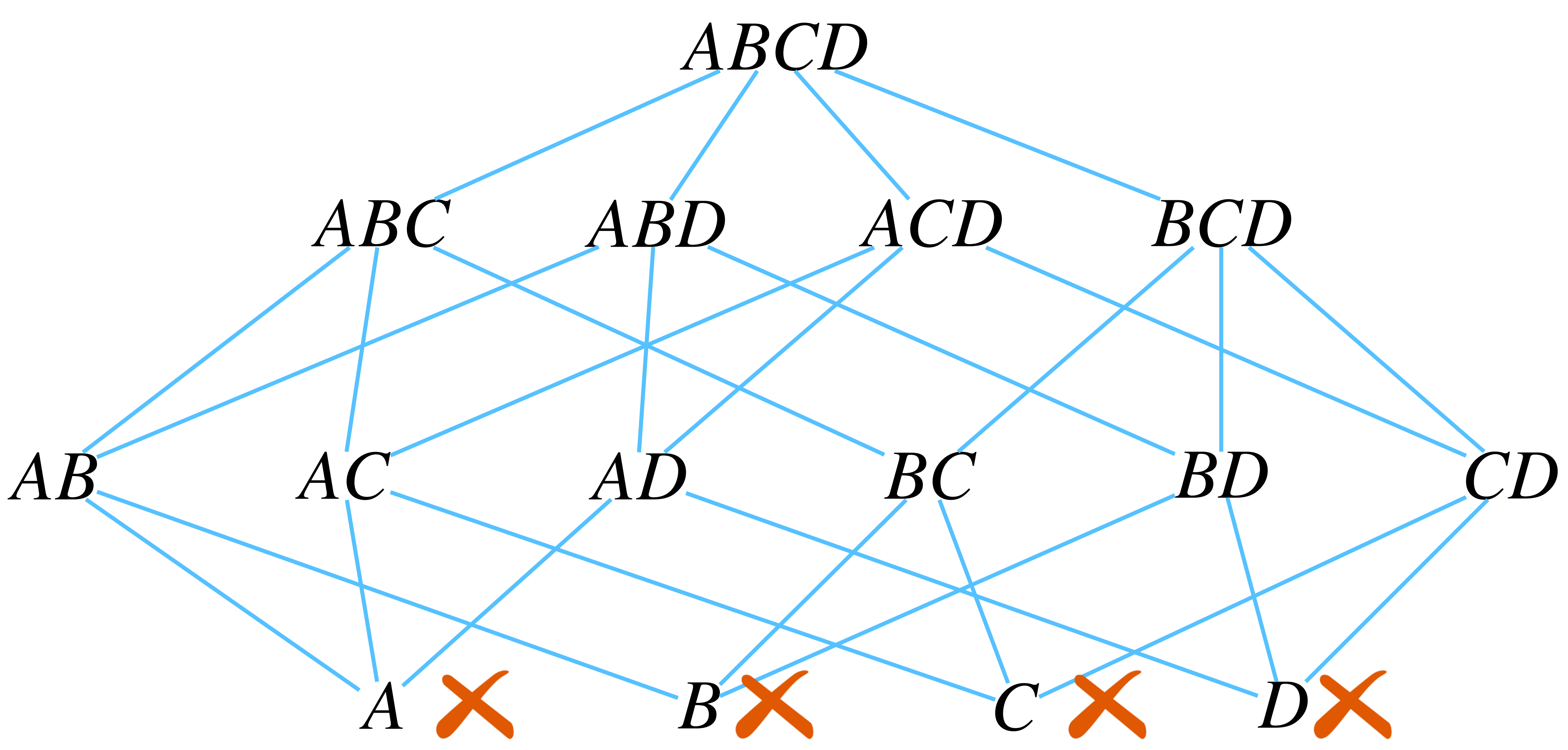

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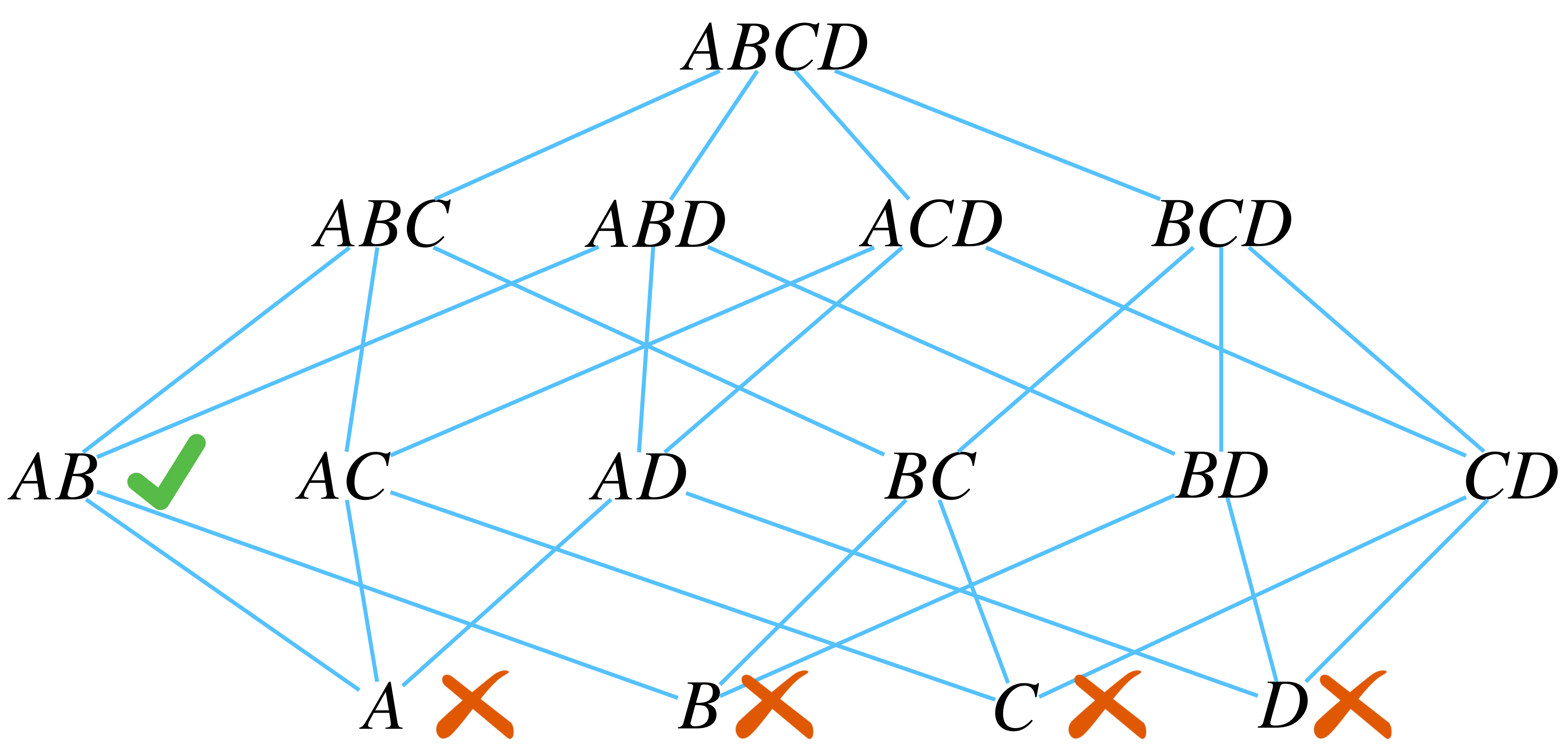

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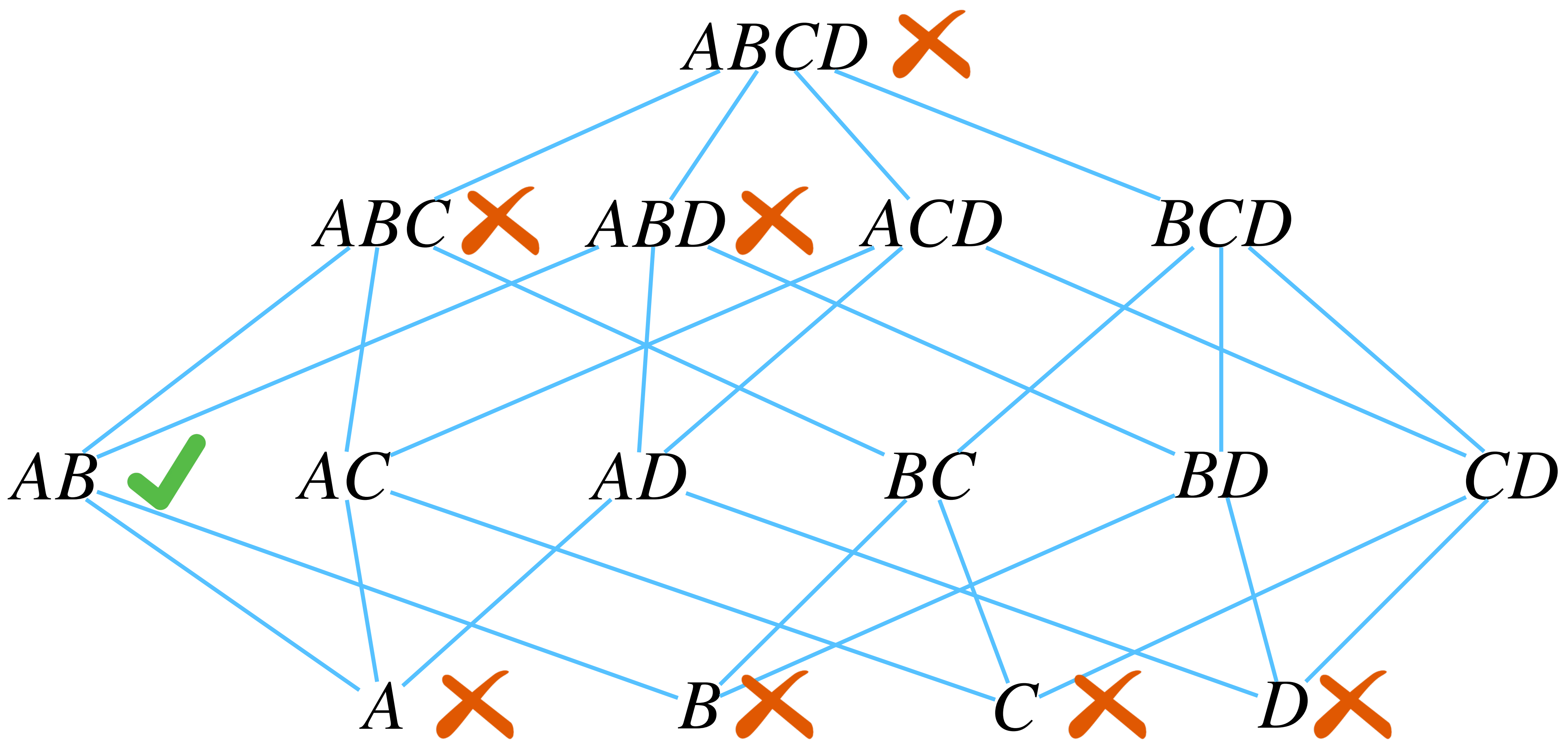

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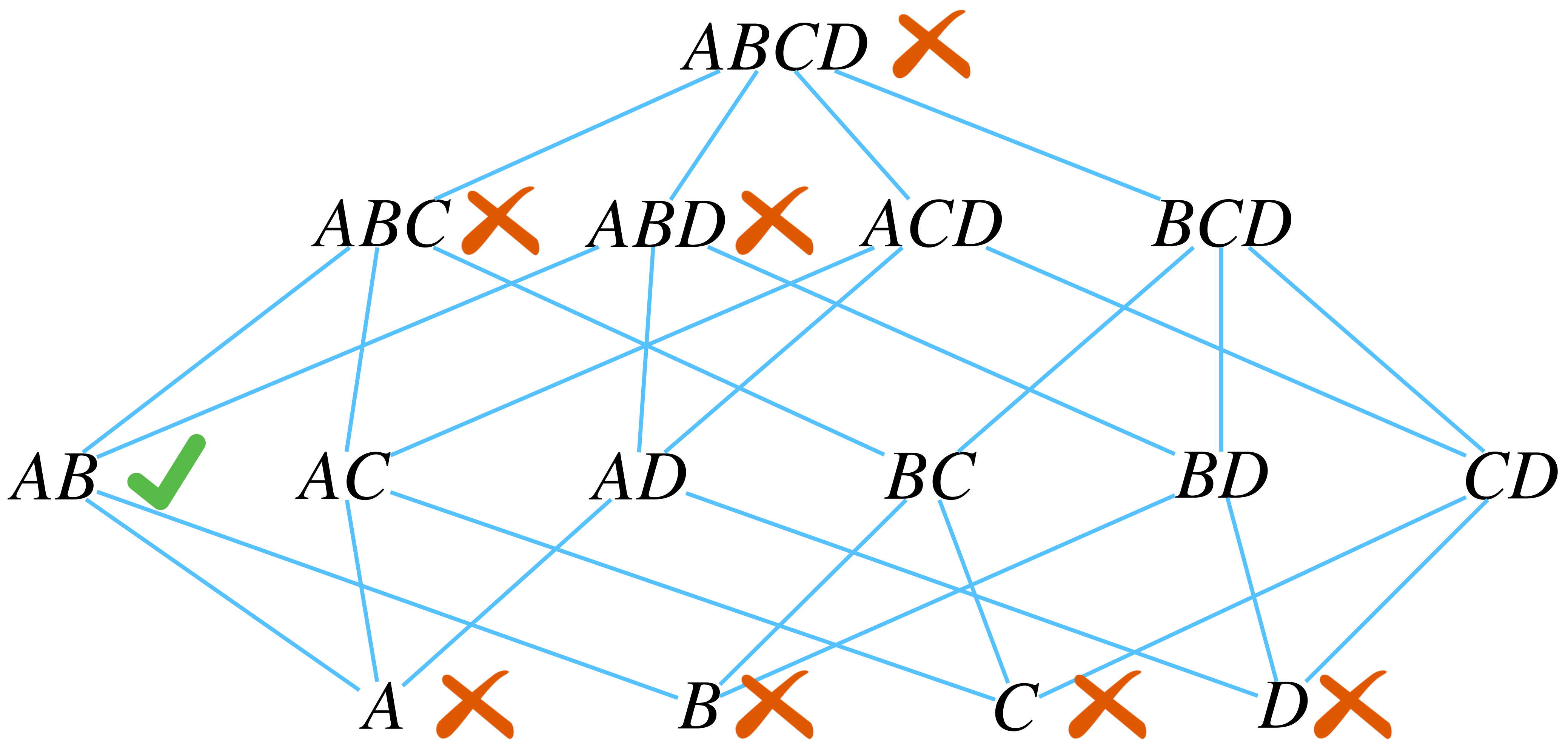

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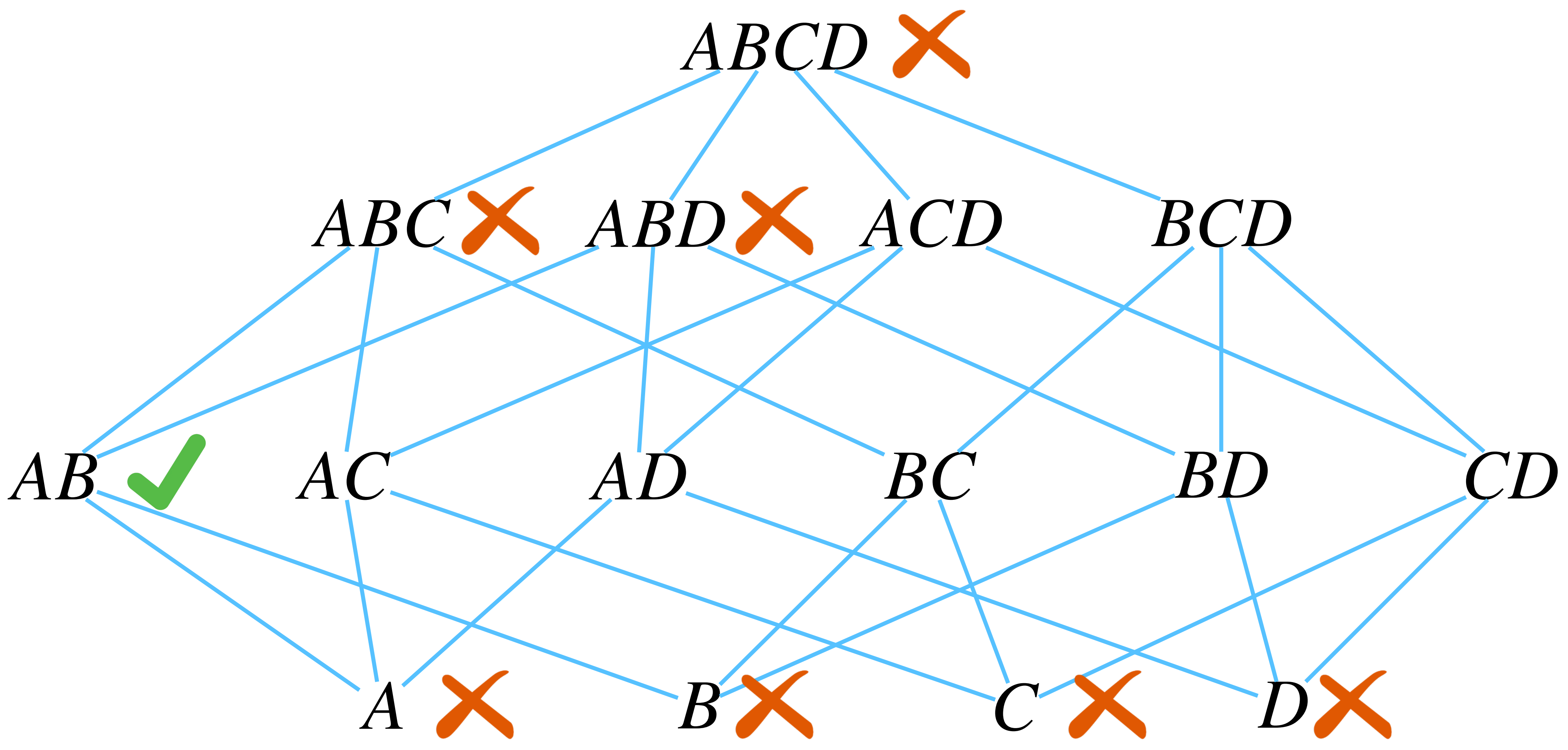

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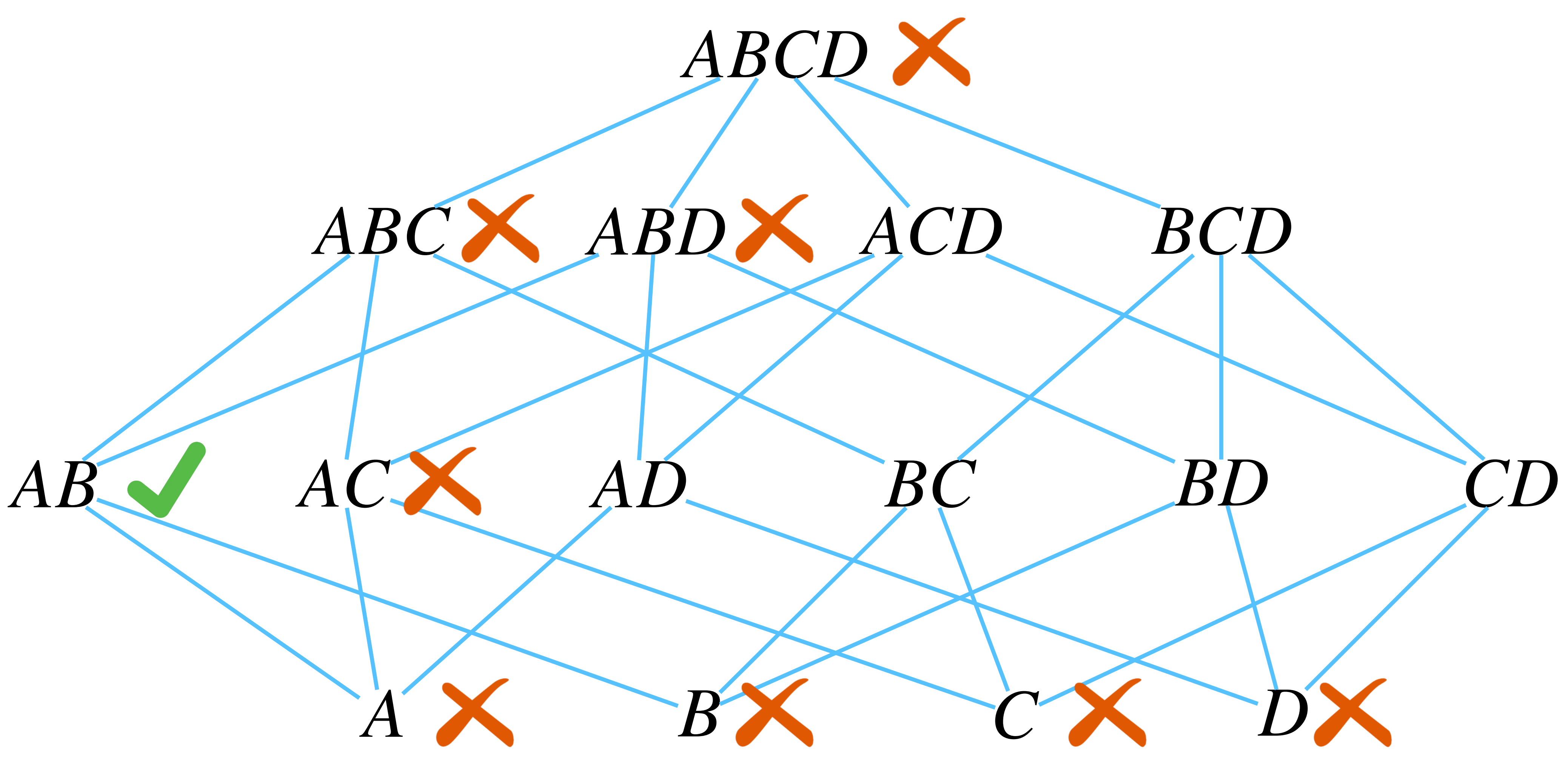

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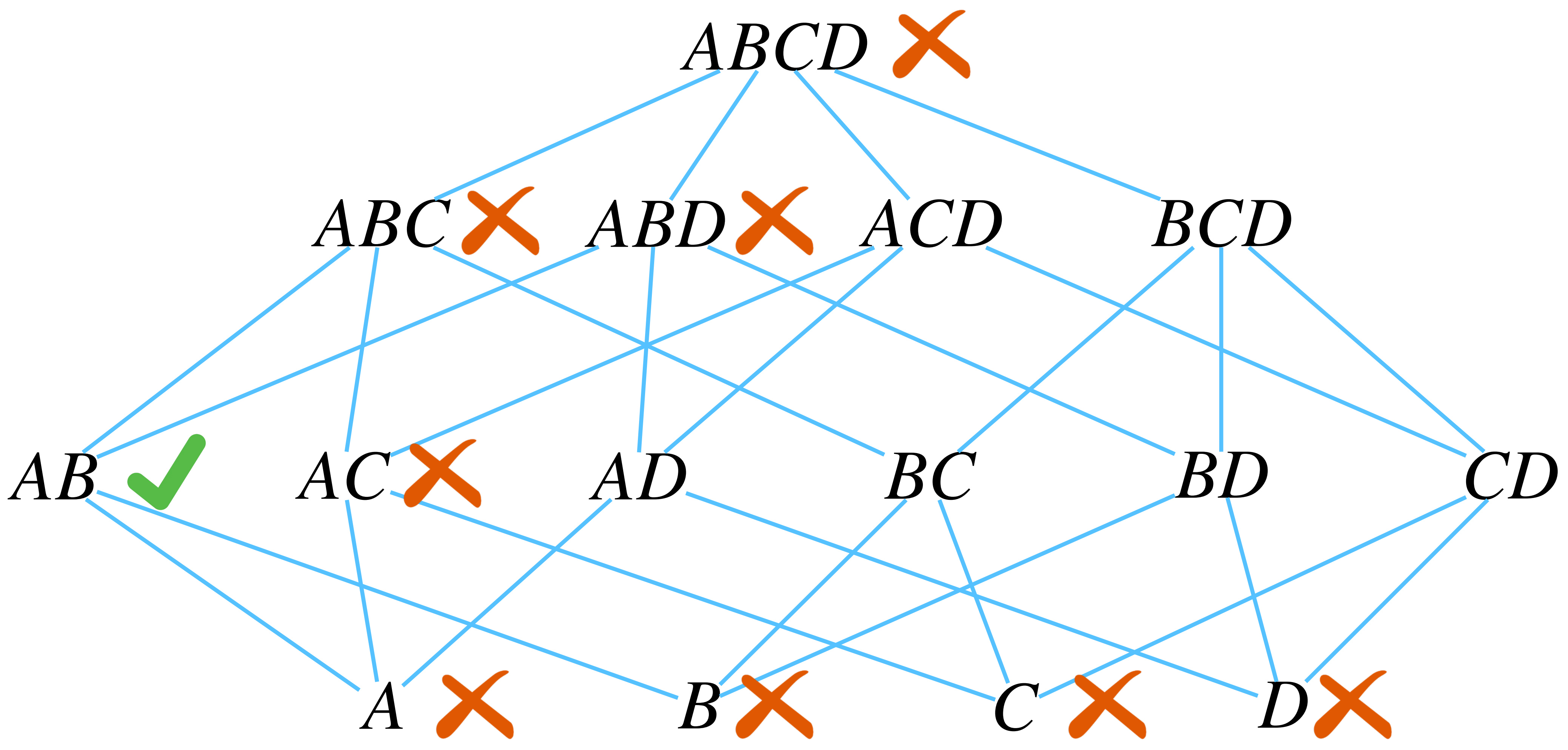
- Is AD a super key?
  - $\{AD\}^+ = ?$



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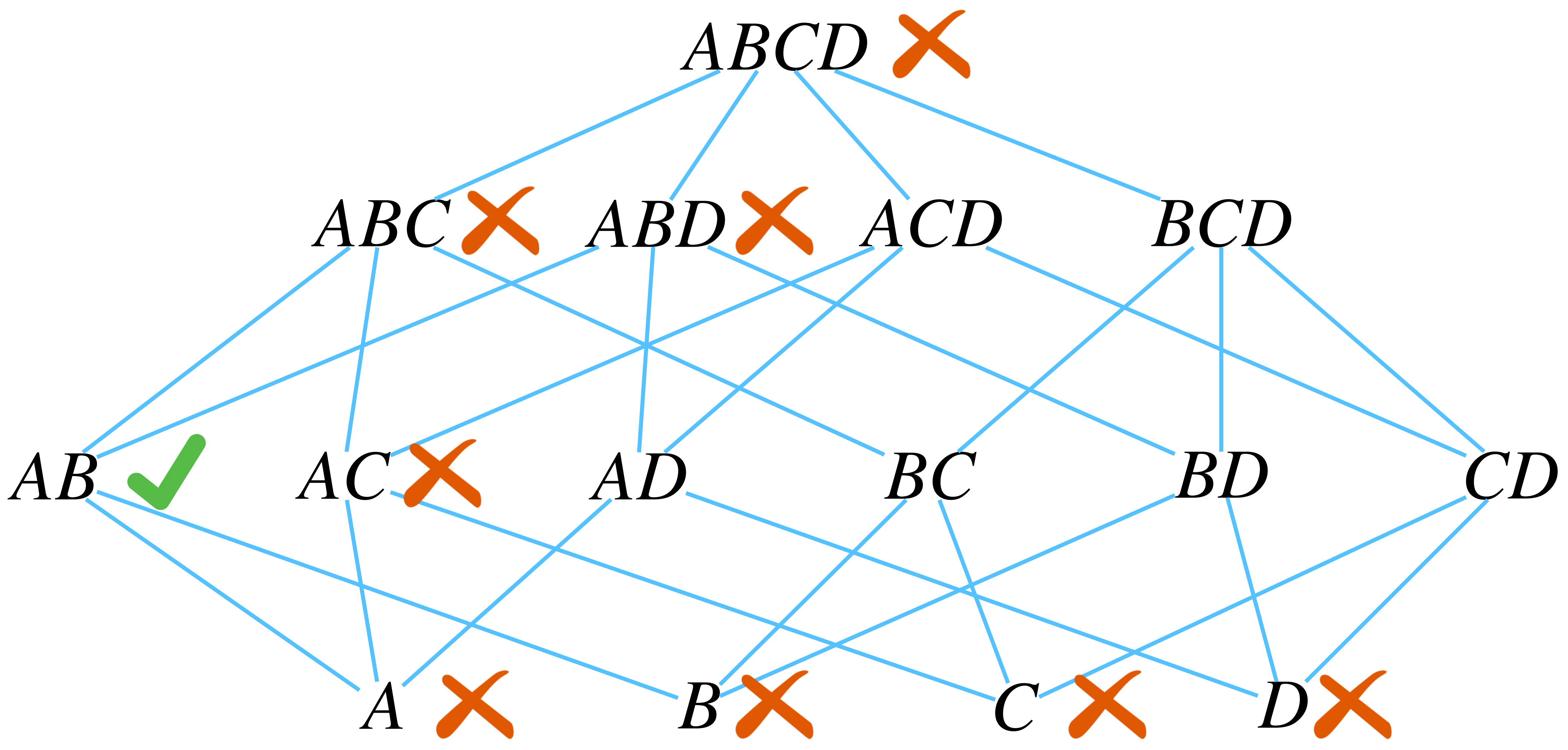

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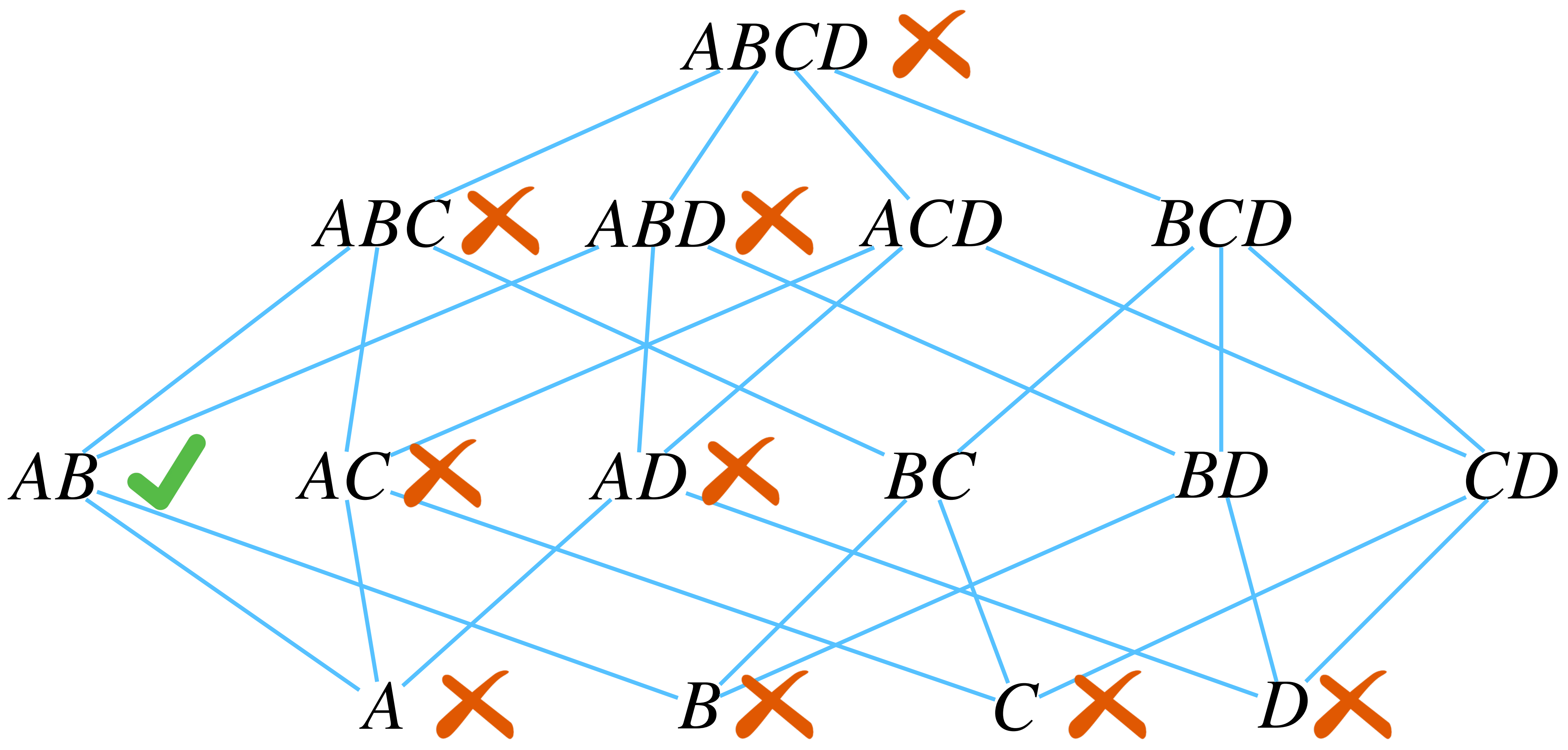

Question 3

- $R = (A, B, C, D), F = \{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$
- List all candidate keys of R.
- Is BC a super key?
  - $\{BC\}^+ = ?$

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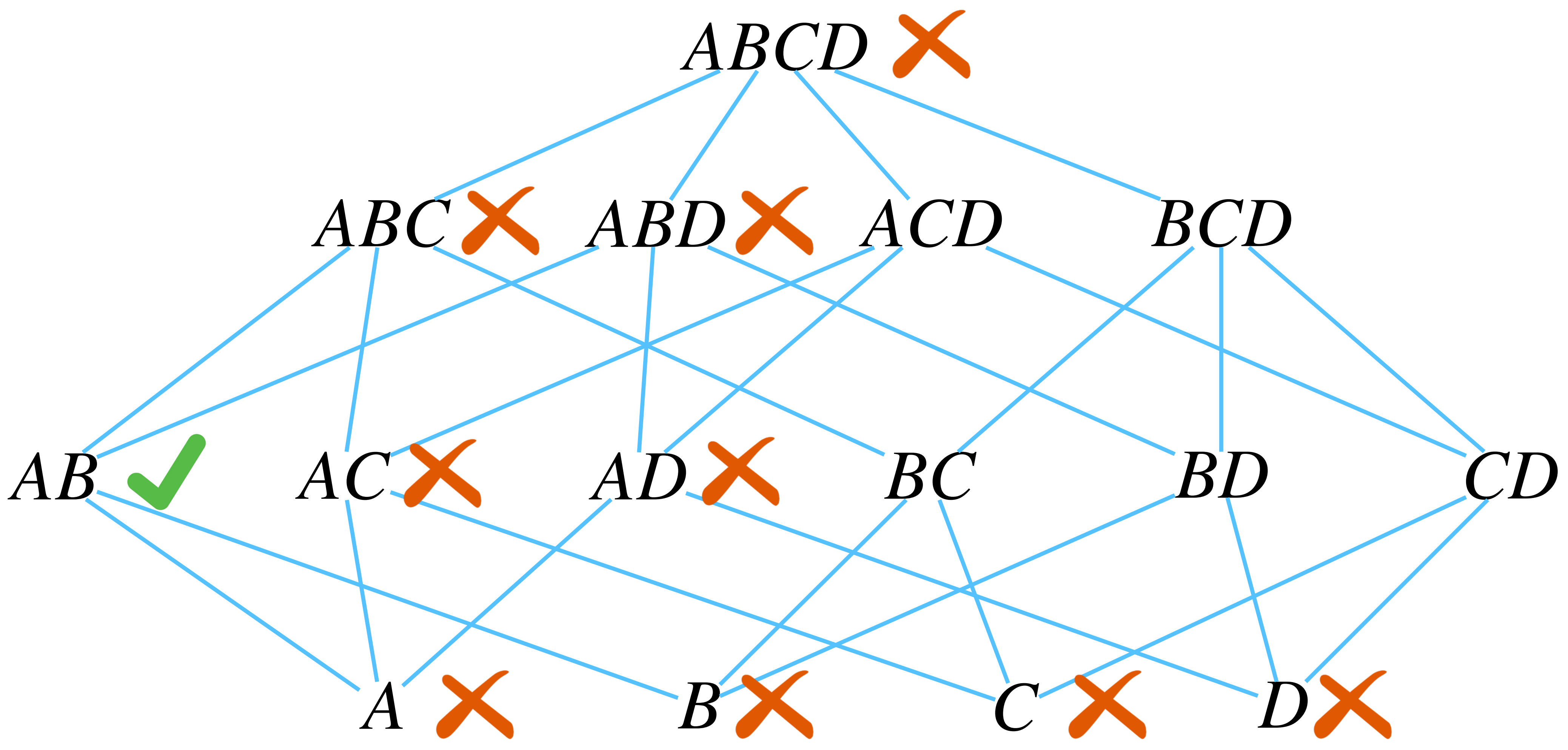

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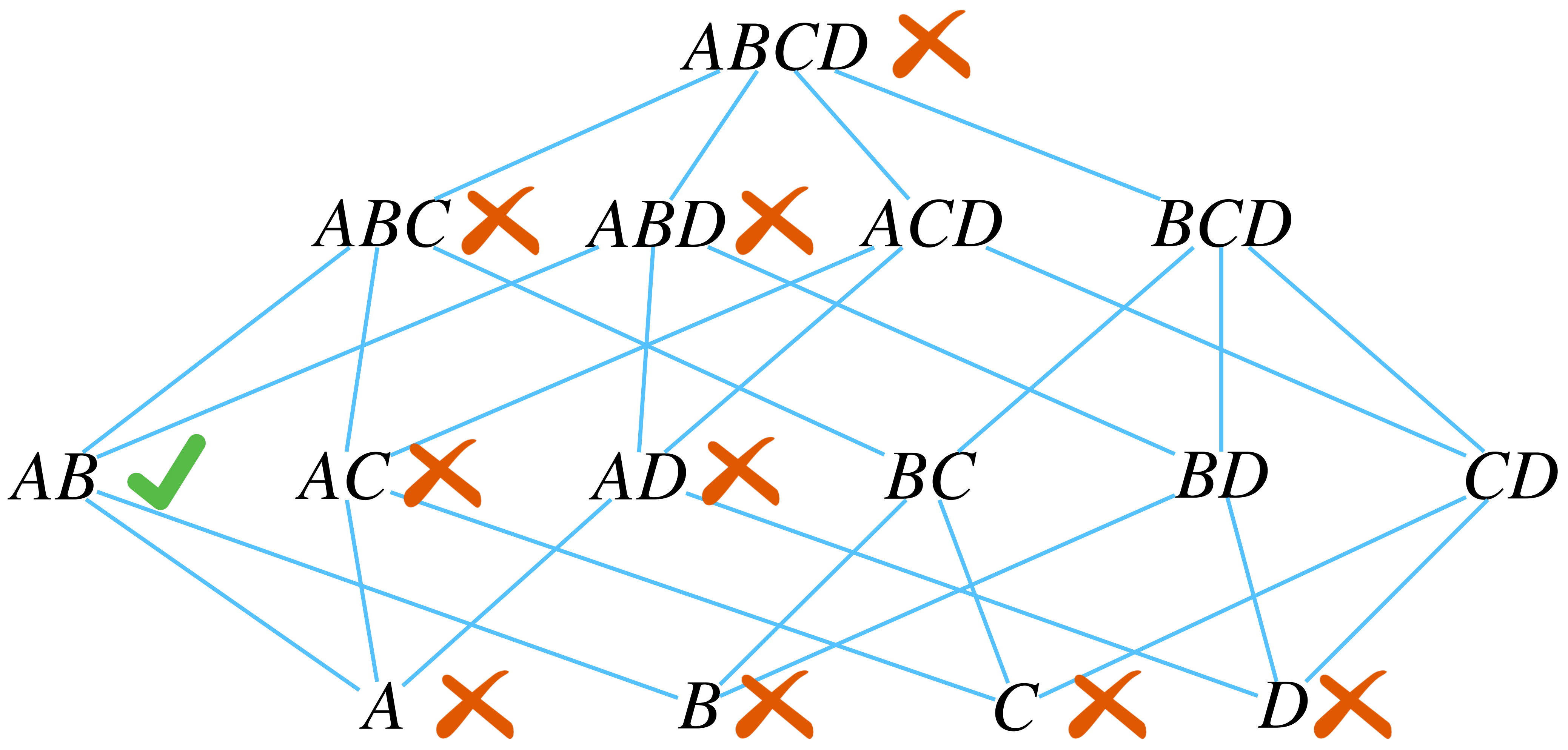

Question 3

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- Is BC a super key?
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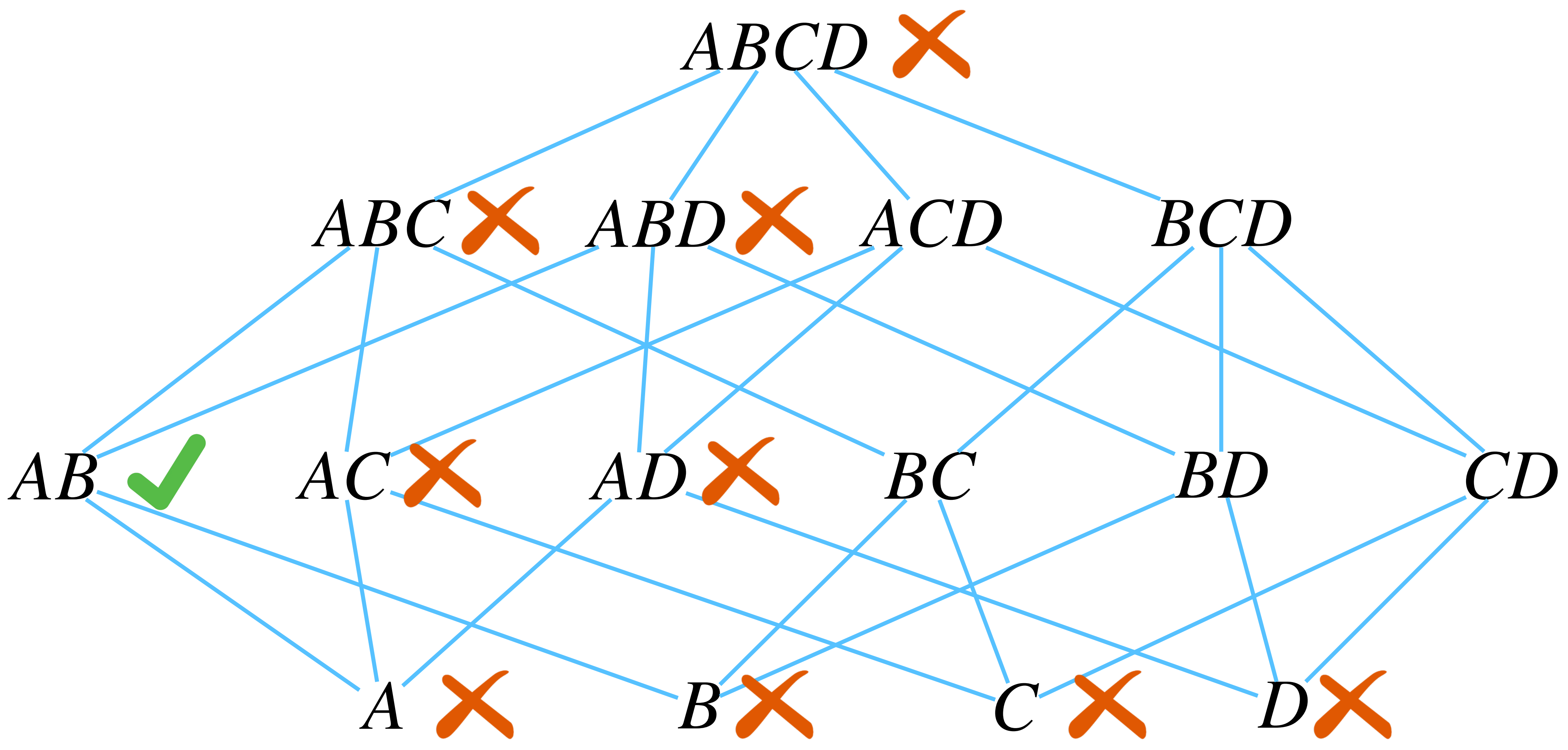

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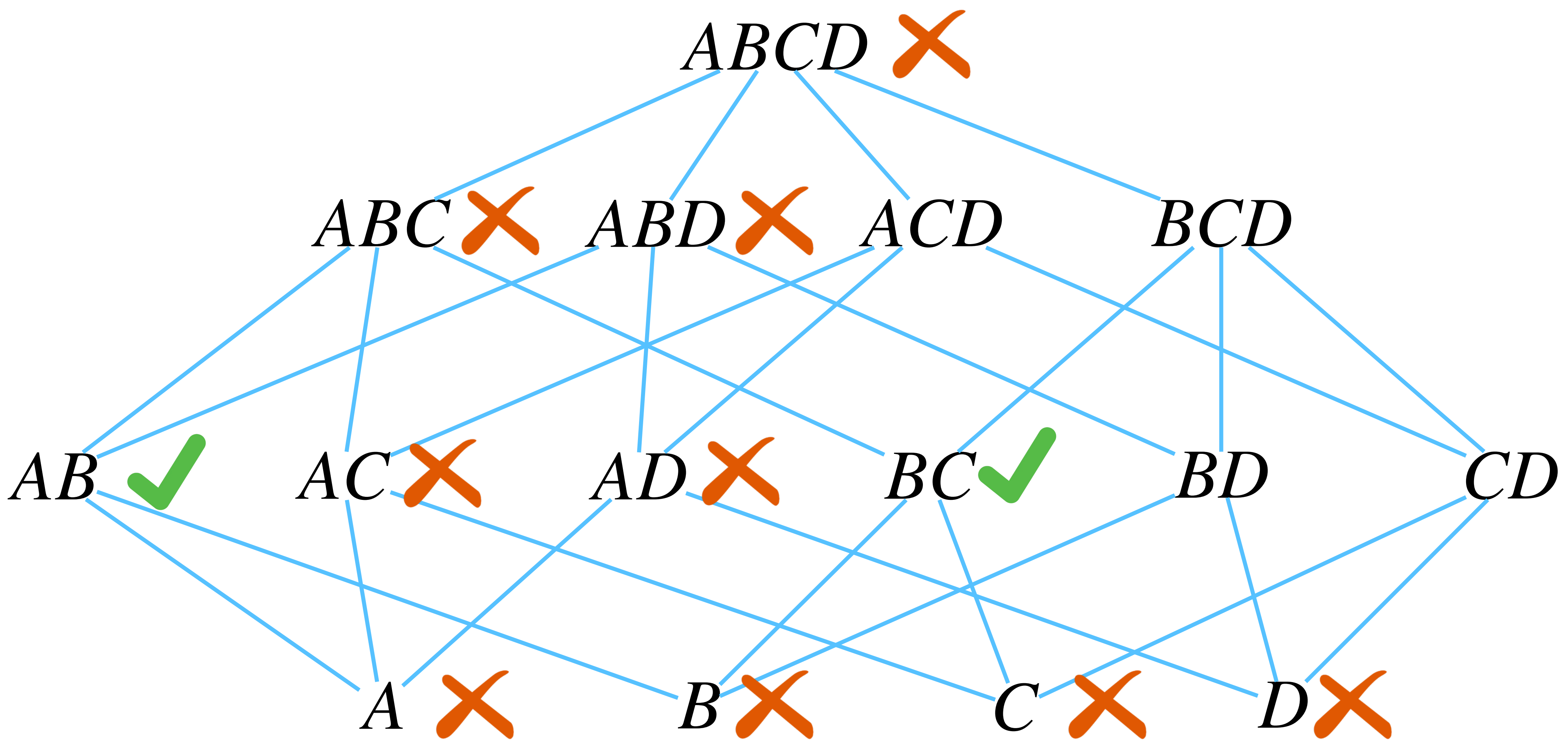

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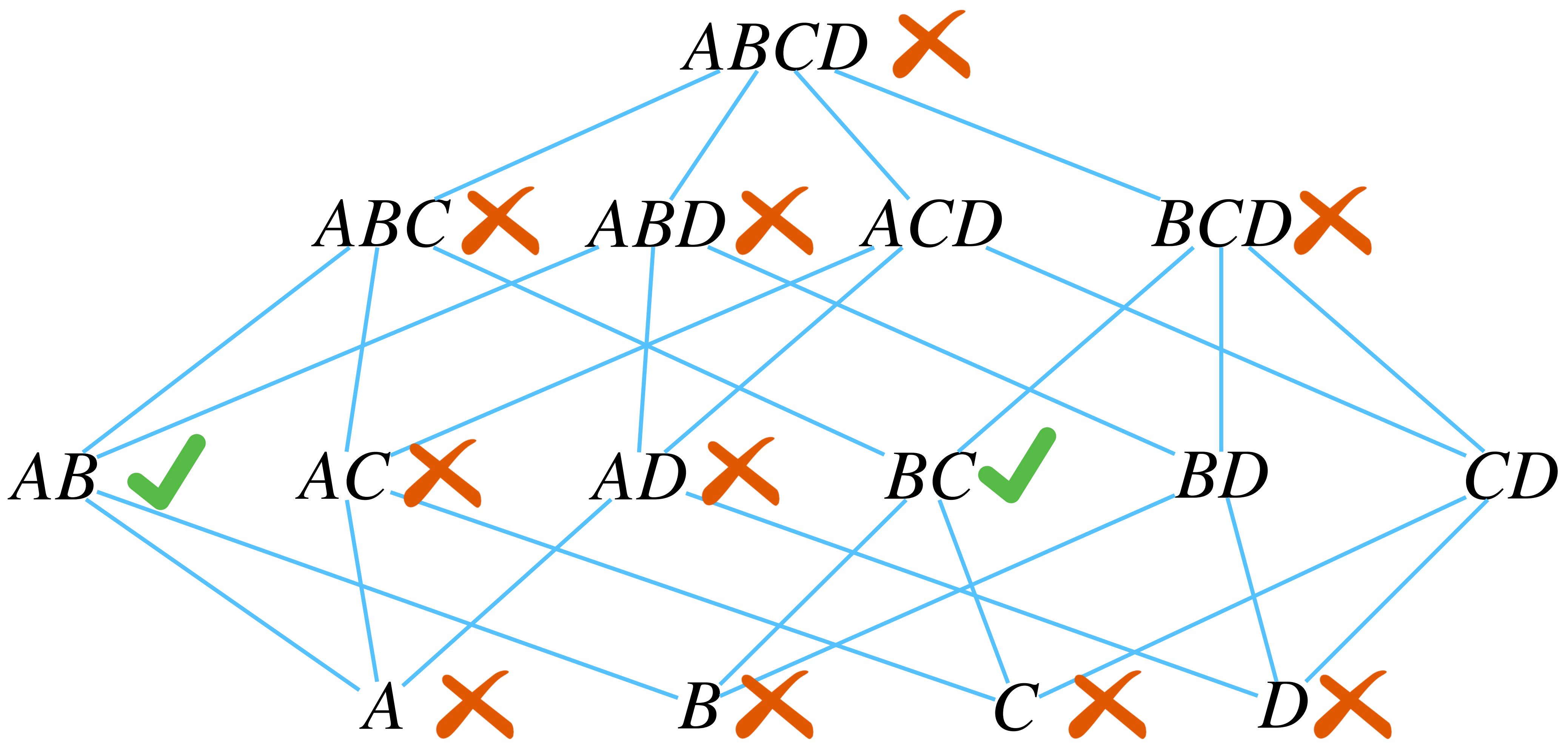

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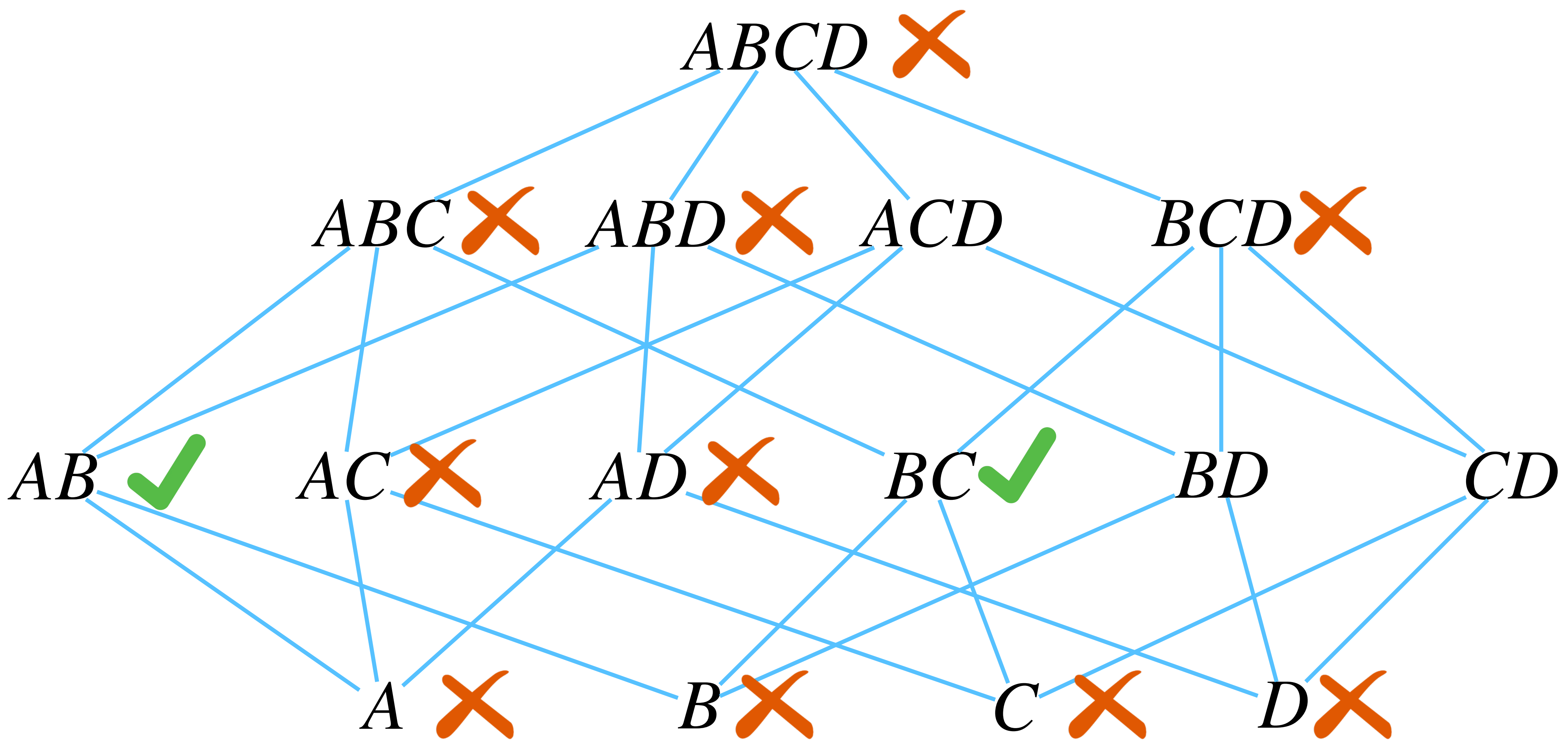
- Is BD a super key?
  - $\{BD\}^+ = ?$



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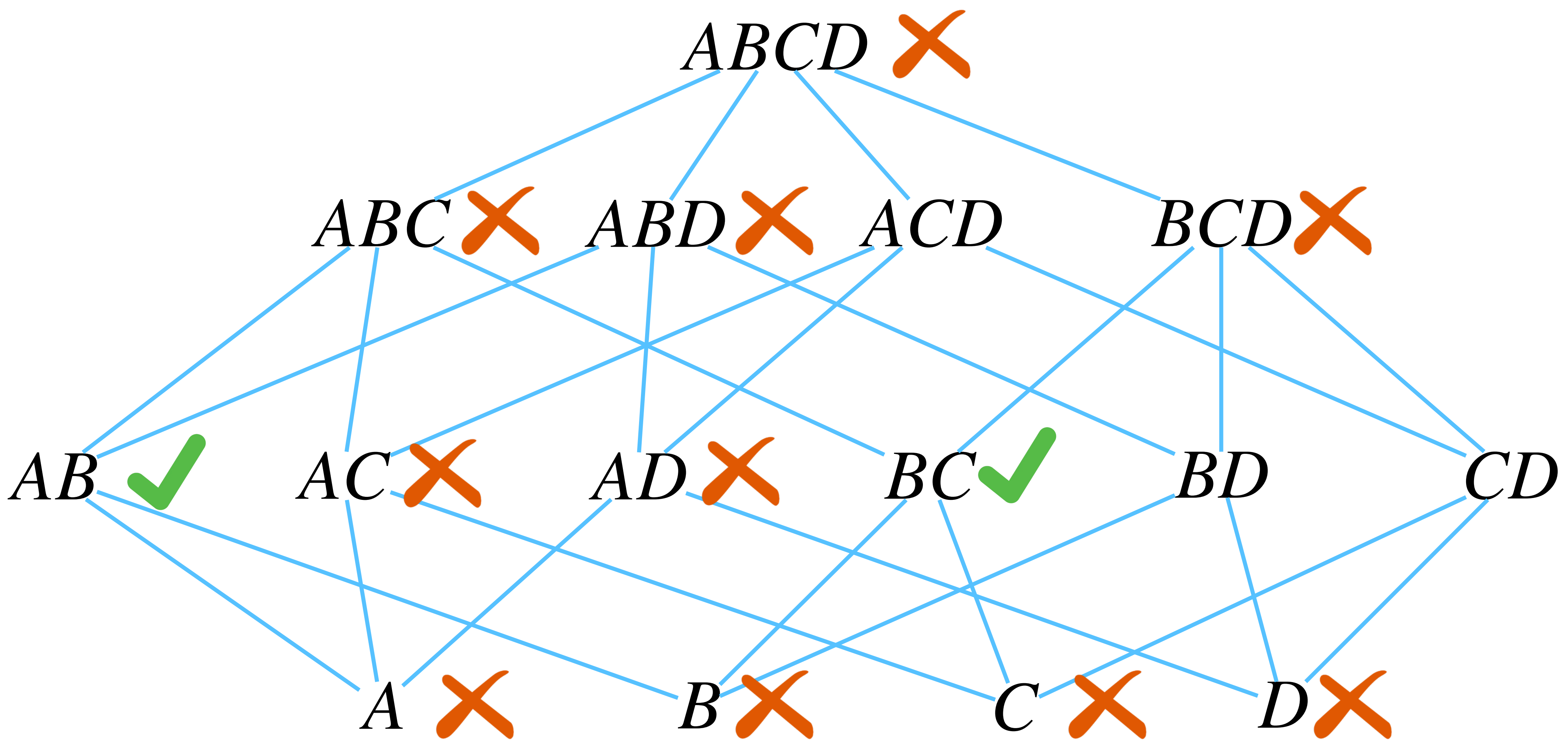

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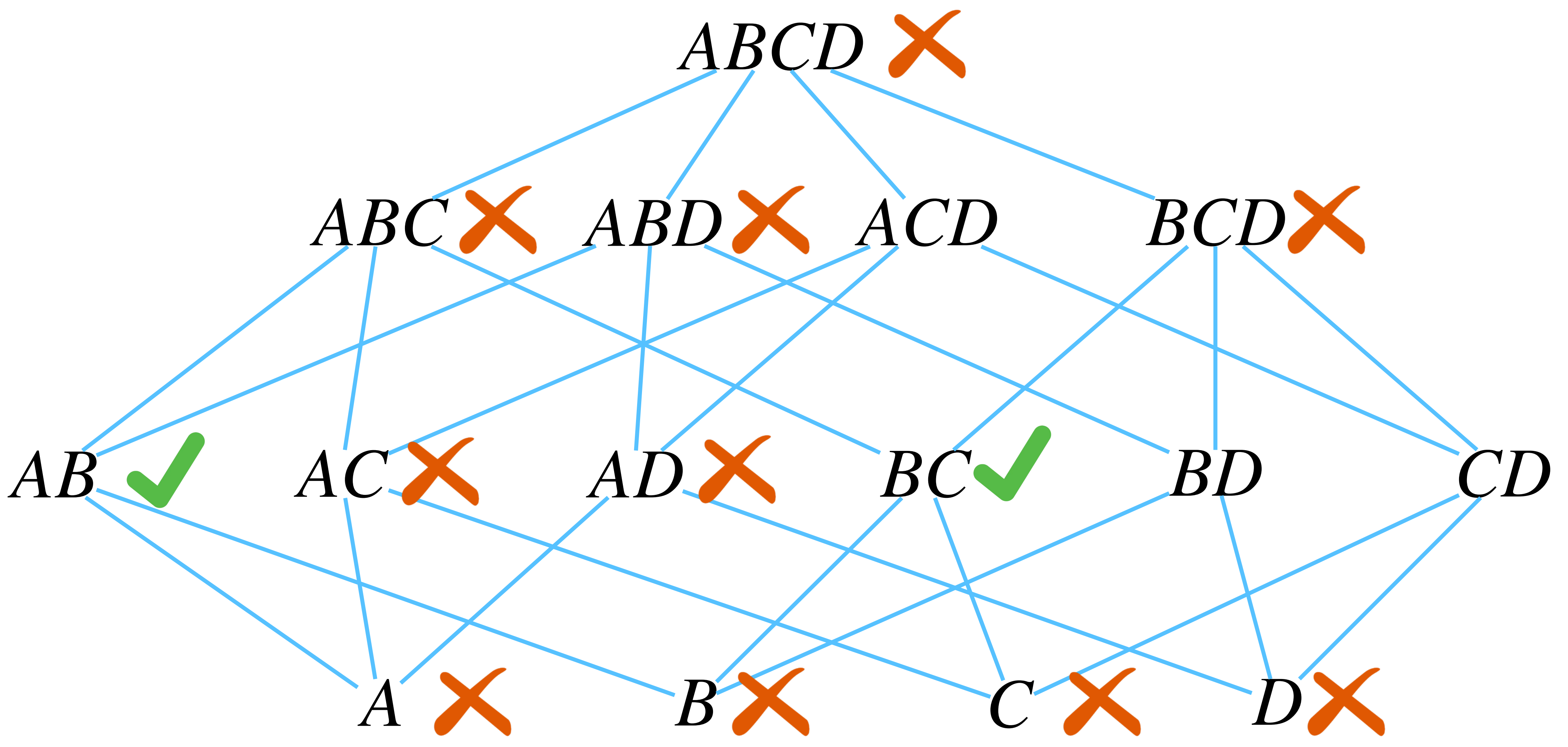

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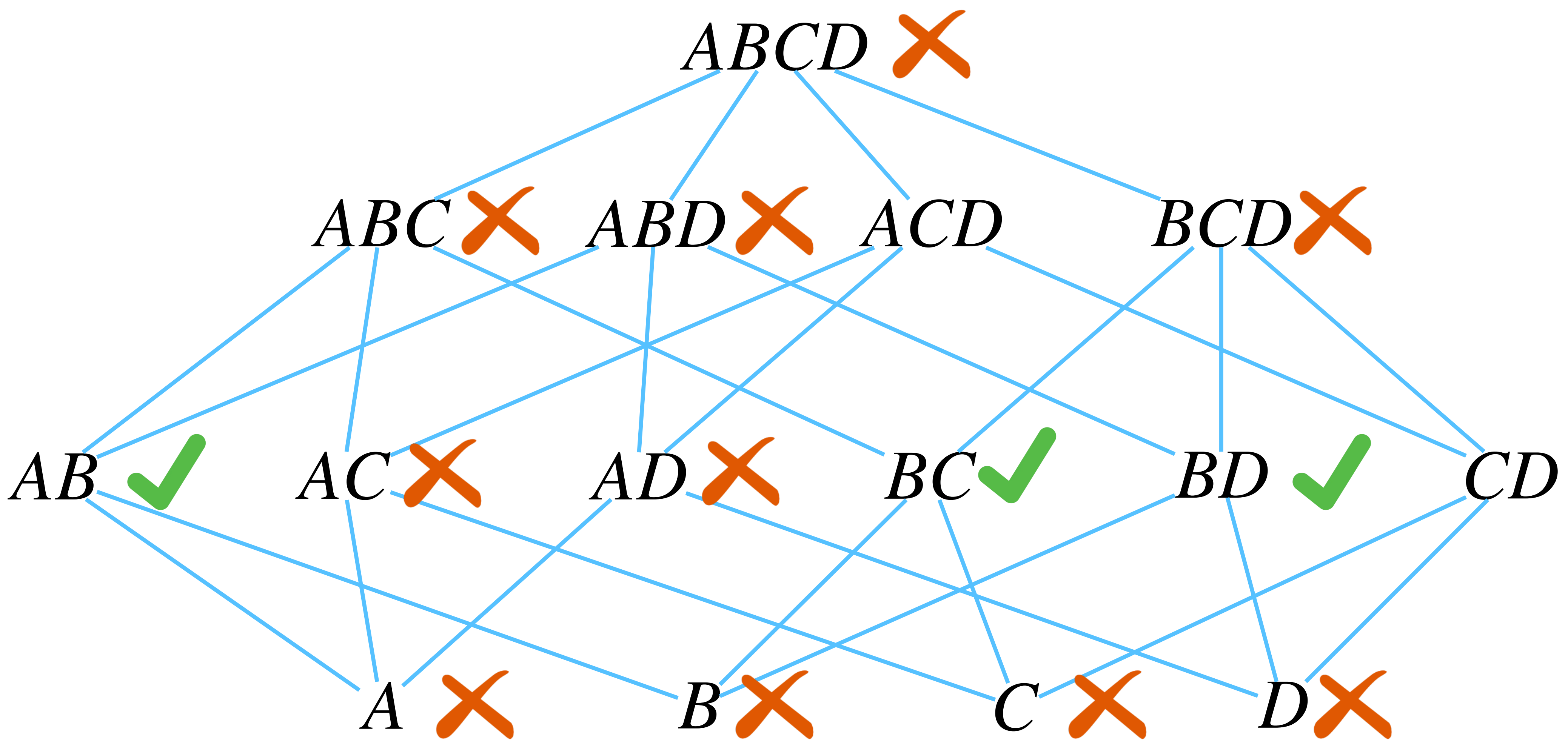

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- Is CD a super key?
  - $\{CD\}^+ = ?$

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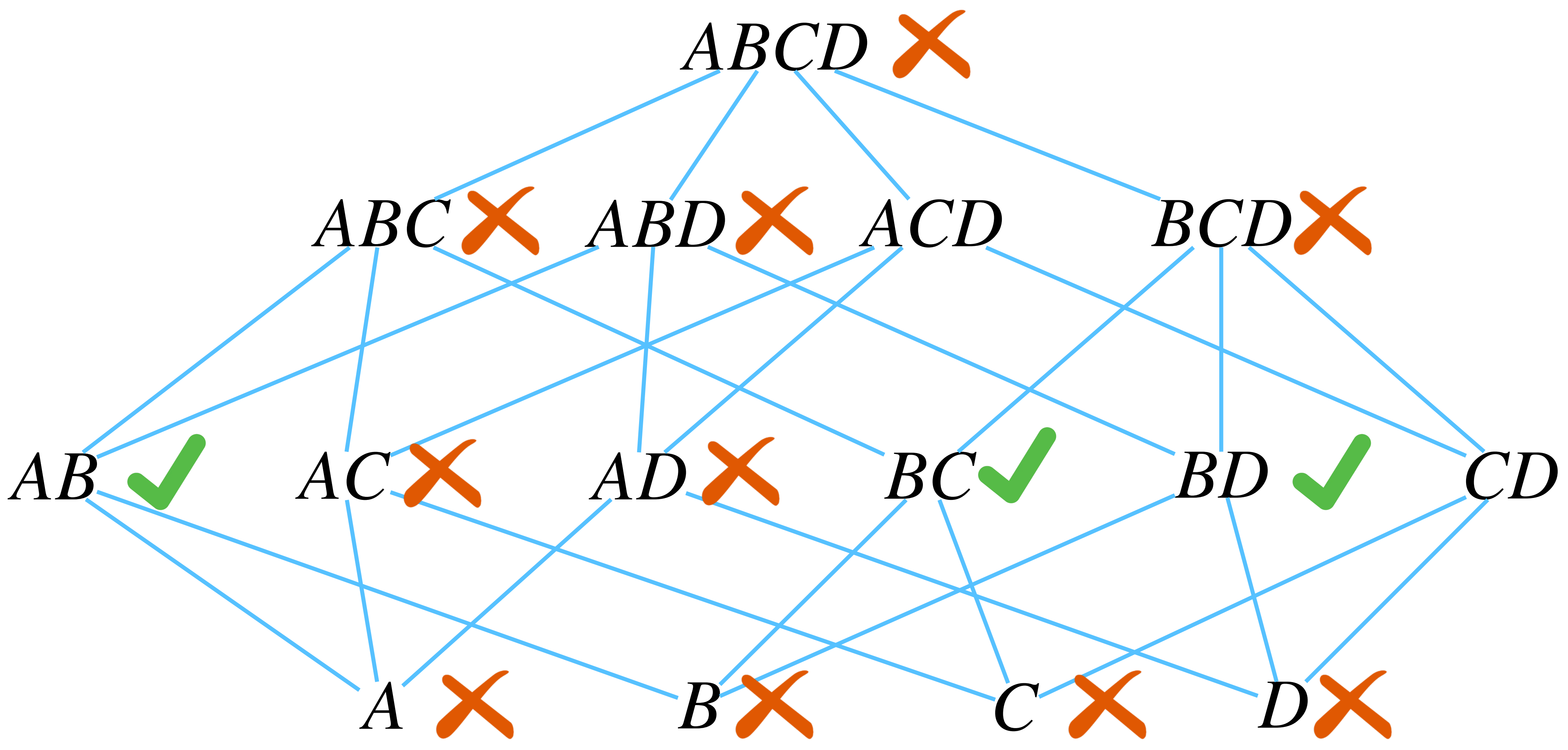

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  - CD is NOT a super key

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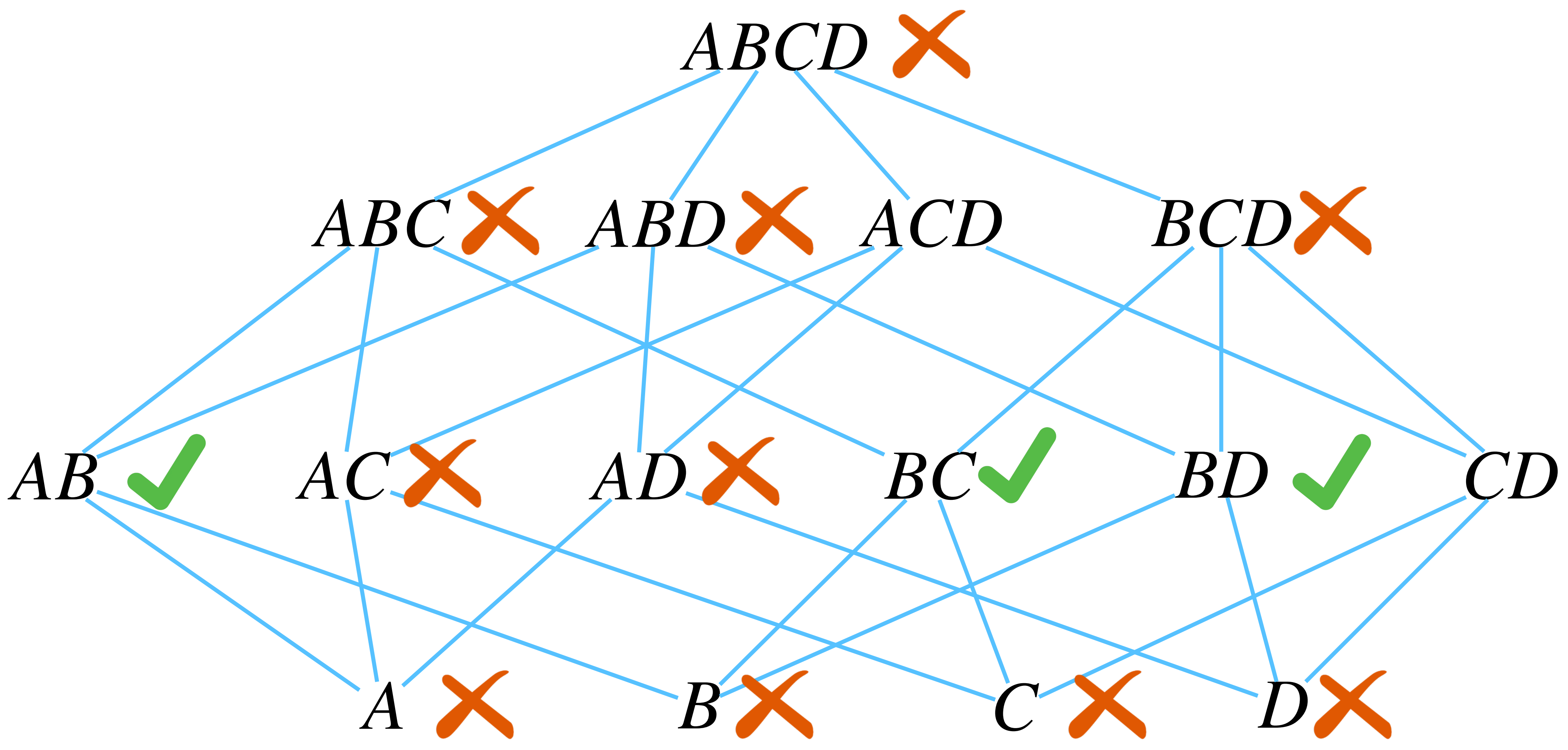

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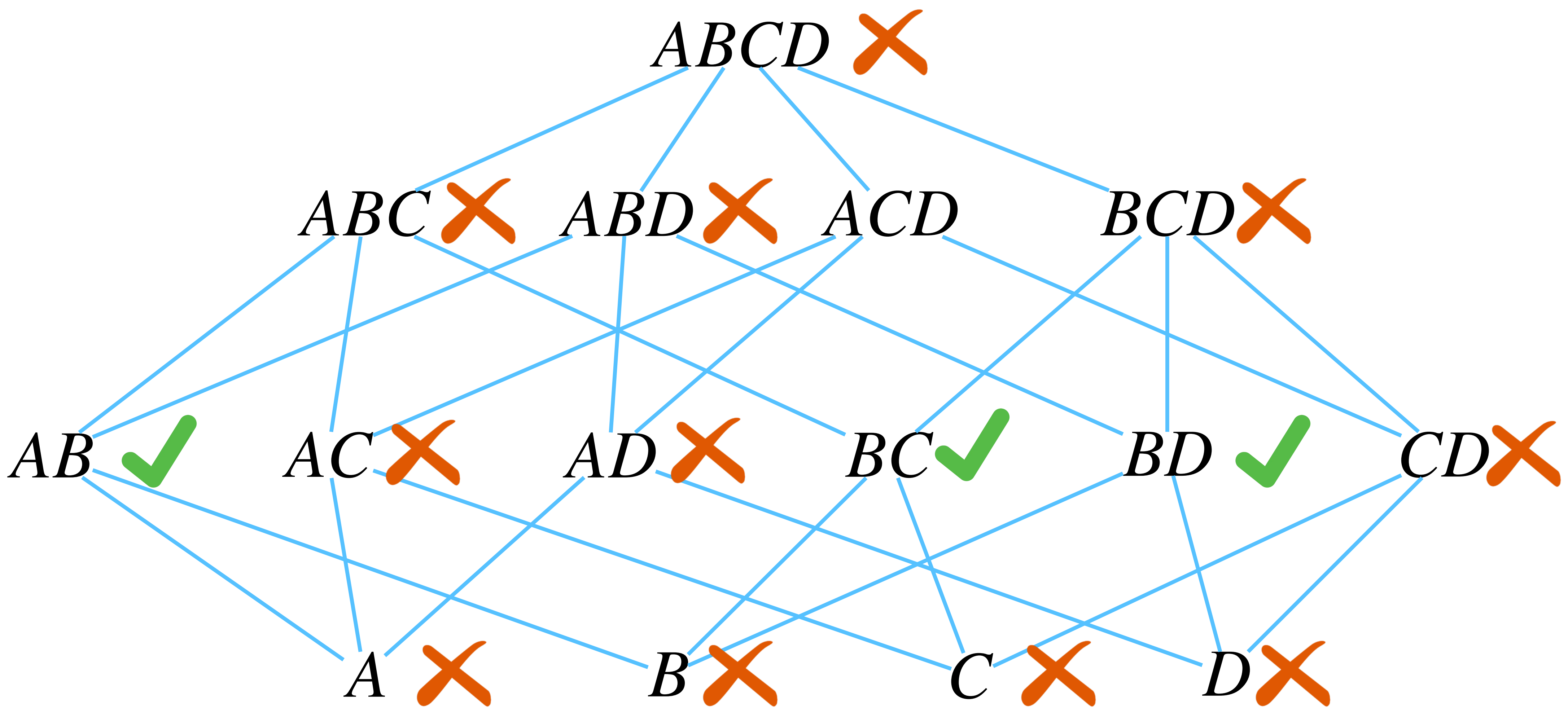

Question 3

- $R = (A, B, C, D), F = \{AB \rightarrow C, C \rightarrow D, D \rightarrow A\}$
- List all candidate keys of R.
- Is ACD a super key?
  - $\{ACD\}^+ = ?$

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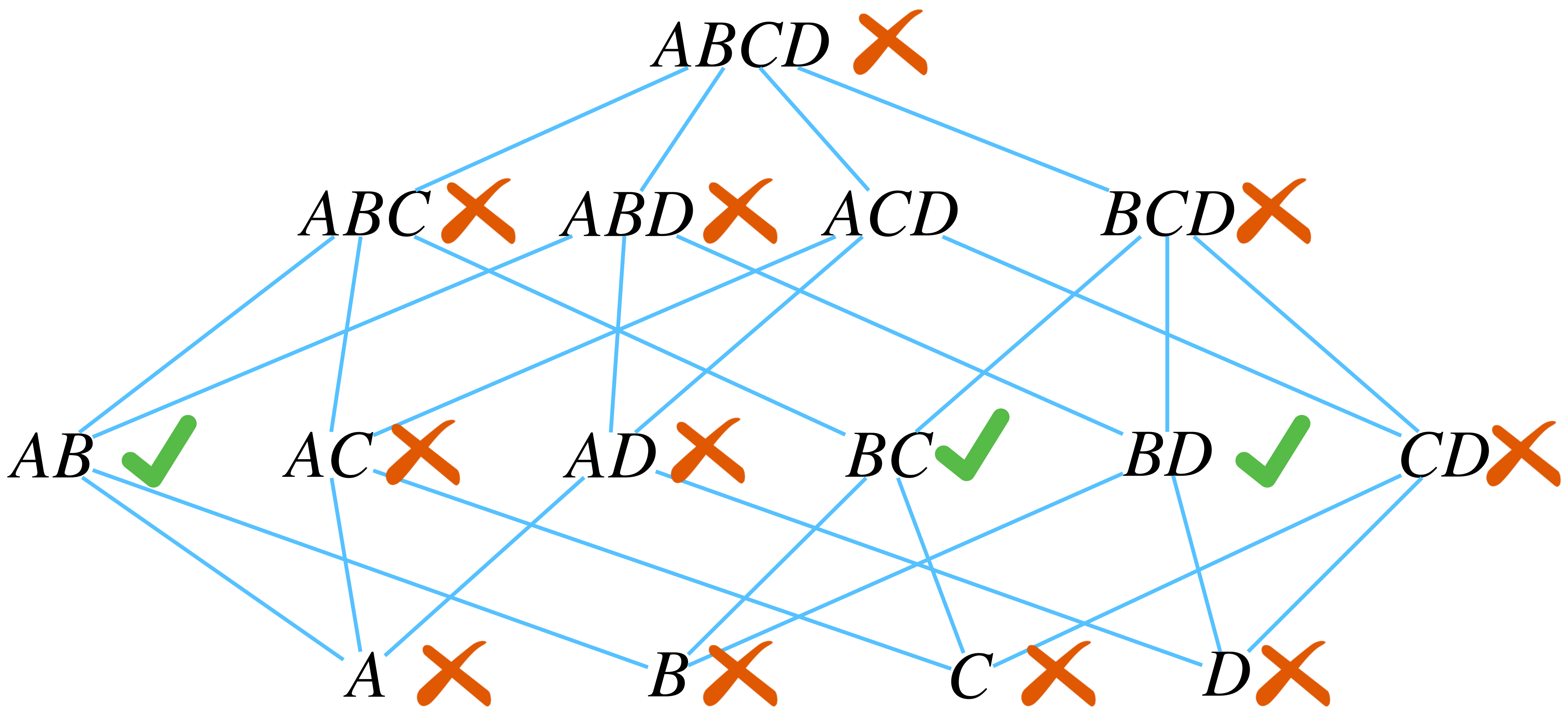

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  - $\{ACD\}^+ = \{A, C, D\} \neq R$
  - ACD s NOT a super key

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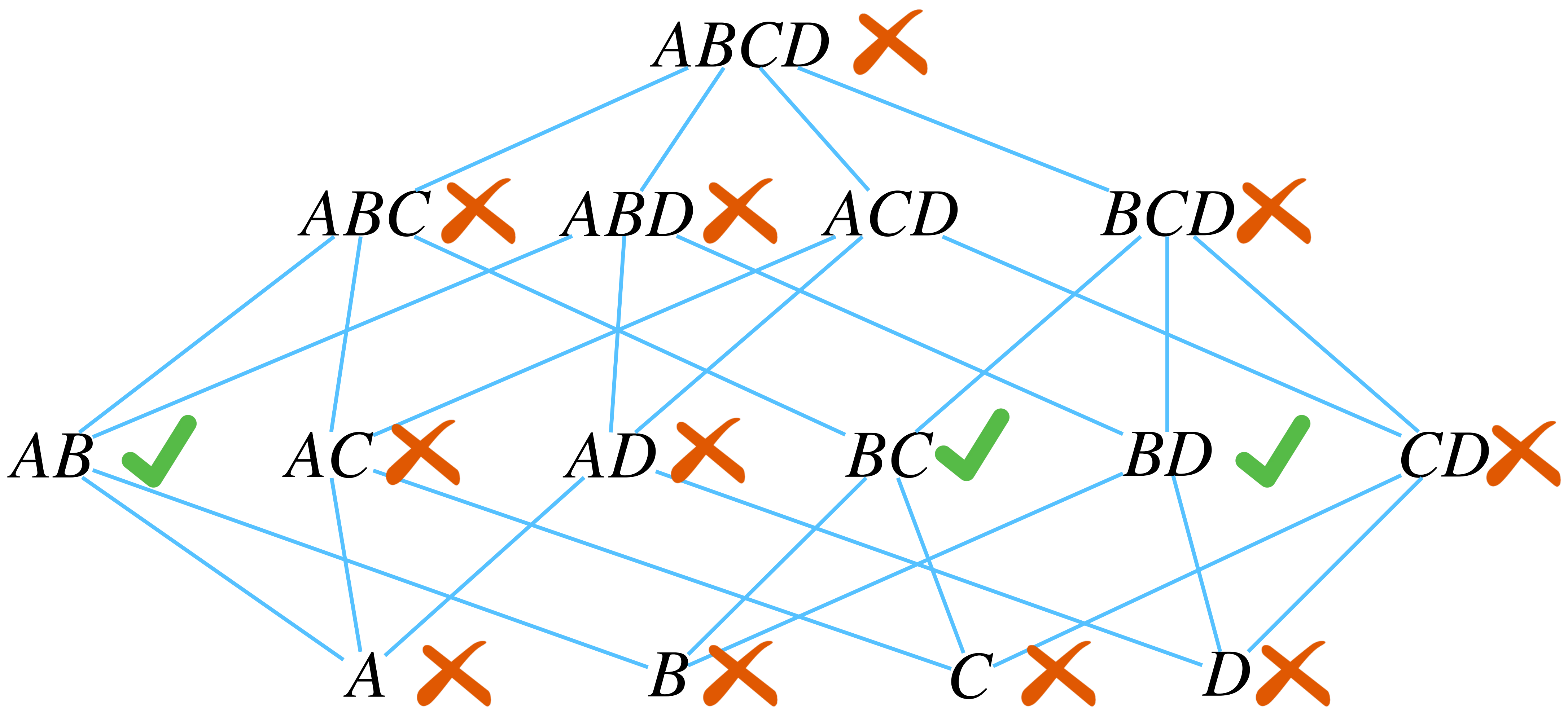

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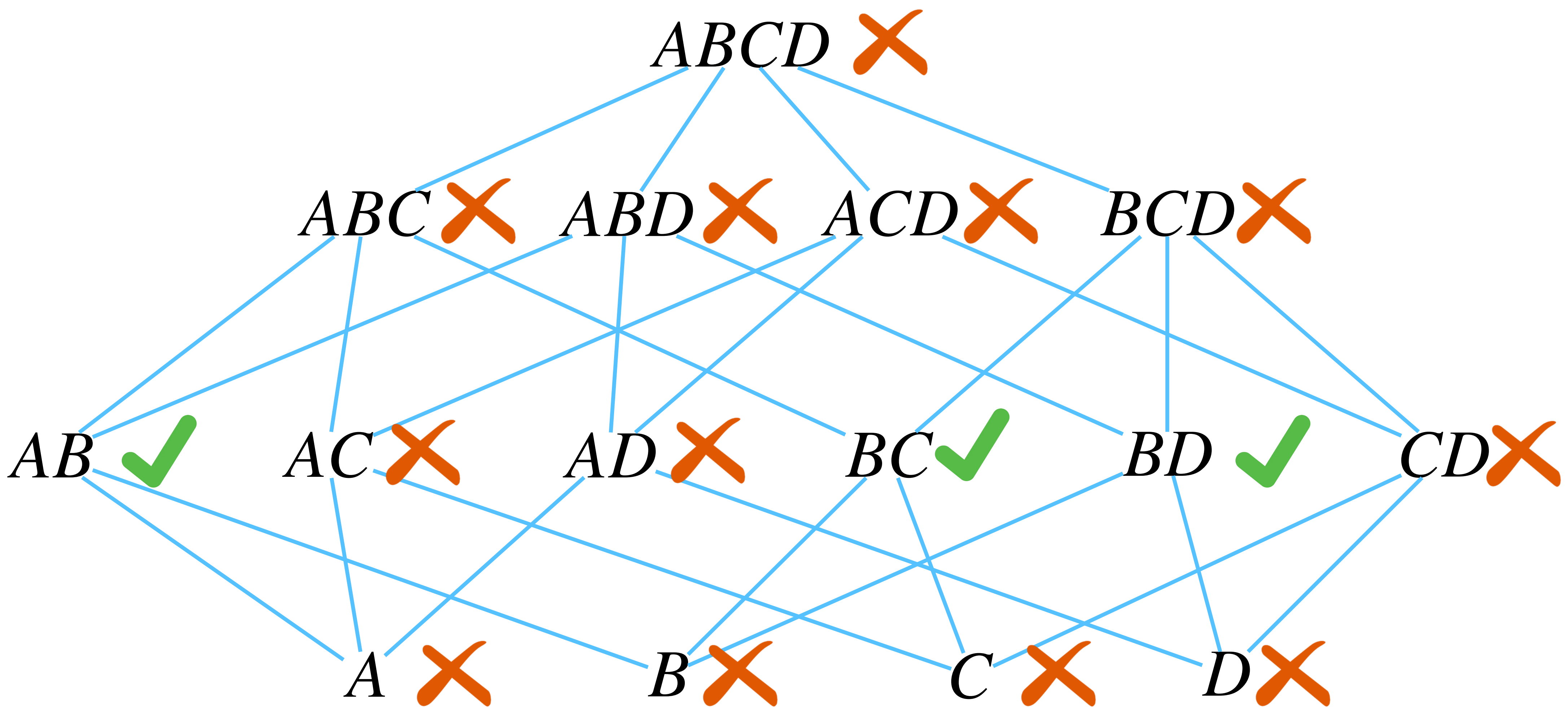

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- Candidate keys of R
  - AB
  - BC
  - BD

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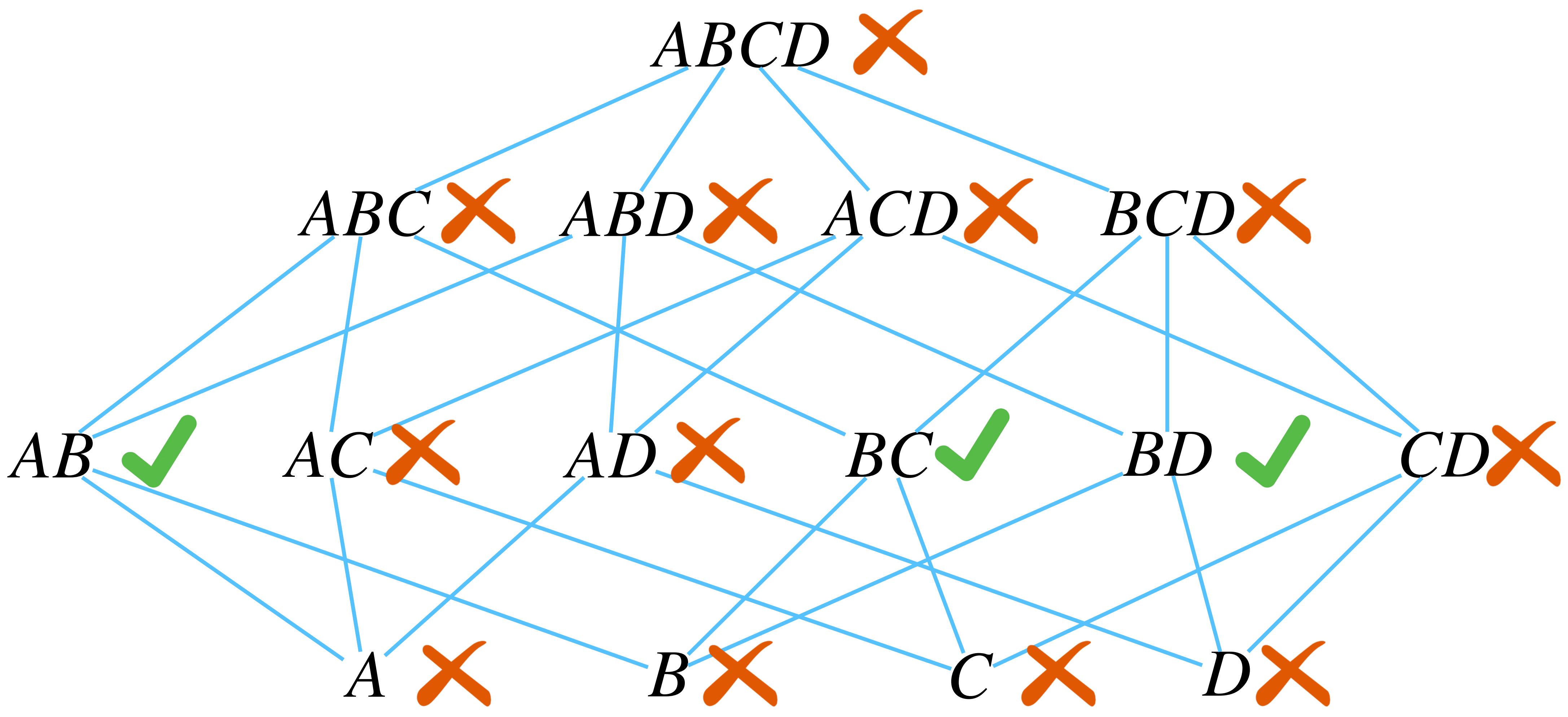

Question 4

- $R = (A, B, C, D), F = \{A \to B, BC \to D\}$
- If R does not satisfy BCNF, perform a BCNF decomposition of R. The decomposition must meet the following requirements:
  - The resulting relations form a lossless-join decomposition.
  - The original functional dependencies are all preserved (dependency preservation).
  - Every decomposed relation is in BCNF.
- If a decomposed relation still violates BCNF, further decompose it until every subrelation is in BCNF.

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
3. Decomposition dependency preserving? - $(F_1 \cup F_2)^+ = F^+$

- $R = (A, B, C, D), F = \{A \rightarrow B, BC \rightarrow D\}$

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- $R = (A, B, C, D), F = \{A \to B, BC \to D\}$
- Is R in BCNF?

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  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \rightarrow B$ violates BCNF

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- Is R in BCNF?
  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \rightarrow B$ violates BCNF
  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF

Question 4

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  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \rightarrow B$ violates BCNF
  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - Let's use $A \rightarrow B$
  - $R_1(A, B)$
  - $R_2(A, C, D)$

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1. R in BCNF? - $\{\alpha\}^+$ covers R
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- Is R in BCNF?
  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \rightarrow B$ violates BCNF
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- Decompose R
  - Let's use $A \rightarrow B$
  - $R_1(A, B)$ — Is R_1 in BCNF?
  - $R_2(A, C, D)$
    - Is R_2 in BCNF?

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- $R = (A, B, C, D), F = \{A \to B, BC \to D\}$
- Is R in BCNF?
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \to D$ violates BCNF
- Decompose R
  - Let's use $A \to B$
  - $R_1(A, B)$
    - Is R_1 in BCNF?
      - $F_1 = \{A \to B, trivials\}$
      - Since $\{A\}^+ = \{A, B\} = R_1$, R_1 is in BCNF
  - $R_2(A, C, D)$
    - Is R_2 in BCNF?
      - $F_2 = \{trivials\}$
      - R_2 is in BCNF

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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \to D$ violates BCNF
- Decompose R
  - Let's use $A \to B$
  - $R_1(A, B)$
  - $R_2(A, C, D)$
    - Is R_2 in BCNF?
      - $F_2 = \{trivials\}$
      - R_2 is in BCNF
- Is R_1 in BCNF?
  - $F_1 = \{A \to B, trivials\}$
  - Since $\{A\}^+ = \{A, B\} = R_1$, R_1 is in BCNF
- Dependency preserving?

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
3. Decomposition dependency preserving? - $(F_1 \cup F_2)^+ = F^+$

- $R = (A, B, C, D), F = \{A \rightarrow B, BC \rightarrow D\}$
- Is R in BCNF?
  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \rightarrow B$ violates BCNF
  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - Let's use $A \rightarrow B$
  - $R_1(A, B)$
    - Is R_1 in BCNF?
      - $F_1 = \{A \rightarrow B, trivials\}$
      - Since $\{A\}^+ = \{A, B\} = R_1$, R_1 is in BCNF
  - $R_2(A, C, D)$
    - Is R_2 in BCNF?
      - $F_2 = \{trivials\}$
      - R_2 is in BCNF
- Dependency preserving?
  - Since $BC \rightarrow D$ disappear, $(F_1 \cup F_2)^+ \neq F^+$, this decomposition is NOT dependency preserving

Question 4

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  - Since $\{A\}^+ = \{A, B\} \neq R$, R is NOT in BCNF, $A \to B$ violates BCNF
  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \to D$ violates BCNF
- Decompose R
  - Let's use $BC \to D$
  - $R_1(B, C, D)$
  - $R_2(B, C, A)$

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - Let's use $BC \rightarrow D$
  - $R_1(B, C, D)$
  - $R_2(B, C, A)$

- Is R_1 in BCNF?
  - $F_1 = \{BC \rightarrow D, trivials\}$
  - Since $\{BC\}^+ = \{B, C, D\} = R_1$, R_1 is in BCNF

- Is R_2 in BCNF?
  - $F_2 = \{A \rightarrow B, trivials\}$
  - Since $\{A\}^+ = \{A, B\} \neq R_2$, R_2 is NOT in BCNF

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - Let's use $BC \rightarrow D$
  - $R_1(B, C, D)$
    - Is R_1 in BCNF?
      - $F_1 = \{BC \rightarrow D, trivials\}$
      - Since $\{BC\}^+ = \{B, C, D\} = R_1$, R_1 is in BCNF
  - $R_2(B, C, A)$
    - Is R_2 in BCNF?
      - $F_2 = \{A \rightarrow B, trivials\}$
      - Since $\{A\}^+ = \{A, B\} \neq R_2$, R_2 is NOT in BCNF
    - Decompose R_2

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
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- Decompose R
  - Let's use $BC \rightarrow D$
  - $R_1(B, C, D)$
  - $R_2(B, C, A)$
- Is R_1 in BCNF?
  - $F_1 = \{BC \rightarrow D, trivials\}$
  - Since $\{BC\}^+ = \{B, C, D\} = R_1$, R_1 is in BCNF
- Is R_2 in BCNF?
  - $F_2 = \{A \rightarrow B, trivials\}$
  - Since $\{A\}^+ = \{A, B\} \neq R_2$, R_2 is NOT in BCNF
- Decompose R_2
  - Let's use $A \rightarrow B$
  - $R_3 = \{A, B\}$
  - $R_4 = \{A, C\}$

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \to D$ violates BCNF
- Decompose R
  - Let's use $BC \to D$
  - $R_1(B, C, D)$
  - $R_2(B, C, A)$

- Is R_1 in BCNF?
  - $F_1 = \{BC \to D, trivials\}$
  - Since $\{BC\}^+ = \{B, C, D\} = R_1$, R_1 is in BCNF

- Is R_2 in BCNF?
  - $F_2 = \{A \to B, trivials\}$
  - Since $\{A\}^+ = \{A, B\} \neq R_2$, R_2 is NOT in BCNF
- Decompose R_2
  - Let's use $A \to B$
  - $R_3 = \{A, B\}$
  - $R_4 = \{A, C\}$

- Is R_3 in BCNF?

- Is R_4 in BCNF?

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - Let's use $BC \rightarrow D$
  - $R_1(B, C, D)$
    - Is R_1 in BCNF?
      - $F_1 = \{BC \rightarrow D, trivials\}$
      - Since $\{BC\}^+ = \{B, C, D\} = R_1$, R_1 is in BCNF
  - $R_2(B, C, A)$
    - Is R_2 in BCNF?
      - $F_2 = \{A \rightarrow B, trivials\}$
      - Since $\{A\}^+ = \{A, B\} \neq R_2$, R_2 is NOT in BCNF
    - Decompose R_2
      - Let's use $A \rightarrow B$
      - $R_3 = \{A, B\}$
        - Is R_3 in BCNF?
          - $F_3 = \{A \rightarrow B, trivials\}$
          - Since $\{A\}^+ = \{A, B\} = R_3$, R_3 is in BCNF
      - $R_4 = \{A, C\}$
        - Is R_4 in BCNF?
          - $F_4 = \{trivials\}$
          - R_4 is in BCNF

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \rightarrow D$ violates BCNF
- Decompose R
  - $R_1(B, C, D)$, $F_1 = \{BC \rightarrow D, trivials\}$
  - $R_3 = \{A, B\}$, $F_3 = \{A \rightarrow B, trivials\}$
  - $R_4 = \{A, C\}$, $F_4 = \{trivials\}$

Question 4

1. R in BCNF? - $\{\alpha\}^+$ covers R
2. BCNF decomposition - $result = (result - R_i) \cup (\alpha\beta) \cup (R_i - \beta)$
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  - Since $\{BC\}^+ = \{B, C, D\} \neq R$, R is NOT in BCNF, $BC \to D$ violates BCNF
- Decompose R
  - $R_1(B, C, D)$, $F_1 = \{BC \to D, trivials\}$
  - $R_3 = \{A, B\}, F_3 = \{A \to B, trivials\}$
  - $R_4 = \{A, C\}, F_4 = \{trivials\}$
- Dependency preserving?
  - Since $F_1 = \{BC \to D\}, F_3 = \{A \to B\}, F_1 \cup F_3 = F$
  - Since $F_1 \cup F_3 = F$, $F_4 = \{trivials\}, (F_1 \cup F_3 \cup F_4)^+ = F^+$
  - This decomposition is dependency preserving

Tutorial 5

END

COMP3278C
Introduction to Database Management Systems

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