Assignment 4

Course: STAT2602 **Due: 10 Dec 2024 (11:59pm)**

Q1. Let $X_1, X_2, \cdots, X_n$ and $Y_1, Y_2, \cdots, Y_m$ be random samples from distributions $N(\theta_1, \theta_3)$ and $N(\theta_2, \theta_4)$, respectively. Assume that $X_1, X_2, \cdots, X_n, Y_1, Y_2, \cdots, Y_m$ are independent. Find the generalized likelihood ratio test statistic for hypotheses:

$$H_0 : \theta_1 = \theta_2 \text{ and } \theta_3 = \theta_4 \quad \text{versus} \quad H_1 : \theta_1 \neq \theta_2 \text{ or } \theta_3 \neq \theta_4.$$

Q2. Let X be $N(\mu, 100)$.

(i) To test $H_0 : \mu = 230$ versus $H_1 : \mu > 230$, what is the rejection region specified by the generalized likelihood ratio test?

(ii) If a random sample of $n = 16$ yielded $\overline{x} = 232.6$, is H_0 accepted at a significance level of $\alpha = 0.10$? What is the p-value?

Q3. Let $X_1, X_2, \cdots, X_n$ be an independent random sample from $N(\mu, \sigma^2)$ with unknown mean μ and σ^2. Show the details to find the generalized likelihood ratio test for hypotheses $H_0 : \mu \geq \mu_0$ versus $H_1 : \mu < \mu_0$.

Q4. Let X_1, X_2 be a random sample of size $n = 2$ from the distribution having p.d.f. $f(x; \theta) = (1/\theta)e^{-x/\theta}$, $0 < x < \infty$, zero elsewhere. Consider hypotheses:

$$H_0 : \theta = 2 \quad \text{versus} \quad H_1 : \theta = 1.$$

Find the most powerful test having a size 0.05 for the above hypotheses.

Q5. Let $X_1, X_2, \cdots, X_{12}$ be an independent random sample for the Poisson distribution with mean λ. Consider the hypotheses:

$$H_0 : \lambda = \frac{1}{2} \quad \text{versus} \quad H_1 : \lambda < \frac{1}{2}.$$

Suppose that the test for the above hypotheses has the rejection region $\{X_1 + X_2 + \cdots + X_{12} \leq 2\}$, and let $K(\lambda)$ be the corresponding power function.

(a) Find the powers $K(1/2)$, $K(1/3)$, $K(1/4)$, $K(1/6)$, and $K(1/12)$.

(b) Sketch the graph of $K(\lambda)$ and state the size of the test.

Q6. Answer the following two questions under the assumption of normal populations with equal variances.

(i) The following are the numbers of sales which a random sample of nine salesmen of industrial chemicals in California and a random sample of six salesmen of industrial chemicals in Oregon made over a fixed period of time:

(California) $x_i : 40, 46, 61, 38, 55, 63, 36, 60, 51$

(Oregon) $y_i : 33, 62, 44, 54, 23, 42$

Using a 5% level of significance, test whether or not Californian salesmen are more efficient in general. Write down the null and alternative hypothesis first.

(ii) Suppose that we wish to investigate whether males and females earn comparable wages in a certain industry. Sample data show that 14 randomly surveyed males earn on the average \$213.5 per week with a standard deviation of \$16.5, while 18 randomly surveyed females earn on the average \$194.1 per week with a standard deviation of \$18.0. Let μ_1 denote the average wage of males and μ_2 the average of females. Using a 5% level of significance, test the null hypothesis that $\mu_1 = \mu_2$ against the alternative that $\mu_1 > \mu_2$.

Q7. Management training programs are often instituted to teach supervisory skills and thereby increase productivity. Suppose a company psychologist administers a set of examinations to each of 10 supervisors before such a training program begins and then administers similar examinations at the end of the program. The examinations are designed to measure supervisory skills, with higher scores indicating increased skill. The results of the tests are shown below:

Supervisor	Pre-Test	Post-Test
1	63	78
2	93	92
3	84	91
4	72	80
5	65	69
6	72	85
7	91	99
8	84	82
9	71	81
10	80	87

Do the data provide evidence that the training program is effective in increasing supervisory skills, as measured by the examination scores? Set up the appropriate hypotheses and test them at 5% significance level. State the assumptions made.

Q8. Let X and Y be the times in days required for maturation of Guardiola seeds from narrow-leaved and broad-leaved parents, respectively. Assume that both X and Y are distributed as normal and they are independent to each other. A sample of size 13 yield $s_X^2 = 9.88$ and the other sample of size 9 yield $s_Y^2 = 4.08$.

Test the hypothesis that the population variances of X and Y are equal. Use $\alpha = 0.05$.