A LIST OF STATISTICAL FORMULAE

1. $M_X(t) = \mathrm{E}(e^{tX})$. $\quad \mathrm{E}(X^r) = \left.\left(\dfrac{\mathrm{d}^r}{\mathrm{d}t^r} M_X(t)\right)\right|_{t=0}$.

2. $\overline{X} = \dfrac{1}{n}\sum_{i=1}^{n} X_i$. $\quad S^2 = \dfrac{1}{n}\sum_{i=1}^{n}\left(X_i - \overline{X}\right)^2$.

3. CLT: $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \approx \mathrm{N}(0,1)$ for large n.

4. Normal population $\Longrightarrow \dfrac{nS^2}{\sigma^2} \sim \chi^2_{n-1}$, $\dfrac{\overline{X} - \mu}{S/\sqrt{n-1}} \sim t_{n-1}$.

5. $\chi^2_\nu \sim \sum_{i=1}^{\nu} Z_i^2$. $\quad t_\nu \sim \dfrac{Z}{\sqrt{\chi^2_\nu/\nu}}$. $\quad F_{\nu_1,\nu_2} \sim \dfrac{\chi^2_{\nu_1}/\nu_1}{\chi^2_{\nu_2}/\nu_2}$.

6. $\mathrm{Bias}(\hat{\theta}) = \mathrm{E}(\hat{\theta}) - \theta$. $\quad \mathrm{E}\left[(\hat{\theta} - \theta)^2\right] = \mathrm{Var}(\hat{\theta}) + \left[\mathrm{Bias}(\hat{\theta})\right]^2$.

7. $I(\theta) = \mathrm{E}\left[\left(\dfrac{\partial \log f(X;\theta)}{\partial\theta}\right)^2\right] = \mathrm{E}\left[-\dfrac{\partial^2 \log f(X;\theta)}{\partial\theta^2}\right]$. $\quad \mathrm{Var}(\hat{\theta}) \geq \dfrac{1}{nI(\theta)}$.

8. $\mathbf{f}(x_1, x_2, \ldots, x_n; \theta) = g\left(u(x_1, x_2, \ldots, x_n), \theta\right) \cdot h(x_1, x_2, \ldots, x_n)$.

9. $\dfrac{\overline{X} - \mu_0}{\sigma_0/\sqrt{n}}$. $\quad \dfrac{\overline{X} - \mu_0}{S/\sqrt{n-1}}$.

10. $\dfrac{\overline{X} - \overline{Y} - \delta}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$. $\quad \dfrac{\overline{X} - \overline{Y} - \delta}{S_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}$, where $S_p^2 = \dfrac{n_1 S_1^2 + n_2 S_2^2}{n_1 + n_2 - 2}$.

11. $\overline{x} \pm z_{\alpha/2}\dfrac{\sigma}{\sqrt{n}}$. $\quad \overline{x} \pm t_{\alpha/2,n-1}\dfrac{s}{\sqrt{n-1}}$. $\quad \left(\dfrac{ns^2}{\chi^2_{\alpha/2,n-1}}, \dfrac{ns^2}{\chi^2_{1-\alpha/2,n-1}}\right)$.

12. $\overline{x} - \overline{y} \pm z_{\alpha/2}\sqrt{\dfrac{\sigma_x^2}{n_1} + \dfrac{\sigma_y^2}{n_2}}$. $\quad \overline{x} - \overline{y} \pm t_{\alpha/2,n_x+n_y-2}\sqrt{\dfrac{n_x s_x^2 + n_y s_y^2}{n_x + n_y - 2}\left(\dfrac{1}{n_x} + \dfrac{1}{n_y}\right)}$.

$$\left(\frac{n_x(n_y-1)s_x^2}{n_y(n_x-1)s_y^2} \cdot \frac{1}{F_{\alpha/2,n_x-1,n_y-1}},\ \frac{n_x(n_y-1)s_x^2}{n_y(n_x-1)s_y^2} \cdot F_{\alpha/2,n_y-1,n_x-1}\right).$$

13. $F_{1-\alpha,m,n} = \dfrac{1}{F_{\alpha,n,m}}$.

14. $-2\ln\Lambda \to \chi^2_d$ as $n \to \infty$.

Cumulative distribution function $\Phi(x)$ for $N(0,1)$

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7703	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990

Upper percentile for the student's t distribution t_ν

ν	60.0%	66.7%	75.0%	80.0%	87.5%	90.0%	95.0%	97.5%	99.0%	99.5%	99.9%
1	0.325	0.577	1.000	1.376	2.414	3.078	6.314	12.706	31.821	63.657	318.31
2	0.289	0.500	0.816	1.061	1.604	1.886	2.920	4.303	6.965	9.925	22.327
3	0.277	0.476	0.765	0.978	1.423	1.638	2.353	3.182	4.541	5.841	10.215
4	0.271	0.464	0.741	0.941	1.344	1.533	2.132	2.776	3.747	4.604	7.173
5	0.267	0.457	0.727	0.920	1.301	1.476	2.015	2.571	3.365	4.032	5.893
6	0.265	0.453	0.718	0.906	1.273	1.440	1.943	2.447	3.143	3.707	5.208
7	0.263	0.449	0.711	0.896	1.254	1.415	1.895	2.365	2.998	3.499	4.785
8	0.262	0.447	0.706	0.889	1.240	1.397	1.860	2.306	2.896	3.355	4.501
9	0.261	0.445	0.703	0.883	1.230	1.383	1.833	2.262	2.821	3.250	4.297
10	0.260	0.444	0.700	0.879	1.221	1.372	1.812	2.228	2.764	3.169	4.144
11	0.260	0.443	0.697	0.876	1.214	1.363	1.796	2.201	2.718	3.106	4.025
12	0.259	0.442	0.695	0.873	1.209	1.356	1.782	2.179	2.681	3.055	3.930
13	0.259	0.441	0.694	0.870	1.204	1.350	1.771	2.160	2.650	3.012	3.852
14	0.258	0.440	0.692	0.868	1.200	1.345	1.761	2.145	2.624	2.977	3.787
15	0.258	0.439	0.691	0.866	1.197	1.341	1.753	2.131	2.602	2.947	3.733
16	0.258	0.439	0.690	0.865	1.194	1.337	1.746	2.120	2.583	2.921	3.686
17	0.257	0.438	0.689	0.863	1.191	1.333	1.740	2.110	2.567	2.898	3.646
18	0.257	0.438	0.688	0.862	1.189	1.330	1.734	2.101	2.552	2.878	3.610
19	0.257	0.438	0.688	0.861	1.187	1.328	1.729	2.093	2.539	2.861	3.579
20	0.257	0.437	0.687	0.860	1.185	1.325	1.725	2.086	2.528	2.845	3.552
21	0.257	0.437	0.686	0.859	1.183	1.323	1.721	2.080	2.518	2.831	3.527
22	0.256	0.437	0.686	0.858	1.182	1.321	1.717	2.074	2.508	2.819	3.505
23	0.256	0.436	0.685	0.858	1.180	1.319	1.714	2.069	2.500	2.807	3.485
24	0.256	0.436	0.685	0.857	1.179	1.318	1.711	2.064	2.492	2.797	3.467
25	0.256	0.436	0.684	0.856	1.178	1.316	1.708	2.060	2.485	2.787	3.450
∞	0.253	0.431	0.674	0.842	1.150	1.282	1.645	1.960	2.326	2.576	3.090