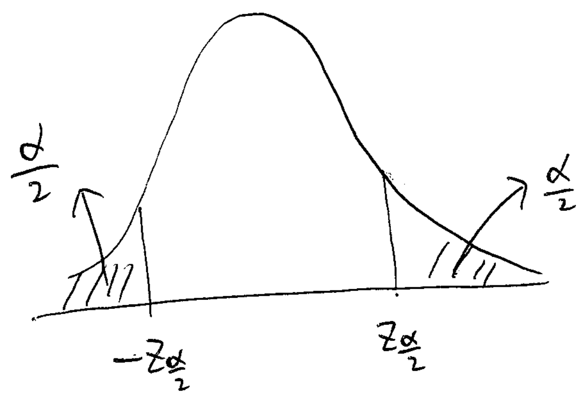
☆ One-sample Case — Mean.

$\underline{X} = \{x_1, \cdots x_n\} \overset{iid}{\sim} N(\mu, \sigma^2)$.

$\boxed{\sigma^2 \text{ is known}}$. $(\sigma^2 = \sigma_0^2)$.

$\bar{x} \sim N(\mu, \frac{\sigma^2}{n})$.

$Z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} \sim N(0, 1)$.



$1-\alpha$ CI of μ is $\left[\bar{x} \pm z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}\right]$.

Tese.	H_0	H_1	Rejection Region.	p-value.
Two-tailed	$\mu = \mu_0$	$\mu \neq \mu_0$	$\left\{ \left\lvert \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \right\rvert \geq z_{\frac{\alpha}{2}} \right\}$	$P\left(\lvert z \rvert \geq \frac{\lvert \bar{x}-\mu_0 \rvert}{\sigma_0/\sqrt{n}} \right)$
left-tailed	$\begin{cases} \mu = \mu_0 \\ \mu \geq \mu_0 \end{cases}$	$\mu < \mu_0$	$\left\{ \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \leq -z_\alpha \right\}$	$P\left(Z \leq \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \right)$
Right-tailed	$\begin{cases} \mu = \mu_0 \\ \mu \leq \mu_0 \end{cases}$	$\mu > \mu_0$	$\left\{ \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \geq z_\alpha \right\}$	$P\left(Z \geq \frac{\bar{x}-\mu_0}{\sigma_0/\sqrt{n}} \right)$

☆ One-sample Case. — Mean.

$\underset{\sim}{X} = \{X_1, \cdots X_n\} \overset{iid}{\sim} N(\mu, \sigma^2)$

$\boxed{\sigma^2 \text{ unknown}}$ $\left\{ \frac{nS^2}{\sigma^2} = \frac{\Sigma(x_i - \bar{x})^2}{\sigma^2} \sim \chi^2_{n-1} \right.$.

$$\frac{\bar{x} - \mu}{S/\sqrt{n-1}} \sim t_{n-1}$$

$1-\alpha$ CI of μ is $\left[\bar{x} \pm t_{\frac{\alpha}{2}, n-1} \frac{S}{\sqrt{n-1}} \right]$

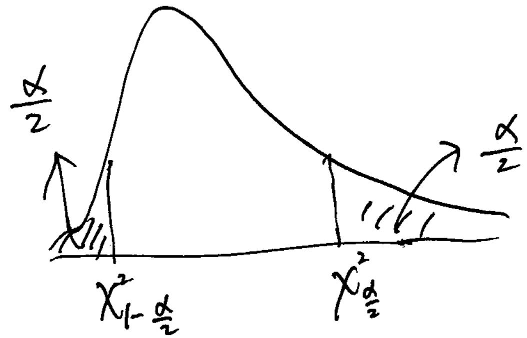
Test	H_0	H_1	Rejection	p-value.
Two-tailed	$\mu = \mu_0$	$\mu \neq \mu_0$	$\left\{ \left\| \frac{\|\bar{x} - \mu_0\|}{S/\sqrt{n-1}} \right\| \geq t_{\frac{\alpha}{2}, n-1} \right\}$	$p\left(\|t_{n-1}\| \geq \left\| \frac{\bar{x} - \mu_0}{S/\sqrt{n-1}} \right\| \right)$
Left-tailed	$\left\{ \begin{array}{l} \mu = \mu_0 \\ \mu \geq \mu_0 \end{array} \right.$	$\mu < \mu_0$	$\left\{ \frac{\bar{x} - \mu_0}{S/\sqrt{n-1}} \leq -t_{\alpha, n-1} \right\}$	$p\left(t_{n-1} \leq \frac{\bar{x} - \mu_0}{S/\sqrt{n-1}} \right)$
Right-tailed	$\left\{ \begin{array}{l} \mu = \mu_0 \\ \mu \leq \mu_0 \end{array} \right.$	$\mu > \mu_0$	$\left\{ \frac{\bar{x} - \mu_0}{S/\sqrt{n-1}} \geq t_{\alpha, n-1} \right\}$	$p\left(t_{n-1} \geq \frac{\bar{x} - \mu_0}{S/\sqrt{n-1}} \right)$.

☆ One - sample Case - Variance.

$\underset{\sim}{X} = \{X_1, \cdots X_n\} \overset{iid}{\sim} N(\mu, \sigma^2)$

$\frac{ns^2}{\sigma^2} \sim \chi^2_{n-1}$.

$1-\alpha$ CI of σ^2 = $\left[\frac{ns^2}{\chi^2_{\frac{\alpha}{2}, n-1}}, \frac{ns^2}{\chi^2_{1-\frac{\alpha}{2}, n-1}}\right]$



Test.	H_0	H_1	Rejection Region
Two-tailed	$\sigma = \sigma_0$	$\sigma \neq \sigma_0$	$\left\{\frac{ns^2}{\sigma_0^2} \leq \chi^2_{1-\frac{\alpha}{2}, n-1}\right\} \cup \left\{\frac{ns^2}{\sigma_0^2} \geq \chi^2_{\frac{\alpha}{2}, n-1}\right\}$
Left-tailed	$\begin{cases} \sigma = \sigma_0 \\ \sigma \geq \sigma_0 \end{cases}$	$\sigma < \sigma_0$	$\left\{\frac{ns^2}{\sigma_0^2} \leq \chi^2_{1-\alpha, n-1}\right\}$
Right-tailed	$\begin{cases} \sigma = \sigma_0 \\ \sigma \leq \sigma_0 \end{cases}$	$\sigma > \sigma_0$	$\left\{\frac{ns^2}{\sigma_0^2} \geq \chi^2_{\alpha, n-1}\right\}$

☆ Two-Sample Case. (Variance known)

$\{X_i\ ;\ i=1,2,\cdots n_1\} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$\{Y_j\ ;\ j=1,2,\cdots n_2\} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$.

σ_1^2, σ_2^2 are known.

$\mu_1 - \mu_2$

$$\begin{cases} \bar{X} \sim N(\mu_1, \frac{\sigma_1^2}{n_1}) \\ \bar{Y} \sim N(\mu_2, \frac{\sigma_2^2}{n_2}) \end{cases}$$

$$\bar{X} - \bar{Y} \sim N(\mu_1 - \mu_2, \frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}).$$

$1-\alpha$ CI of $\mu_1 - \mu_2$ is

$\cancel{[\bar{X} - \bar{Y} \pm}$

$$\frac{(\bar{X} - \bar{Y}) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \sim N(0, 1).$$

$$\left[\bar{X} - \bar{Y} \pm Z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

Tese.	H_0	H_1	Rejection Region.
Two-tailed	$\mu_1 - \mu_2 = \delta$.	$\mu_1 - \mu_2 \neq \delta$	$\left\{\frac{\lvert\bar{X} - \bar{Y} - \delta\rvert}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \geq Z_{\frac{\alpha}{2}}\right\}$.
Left-tailed	$\begin{cases} \mu_1 - \mu_2 = \delta. \\ \mu_1 - \mu_2 \geq \delta. \end{cases}$	$\mu_1 - \mu_2 < \delta$	$\left\{\frac{[\bar{X} - \bar{Y} - \delta]}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \leq -Z_\alpha\right\}$.
Right-tailed	$\begin{cases} \mu_1 - \mu_2 = \delta \\ \mu_1 - \mu_2 \leq \delta \end{cases}$	$\mu_1 - \mu_2 > \delta$	$\left\{\frac{\bar{X} - \bar{Y} - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} \geq Z_\alpha\right\}$.

★ Two-sample Case (Variance unknown).

$\{X_i;\ i=1,2,\cdots n_1\} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$\{Y_j;\ j=1,2,\cdots n_2\} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$.

$\sigma_1^2 = \sigma_2^2 = \sigma^2$ unknown

$$\frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \sim N(0,1).$$

$$\frac{n_1 S_1^2 + n_2 S_2^2}{\sigma^2} \sim \chi^2_{n_1+n_2-2}.$$

$$W = \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sigma\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \Bigg/ \sqrt{\frac{n_1 S_1^2+n_2 S_2^2}{\sigma^2} \Big/ (n_1+n_2-2)}$$

$$= \frac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\ \sqrt{\frac{n_1S_1^2+n_2S_2^2}{n_1+n_2-2}}}$$

$W \sim t_{n_1+n_2-2}$.

$$\begin{cases} W = \dfrac{(\bar{X}-\bar{Y})-(\mu_1-\mu_2)}{\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\cdot S_p} \\ S_p^2 = \dfrac{n_1S_1^2+n_2S_2^2}{n_1+n_2-2} = \dfrac{\sum_{i=1}^{n_1}(X_i-\bar{X})^2+\sum_{j=1}^{n_2}(Y_j-\bar{Y})^2}{n_1+n_2-2}. \end{cases}$$

$1-\alpha$ CI of $\mu_1-\mu_2$ is

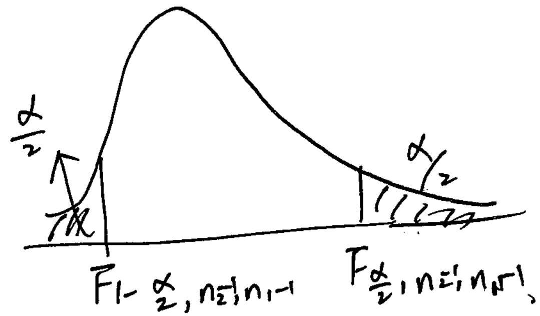
$$\left[\bar{X}-\bar{Y} \pm t_{\frac{\alpha}{2}, n_1+n_2-2}\cdot\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}\cdot S_p\right]$$

Test	H_0	H_1	Rejection Region
Two-tailed	$\mu_1-\mu_2=\delta$	$\mu_1-\mu_2\neq\delta$	$\left\{\dfrac{\lvert\bar{X}-\bar{Y}-\delta\rvert}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \geq t_{\frac{\alpha}{2}, n_1+n_2-2}\right\}$
Left-tailed	$\begin{cases}\mu_1-\mu_2=\delta\\ \mu_1-\mu_2\geq\delta\end{cases}$	$\mu_1-\mu_2<\delta$	$\left\{\dfrac{\bar{X}-\bar{Y}-\delta}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \leq -t_{\alpha, n_1+n_2-2}\right\}$
Right-tailed	$\begin{cases}\mu_1-\mu_2=\delta\\ \mu_1-\mu_2\leq\delta\end{cases}$	$\mu_1-\mu_2>\delta$	$\left\{\dfrac{\bar{X}-\bar{Y}-\delta}{S_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}} \geq t_{\alpha, n_1+n_2-2}\right\}$.

☆ Two Sample Case – Variance.

$\{X_i ;\ i=1,2,\cdots,n_1\} \overset{iid}{\sim} N(\mu_1, \sigma_1^2)$

$\{Y_j ;\ j=1,2,\cdots,n_2\} \overset{iid}{\sim} N(\mu_2, \sigma_2^2)$.



$$\begin{cases} \dfrac{n_1 S_1^2}{\sigma_1^2} \sim \chi^2_{n_1-1} \\ \dfrac{n_2 S_2^2}{\sigma_2^2} \sim \chi^2_{n_2-1}. \end{cases} \qquad \left[\frac{n_2 S_2^2}{\sigma_2^2 (n_2-1)}\right] \Big/ \left[\frac{n_1 S_1^2}{\sigma_1^2 (n_1-1)}\right] \sim F_{n_2-1, n_1-1}.$$

$1-\alpha$ CI of $\dfrac{\sigma_1^2}{\sigma_2^2}$ is $\left[\dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_2^2} F_{1-\frac{\alpha}{2}, n_2-1, n_1-1} ,\ \dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_2^2} \cdot F_{\frac{\alpha}{2}, n_2-1, n_1-1} \right]$

Test	H_0	H_1	Rejection Region
Two-tailed	$\sigma_1 = \sigma_2$	$\sigma_1 \neq \sigma_2$	$\left\{\dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_2^2} \geq F_{\frac{\alpha}{2}, n_1-1, n_2-1}\right\} \cup \left\{\dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_2^2} \leq F_{1-\frac{\alpha}{2}, n_1-1, n_2-1}\right\}$
Left-tailed	$\begin{cases}\sigma_1 = \sigma_2 \\ \sigma_1 \geq \sigma_2\end{cases}$	$\sigma_1 < \sigma_2$	$\left\{\dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_2^2} \leq F_{1-\alpha, n_1-1, n_2-1}\right\}$
Right-tailed	$\begin{cases}\sigma_1 = \sigma_2 \\ \sigma_1 \leq \sigma_2\end{cases}$	$\sigma_1 > \sigma_2$	$\left\{\dfrac{n_1(n_2-1) S_1^2}{n_2(n_1-1) S_1^2} \geq F_{\alpha, n_1-1, n_2-1}\right\}$.

$$H_0: \sigma_1 = \sigma_2 \quad \text{v.s.} \quad H_1: \sigma_1 \neq \sigma_2 .$$

Solution. $\Lambda = \dfrac{L(\Omega_0)}{L(\Omega)}$.

$$\Omega_0 = \left\{ (\mu_1, \mu_2, \sigma_1, \sigma_2) : \sigma_1 > 0 \overset{\boxed{\sigma_1 = \sigma_2}}{\wedge} \sigma_2 > 0 ,\ \mu_1 \in \mathbb{R} ,\ \mu_2 \in \mathbb{R} \right\}$$

$$\Omega = \left\{ (\mu_1, \mu_2, \sigma_1, \sigma_2) : \sigma_1 > 0 ,\ \sigma_2 > 0 ,\ \mu_1 \in \mathbb{R} ,\ \mu_2 \in \mathbb{R} \right\}$$

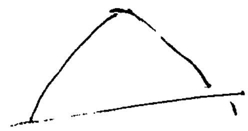
Likelihood: $\theta = (\mu_1, \mu_2, \sigma_1, \sigma_2)$

$$☆\ L(\mu_1, \mu_2, \sigma_1, \sigma_2) = \left(\frac{1}{\sqrt{2\pi\sigma_1^2}} \right)^{n_1} \exp\left\{ - \frac{\sum_{i=1}^{n_1} (X_i - \mu_1)^2}{2\sigma_1^2} \right\} \left(\frac{1}{\sqrt{2\pi\sigma_2^2}} \right)^{n_2} \exp\left\{ - \frac{\sum_{j=1}^{n_2} (Y_j - \mu_2)^2}{2\sigma_2^2} \right\}$$

$$l(\theta) = C - n_1 \log \sigma_1 - n_2 \log \sigma_2 - \frac{\sum (X_i - \mu_1)^2}{2\sigma_1^2} - \frac{\sum (Y_j - \mu_2)^2}{2\sigma_2^2} .$$

On Ω_0,

$$\begin{cases} \dfrac{\partial l}{\partial \sigma} = - \dfrac{n_1 + n_2}{\sigma} + \dfrac{1}{\sigma^3} \left\{ \sum (X_i - \mu_1)^2 + \sum (Y_j - \mu_2)^2 \right\} = 0 . \\[2ex] \dfrac{\partial l}{\partial \mu_1} = - \dfrac{\sum (X_i - \mu_1)}{2\sigma^2} (-2) = \dfrac{\sum (X_i - \mu_1)}{\sigma^2} = 0 \\[2ex] \dfrac{\partial l}{\partial \mu_2} = \dfrac{\sum (Y_j - \mu_2)}{\sigma^2} = 0 . \end{cases}$$



$$\Rightarrow \begin{cases} \hat{\mu}_1 = \bar{X} \\ \hat{\mu}_2 = \bar{Y} \\ \hat{\sigma}^2 = \dfrac{1}{n_1 + n_2} \left\{ \sum (X_i - \overset{\bar{X}}{\cancel{\hat{\mu}_1}})^2 + \sum (Y_j - \overset{\bar{Y}}{\hat{\mu}_2})^2 \right\} \quad \Omega_0 \end{cases}$$

$$= \frac{1}{n_1 + n_2} \left(n_1 S_1^2 + n_2 S_2^2 \right)$$

On Ω, $\begin{cases} \frac{\partial l}{\partial \sigma_1} = -\frac{n_1}{\sigma_1} + \frac{1}{\sigma_1^3} \sum (X_i - \mu_1)^2 \\ \frac{\partial l}{\partial \sigma_2} = -\frac{n_2}{\sigma_2} + \frac{1}{\sigma_2^3} \sum (Y_j - \mu_2)^2 \\ \frac{\partial l}{\partial \mu_1} = \sim . \\ \frac{\partial l}{\partial \mu_2} = \sim . \end{cases} \Rightarrow \begin{cases} \hat{\mu}_1 = \bar{X} \\ \hat{\mu}_2 = \bar{Y} \\ \hat{\sigma}_1^2 = \frac{1}{n_1} \sum (X_i - \bar{X})^2 \\ \hat{\sigma}_2^2 = \frac{1}{n_2} \sum (Y_j - \bar{Y})^2 \end{cases}$

$$\begin{cases} \hat{\sigma}_1^2 = S_1^2 \\ \hat{\sigma}_2^2 = S_2^2 \end{cases}$$

$$\Lambda = \frac{L(\Omega_0)}{L(\Omega)} =$$

$$L(\Omega_0) = C \cdot (\hat{\sigma}^2)^{-\frac{n_1+n_2}{2}} \exp\left\{ -\frac{1}{2\hat{\sigma}^2} \left[\sum (X_i - \bar{X})^2 + \sum (Y_j - \bar{Y})^2 \right] \right\}$$

$$= C \cdot (\hat{\sigma}^2)^{-\frac{n_1+n_2}{2}} \exp\left\{ -\frac{n_1+n_2}{2} \right\}$$

$$L(\Omega) = C \cdot (\hat{\sigma}_1^2)^{-\frac{n_1}{2}} \cdot (\hat{\sigma}_2^2)^{-\frac{n_2}{2}} \exp\left(-\frac{n_1+n_2}{2} \right)$$

$$\Lambda = \frac{L(\Omega_0)}{L(\Omega)} = \frac{(\hat{\sigma}^2)^{-\frac{n_1+n_2}{2}}}{(\hat{\sigma}_1^2)^{-\frac{n_1}{2}} (\hat{\sigma}_2^2)^{-\frac{n_2}{2}}} = \left(\frac{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2}}{S_1^2} \right)^{-\frac{n_1}{2}} \left(\frac{\frac{n_1 S_1^2 + n_2 S_2^2}{n_1+n_2}}{S_2^2} \right)^{-\frac{n_2}{2}}$$

$$= C \cdot \left(\frac{\frac{n_2 S_2^2}{S_1^2} + n_1}{n_1+n_2} \right)^{-\frac{n_1}{2}} \left(\frac{n_1 \frac{S_1^2}{S_2^2} + n_2}{n_1+n_2} \right)^{-\frac{n_2}{2}}$$

$$= C^* \cdot \left(\frac{n_2 S_2^2}{S_1^2} + n_1 \right)^{-\frac{n_1}{2}} \left(n_1 \frac{S_1^2}{S_2^2} + n_2 \right)^{-\frac{n_2}{2}}$$

$$= C^* \cdot \left(n_1 \frac{S_1^2}{S_2^2} + n_2 \right)^{-\frac{n_1}{2}} \left(\frac{S_1^2}{S_2^2} \right)^{+\frac{n_1}{2}} \left(n_1 \frac{S_1^2}{S_2^2} + n_2 \right)^{-\frac{n_2}{2}}$$

$$= \left(n_1 \frac{S_1^2}{S_2^2} + n_2 \right)^{-\frac{n_1+n_2}{2}} \cdot \left(\frac{S_1^2}{S_2^2} \right)^{\frac{n_1}{2}}$$

$$\Lambda = \frac{L(\Omega_0)}{L(\Omega)} = \frac{\left(\frac{S_1^2}{S_2^2}\right)^{\frac{n_1}{2}}}{\left(n_1 \frac{S_1^2}{S_2^2} + n_2\right)^{\frac{n_1+n_2}{2}}}$$

$$G(w) = \frac{w^{\frac{n_1}{2}}}{(n_1 w + n_2)^{\frac{n_1+n_2}{2}}}$$

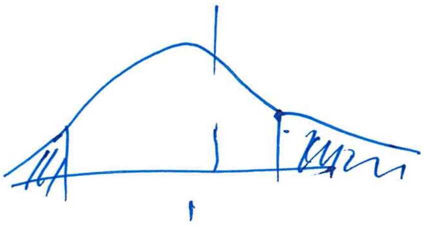
$$\ln G(w) = \frac{n_1}{2} \lg w - \frac{n_1+n_2}{2} \lg(n_1 w + n_2)$$

$$\frac{\partial l}{\partial w} = \frac{n_1}{2}\frac{1}{w} - \frac{n_1+n_2}{2}\frac{n_1}{n_1 w + n_2} = 0$$

$$n_1(n_1 w + n_2) - (n_1+n_2) n_1 w = 0$$

$$\cancel{n_1^2 w} + n_1 n_2 - \cancel{n_1^2 w} - n_1 n_2 w = 0$$

$$w = 1$$



$\frac{S_1^2}{S_2^2}$ large or small.

$$\frac{n_1 S_1^2/(n_1-1)}{n_2 S_2^2/(n_2-1)} \sim F_{n_1-1,\ n_2-1}$$

5.4. proof. $1-\alpha = P(F_{1-\alpha,m,n} \le X)$

$F_{1-\alpha,m,n}$

$$1-\alpha = P\left(\frac{1}{F_{1-\alpha,m,n}} \geq \frac{1}{X}\right)$$

$$\alpha = P\left(\frac{1}{F_{1-\alpha,m,n}} \le \frac{1}{X}\right) \sim F_{n,m}$$

$$\frac{1}{F_{1-\alpha,m,n}} = F_{\alpha,n,m}.$$

☆ $H_0: p_i = p_{i,0} \quad i=1,2,\cdots m.$

$H_1: p_i \neq p_{i,0}$ for at least one $i=1,2,\cdots,m.$

$$P(Y_i = y_i) = \frac{n!}{y_1!\cdots y_m!} p_1^{y_1} p_2^{y_2} \cdots p_m^{y_m}.$$

$$L(\Omega_0) = \frac{n!}{y_1!\cdots y_m!} p_{1,0}^{y_1} p_{2,0}^{y_2} \cdots p_{m,0}^{y_m}.$$

MLE of $p_i (i=1,2,\cdots m)$ should be

$$\hat{p}_i = \frac{y_i}{n}.$$

So $$L(\Omega) = \frac{n!}{y_1!\cdots y_m!} \left(\frac{y_1}{n}\right)^{y_1} \left(\frac{y_2}{n}\right)^{y_2} \cdots \left(\frac{y_m}{n}\right)^{y_m}.$$

$$\Lambda = \frac{L(\Omega_0)}{L(\Omega)} = \prod_{i=1}^{m} \left(\frac{p_{i,0}}{y_i} n\right)^{y_i}$$

$$\begin{cases} O_i = Y_i \\ E_i = n \cdot p_{i,0} \end{cases} \qquad \Lambda = \prod_{i=1}^{m} \left(\frac{E_i}{O_i}\right)^{O_i}.$$

$$-2\log\Lambda = -2 \cdot \sum_{i=1}^{m} O_i \log\left(\frac{E_i}{O_i}\right)$$

$$= 2 \sum_{i=1}^{m} O_i \log\left(\frac{O_i}{E_i}\right) \sim \chi^2_{m-1}.$$

On Ω, $\dim(\Omega) = m-1.$

On Ω_0, $\dim(\Omega_0) = 0.$

$$df = m-1.$$

$\boxed{\text{Approximation}}$ Taylor Expansion.

$$f(x) = x\log\left(\frac{x}{x_0}\right) = x\log x - x\log x_0.$$

$$= (x-x_0) + \frac{1}{2}(x-x_0)^2 \frac{1}{x_0} + \cdots$$

$$f'(x) = 1 + \log x - \log x_0$$

$$f''(x) = \frac{1}{x}$$

$$\begin{cases} f(x_0) = 0. \\ f'(x_0) = 1 \\ f''(x_0) = \frac{1}{x_0} \end{cases}$$

So $$-2\log\Lambda \approx \sum_{i=1}^{m} 2\left\{(O_i - E_i) + \frac{1}{2}(O_i - E_i)^2 \cdot \frac{1}{E_i} + \cdots\right\}$$

$$\approx \sum_{i=1}^{m} \frac{(O_i - E_i)^2}{E_i}$$

$\left(\sum_{i=1}^{m} (O_i - E_i) = 0.\ \text{Always!} \right)$