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Solar position algorithm for solar radiation applications

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Abstract

There have been many published articles describing solar position algorithms for solar radiation applications. The best uncertainty achieved in most of these articles is greater than ±0.01° in calculating the solar zenith and azimuth angles. For some, the algorithm is valid for a limited number of years varying from 15 years to a hundred years. This report is a step by step procedure for implementing an algorithm to calculate the solar zenith and azimuth angles in the period from the year −2000 to 6000, with uncertainties of ±0.0003°. The algorithm is described in a book written by Jean Meeus in 1998. This report is written in a step by step format to simplify the complicated steps described in the book, with a focus on the sun instead of the planets and stars in general. It also introduces some changes to accommodate for solar radiation applications. The changes include changing the direction of measuring azimuth angles to be measured from north and eastward instead of being measured from south and eastward, and the direction of measuring the observer's geographical longitude to be measured as positive eastward from Greenwich meridian instead of negative. This report also includes the calculation of incidence angle for a surface that is tilted to any horizontal and vertical angle, as described by Iqbals in 1983.

Keywords: Global solar irradiance; Solar zenith angle; Solar azimuth angle; VSOP87 theory; Universal time; ΔUT1

1. Introduction

With the continuous technological advancements in solar radiation applications, there will always be a demand for smaller uncertainty in calculating the solar position. Many methods to calculate the solar position have been published in the solar radiation literature, nevertheless, their uncertainties have been greater than ±0.01° in solar zenith and azimuth angle calculations, and some are only valid for a specific number of years (Blanco-Muriel et al., 2001). For example, Michalsky's calculations are limited to the period from 1950 to 2050 with uncertainty of greater than ±0.01° (Michalsky, 1988), and the calculations of Blanco-Muriel et al.'s are limited to the period from 1999 to 2015 with uncertainty greater than >±0.01° (Blanco-Muriel et al., 2001).

An example emphasizing the importance of reducing the uncertainty of calculating the solar position to lower than ±0.01°, is the calibration of pyranometers that measure the global solar irradiance. During the calibration, the responsivity of the pyranometer is calculated at zenith angles from 0° to 90° by dividing its output voltage by the reference global solar irradiance (G), which is a function of the cosine of the zenith angle ($\cos\theta$). Fig. 1 shows the magnitude of errors that the 0.01° uncertainty in θ can contribute to the calculation of $\cos\theta$, and consequently G that is used to calculate the responsivity. Fig. 1 shows that the uncertainty in $\cos\theta$ exponentially increases as θ reaches 90° (e.g. at θ equal to 87°, the uncertainty in $\cos\theta$ is 0.7%, which can result in an uncertainty of 0.35% in calculating G; because at such large zenith angles the normal incidence irradiance is approximately equal to half the value of G).

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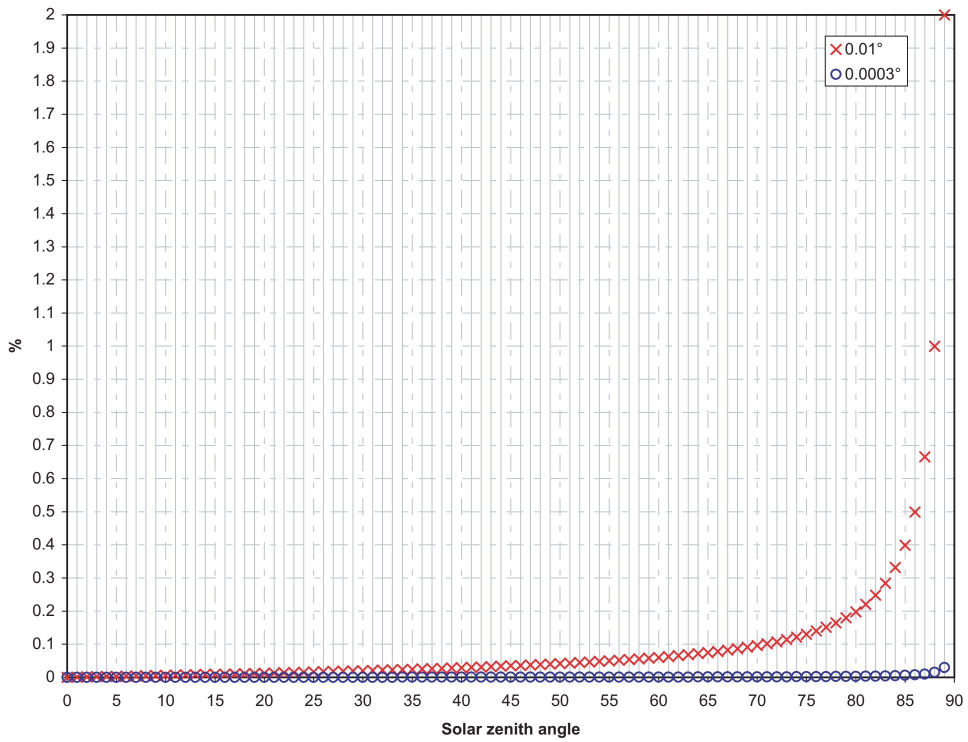

Fig. 1. Uncertainty of cosine the solar zenith angle resulting from 0.01° and 0.0003° uncertainty in the angle calculation.

The solar position calculations, using the algorithms mentioned above, are based on predicting the earth irregular rotation around the sun using historical observations. This causes users to develop different algorithms with different sets of coefficients for every specific number of years, consequently the scientific community will have multiple algorithms that may cause confusion and inconsistency.

From this arises the need to use one solar position algorithm that has one set of coefficients, that are valid for a long period of time, and has lower uncertainty for users that are interested in measuring the global solar irradiance with smaller uncertainties in the full zenith angle range from 0° to 90°.

In this report we describe a procedure for a Solar Position Algorithm (SPA) to calculate the solar zenith and azimuth angle with uncertainties equal to ±0.0003° in the period from the year −2000 to 6000 (Meeus, 1998). Fig. 1 shows that the uncertainty of the reference global solar irradiance, resulting from ±0.0003° in calculating the solar zenith angle in the range from 0° to 90° is negligible. The procedure is adopted from *The Astronomical Algorithms* book, which is based on the Variations Sèculaires des Orbites Planètaires Theory (VSOP87) that was developed by Bretagnon in 1982 then modified in 1987 by Bretagnon and Francou (Meeus, 1998). In this report, we summarize the complex algorithm elements scattered throughout the book to calculate the solar position, and introduce some modification to the algorithm to accommodate solar radiation applications. For example, in *The Astronomical Algorithms*, the azimuth angle is measured westward from south, but for solar radiation applications, it is measured eastward from north. Also, the observer's geographical longitude is considered positive west, or negative east from Greenwich, while for solar radiation applications, it is considered negative west, or positive east from Greenwich.

In Section 2, we describe the time scales because of the importance of using the correct time in the SPA. In this section, the term ΔUT1 is added algebraically to the time to correct for the Earth irregular rotational rate. Using the SPA without ΔUT1, at −105° longitude and 40° latitude, introduces errors of 0.001° and 0.01° in zenith and azimuth angle calculations at solar noon, consequently, and 0.003° in zenith and azimuth angle calculations at sunrise and sunset. The magnitude of these errors may vary, depending on the location.

In Section 3, we provide a step by step procedure to calculate the solar position described in *The Astronomical Algorithms* (Meeus, 1998), and calculate the solar incidence angle for an arbitrary surface orientation using the methods described in *An Introduction to Solar Radiation* (Iqbal, 1983), then in Section 4 we evaluate the SPA against the *Astronomical Almanac* (AA) data for the years 1994–96, and 2004.

Users can obtain a detailed technical report about the algorithm from the National Renewable Energy Laboratory (Reda and Andreas, 2003). The technical report includes examples with calculated values to give the users confidence in their calculations. It also includes *C* source code of the algorithm with its tables in ASCII format.

The users should note that this report is used to calculate the solar position for solar radiation applications only, and that it is purely mathematical and not meant to teach astronomy or to describe the Earth rotation. For more description about the astronomical nomenclature that is used through out the report, the user is encouraged to review the definitions in the AAs, *US Naval Observatory Literature*, or other astronomical reference.

2. Time scale

The following are the internationally recognized time scales:

- The universal time (UT), or Greenwich civil time, is based on the Earth's rotation and counted from 0-h at midnight; the unit is mean solar day (Meeus, 1998). UT is the time used to calculate the solar position in the described algorithm. It is sometimes referred to as UT1.
- The international atomic time (TAI) is the duration of the system international second (SI-second) and based on a large number of atomic clocks (The Astronomical Almanac, 2004).
- The coordinated universal time (UTC) is the basis of most radio time signals and the legal time systems. It is kept to within 0.9 s of UT1 (UT) by introducing one second step to its value (leap second); to date the steps are always positive and introduced, irregularly, based on observation.
- The terrestrial dynamical or terrestrial time (TDT or TT) is the time scale of ephemeredes for observations from the Earth surface.

The following equations describe the relationship between the above time scales (in seconds):

$$\mathrm{TT} = \mathrm{TAI} + 32.184, \tag{1}$$

$$\mathrm{UT} = \mathrm{TT} - \Delta T, \tag{2}$$

where ΔT is the difference between the Earth rotation time and the TT. It is derived from observation only and reported yearly in the AA.

$$\mathrm{UT} = \mathrm{UT1} = \mathrm{UTC} + \Delta\mathrm{UT1}, \tag{3}$$

where ΔUT1 is a fraction of a second, positive or negative value, that is added to the UTC to adjust for the Earth irregular rotational rate. It is derived from observation, but predicted values are transmitted in code in some time signals, e.g. weekly by the US Naval Observatory (USNO).

3. Procedure

The following site parameters and variables are required for this procedure: Date, UTC, ΔUT1, longitude, latitude, elevation, and the annual local pressure and temperature. The slope and azimuth rotation are also required to calculate the incidence angle for a surface oriented in any direction.

3.1. Calculate the Julian and Julian Ephemeris Day (JDE), century, and millennium

The Julian date starts on January 1, in the year—4712 at 12:00:00 UT (Meeus, 1998). The Julian Day (JD) is calculated using UT and the JDE is calculated using TT. In the following steps, note that there is a 10-day gap between the Julian and Gregorian calendar where the Julian calendar ends on 4 October 1582 (JD = 2 299 160), and the Gregorian calendar starts on 15 October 1582.

3.1.1 Calculate the JD,

$$\begin{aligned}\mathrm{JD} = {} & \mathrm{INT}(365.25 * (Y + 4716)) \\ & + \mathrm{INT}(30.6001 * (M + 1)) + D + B \\ & - 1524.5,\end{aligned} \tag{4}$$

where, INT is the integer of the calculated terms (e.g. $8.7 = 8$, $8.2 = 8$, and $-8.7 = -8$ etc.); Y is the year (e.g. 2001, 2002, etc.), M is the month of the year (e.g. 1 for January, etc.). Note that if $M > 2$, then Y and M are not changed, but if $M = 1$ or 2, then $Y = Y - 1$ and $M = M + 12$; D is the day of the month with decimal time (e.g. for the second day of the month at 12:30:30 UT, $D = 2.521180556$); B is equal to 0, for the Julian calendar {i.e. by using $B = 0$ in Eq. (4) for $\mathrm{JD} \leqslant 2\,299\,160$}, and equal to $(2 - A + \mathrm{INT}(A/4))$ for the Gregorian calendar {i.e. for $\mathrm{JD} > 2\,299\,160$}, where A equals $\mathrm{INT}(Y/100)$.

For users who wish to use their local time instead of UT, change the time zone to a fraction of a day (by dividing it by 24), then subtract the result from JD. Note that the fraction is subtracted from JD calculated before the test for $B < 2\,299\,160$ to maintain the Julian and Gregorian periods.

3.1.2 Calculate the JDE,

$$\mathrm{JDE} = \mathrm{JD} + \frac{\Delta T}{86400}. \tag{5}$$

3.1.3 Calculate the Julian century (JC) and the Julian Ephemeris Century (JCE) for the 2000 standard epoch,

$$\mathrm{JC} = \frac{\mathrm{JD} - 2451545}{36525}, \tag{6}$$

$$\mathrm{JCE} = \frac{\mathrm{JDE} - 2451545}{36525}. \tag{7}$$

3.1.4 Calculate the Julian Ephemeris Millennium (JME) for the 2000 standard epoch,

$$\mathrm{JME} = \frac{\mathrm{JCE}}{10}. \tag{8}$$

3.2. Calculate the Earth heliocentric longitude, latitude, and radius vector (L, B, and R)

"Heliocentric" means that the Earth position is calculated with respect to the center of the sun.

3.2.1 For each row of Table 1, calculate the term $L0_i$ (in radians),

$$L0_i = A_i * \cos(B_i + C_i * \mathrm{JME}), \tag{9}$$

where, i is the ith row for the term $L0$ in Table 1; A_i, B_i, and C_i are the values in the ith row and A, B, and C columns in Table 1, for the term $L0$ (in radians).

3.2.2 Calculate the term $L0$ (in radians),

$$L0 = \sum_{i=0}^{n} L0_i, \tag{10}$$

where n is the number of rows for the term $L0$ in Table 1.

3.2.3 Calculate the terms $L1$, $L2$, $L3$, $L4$, and $L5$ by using Eqs. (9) and (10) and changing the 0 to 1, 2, 3, 4, and 5, and by using their corresponding values in columns A, B, and C in Table 1 (in radians).

3.2.4 Calculate the Earth heliocentric longitude, L (in radians),

$$L = \frac{L0 + L1 * \mathrm{JME} + L2 * \mathrm{JME}^2 + L3 * \mathrm{JME}^3 + L4 * \mathrm{JME}^4 + L5 * \mathrm{JME}^5}{10^8}. \tag{11}$$

3.2.5 Calculate L in degrees,

$$L(\text{in degrees}) = \frac{L(\text{in radians}) * 180}{\pi}, \tag{12}$$

where π is approximately equal to 3.1415926535898.

3.2.6 Limit L to the range from 0° to 360°. That can be accomplished by dividing L by 360 and recording the decimal fraction of the division as F. If L is positive, then the limited $L = 360 * F$. If L is negative, then the limited $L = 360 - 360 * F$.

3.2.7 Calculate the Earth heliocentric latitude, B (in degrees), by using Table 1 and steps 3.2.1 through 3.2.5 and by replacing all the Ls by Bs in all equations. Note that there are no $B2$ through $B5$, consequently, replace them by zero in steps 3.2.3 and 3.2.4.

3.2.8 Calculate the Earth radius vector, R (in Astronomical Units, AU), by repeating step 3.2.7 and by replacing all Ls by Rs in all equations. Note that there is no $R5$, consequently, replace it by zero in steps 3.2.3 and 3.2.4.

3.3. Calculate the geocentric longitude and latitude (Θ and β)

"Geocentric" means that the sun position is calculated with respect to the Earth center.

3.3.1 Calculate the geocentric longitude, Θ (in degrees),

$$\Theta = L + 180. \tag{13}$$

3.3.2 Limit Θ to the range from 0° to 360° as described in step 3.2.6.

3.3.3 Calculate the geocentric latitude, β (in degrees),

$$\beta = -B. \tag{14}$$

3.4. Calculate the nutation in longitude and obliquity ($\Delta\psi$ and $\Delta\varepsilon$)

3.4.1 Calculate the mean elongation of the moon from the sun, X_0 (in degrees),

$$X_0 = 297.85036 + 445267.111480 * \mathrm{JCE} - 0.0019142 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{189474}. \tag{15}$$

3.4.2 Calculate the mean anomaly of the sun (Earth), X_1 (in degrees),

$$X_1 = 357.52772 + 35999.050340 * \mathrm{JCE} - 0.0001603 * \mathrm{JCE}^2 - \frac{\mathrm{JCE}^3}{300000}. \tag{16}$$

Table 1
Earth periodic terms

Term	Row number	A	B	C
L0	0	175 347 046	0	0
	1	334 1656	4.6692568	6283.07585
	2	34 894	4.6261	12566.1517
	3	3497	2.7441	5753.3849
	4	3418	2.8289	3.5231
	5	3136	3.6277	77713.7715
	6	2676	4.4181	7860.4194
	7	2343	6.1352	3930.2097
	8	1324	0.7425	11506.7698
	9	1273	2.0371	529.691
	10	1199	1.1096	1577.3435
	11	990	5.233	5884.927
	12	902	2.045	26.298
	13	857	3.508	398.149
	14	780	1.179	5223.694
	15	753	2.533	5507.553
	16	505	4.583	18849.228
	17	492	4.205	775.523
	18	357	2.92	0.067
	19	317	5.849	11790.629
	20	284	1.899	796.298
	21	271	0.315	10977.079
	22	243	0.345	5486.778
	23	206	4.806	2544.314
	24	205	1.869	5573.143
	25	202	2.4458	6069.777
	26	156	0.833	213.299
	27	132	3.411	2942.463
	28	126	1.083	20.775
	29	115	0.645	0.98
	30	103	0.636	4694.003
	31	102	0.976	15720.839
	32	102	4.267	7.114
	33	99	6.21	2146.17
	34	98	0.68	155.42
	35	86	5.98	161000.69
	36	85	1.3	6275.96
	37	85	3.67	71430.7
	38	80	1.81	17260.15
	39	79	3.04	12036.46
	40	71	1.76	5088.63
	41	74	3.5	3154.69
	42	74	4.68	801.82
	43	70	0.83	9437.76
	44	62	3.98	8827.39
	45	61	1.82	7084.9
	46	57	2.78	6286.6
	47	56	4.39	14143.5
	48	56	3.47	6279.55
	49	52	0.19	12139.55
	50	52	1.33	1748.02
	51	51	0.28	5856.48
	52	49	0.49	1194.45
	53	41	5.37	8429.24
	54	41	2.4	19651.05
	55	39	6.17	10447.39
	56	37	6.04	10213.29

(*continued on next page*)

Table 1 (*continued*)

Term	Row number	*A*	*B*	*C*
	57	37	2.57	1059.38
	58	36	1.71	2352.87
	59	36	1.78	6812.77
	60	33	0.59	17789.85
	61	30	0.44	83996.85
	62	30	2.74	1349.87
	63	25	3.16	4690.48
L1	0	628 331 966 747	0	0
	1	206 059	2.678235	6283.07585
	2	4303	2.6351	12566.1517
	3	425	1.59	3.523
	4	119	5.796	26.298
	5	109	2.966	1577.344
	6	93	2.59	18849.23
	7	72	1.14	529.69
	8	68	1.87	398.15
	9	67	4.41	5507.55
	10	59	2.89	5223.69
	11	56	2.17	155.42
	12	45	0.4	796.3
	13	36	0.47	775.52
	14	29	2.65	7.11
	15	21	5.34	0.98
	16	19	1.85	5486.78
	17	19	4.97	213.3
	18	17	2.99	6275.96
	19	16	0.03	2544.31
	20	16	1.43	2146.17
	21	15	1.21	10977.08
	22	12	2.83	1748.02
	23	12	3.26	5088.63
	24	12	5.27	1194.45
	25	12	2.08	4694
	26	11	0.77	553.57
	27	10	1.3	3286.6
	28	10	4.24	1349.87
	29	9	2.7	242.73
	30	9	5.64	951.72
	31	8	5.3	2352.87
	32	6	2.65	9437.76
	33	6	4.67	4690.48
L2	0	52 919	0	0
	1	8720	1.0721	6283.0758
	2	309	0.867	12566.152
	3	27	0.05	3.52
	4	16	5.19	26.3
	5	16	3.68	155.42
	6	10	0.76	18849.23
	7	9	2.06	77713.77
	8	7	0.83	775.52
	9	5	4.66	1577.34
	10	4	1.03	7.11
	11	4	3.44	5573.14
	12	3	5.14	796.3
	13	3	6.05	5507.55
	14	3	1.19	242.73
	15	3	6.12	529.69
	16	3	0.31	398.15

Table 1 (*continued*)

Term	Row number	*A*	*B*	*C*
	17	3	2.28	553.57
	18	2	4.38	5223.69
	19	2	3.75	0.98
L3	0	289	5.844	6283.076
	1	35	0	0
	2	17	5.49	12566.15
	3	3	5.2	155.42
	4	1	4.72	3.52
	5	1	5.3	18849.23
	6	1	5.97	242.73
L4	0	114	3.142	0
	1	8	4.13	6283.08
	2	1	3.84	12566.15
L5	0	1	3.14	0
B0	0	280	3.199	84334.662
	1	102	5.422	5507.553
	2	80	3.88	5223.69
	3	44	3.7	2352.87
	4	32	4	1577.34
B1	0	9	3.9	5507.55
	1	6	1.73	5223.69
R0	0	100013989	0	0
	1	1670700	3.0984635	6283.07585
	2	13956	3.05525	12566.1517
	3	3084	5.1985	77713.7715
	4	1628	1.1739	5753.3849
	5	1576	2.8469	7860.4194
	6	925	5.453	11506.77
	7	542	4.564	3930.21
	8	472	3.661	5884.927
	9	346	0.964	5507.553
	10	329	5.9	5223.694
	11	307	0.299	5573.143
	12	243	4.273	11790.629
	13	212	5.847	1577.344
	14	186	5.022	10977.079
	15	175	3.012	18849.228
	16	110	5.055	5486.778
	17	98	0.89	6069.78
	18	86	5.69	15720.84
	19	86	1.27	161000.69
	20	85	0.27	17260.15
	21	63	0.92	529.69
	22	57	2.01	83996.85
	23	56	5.24	71430.7
	24	49	3.25	2544.31
	25	47	2.58	775.52
	26	45	5.54	9437.76
	27	43	6.01	6275.96
	28	39	5.36	4694
	29	38	2.39	8827.39
	30	37	0.83	19651.05
	31	37	4.9	12139.55
	32	36	1.67	12036.46
	33	35	1.84	2942.46
	34	33	0.24	7084.9
	35	32	0.18	5088.63
	36	32	1.78	398.15

(*continued on next page*)

Table 1 (*continued*)

Term	Row number	A	B	C
	37	28	1.21	6286.6
	38	28	1.9	6279.55
	39	26	4.59	10447.39
R1	0	103 019	1.10749	6283.07585
	1	1721	1.0644	12566.1517
	2	702	3.142	0
	3	32	1.02	18849.23
	4	31	2.84	5507.55
	5	25	1.32	5223.69
	6	18	1.42	1577.34
	7	10	5.91	10977.08
	8	9	1.42	6275.96
	9	9	0.27	5486.78
R2	0	4359	5.7846	6283.0758
	1	124	5.579	12566.152
	2	12	3.14	0
	3	9	3.63	77713.77
	4	6	1.87	5573.14
	5	3	5.47	18849
R3	0	145	4.273	6283.076
	1	7	3.92	12566.15
R4	0	4	2.56	6283.08

3.4.3 Calculate the mean anomaly of the moon, X_2 (in degrees),

$$X_2 = 134.96298 + 477198.867398 * \mathrm{JCE} + 0.0086972 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{56\,250}. \quad (17)$$

3.4.4 Calculate the moon's argument of latitude, X_3 (in degrees),

$$X_3 = 93.27191 + 483202.017538 * \mathrm{JCE} - 0.0036825 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{327\,270}. \quad (18)$$

3.4.5 Calculate the longitude of the ascending node of the moon's mean orbit on the ecliptic, measured from the mean equinox of the date, X_4 (in degrees),

$$X_4 = 125.04452 - 1934.136261 * \mathrm{JCE} + 0.0020708 * \mathrm{JCE}^2 + \frac{\mathrm{JCE}^3}{450\,000}. \quad (19)$$

3.4.6 For each row in Table 2, calculate the terms $\Delta\psi_i$ and $\Delta\varepsilon_i$ (in 0.0001 of arc seconds),

$$\Delta\psi_i = (a_i + b_i * \mathrm{JCE}) * \sin\left(\sum_{j=0}^{4} X_j * Y_{i,j}\right), \quad (20)$$

$$\Delta\varepsilon_i = (c_i + d_i * \mathrm{JCE}) * \cos\left(\sum_{j=0}^{4} X_j * Y_{i,j}\right), \quad (21)$$

where, a_i, b_i, c_i, and d_i are the values listed in the ith row and columns a, b, c, and d in Table 2; X_j is the jth X calculated by using Eqs. (15)–(19); $Y_{i,j}$ is the value listed in ith row and jth Y column in Table 2.

3.4.7 Calculate the nutation in longitude, $\Delta\psi$ (in degrees),

$$\Delta\psi = \frac{\sum_{i=0}^{n} \Delta\psi_i}{36\,000\,000}, \quad (22)$$

where n is the number of rows in Table 2.

3.4.8 Calculate the nutation in obliquity, $\Delta\varepsilon$ (in degrees),

$$\Delta\varepsilon = \frac{\sum_{i=0}^{n} \Delta\varepsilon_i}{36\,000\,000}, \quad (23)$$

where n is the number of rows in Table 2.

3.5. Calculate the true obliquity of the ecliptic, ε (in degrees)

3.5.1 Calculate the mean obliquity of the ecliptic, ε_0 (in arc seconds),

$$\varepsilon_0 = 84381.448 - 4680.93U - 1.55U^2 + 1999.25U^3 - 51.38U^4 - 249.67U^5 - 39.05U^6 + 7.12U^7 + 27.87U^8 + 5.79U^9 + 2.45U^{10}, \quad (24)$$

where $U = \mathrm{JME}/10$.

Table 2
Periodic terms for the nutation in longitude and obliquity

Coefficients for sin terms					Coefficients for $\Delta\psi$		Coefficients for $\Delta\epsilon$	
$Y0$	$Y1$	$Y2$	$Y3$	$Y4$	a	b	c	d
0	0	0	0	1	−171 996	−174.2	92 025	8.9
−2	0	0	2	2	−13 187	−1.6	5736	−3.1
0	0	0	2	2	−2274	−0.2	977	−0.5
0	0	0	0	2	2062	0.2	−895	0.5
0	1	0	0	0	1426	−3.4	54	−0.1
0	0	1	0	0	712	0.1	−7	
−2	1	0	2	2	−517	1.2	224	−0.6
0	0	0	2	1	−386	−0.4	200	
0	0	1	2	2	−301		129	−0.1
−2	−1	0	2	2	217	−0.5	−95	0.3
−2	0	1	0	0	−158			
−2	0	0	2	1	129	0.1	−70	
0	0	−1	2	2	123		−53	
2	0	0	0	0	63			
0	0	1	0	1	63	0.1	−33	
2	0	−1	2	2	−59		26	
0	0	−1	0	1	−58	−0.1	32	
0	0	1	2	1	−51		27	
−2	0	2	0	0	48			
0	0	−2	2	1	46		−24	
2	0	0	2	2	−38		16	
0	0	2	2	2	−31		13	
0	0	2	0	0	29			
−2	0	1	2	2	29		−12	
0	0	0	2	0	26			
−2	0	0	2	0	−22			
0	0	−1	2	1	21		−10	
0	2	0	0	0	17	−0.1		
2	0	−1	0	1	16		−8	
−2	2	0	2	2	−16	0.1	7	
0	1	0	0	1	−15		9	
−2	0	1	0	1	−13		7	
0	−1	0	0	1	−12		6	
0	0	2	−2	0	11			
2	0	−1	2	1	−10		5	
2	0	1	2	2	−8		3	
0	1	0	2	2	7		−3	
−2	1	1	0	0	−7			
0	−1	0	2	2	−7		3	
2	0	0	2	1	−7		3	
2	0	1	0	0	6			
−2	0	2	2	2	6		−3	
−2	0	1	2	1	6		−3	
2	0	−2	0	1	−6		3	
2	0	0	0	1	−6		3	
0	−1	1	0	0	5			
−2	−1	0	2	1	−5		3	
−2	0	0	0	1	−5		3	
0	0	2	2	1	−5		3	
−2	0	2	0	1	4			
−2	1	0	2	1	4			
0	0	1	−2	0	4			
−1	0	1	0	0	−4			
−2	1	0	0	0	−4			
1	0	0	0	0	−4			
0	0	1	2	0	3			

(*continued on next page*)

Table 2 (*continued*)

Coefficients for sin terms					Coefficients for $\Delta\psi$		Coefficients for $\Delta\epsilon$	
*Y*0	*Y*1	*Y*2	*Y*3	*Y*4	*a*	*b*	*c*	*d*
0	0	−2	2	2	−3			
−1	−1	1	0	0	−3			
0	1	1	0	0	−3			
0	−1	1	2	2	−3			
2	−1	−1	2	2	−3			
0	0	3	2	2	−3			
2	−1	0	2	2	−3			

3.5.2 Calculate the true obliquity of the ecliptic, ε (in degrees),

$$\varepsilon = \frac{\varepsilon_0}{3600} + \Delta\varepsilon. \tag{25}$$

3.6. Calculate the aberration correction, $\Delta\tau$ (in degrees)

$$\Delta\tau = -\frac{20.4898}{3600 * R}, \tag{26}$$

where R is computed in step 3.2.8.

3.7. Calculate the apparent sun longitude, λ (in degrees)

$$\lambda = \Theta + \Delta\psi + \Delta\tau. \tag{27}$$

3.8. Calculate the apparent sidereal time at Greenwich at any given time, v (in degrees)

3.8.1 Calculate the mean sidereal time at Greenwich, v_0 (in degrees),

$$v_0 = 280.46061837 + 360.98564736629 * (\mathrm{JD} - 2451545) + 0.000387933 * \mathrm{JC}^2 - \frac{\mathrm{JC}^3}{38710000}. \tag{28}$$

3.8.2 Limit v_0 to the range from 0° to 360° as described in step 3.2.6.

3.8.3 Calculate the apparent sidereal time at Greenwich, v (in degrees),

$$v = v_0 + \Delta\psi * \cos(\varepsilon). \tag{29}$$

3.9. Calculate the geocentric sun right ascension, α (in degrees)

3.9.1 Calculate the sun right ascension, α (in radians),

$$\alpha = \mathrm{Arc\,tan2}\left(\frac{\sin\lambda * \cos\varepsilon - \tan\beta * \sin\varepsilon}{\cos\lambda}\right), \tag{30}$$

where Arc tan2 is an arctangent function that is applied to the numerator and the denominator (instead of the actual division) to maintain the correct quadrant of the α where α is in the range from $-\pi$ to π.

3.9.2 Calculate α in degrees using Eq. (12), then limit it to the range from 0° to 360° using the technique described in step 3.2.6.

3.10. Calculate the geocentric sun declination, δ (in degrees)

$$\delta = \mathrm{Arc\,sin}(\sin\beta * \cos\varepsilon + \cos\beta * \sin\varepsilon * \sin\lambda), \tag{31}$$

where δ is positive or negative if the sun is north or south of the celestial equator, respectively. Then change δ to degrees using Eq. (12).

3.11. Calculate the observer local hour angle, H (in degrees)

$$H = v + \sigma - \alpha, \tag{32}$$

where σ is the observer geographical longitude, positive or negative for east or west of Greenwich, respectively.

Limit H to the range from 0° to 360° using step 3.2.6 and note that it is measured westward from south in this algorithm.

3.12. Calculate the topocentric sun right ascension α' (in degrees)

"Topocentric" means that the sun position is calculated with respect to the observer local position at the Earth surface.

3.12.1 Calculate the equatorial horizontal parallax of the sun, ξ (in degrees),

$$\xi = \frac{8.794}{3600 * R}, \tag{33}$$

where R is calculated in step 3.2.8.

3.12.2 Calculate the term u (in radians),

$$u = \operatorname{Arc}\tan(0.99664719 * \tan\varphi), \tag{34}$$

where φ is the observer geographical latitude, positive or negative if north or south of the equator, respectively. Note that the 0.99664719 number equals $(1-f)$, where f is the Earth's flattening.

3.12.3 Calculate the term x,

$$x = \cos u + \frac{E}{6\,378\,140} * \cos\varphi, \tag{35}$$

where E is the observer elevation (in meters). Note that x equals $\rho * \cos\varphi'$ where ρ is the observer's distance to the center of Earth, and φ' is the observer's geocentric latitude.

3.12.4 Calculate the term y,

$$y = 0.99664719 * \sin u + \frac{E}{6\,378\,140} * \sin\varphi, \tag{36}$$

note that y equals $\rho * \sin\varphi'$.

3.12.5 Calculate the parallax in the sun right ascension, $\Delta\alpha$ (in degrees),

$$\Delta\alpha = \operatorname{Arc}\tan2\left(\frac{-x * \sin\xi * \sin H}{\cos\delta - x * \sin\xi * \cos H}\right). \tag{37}$$

Then change $\Delta\alpha$ to degrees using Eq. (12).

3.12.6 Calculate the topocentric sun right ascension α' (in degrees),

$$\alpha' = \alpha + \Delta\alpha. \tag{38}$$

3.12.7 Calculate the topocentric sun declination, δ' (in degrees),

$$\delta' = \operatorname{Arc}\tan2\left(\frac{(\sin\delta - y * \sin\xi) * \cos\Delta\alpha}{\cos\delta - y * \sin\xi * \cos H}\right). \tag{39}$$

3.13. Calculate the topocentric local hour angle, H′ (in degrees)

$$H' = H - \Delta\alpha. \tag{40}$$

3.14. Calculate the topocentric zenith angle, θ (in degrees)

3.14.1 Calculate the topocentric elevation angle without atmospheric refraction correction, e_0 (in degrees),

$$e_0 = \operatorname{Arc}\sin(\sin\varphi * \sin\delta' + \cos\varphi * \cos\delta' * \cos H'). \tag{41}$$

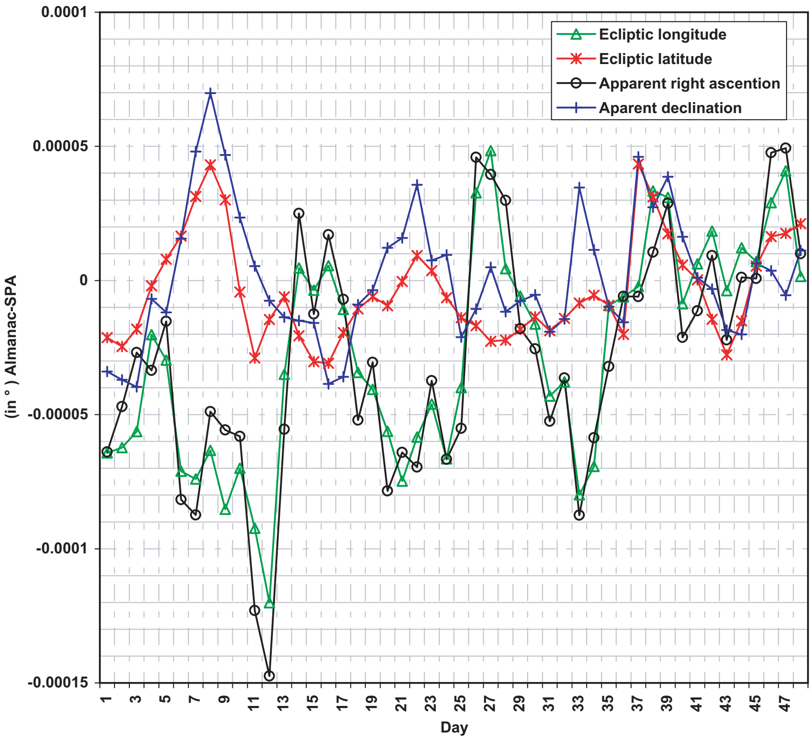

Fig. 2. Difference between the Almanac and SPA for the ecliptic longitude, ecliptic latitude, apparent right ascension, and apparent declination on the second day of each month at 0-TT for the years 1994–96, and 2004.

Then change e_0 to degrees using Eq. (12).

3.14.2 Calculate the atmospheric refraction correction, Δe (in degrees),

$$\Delta e = \frac{P}{1010} * \frac{283}{273 + T} * \frac{1.02}{60 * \tan\left(e_0 + \frac{10.3}{e_0 + 5.11}\right)}, \tag{42}$$

where, P is the annual average local pressure (in mbar); T is the annual average local temperature (in °C); e_0 is in degrees. Calculate the tangent argument in degrees, then convert to radians if required by calculator or computer.

3.14.3 Calculate the topocentric elevation angle, e (in degrees),

$$e = e_0 + \Delta e. \tag{43}$$

3.14.4 Calculate the topocentric zenith angle, θ (in degrees),

$$\theta = 90 - e. \tag{44}$$

3.15. Calculate the topocentric azimuth angle, Φ (in degrees):

3.15.1 Calculate the topocentric astronomers azimuth angle, Γ (in degrees),

$$\Gamma = \mathrm{Arc}\tan 2\left(\frac{\sin H'}{\cos H' * \sin\varphi - \tan\delta' * \cos\varphi}\right). \tag{45}$$

Change Γ to degrees using Eq. (12), then limit it to the range from 0° to 360° using step 3.2.6. Note that Γ is measured *westward* from *south*.

3.15.2 Calculate the topocentric azimuth angle, Φ for navigators and solar radiation users (in degrees),

$$\Phi = \Gamma + 180. \tag{46}$$

Limit Φ to the range from 0° to 360° using step 3.2.6. Note that Φ is measured *eastward* from *north*.

3.16. Calculate the incidence angle for a surface oriented in any direction, I (in degrees)

$$I = \mathrm{Arc}\cos(\cos\theta * \cos\omega + \sin\omega * \sin\theta * \cos(\Gamma - \gamma)), \tag{47}$$

where, ω is the slope of the surface measured from the horizontal plane; γ is the surface azimuth rotation angle, measured from south to the projection of the surface normal on the horizontal plane, positive or negative if oriented east or west from south, respectively.

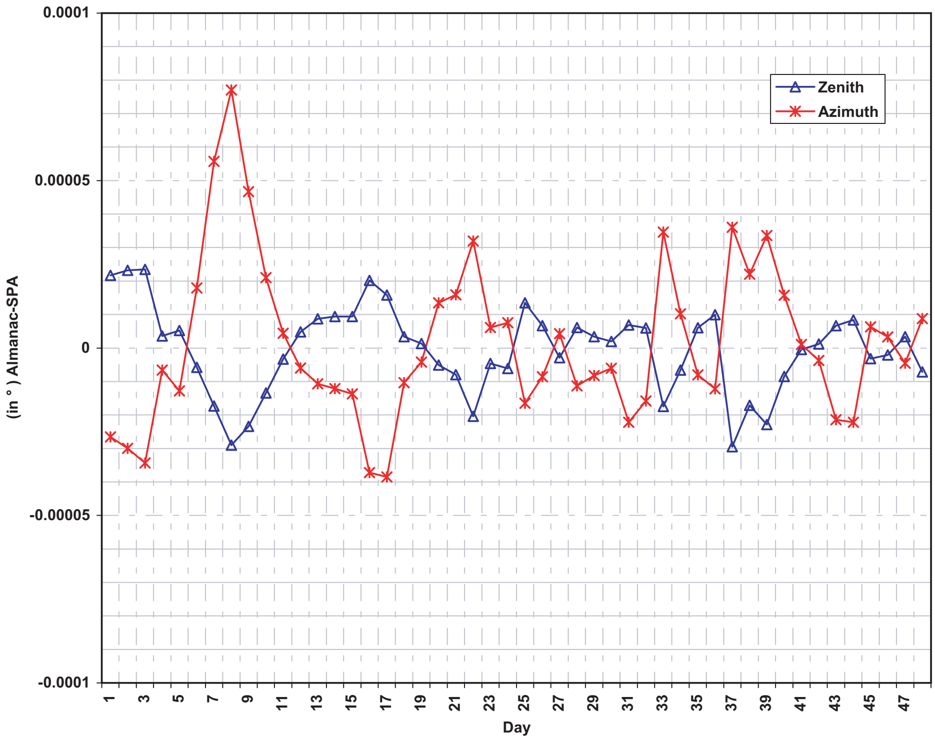

Fig. 3. Difference between the Almanac and SPA for the solar zenith and azimuth angles on the second day of each month at 0-TT for the years 1994–96, and 2004.

4. SPA evaluation and conclusion

Because the solar zenith, azimuth, and incidence angles are not reported in the AA, the following sun parameters are used for the evaluation: The main parameters (ecliptic longitude and latitude for the mean Equinox of date, apparent right ascension, apparent declination), and the correcting parameters (nutation in longitude, nutation in obliquity, obliquity of ecliptic, and true geometric distance). Exact trigonometric functions are used with the AA reported sun parameters to calculate the solar zenith and azimuth angles, therefore it is adequate to evaluate the SPA uncertainty using these parameters. To evaluate the uncertainty of the SPA, we arbitrarily chose the second day of each month, for each of the years 1994–96, and 2004, at 0-h terrestrial time (TT). We were restricted to the 0-h because it is the only time recorded in the AA. Fig. 2 shows that the maximum difference between the AA and SPA main parameters is $-0.00015°$. Fig. 3 shows that the maximum difference between the AA and SPA for calculating the zenith or azimuth angle is $0.00003°$ and $0.00008°$, respectively. The differences root mean square, RMS, were also calculated and found to be less than $0.000017°$. This implies that the SPA, using the VSOP87 theory that was developed in 1987, is well within the stated uncertainty of $\pm 0.0003°$ up to the year 2004.

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References

Blanco-Muriel, M., Alarcon-Padilla, D.C., Lopea-Moratalla, T., Lara-Coira, M., 2001. Computing the solar vector. J. Solar Energy 70 (5), 431–441.

Meeus, J., 1998. Astronomical Algorithms, second ed. Willmann-Bell, Inc., Richmond, Virginia, USA.

Michalsky, J.J., 1988. The astronomical Almanac's algorithm for approximate solar position (1950–2050). J. Solar Energy 40 (3), 227–235.

Iqbal, M., 1983. An Introduction to Solar Radiation. New York, pp. 23–25.

Reda, I., Andreas, A., 2003. Solar position algorithm for solar radiation applications. Technical report: NREL/TP-560-34302. Golden, USA, http://www.nrel.gov.

The Astronomical Almanac. Norwich:2004.

The US Naval Observatory. Washington, DC, http://www.usno.navy.mil.