Coding Theory

Here, I am demonstrating my final product so to speak. The goal being put together a coherent paper about a topic in information theory that mattered to me. I chose channel capacity of fiber optics, as I am concerned over the exponential growth and increased demand on fiber infrastructure caused by more ‘smart’ devices or the IoT (internet of things).

Portfolio My Variation

In my variation we are asked to consider how larger size of stocks to choose from effects our models predicted optimal combination of stocks.

Game Theory

We are to demonstrate basic game theory application.

Stock Portfolio

We developed a model based on the Markowitz equation. Our model had to choose an optimal combination of stocks from a finite starting pool, based on several controllable constraints including the extent of risk one desired to take.

Markowitz Portfolio Optimization

Mulberry and Lemay RoundAbout

A very popular intersection in Fort Collins, CO. The city ruled against installing a roundabout on grounds the cost would be prohibitive. We were asked to concur or refute this claim based on traffic engineer data collected. We as a group decided on a discrete time model. This model would divide the roundabout into several sectors, each sector was allotted one car plus ten foot clearance. Based on several map views we measured the approximation of the intersection and I used unit analysis to determine a reasonable speed in the intersection. After several other adjustments we had a model to which we could answer the question, can 50,000 cars/day travel through a single lane round about in the Mulberry and Lemay intersection.

Channel Capacity

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May 14, 2015

1 Introduction

How do you regard a person who signal handedly changed the understanding of information and how we look at it? As Dr. Claude E. Shannon. Shannon received his Ph.D. in mathematics from MIT in 1940. His big idea though would not occur until he got his feet wet in work at Bell Laboratories. This was a paper published 8 years later, in 1948. The paper was titled: A mathematical theory of communication. It is the ideas present in this paper which help define information theory and quantify communication.[5] So we know who is of interest, but what and why?

Increasingly large amounts of data are becoming staple in the business diet and 21st century in general. We are already facing numerous analytical problems associated with this so called 'big data' rush. One such problem is that of how to transmit that data. As computing capacity grows exponentially so too, does the data those computers produce. We have been studying for the last 60 or so years this discipline of information theory. What we need to know is what does information theory have to offer us in means of solutions to this transmission of 'big data'? Can our current infrastructure be maintained? If so, for how long?

2 Motivation

Today, you use the internet for everything. This is delivered to you with a combination of wired and wireless technologies. Of interest to us in this paper is fiber optics. This represents the hard wire backbone of the internet. Assuredly your communication will pass through one of these cables at some point. Yet, we dont give it much attention. For those in the know however, they are becoming increasingly concerned with what the capacity of fiber optics is. In addition to how and if at all, it can be improved.[3]

3 Problem Formulation

In this paper we will consider a couple of solutions to this. One is brought to us by none other than Shannon by the application of Dr. Jau Tang. The other is a more modern take using The treatment of a fiber optics channel as a time-varying frequency selective channel[1]. In other words, a channel that is based in time variation of frequency as opposed to Tang whom extends on Shannons' classic work demonstrating channel capacity as a relationship of signal to noise.

Before identifying what the channel capacity of fiber optics is, we must first define what the channel capacity is.

A more common form of Shannons channel capacity is provided by our modern author, Jau Tang. For a linear transmission system with a signal-to-noise ratio $\frac{S}{N}$, the optimal channel capacity is given by [6]

$$C = log_2(1 + \frac{S}{N}) \tag{1}$$

In English the channel capacity is the upper bound on the information rate of a particular channel.[7]

On the other hand we have this approach from Rajan, that states.

$$C(Y;X) = \max_{p(A)} I(Y;X)$$

Figure 1: Where I(Y;X) is the mutual information between Y and X [1]

Figure 1 and equation 1 illustrates our two distinct approaches to this particular problem. I will present the time varying equation from Rajan's work later in the paper; however, analysis will not be conducted. Instead, the two equations above will be presented and there derivation from our class theory will be analyzed.

4 Theory

In this section I will introduce the necessary theory behind the two separate approaches to channel capacity. First, will be the non-Shannon capacity as provided by Rajan. Second, will be the Shannon based capacity as described by Tang.

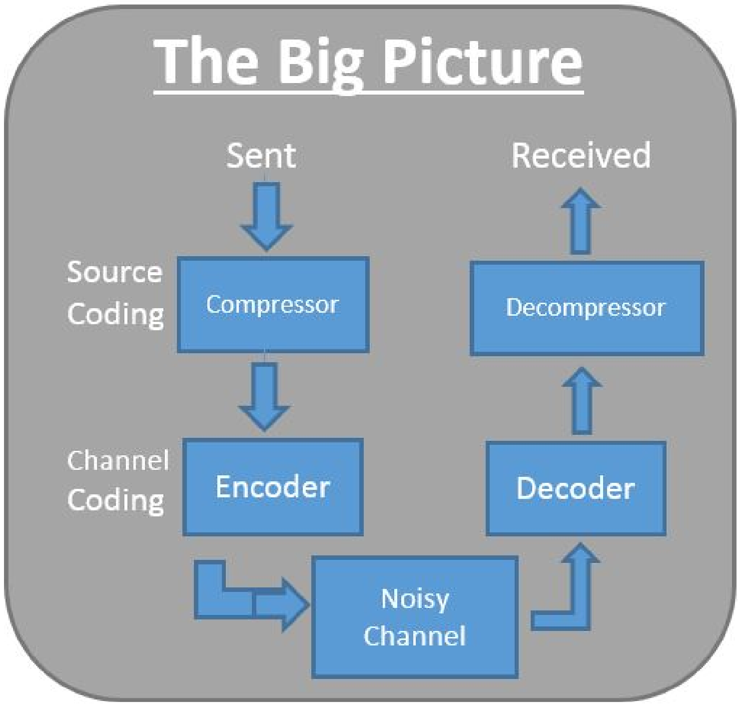

Figure 2: [4]

4.1 Background

Figure 2 does a great job at illustrating what the components of information theory deal with. The focus of this paper is on the Noisy Channel. I will encompass the necessary relationships with the other components as is necessary to describe the channel capacity.

To really gain some understanding of what is all needed in order to answer the bigger picture question about fiber optics channel capacity we need to develop a suitable launching point. Such a point would need to be founded in noisy channel theory and be able to expand to accommodate some of below listed limitations present for fiber optics channels.

- Attenuation
- Phase Conjugation
- Nonlinear system
- Time Domain
- Wave Division Multiplex (WDM)
- Dispersion
- Additive White Gaussian Noise (AWGN)
- Power
- Amplification

All channels in the world posses some noise. The noise is the background "junk" that is not part of our message and therefore acts as a hinderence on capacity of the channel. This noise can come from a variety of sources that include the following short list.

- Temperature
- Manufacturing quality
- Manufacturing Defects
- Background electronics
- etc...

In order to accurately model fiber optics as a channel we must address both the noise and complexities of implementing this particular channel. To do so, we need to return to the basics of channel models in order to have a starting point. Given that models of this nature have been tried and failed to accurately account for the above facts, it must be noted the significance of this STARTING POINT.

4.2 Textbook

What we know from our earlier studies of channels is a great starting point. We learned in the course to account for several key characteristics in channel functioning. First, we discussed the notion of error-free communication and determined that no such communication was possible. The probabilities are simply telling a different story. We see that we can reduce the likelihood of errors but cannot dispose of them entirely. As such consideration of bit flip is very important and thusly a consideration of the probabilities of such errors occurring must be present in any model of communication channels. In addition, we discuss the notion of measuring the certainty in the information sent. We call this the entropy. The entropy is understood to me from this example. A rare event contains higher entropy, as our uncertainty is low, that, what we received was incorrect.

So with these two channel characteristics we can approach the idea of the channel capacity. A great example of this comes from David Mackay's book: Information Theory, Inference, and learning algorithms. Pg 149 describes the way we can go about measuring the amount of data (capacity) of our channel. We first consider "a particular input ensemble X, we can measure how much information the output conveys about the input by the mutual information:" [4]

$$I(X;Y) \equiv H(X) - H(X|Y) = H(Y) - H(Y|X) \tag{2}$$

Of course there is some explanation needed here. First we should address the mutual information definition. "It measures the average reduction in uncertainty about x that results from learning the value of y; or vice versa." This is analogous to a co variance matrix, we need a way to consider how the two (input,output) relate to one another and in particular in terms of the uncertainty. The essence of what we are trying to accomplish here is to get to the importance of the input.

Indeed the maximum mutual information is one way to think of the channel capacity. David provides the following definition for a capacity of channel Q:

$$C(Q) = maxI(X : Y) \tag{3}$$

The max is the optimal input distribution. This relates to our earlier principle of probability distributions. This also, connects the relationship between mutual information and channel capacity. So of course, we will assume we have chosen such a distribution when making this models. This is quite a reasonable assumption based on the numerable efficient codes we discussed in the duration of this course. Finally, this segways perfectly into our non shannon approach depicted by Rajan and his peers.

5 Non Shannon

In the modern approach to channel capacity we use the power of a non linear shrödinger equation. This approach claims to add new bounds on the dispersive fiber optic channel.[1] In order to do so the author has several key assumptions.

5.1 Assumptions

- Linear Dispersion
- Sufficiently large traveling distances

He uses this approach to create a bound, that we can consider as the capacity for fiber optics.

5.2 Non linear Shrödinger equation (NLSE)

The channel capacity is defined as

$$C(Y;X) = \max_{p(A)} I(Y;X) \,, \tag{37}$$

where $I(Y;X)$ is the mutual information between Y and X, defined as

$$I(Y;X) = E\left\{ \log\left(\frac{p(Y/X)}{E\{p(Y/X)\}} \right) \right\}. \tag{38}$$

The maximization in (37) is carried out over the probability densities of the transmitted signal x_j subject to the power constraint

$$\sum_j \left| x_j \right|^2 = P \,, \tag{39}$$

where P is the total available power.

Figure 3: The Channel capacity according to Rajan [1]

In the linear regime where the classical discrete-time intersymbol interference channel model holds, the channel capacity is given as [6]

$$C_G = \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln\left(1 + \frac{P\left|R_h(e^{-j\theta})\right|^2}{\sigma_w^2}\right) d\theta \,, \tag{40}$$

where R_h is the discrete Fourier transform of h_n and P the launch power. The channel capacity in (40) is achieved only when the input symbols x_k are i.i.d. Gaussian random variables. In many communications systems this condition may not hold. Practical lower bounds however exist to evaluate the capacity of these systems [7]. One such bound, suitable for on-off keying (OOK) WDM systems is given as

$$C_{LWMF} = C_b\left(\frac{P}{\sigma_w^2} \frac{1}{2\pi} \int_{-\pi}^{\pi} \ln\left|R_h(e^{-j\theta})\right|^2 d\theta\right) \tag{41}$$

where C_b is the capacity of the binary-input Gaussian memoryless channel [7]. Equations (40) and (41) set the capacity limits on the strictly dispersive channel (i.e.,

Figure 4: Discrete-time [1]

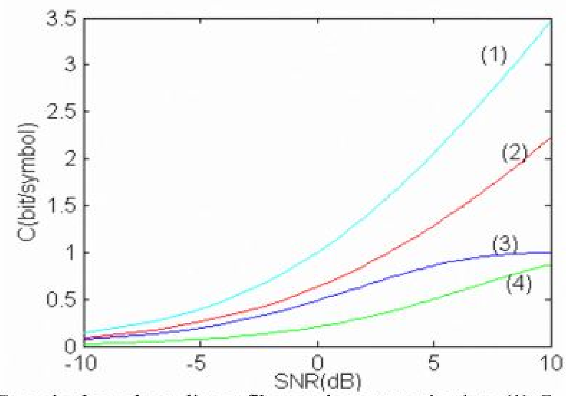

Fig. 3. Capacity bounds on linear fiber optic communication. (1) C_{awg}, (2) C_G, (3) C_b, (4)C_{LWMF}.

Figure 5: Plot from Rajan [1]

This is a great deal of information to digest. Initially we can recognize our important emphasis we place on mutual information. This particular equation (37) nicely relates the power to this input X. This is good as it was one of our concerns presented earlier about model fiber optics.

Now the rest of the content is a little above my pay grade so to speak. What is possible to take out of this is that we are accounting for increasingly complex scenarios. This includes making the system continuous instead of treating it as a discrete system. As well as trying to accommodate time variation and again the probability distributions.

In addition the main point being made is that the bound shown in equation (41) yields some interesting results about the capacity. Figure 6. shows us how the capacity is nearly 10 times that of today's achievable rates. In addition you can see clear themes from the theory I presented earlier. While it is not immediately evident if this capacity is accurate, such a finding would prove useful in testing the fiber optic channels against this theoretical capacity.

So, given what we have been able to learn about the capacity from a time dependent NLSE, what can we find from our other author?

6 Shannon

6.1 Dispersion

In this section I would like to briefly identify a key component in describing channel capacity. In order to accurately talk about channel capacity one must discuss how you are treating the dispersion in your assumptions or model descriptions.

To begin, Dispersion is not a particularly complex idea. So I will include some graphics and notes describing what the graphics are depicting.

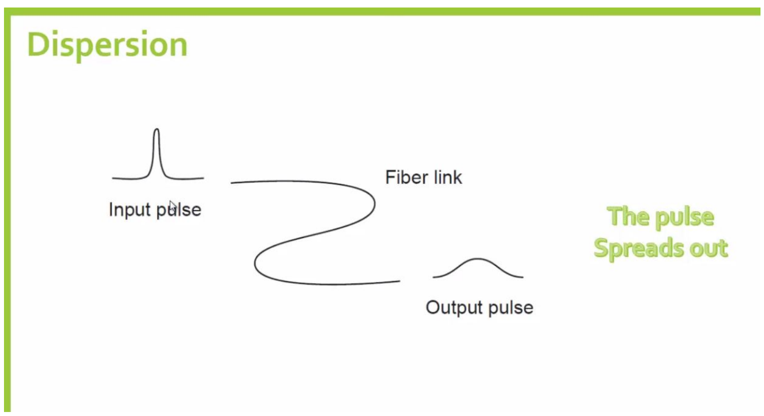

Figure 6: Dispersion is the broadening of the input pulse (in time) as the pulse travels through the optical fiber.Note the difference in wavelength.[2]

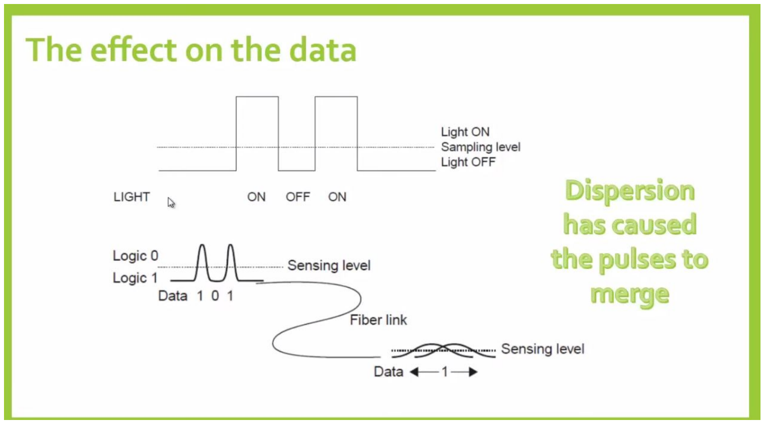

Figure 7: The effects dispersion has on Data Transmission.Where the effects are you cannot accurately discern one pulse from another, causing an 'erasure' or 'bitflip' in data transmission terms.[2]

So what you should take away from this is the importance of identifying how you will treat the dispersion in your solution to channel capacity. The next approach is by an author named Jau Tang, and utilizes shannon's work.

6.2 Shannon via Tang

We will first identify the channel capacity as interpreted by Jau Tang using Shannons work. Tang broke fiber optics into three categories. Those with no dispersion, those with constant dispersion, and those with length-dependent dispersion.[6]

6.3 No Dispersion

$$\mathbf{SC}_{\mathrm{MAX}} \approx \log_2 \left[1 + \left(\frac{32\pi^2}{81} \right)^{\frac{1}{3}} \left(N_S^2 N_c \gamma_L P_W \omega_c \right)^{-\frac{2}{3}} \right]$$

Figure 8: SC_{Max} with No Dispersion

$$P_{\mathrm{MAX}} \approx \left(\frac{4\pi^2}{3} \right)^{\frac{1}{3}} \left(N_S N_c^2 \gamma_L^2 \right)^{-\frac{1}{3}} \left(P_W \omega_c \right)^{\frac{1}{3}}.$$

Figure 9: P_{Max} with No Dispersion

6.4 Constant Dispersion

$$\mathbf{SC}_{\mathrm{MAX}} \approx \log_2 \left[1 + \frac{2}{3} N_s^{-1} (\gamma_L P_W \omega_c)^{-\frac{2}{3}} \right. \\ \left. \times \left(\frac{2\alpha}{\pi\beta\omega_c^2} \ln \left[\frac{\beta N_c^2 \omega_c^2}{4\alpha} \right] \right)^{-\frac{1}{3}} \right]$$

Figure 10: SC_{Max} with Constant Dispersion

$$P_{\mathrm{MAX}} \approx \left[\frac{2\alpha\gamma_L^2}{\pi\beta\omega_c^2} \ln \left(\frac{\beta N_c^2 \omega_c^2}{4\alpha} \right) \right]^{-\frac{1}{3}} \left(P_W \omega_c \right)^{\frac{1}{3}}.$$

Figure 11: P_{Max} with Constant Dispersion

6.5 Length-dependent Dispersion

$$\mathbf{SC}_{\mathrm{MAX}} \approx \log_2 \left[1 + \frac{2}{3} N_s^{-1} (\gamma_L P_W \omega_c)^{-\frac{2}{3}} \left(\frac{4\alpha}{\pi \zeta \beta \omega_c^2} \right)^{-\frac{1}{3}} \right.$$
$$\left. \times \left(\ln \left(\frac{\zeta \beta N_c^2 \omega_c^2}{\alpha} \right) - 1 \right)^{-\frac{1}{3}} \right]$$

Figure 12: SC_{Max} with Length Dependent Dispersion

$$P_{\mathrm{MAX}} \approx (P_W \omega_c)^{\frac{1}{3}} \left(\frac{4\alpha \gamma_L^2}{\pi \zeta \beta \omega_c^2} \right)^{-\frac{1}{3}} \left[\ln \left(\frac{\zeta \beta N_c^2 \omega_c^2}{4\alpha} \right) - 1 \right]^{-\frac{1}{3}}$$

Figure 13: P_{Max} with Length Dependent Dispersion

Similarly to our other author there is a great deal of content in these equations. I will try my best to shed some light on them and clarify what Tang is trying to accomplish. N_c times w_c provide the bandwidth. N_s represents a multispan (multiple fiber channels) system. MuL is accounting for the refractive index change do to the kerr effect. Diving deeper into these equations is a great deal more work than I am capable of for this paper. The point with the elements I have identified is the concept of accounting for the added complexities. While conceptually this is easy to understand, in practice the math is many years worth of study.

If however, we take a step back and look into the bigger picture what we see is a very clear central theme. The shannon capacity in the form

$$C = log_2(1 + \frac{S}{N}) \tag{4}$$

This is distinctly similar to what Shannon published in his 1948 paper seen here [5]

$$C \leq W \log\left(1 + \frac{P}{N}\right) \leq W \log\left(1 + \frac{S}{N}\right)$$

Figure 14: Capacity of discrete System

What is of importance here is the note of the bandwidth (W) and the peak power (S). Here Shannon is providing the capacity for an as of yet undiscovered ensemble. Where an ensemble is a type of code. In other words, we need this particular code to even attain this theoretical capacity. So yes, we are indeed limited to not just to the nature of the channel, but even the code through the channel.

So we can see that for a finite (discrete) system we are able to establish a theoretical bound. While Tang and others are working to increase the precision of this bound we still need to address one additional question? What about continuous systems? Is it possible to find such a bound?

Thankfully Shannon was here to help with this as well. According to Shannon, a continuous system would require infinite bandwidth which is not possible. Yet, he makes a clever move to evade that train of thought. Instead he points out that we need not worry of a truly continuous source. We should instead consider the continuous system to be approximate up to a certain tolerance. It is thusly this tolerance we should specify to attain transmission of a continuous source. [5]

7 Summary

So what can we say we have learned about channel capacity of fiber optic cables? Well, not as much as I thought I would have when I started out on this project. Thankfully there is several lessons at hand. The first and most promising lesson is that making a simple math model accommodate the complexities of the 'real world' takes a great deal of mathematical understanding. However, we must start with the basics and slowly build onto the model.

Second, when asking about the channel capacity we have several key considerations to make. These include the optimal selection of code, the limitations of transportation medium (fiber optic), and the general noise in the system. The greater the limitations of the medium, such as manufacturing error, attenuation, and noise from environment the greater the negative effect on channel capacity. We should hope for our sake that the fibers in the ground already had engineers concerned with this matters. Otherwise, we may find great variability and inconsistency among channels of one distributer/manufacturer to another.

While many including the authors presented in this paper are working tediously to try and answer the channel capacity of fiber optics, the answer was started many years ago in 1948 by Claude E Shannon. We may have little regard for what this truly means in our day to day lives but the work this man contributed to us has made everything we use daily possible. He is for all purposes the father of information theory and his ideas are laden throughout the contents of this paper.

8 Conclusions

Between the competing ideas, we see that neither is the victor in describing channel capacity. This is evident by the fact that we have additional recommendations and limitations present from both authors. They do provide us with ideas on how to improve on the infrastructure already in place.

Additional room for research would include, making additional efforts to alternative approaches for the information theory models. Taking time to consider cheap repairs. Finally, research and develop the next generation fiber optics that will outlast at least 100 years of exponential data growth.

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Portfolio Optimization: New Data Set

May 12, 2015

Properly Labeled Graph (PLG)

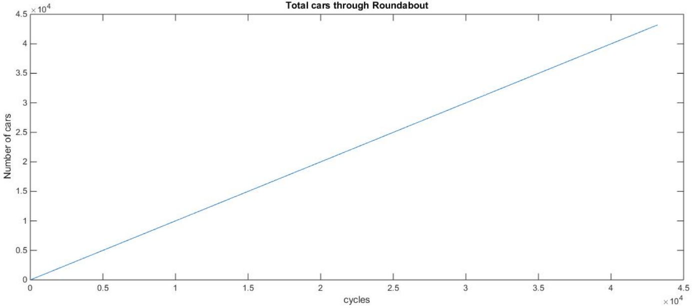

Figure: Cars in 24 hrs

Added Value

- Building a model always involves starting with a simple ideal (easy to formulate) problem and then adding each complexity as time and money allow
- Math models are inescapably dependent on computers. Computers require coding theory and knowledge to implement. Learn coding theory.
- Interpreting your model results will take time and understanding. The more you design you model with that in mind the more likely you will be to easily understand what your output means.

Proposed Investment Portfolio

I was able to work with Dr. Kirby and acquire some new data sets for the portfolio. With the new data set I reran my original program to solve the Markowitz solution. What I found was hardly a reason to drop out of school and start an investment firm.

('JPM')	0	0	0	0	0	0
('VALE-P')	0.084378	0.04827	0.046778	0	0	0
('YHOO')	0	0	0	0	0	0
('EXEL')	0	0	0	0	0	0
('RIG')	0.006038	0	0	0	0	0
('TWTR')	0.644797	0.586313	0.519717	0.398212	0	0
('WFC')	0.005713	0	0.029808	0.024626	0.109301	0
('AMAT')	0	0	0	0	0	0
('PHH')	0	0.002176	0	0	0.074631	0
('HAL')	0	0	0	0	0	0
('EMC')	0	0.024462	0	0.030819	0	0
('CX')	0.022527	0.018936	0.108957	0.301182	0	0
('BOJA')	0.034745	0.021986	0	0	0	0
('JCP')	0.00219	0	0	0	0	0
Reward	1.005666	1.012811	1.022711	1.026005	1.040806	1.098802
Risk	0.006774	0.015291	0.024809	0.00302	0.004157	0.001828
Days ago	90	60	30	14	7	2

Project 4: Game Theory and Application

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May 7, 2015

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1 Assignment

M435 Project 4: Game Theory

1.1 Problem 1

The bomber sortie problem can be represented by the following payoff matrix, where the column player is the bomb placement and the row player is which plane the defender should attack:

	Bomber	Support
Bomber	.8	.9
Support	1	.5

- v is .8333
- $\bar{x}$ = .6667, .3333
- $\bar{y}$ = 0.8333, 0.1667

Interpretation

These values show that $\frac{2}{3}$ of the time the bomb should be placed on the bomber and $\frac{1}{3}$ of the time the bomb should be placed on the support plane. For the defender, there is a probability of $\frac{5}{6}$ that attacking the bomber is the best strategy and $\frac{1}{6}$ that attacking the support plane is the best strategy. The value of game(expected value) is 0.8333.

1.2 Problem 2

Use maximin game theory to determine a portfolio using all but your last row of data. How does the portfolio compare with the Markowitz portfolio? Which gives you the most portfolio?

1	0	'goog'		0	0.057345
2	0	'aapl'		0	0
3	0	'msft'		0	0
4	0.7331	'gspc'		0	0.689574
5	0	'djia'		0	0
6	0	'hon'		0	0.006244
7	0.1826	'ibm'		0	0.099211
8	0	'wmt'		0	0
9	0.0743	'ual'		0	0.120419
10	0.01	'ford'		1	0.027208
11	0	'ba'		0	0
12				mu = 2^-3	mu=2^10
13	Game Thoery			Markowitz	
14	Risk	1.60E-05		0.002297	1.46E-05
15	Reward	0.9992		1.0062	0.999312

Figure 1: What you see here is that Markowitz is the superior model. Even with the lowest risk possible, Markowitz losses you less money. At the highest risk, Markowitz is yielding money.

1.3 Problem 3

This problem can be represented by the matrix below where one of the prisoners is the row player and the other is the column player.

	Testify	Silent
Testify	(-10,-10)	(0,-15)
Silent	(-15,0)	(-5,-5)

This can then be broken up into two zero sum games: the row player and the column player each against imaginary opponents.
The row player:

	Testify	Silent
Testify	-10	-15
Silent	0	-5

The column player:

	Testify	Silent
Testify	-10	-15
Silent	0	-5

Since the games for each individual player are the same, we expect that each player should have the same strategy. Running the code for this matrix, we get that with 100% certainty each player should testify against the other. Moreover, the value of the game is -10.

1.4 Problem 4

The Cold War game of Chicken can be represented by the following payoff matrix:

	Persist	Back down
Persist	(-1,-1)	(1,0)
Back down	(0,1)	(.5,.5)

In this game we chose mutual destruction as the worst possible outcome, so we gave it a value of -1. Winning is the most desirable outcome, so we gave it a value of 1. Losing is a bad outcome, but not nearly as bad as mutual destruction, so we assigned it a value of 0. Drawing is somewhere in between winning and losing, so it was assigned a value of .5.

This can then be broken up into two zero sum games: the row player and the column player each against imaginary opponents.

The row player:

	Persist	Back down
Persist	-1	0
Back down	1	.5

The column player:

	Persist	Back down
Persist	-1	0
Back down	1	.5

Since the games for each individual player are the same, we expect that each player should have the same strategy. Running the code for this matrix, we get that with 100% certainty each player should back down every time.

1.5 Problem 5

The following is the payoff matrix for the battle of the sexes game:

	Opera	Fight
Opera	(5,4)	(2,2)
Fight	(0,0)	(3,5)

In this matrix, the first number of the pair represents the wife and the second represents the husband. We figured that the wife would be less happy at the fight than the husband would be at the opera. This game can be broken down into two games as before.

The wife:

	Opera	Fight
Opera	5	2
Fight	0	3

The husband:

	Opera	Fight
Opera	4	0
Fight	2	5

Running these zero-sum games through the code we get that the wife should choose the fight with 83% certainty and the husband should choose the opera with 71% certainty. This is interesting, because neither chooses the event that they would go to if they were to go alone, however they are both willing to sacrifice in order to go together.

1.6 Problem 6

There is a Game Theory homework posted by Dr.Kirby. Two students, Yiming and Eric, both found the standard solution by Google and they knew each other did find the solution successfully. They could choose to plagiarize, however, if they both plagiarize, Dr.Kirby will detect that they have plagiarized and they would both receive 0 point. If they choose not to plagiarize, they would both receive 80 points. Otherwise, if, say, Eric chooses to plagiarize and Yiming does not, Eric would receive 100 points and Yiming receives 60 points because what Eric did is so excellent and Yiming's homework looks mediocre compared with Eric's!

The following is the payoff matrix for the homework plagiarism game: (Row player is Yiming and column player is Eric)

	Plagiarize	Not
Plagiarize	(0,0)	(60,100)
Not	(100,60)	(80,80)

This game can be broken down into two games as before.

Yiming:

	Plagiarize	Not
Plagiarize	0	60
Not	100	80

Eric:

	Plagiarize	Not
Plagiarize	0	60
Not	100	80

Running the code for this matrix, we get that with 100% certainty both Yiming and Eric would choose not to plagiarize every time. The value of the game is 60.

Moreover, it's not difficult to find that the strategy we got using Game Theory matches the strategy that we would pick intuitively, for the reason that the potential lose of plagiarism is far larger than the gain. Therefore, it is proper to apply the minimax method to this situation.

Markowitz Portfolio Optimization
Theory and Application

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April 23, 2015

1 Introduction

Suppose you have a data matrix comprised of several stock options over a set period of time. How do you choose the optimal collection of stocks such that you maximize your returns for a given level of risk? What Markowitz found was an elegant equation.

$$Min - \sum_j x_j(E(R_j)) + \mu \sum_{j,k} x_i x_j (E(\hat{R}_j \hat{R}_k)) \tag{1}$$

What we realized rather quickly is there does not exist a closed form solution to this problem. Instead we use the tried and tested linear approximation. By transforming this problem into Matrix multiplication, we are able to quickly and (with desired accuracy) approximate the optimal solution, using only linear algebra.

2 Problem Formulation

Given that Markowitz did the heavy lifting all that remains for us is to create a model that can take in data from Market analysis, transform that data into a quadratic problem, then optimize the approximation to the solution to a determined stopping point or (tolerance). To do so, we break this task into several sub steps.

1. Develop a solution technique for the optimization of quadratic programs
2. Cast the Markowitz portfolio problem as a quadratic program
3. Explore the expected returns for allowable risks.

2.1 Assumptions

- Only 11 investment opportunities from an assigned data matrix
- Allowable solutions are those determined to be within an allowable range of the true solution
- Customer's allowable risks can be captured by a single scalar parameter

2.2 Variables

All of the return data is held within a $M \times N$ matrix, $\mathcal{X}$, whose columns are different stocks and whose rows represent the change in the stocks value through time. The decision variable, x, will be the distribution of funds across the considered stocks. This will be a vector whose components are non-negative and sum to unity. The determination of this is the goal of this project. Additionally, there will be a risk parameter, μ, the purpose of which will be described later.

2.3 Interpretation as a quadratic program

The notion of risk and reward must be formalized quantitatively. A simple model for the reward is to consider the mean over time of each stock and the relative quantity of that stock bought. This will be achieved by introducing a vector whose components are the mean returns of each financial instrument.

$$c_i = \frac{1}{M}\sum_{j=1}^{M}\mathcal{X}_{ji}, \qquad 1 \le i \le N \tag{2}$$

Therefore the reward will be $\hat{R} = c^T x$. A reasonable characterization of risk is the variance of the expected reward.

$$\begin{aligned} Var(\hat{R}) &= E\left[\left(\hat{R} - E(\hat{R})\right)^2\right] \\ &= E\left[\left(\sum_j x_j(R_j - E(R_j))\right)^2\right] \end{aligned}$$

then by substituting $(R_j - E(R_j)$ by $\tilde{R}_j$ the variance may be simplified to a form where the decision variable may be extracted.

$$\begin{aligned} Var(\hat{R}) &= E\left[\left(\sum_j x_j\tilde{R}_j\right)^2\right] \\ &= \sum_j\sum_k x_j x_k E(\tilde{R}_j\tilde{R}_k) \\ &= x^T Q x \end{aligned} \tag{3}$$

Where the expectation of the mean subtracted financial instruments has been wrapped up in the matrix Q. This problem may now be formulated as an optimization problem.

$$minimize \qquad \frac{1}{2}x^T Q x - c^T x \tag{4}$$

Finally the constraints of the problem must be applied. Since the decision variable is the relative distribution of funds its components must sum to one and be non-negative. The first may be enforced by the use of Lagrange multipliers and the second may be achieved by barrier methods. This amounts to including a monotonic term that grows very large for small values of the components. Here a logarithmic term will be used. Some optimal solutions will have components that are zero which can be handled by decreasing the effect of this term as the solution progresses. The inclusion of these terms results in the final quadratic program.

$$minimize \qquad \frac{1}{2}x^TQx - c^Tx + y^T(b - Ax) - \beta \sum_i ln(x_i) \tag{5}$$

Where y is the vector of Lagrange multipliers.

2.4 Iterative solution of the quadratic program

Taking partial derivatives of equation (4) with respect to the unknowns x and y yields a system of equations.

$$\begin{aligned} Qx - c - A^Ty - \beta\bar{X}^{-1}e = 0 \\ b - Ax = 0 \end{aligned} \tag{6}$$

Here, $\bar{X}$ is a diagonal matrix of the components of x and e is a column vector of ones. An inverted matrix will become difficult to deal with and as such a substitution may be made by introducing the variable z.

$$\begin{aligned} z &= \beta\bar{X}^{-1}e \\ \bar{X}z &= \beta e \\ \bar{X}Ze &= \beta e \end{aligned} \tag{7}$$

The non-linearity of this system does not lend it direct solution techniques, instead an initial guess will be made and the solution iteratively approached. The vectors x, y, z will be assigned some initial value and then replaced by $x + \Delta x, y + \Delta y, z + \Delta z$ in the system where $\Delta x, \Delta y, \Delta z$ are now the unknowns.

$$\begin{aligned} Q(x + \Delta x) - c - A^T(y + \Delta y) - (z + \Delta z) &= 0 \\ b - A(x + \Delta x) &= 0 \\ \bar{X}Ze + \Delta\bar{X}Ze + \bar{X}\Delta Ze + \Delta\bar{X}\Delta Ze &= \beta e \end{aligned} \tag{8}$$

Then by recognizing that the term $\Delta\bar{X}\Delta Ze$ should be small compared to all other terms, it may be dropped to linearize the system thus obtaining its final form.

$$\begin{aligned} Q\Delta x - A^T\Delta y - \Delta z &= -Qx + c + A^Ty + z \\ A\Delta x &= Ax - b \\ \Delta\bar{X}Ze + \bar{X}\Delta Ze &= \beta e - \bar{X}Ze \end{aligned} \tag{9}$$

This system may be written succinctly in block matrix form.

$$\begin{pmatrix} A & 0 & 0 \\ -Q & A' & -I \\ Z & 0 & \bar{X} \end{pmatrix} \begin{pmatrix} \Delta x \\ \Delta y \\ \Delta z \end{pmatrix} \begin{pmatrix} b - Ax \\ c + Qx - A'y + z \\ \beta * e - \bar{X} * Z * e \end{pmatrix}$$

From this form progressively better solutions may be found by solving for $\Delta x, \Delta y, \Delta z$, updating x, y, z, and repeating until the prescribed tolerance is reached. This is done by summing the norms of the right hand side vectors and comparing to the tolerance.

$$||b - Ax||_2 + ||c + Qx - A'y + z||_2 + ||\beta * e - \bar{X} * Z * e||_2 < TOL \tag{10}$$

Finally, to ensure that the stepping of the variables forward does not result in negative values of x the values at each iteration will be updated with a term reminiscent of successive under relaxation i.e. $x_{new} = x + \epsilon^* \Delta x$ where the step size ϵ^* is calculated as such.

$$\begin{aligned} \epsilon &= \tfrac{1}{\min\limits_i \Delta\theta_i / \theta_i} \\ \epsilon^* &= min(\epsilon, 0.9) \end{aligned} \tag{11}$$

3 Model Development

In the application of this quadratic program framework to the Markowitz portfolio optimization the matrix Q may be pre-multiplied by some positive real scalar μ to vary its relative importance to the problem. Large values of μ then force the risk to play a greater role in the optimization thus returning a lower risk portfolio. The full effects of this risk parameter will be explored in later sections.

4 Results

4.1 Problem 3

The above table shows that increasing the risk also increases the reward. However, there is a huge caveat. What is demonstrated with Figure 1, is that for a small risk you are actually looking at taking on a loss. Where as the highest risk is only a return of .5 percent. This is not exceptionally exciting, a typical savings account can match those returns. A CD can easily return 1.3 percent.

μ	0.125	0.25	0.5	1	2	4	8	16	32	64	128	256	512	1024
goog	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.040	0.055	0.235	0.206	0.204	0.237	0.237
aapl	0.000	0.000	0.000	0.407	0.637	0.753	0.640	0.347	0.100	0.039	0.014	0.012	0.012	0.012
msft	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.016	0.012	0.016	0.026	0.028	0.027
gspc	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.422	0.303	0.348	0.394	0.393	0.402
djia	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.014	0.070	0.087	0.067	0.050	0.047
hon	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.021	0.051	0.073	0.062	0.047	0.032
ibm	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.044	0.056	0.065	0.054	0.038	0.036
wmt	0.000	0.000	0.000	0.000	0.000	0.000	0.043	0.031	0.045	0.028	0.012	0.019	0.039	0.048
ual	0.000	0.000	0.000	0.000	0.000	0.000	0.141	0.471	0.199	0.109	0.111	0.084	0.064	0.065
ford	1.000	1.000	1.000	0.593	0.363	0.247	0.175	0.111	0.069	0.078	0.047	0.045	0.055	0.057
ba	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.015	0.019	0.021	0.032	0.037	0.037
Risk	0.00233	0.00233	0.00233	0.00080	0.00036	0.00024	0.00017	0.00009	0.00003	0.00002	0.00002	0.00002	0.00002	0.00002
Reward	1.0051	1.0051	1.0051	1.0040	1.0034	1.0031	1.0026	1.0018	1.0002	1.0000	0.9997	0.9996	0.9996	0.9996

4.2 Problems 4 and 5

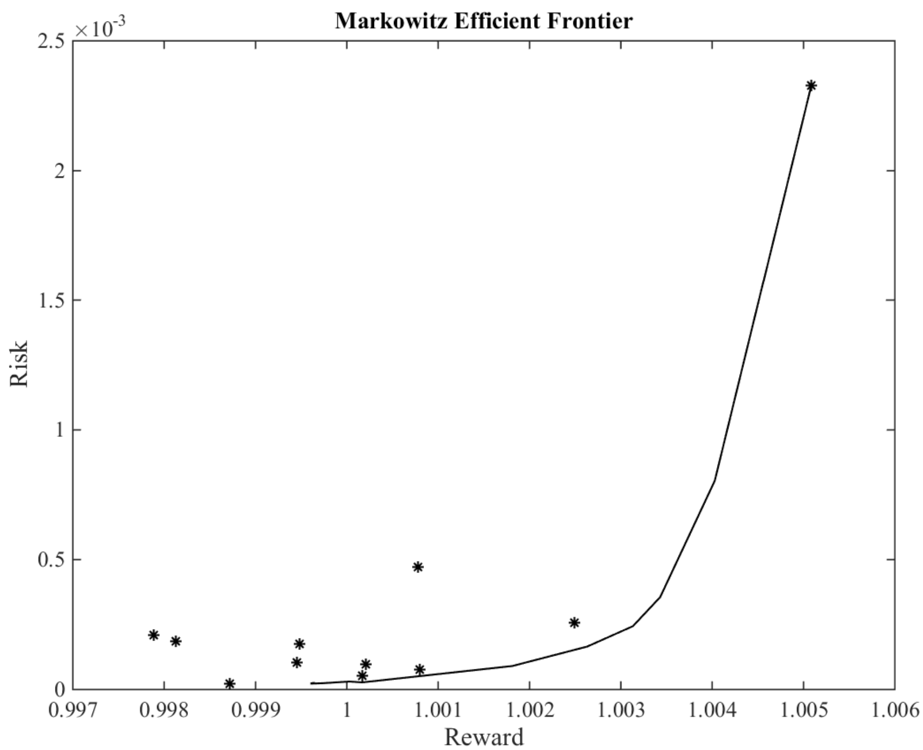

Figure 1: The solid line indicates the notable efficient frontier from standard Markowitz theory. Only optimal portfolios will lie on this border. Asterisks denote the expected returns from investing in only single financial instruments.

Figure 1 illustrates the famous efficient frontier provided by Markowitz portfolio theory. This curve represents the highest yielding portfolios for each allowable risk. The individual financial instruments (shown as asterisks) all lie on the lower yielding side of the curve as investing in only a single stock has a lower return than a diverse portfolio. The only exception to this is at the highest risk Markowitz theory shows that only the Ford stock should be purchased.

5 Robustness

The easiest way to test our model is to try other quadratic functions, and note the amount of steps to converge to the correct solution. Checking the vector $\vec{x}$ will show the two zero's for the quadratic function.

6 Model Evaluation

6.1 Added Value

There is more to investing than looking at the average return of an instrument. The variance of the return is very important to consider, even though it is not typically thought of. Using this model, the return variance is taken into consideration. This model adds an extra important variable to the decision making process when investing in the stock market.

6.2 Model Strengths

- Well written to easily use
- Easy to adjust tolerance
- Easy to keep track of amount of runs

6.3 Model Weaknesses

- Data used does not necessarily represent whole market data

7 Extensions of the Model

The most important extensions to the model are predictive methods. Three such methods have been implemented showing interesting deviations from the standard model. First, a naive method is simply to ignore some of the old data under the notion that it will have less bearing on the present behavior. In the sample calculated the first quarter of the data has been ignored. A second more interesting approach is to try extrapolating the return data and using that augmented matrix to perform the Markowitz analysis on. The third method is simply a combination of the first two. For a fixed risk of $\mu = 1$ the various compare as shown in the table below.

Case	Standard	Truncated	Extrapolated	Combined
goog	0.000	0.000	0.000	0.000
aapl	0.407	0.535	0.157	0.209
msft	0.000	0.000	0.000	0.000
gspc	0.000	0.000	0.000	0.000
djia	0.000	0.000	0.000	0.000
hon	0.000	0.000	0.000	0.000
ibm	0.000	0.000	0.000	0.000
wmt	0.000	0.000	0.000	0.000
ual	0.000	0.000	0.000	0.000
ford	0.593	0.465	0.843	0.791
ba	0.000	0.000	0.000	0.000

As the risk parameter is raised the different methods all start to homogenize and yield very similar answers. These methods were tested by withholding some of the most recent data and then evaluating the return that would have been obtained. For a risk of $\mu = 6$ the combined method yielded the best return at 1.0033 with a portfolio as shown.

μ	1
goog	0.0000
aapl	0.2507
msft	0.0000
gspc	0.0000
djia	0.0000
hon	0.0000
ibm	0.0000
wmt	0.1501
ual	0.1876
ford	0.4116
ba	0.0000

These extensions require much more testing and validation.

7.1 Extrapolation Code

While truncation is a simple task, the details of extrapolation are worthy of mention. The code simply augments the return data matrix $\mathcal{X}$.

```
load('Portfolio.mat');
[M, N] = size(X);
L = 20;
leastSquares = zeros(2,N);
Alsqr = ones(L,2);
for k=1:L
    Alsqr(k,1) = k;
end
for k=1:N
    blsqr = X(end-L+1:end,k);
    leastSquares(:,k) = lsqr(Alsqr, blsqr);
end
for k=1:L/2
    for i=1:N
        X(M+k,i) = k*leastSquares(1,i)+leastSquares(2,i);
    end
end
```

8 Conclusions

Through this investigation of optimal portfolio selections we have found that it is not possible to yield great returns with this method. Our returns, while mostly positive, were less than 1% for all cases. We believe this is caused in part by the data used to test our model. In further studies it would be advantageous to choose stocks that represent all sectors of the market.

Mulberry and Lemay RoundAbout

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[1]Breckenridge

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1 Introduction

At the intersection, Lemay and Mulberry in Fort Collins Colorado, a large volume of cars travel through the intersection every day. This heavy use has raised an important question, can we design another intersection that will improve the flow of traffic? In 2001, the City of Fort Collins explored the possibility of improving the intersection by constructing a large traffic circle. After reviewing several bids from potential construction firms, the city council rejected the bids due to the cost of the remodel. This particular intersection has been estimated to host 47,000 vehicles in an exceptionally high volume day. So the possibility of a lowered commute time would be very useful for the daily commuters, truckers and local businesses that use the intersection. Can a traffic circle keep the flow of traffic constant at a high enough rate to decrease the time spent at the Mulberry and Lemay intersection? Will a single lane traffic circle provide enough volume to effectively decrease the commute time for the vehicles using the intersection?

2 Problem Formulation

Interpreting the problem of creating a traffic circle as an engineer or mathematician is a much more complex scenario. To simulate a traffic circle that can run 47,000 vehicles in a 24 hour period several scenarios need to be confronted. Of the vehicles that travel through the traffic circle in a day, how many are turning to head a new direction? How many are staying on the same road? Lastly how can our model incorporate this flow in order to model a volume of 47,000?

2.1 Assumptions

- No accidents will occur between vehicles
- No delay will occur when vehicles merge behind vehicles in traffic circles
- No delay will occur when vehicles merge out of traffic circle
- No vehicles will pass their intended exit to continue around the traffic circle
- Each vehicles speed is constant at 10mph in the traffic circle
- Vehicles cannot enter the traffic circle traveling in the wrong direction.

2.2 Variable Definitions

First our group started with the flow of the vehicles that are traveling around the circle. We didnt want to start out complex with any of our assumptions or our variables, so the flow of vehicles coming into the intersection is equal to the flow out of the intersection. While this parameter is simple we at least had a constant to build on when we created the model. To further simplify the traffic circle we made multiple sections that the vehicles could occupy. Next the cars were assumed to be traveling at the same constant speed around the circle, progressing one section at a time. Using data found on the City of Fort Collins website each vehicle starts into the circle with an exit already specified. Also each vehicle would not be able to enter the traffic circle if another vehicle was occupying the space in the traffic circle in front of that entrance. Lastly, a cycle is run every 2 seconds, which was calculated based on several assumptions including speed, vehicle length and distance between vehicles. With that information the time for each vehicle to travel around the circle is calculated and the duration of each cycle is set to match.

3 Model Development

3.1 Hypothesis

Can a single lane roundabout, with a 10mph speed limit, flowing at the provided rates as observed by the City Of Fort Collins engineers (see City of Fort Collins, 31 May 2011) sustain a rate of 50,000 cars per day?

3.2 The Roundabout

The idea we used for constructing the roundabout was similar to a notion of a clock the progresses in discrete increments. Each tick represents what we coined a 'cycle'. As the cycles increment the system evolves to create the effect of rotating the 'slots'. These rotating slots are, in effect, our roundabout.

3.3 Entry and Exit

Our early notions where influenced by Kirchhoff's current Law. Our analogy being the vehicles going in are equal to the vehicles going out. With this in mind we set forth on designing a Que structure to govern the populating of our system vehicles. Each entry would contain Que's. The Que's would populate with the value of the vehicles exit number [1-4]. In this manner we would simply need asses the system at each cycle for exit needs based on slot match to exit value. Initially our Que's where generating exit numbers at random. Our revision was to create a more accurate exit ratio by using the data collected from city engineers. By doing so, we assured our system would generate the Que's with inflow and leave the roundabout with outflow consistent with the observed ratios.

3.4 Speed

In order to devise an accurate system we needed to ensure we had a reasonable speed. To account for this, we used a Google maps of the Mulberry and Lemay intersection. Using the scaling provided on the map, we approximated the radius of our hypothetical roundabout. This showed a rough fitting roundabout of approximately 50ft would fill the intersection. We have 12 'slots' for the entire roundabout. With an assumed average of 15ft per car and 1 car length between cars, you arrive at an intersection perimeter of 360 ft. This implies we need a radius of 58ft with those assumptions. Therefore our model should fit into the intersection with the above assumptions. With that, equation 1, 2, 3, 4, 5, 6, and 7 demonstrate how you arrive at the rate of 2sec/cycle.

4 Results

We where able to reject our hypothesis. Figure 5 shows that there is a good flow occurring. However, the end result was that we were unable to attain 50,000 cars in a 24hr day. We averaged around 43,000 cars in 24hrs. This is a few shy of the goal of 50,000.

Accurately, we where able to reflect the entry and exit rates but in doing so we feel rather confident in our recommendation that a single lane roundabout will not handle the volume of traffic as was required.

Fig 1 shows the number of cars inside the roundabout at any given time of the cycle (up to 100 cycles). This demonstrates how cars are entering and leaving the roundabout. However, it should be noted that for this plot, the flow of traffic was half of the 50k car goal.

Fig 2 is similar to the first figure except it shows the traffic flow at a rate high enough to produce 50k cars per day. There isn't much difference because the size and number of exits is the limitation here. There is an anomaly in e middle that is simply a result of the traffic pattern.

Figures 3 and 4 show the number of cars waiting in to enter the roundabout from each direction. Figure 3 is when traffic is slowed to 25k cars, and shows only a couple cars have to wait at times. Figure 4 shows what happens when 50k cars is attempted. It can be seen that the number of cars waiting increases dramatically with WB Mulberry eventually maxing out our queue at 50 cars.

5 Robustness

The easiest way to test our model is to set an arbitrary amount of vehicles and run the program. In looking at the plots you will be able to determine

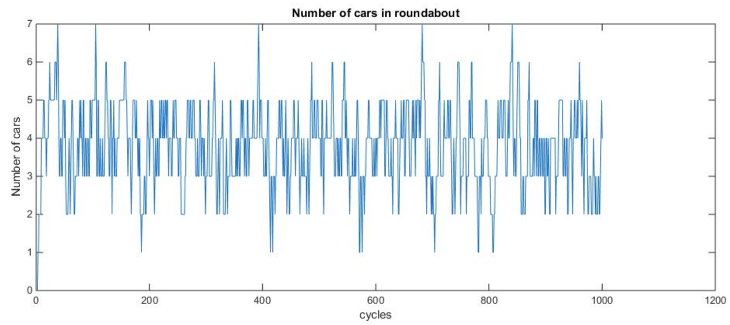

Figure 1: Cars in Round About

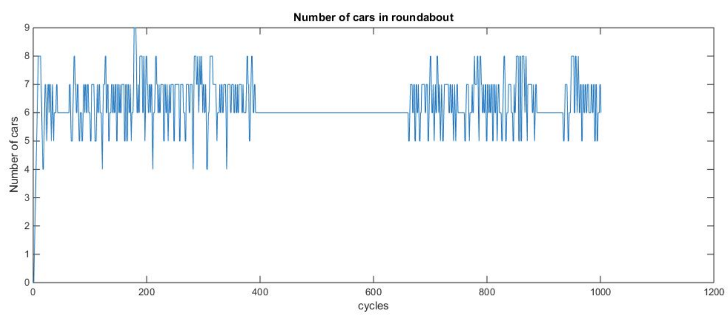

Figure 2: Cars in Round About

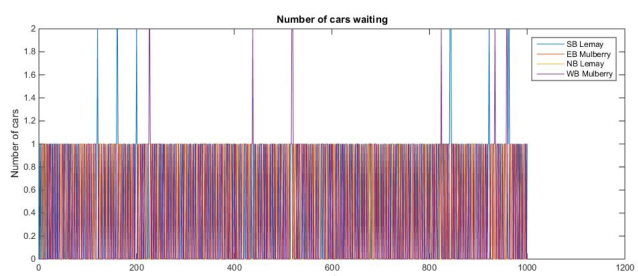

Figure 3: Cars Waiting To Enter Roundabout

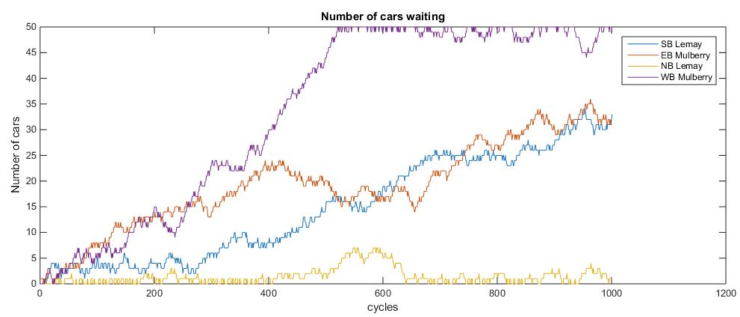

Figure 4: Cars Waiting To Enter Roundabout

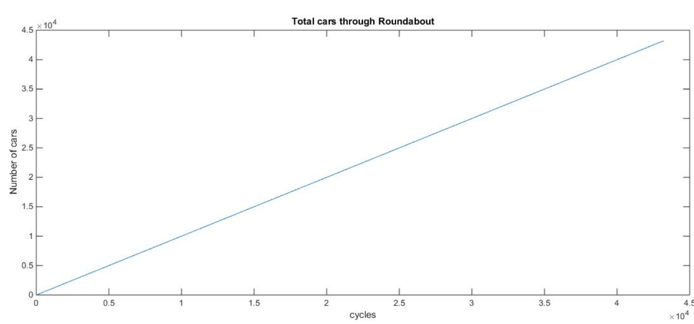

Figure 5: Total Cars in 24 HR period

if the system function as expected. (flow occurs, volatile function) OR if the system would fill and stagnate (no flow, flat horizontal line). You can also try variable cycle lengths, which would represent different delays in traffic motion. Changing the size of the roundabout could also be interesting but should be with the context of the intersection and its physical limitations. Finally, you can test flow by keeping track of the variable called all cars.

6 Model Evaluation

6.1 Added Value

What did we learn that was nontrivial?

The modeling of traffic has many moving parts (pun intended). Even our simple model illustrates the flow is critical. The simplest of interruptions in the flow preclude a total backup that saturates the system and takes a long time to clear.

Could we make predictions?

Based on the rates we found from the city we had reasonable expectations regarding what direction(s) would back up the quickest. In accordance with the data we suspected Mulberry would back up the quickest. This was confirmed with plot (fig 3).

6.2 Model Strengths

- Over Simplified for ease of use
- Discrete
- Easy to adjust Entry and Exit Rates

6.3 Model Weaknesses

- Single Lane
- Many assumptions
- Not continuous
- Static Entry and Exit Rates

7 Extensions of the Model

The pay off for us would be to attain the desired 50,000 cars per day. This would not be possible without a second lane. Overcoming the complexities associated with the rules for progressing the state of our system would provide some valuable insight into the possibility of attaining the desired flow rate. Furthermore, the flow rate give us a larger total capacity, which would answer the original question of improving the intersection with a traffic circle. In addition, addressing the issue of the vehicle size limitation would be valuable, as this too would affect the rate of vehicles per day.

8 Engineer Paper

Our Objectives are as follows:

- Use a discrete system to model a simple single lane traffic circle.
- Simplify the model to 4 exits
- Progress the model one slot increments = 1 cycle
- Each increment is the size to contain one and only one vehicle
- Assume no large vehicles (i.e. semi-trucks)
- Utilize the traffic averages as was recorded by the Fort Collins Traffic engineers To govern entry and exit rates

We have designed our model with the traffic control mechanism as a yield sign. The yield is to incoming traffic. Only if space is available can vehicles enter. We have modularized the program using an object oriented approach. We have two classes. A main and Que class. The main runs the increment , car location, and count of current round about population.

We use the notion that all vehicles will know their exit. Therefore we generate the exit number assigned to the vehicle in the Que. The Que is a simple array that holds 10 vehicles. The flow regulation into the circle was based on the observed data. We gathered the percentages of entrys and exits from each of the four directions.

The main module is responsible for mediating the production of cars in the Que, such that, the production complies with the percentages. To accomplish this we set constants with the percentages and compared them with the observed variables. These variables are the system state measurements for exit and entries as the system increments, their state changes. As their state changes, the new state of system is compared against the stored constants. The effect being we are able to adjust the Que production and flow to reflect the requirements of the observed data. In order to change the system for new observed data, you simply need re-enter the constant values to account for the new observed entry and exit rates.

Our model is unable to accommodate large vehicles. It assumes that all vehicles need the same space. Dealing with the variable space size was not an aspect we wanted to focus on as we built in the complexity of the model. One aspect we did want to focus on was correctly matching the average flow in accordance to the city data. Finally given this flow, yield and assumed car size/speed we where able to determine maximum vehicles per day.

When you assume 10 MPH you find a rate of 2sec/cycle. This means a model of one day (24hrs) is 43,200 cycles. If you wish to change the assumed speed you may adjust equation 3 to reflect the newly chosen speed.

9 Conclusions

The value added to the community from our model is that we have determined with reasonable certainty that a single lane roundabout, operating under our stated assumptions would fail to sustain a 50,000 cars per day flow rate. This failure is highly indicative of a more complex configuration required to sustain this rate. This complexity will most certainly add to construction, maintenance, and operational cost. In addition, a two lane roundabout would be unique in the city and may flow even more poorly than our single lane assumptions due to driver comfort in the roundabout. That particular intersection is a popular access for visitors to the university. Visiting drivers would be likely also unfamiliar with such a design, adding to congestion and collision rates.

Additional room for research would include adding an additional lane. Also, increasing the radius of the Roundabout. Adjusting the assumed time and distances for vehicles.

10 Equations

$$1car + 1carlength = 24(cars + length)perroundabout \tag{1}$$

$$14ft/car * 2 * 12cycles/roundabout = \tag{2}$$

$$2 * \pi * X = 360ft \tag{3}$$

$$2 * \pi * 58ft \approx 360ft \tag{4}$$

$$\frac{360\,\text{ft}}{1\,\text{roundabout}}\,\frac{1\,\text{sec}}{15\,\text{ft}} = 24\,\text{sec/roundabout} \tag{5}$$

$$\frac{24\,\text{sec}}{4\,\text{exits}} = 6\,\text{sec/exit} \tag{6}$$

$$\frac{6\,\text{sec}}{1\,\text{exit}}\,\frac{1\,\text{exit}}{3\,\text{cycles}} = 2\,\text{sec/cycle} \tag{7}$$

11 Program

```matlab
%% Main program
function main()

% initialize roundabout
R=zeros(1,12);

totalCars=1000;

% initialize % constants
ratioC = [19.5 25.2 15 40.4;16.3 31.5 20 31.50];
% initialize % running
r=min(size(ratioC));
c=max(size(ratioC));
ratioR = zeros(r,c);

% initialize running with random values
for i=1:r
    for j=1:c
        ratioR(i,j)=ratioC(i,j)+rand(1,1)-0.5;
    end
end

% initialize queues with locations
Q1 = Queue(1);
Q2 = Queue(4);
Q3 = Queue(7);
Q4 = Queue(10);

Qs=[Q1,Q2,Q3,Q4];

% test
% x=rand(1,1);

cycles = 43200;

s=max(size(Qs));
Counts=zeros(cycles,(2*s)+1);
allCarsTotal=zeros(cycles,1);
allCars=0;

for k=1:(cycles+1)

    % tasks to complete each cycle
    rotate      % rotate cars around roundabout\
    exitRnd     % check if any can exit
    enterRnd    % move cars into roudabout

f=1;    % flow rate (cycles/car)

if floor(k/f)==k/f
    addCars     % put more cars in queues
end
    % Get counts
    Counts(k,1)=getRoundCount;
    for j=1:max(size(Qs))
        Counts(k,j+1)=Qs(1,j).getWaiting;
    end
end

%x=(1:cycles,i=1);
figure
R=Counts(1:end,1);
plot(R);
%axis([1 200 0 10])
title('Number of cars in roundabout');
xlabel('cycles');
ylabel('Number of cars');

qcount=Counts(1:end,2:s+1);
figure
plot(qcount);
title('Number of cars waiting');
xlabel('cycles');
ylabel('Number of cars');
legend('SB Lemay','EB Mulberry','NB Lemay','WB Mulberry');
%axis([1 200 0 11])

% qwait=Counts(1:end,(s+2):end);
% figure
% plot(qwait);
% title('Wait time');
% xlabel('cycles');
% ylabel('Number of cycles');
% legend('SB Lemay','EB Mulberry','NB Lemay','WB Mulberry');
%axis([1 200 0 11])

figure
plot(allCarsTotal)
title('Total cars through Roundabout');
xlabel('cycles');
ylabel('Number of cars');

%% functions used in main

% recalculate running %'s
    function incCars(q,exit)

        % get counts in each direction
        dirCount=zeros(r,c);
        for i=1:r
            for j=1:c
                x=totalCars*ratioR(i,j)/100;
                dirCount(i,j)=round(x);
            end
        end
        % inc incomming
        dirCount(1,q)=dirCount(1,q)+1;
        % inc outgoing
        dirCount(2,exit)=dirCount(2,exit)+1;
        % inc total cars
        totalCars=totalCars+1;

        for i=1:r
            for j=1:c
                ratioR(i,j)=dirCount(i,j)/totalCars*100;
            end
        end
    end

% get next incomming
    function inQ=getNextIn()
        maxDiff=0;
        inQ=1;
        for i=1:length(ratioC)
            diff=ratioC(1,i)-ratioR(1,i);
            if diff>maxDiff
                maxDiff=diff;
                inQ=i;
            end
        end
    end

% get next exit
    function outQ=getNextOut()
        maxDiff=0;
        outQ=1;
        for i=1:length(ratioC)
            diff=ratioC(2,i)-ratioR(2,i);
            if diff>maxDiff
                maxDiff=diff;
                outQ=i;
            end
        end
    end


% get count of cars inside roundabout
    function count=getRoundCount()
        count = 0;
        for i=1:max(size(R))
            if R(1,i)>0
             count=count+1;
            end
        end
    end

% add cars to queues
    function addCars()
 %       exits=max(size(Qs));
 %       for i=1:exits    % for each exit point
            q=getNextIn();
            e=getNextOut();
            eLoc=Qs(1,e).getLoc();
            Qs(1,q).addCar(eLoc);
            incCars(q,e);
 %       end
    end


% enter cars into roundabout
    function enterRnd()
        % check on-comming traffic
        s=max(size(Qs));
        for i=1:s
            loc=Qs(1,i).getLoc;
            if oncomming(loc) > 0
                R(1,loc)=Qs(1,i).getNext();
                Counts(k,s+1+i)=0; % reset wait
            else
                if k>1
                    Counts(k,s+1+i)=Counts(k-1,s+1+i)+1;
                end
            end
        end
        allCars=allCars+1;
        allCarsTotal(k,1)=allCars;
    end

% check oncomming traffic
    function clear = oncomming(q)
        clear = 0;
        if q==min(size(Qs));
            if R(1,q)==0 && R(1,max(size(R)))==0
                clear = 1;
            end
        else
            if R(1,q)==0 && R(1,q-1)==0
                clear = 1;
            end
        end
    end


% check if any cars are at their exit
    function exitRnd()
        for i=1:max(size(Qs))    % for each exit point
            % get location
            loc=Qs(1,i).getLoc;
              % check if car = exit point
            if R(1,loc)== loc;
                R(1,loc)=0;
            end
        end
    end
```

```matlab
% rotate cars around roundabout
    function rotate()
        temp=R(1,12);

        for i=1:(max(size(R))-1)
            R(1,13-i)=R(1,12-i);
        end
        R(1,1)=temp;

    end
end
```

```matlab
 classdef Queue < handle

    properties (GetAccess='private',SetAccess='private')
        location    % location of queue in roundabout
        waiting     % number of cars waiting
        Q
    end

    % public methods
    methods
        % creation of new queue
        function self=Queue(loc)
            self.Q=zeros(1,50); % create empty queue
            self.location=loc;  % set location
            self.waiting=0;     % initialize waiting
        end

        % return number of cars waiting
        function w = getWaiting(self)
            w = self.waiting;
        end

        % return location of queue
        function l = getLoc(self)
            l = self.location;
        end

        % return the next car in line
        function n = getNext(self)
            n = self.Q(1,1);            % return next car
            for i=1:(max(size(self.Q))-1)   % incriment cars up
                self.Q(1,i)=self.Q(1,i+1);
            end
            if self.waiting > 0
                self.waiting=self.waiting-1;   % decriment cars waiting
            end
        end

%         % add car to line
%         function addCar(self,qs)
%             % get total number of exits
%             exits=max(size(qs));
%             if self.waiting < max(size(self.Q))
%                 % get random q to exit at
%                 q=round((exits-1)*rand(1,1))+1;
%                 % get location of that q
%                 loc=qs(1,q).getLoc;
%                 % put that location next in line
%                 self.Q(1,self.waiting+1) = loc;
%                 self.waiting=self.waiting+1; %incriment waiting
%             end
%         end

 % add car to line
        function addCar(self,exit)
            if self.waiting < max(size(self.Q))
                self.Q(1,self.waiting+1)=exit;
                self.waiting=self.waiting+1; %incriment waiting
            end
        end

    end

end
```

References

[1] "Intersection Turning Movement Report." City of Fort Collins Public Records. City of For Collins, 31 May 2011. Web. 26 Mar. 2015.