

ANSC 446, IB 416, Problem Set 1 Answers

1. A population of flies includes 400 females, with 250 wild-type and 150 recessive mutants, and 600 males, with 375 wild-type and 225 mutants.
 - a. If a single fly is sampled from this population, what is the probability that it is wild-type?
 - b. If a single fly is sampled from this population, what is the probability that it is a female?
 - c. If a single female fly is sampled from this population, what is the probability that it is wild-type?
 - d. If a single wild-type fly is sampled from this population, what is the probability that it is a female?
 - e. If a single fly is sampled from this population, what is the probability that it is wild-type female?
 - f. If two flies are sampled from this population, what is the probability that they are one wild-type female and one mutant male (in any order)?
 - g. If nine flies were sampled from this population, what is the probability that four were wild-type females, three were mutant females and 2 were mutant males?
 - h. If one male fly and one female fly were sampled from this population and mated, what is the probability that their first offspring would be a mutant?

	Wild Types	Mutants	Total
Females	250	150	400
Males	375	225	600
Total	625	375	1000

- a. $P(\text{wild type}) = 625/1000 = 5/8$
- b. $P(\text{female}) = 400/1000 = 2/5$
- c. $P(\text{wild type/female}) = 250/400 = 5/8$
- d. $P(\text{female/wild type}) = 250/625$
- e. $P(\text{wild type female}) = P(\text{wild type/female}) * P(\text{female}) = (250/400)*(400/1000) = 1/4$
- f. $\frac{n!}{s!t!} P(\text{wild type female})^s * P(\text{mutant male})^t =$
 $\frac{2!}{1!1!} (1/4)^1 * (225/1000)^1 = .1125 = 9/80$
- g. $\frac{9!}{4!3!2!} (1/4)^4 * (150/1000)^3 * (225/1000)^2 = 8.41 \cdot 10^{-4}$

h. $P(\text{mutant}) = P(\text{mutant/all 6 mating types}) * P(\text{mating types})$

$$P(aa/\text{female}) = 150/400 = .375$$

$$P(aa/\text{male}) = 225/600 = .375$$

-unable to detect heterozygotes, so...

$$q_a = \sqrt{.375} = .61$$

$$q_A = 1 - .61 = .39$$

$$P(Aa/\text{male}) = P(Aa/\text{female}) = 2pq = 2 * q_a * q_A = 2 * .61 * .39 = .48$$

$$P(m/AA*AA)P(AA*AA) = 0$$

$$P(m/AA*Aa)P(AA*Aa) = 0$$

$$P(m/AA*aa)P(AA*aa) = 0$$

$$P(m/Aa*Aa)P(Aa*Aa) = [1/4](.48)(.48) = .0576$$

$$P(m/Aa*aa)P(Aa*aa) = [1/2](.48)(.375) + [1/2](.375)(.48) = .18$$

$$P(m/aa*aa)P(aa*aa) = [1](.375)(.375) = .140625$$

$$= .378$$

2. Coat color in Angus cattle is controlled by a single pair of alleles. The **B** allele is dominant and produces black color. The **b** allele is recessive and produces red color when homozygous. Consider a mating between two heterozygous black Angus.

- What is the probability of a black offspring?
- What is the probability of a red offspring?
- What is the probability that the first three offspring alternate in color (black-red-black or red-black-red)?
- How many birthing orders could produce two black and one red offspring?
- What is the probability among three offspring that two are black and one is red?

	B	b
B	BB	Bb
b	Bb	bb

a. **3/4**

b. **1/4**

$$c. (3/4)(1/4)(3/4) = 9/64 \quad (1/4)(3/4)(1/4) = 3/64 \quad (9/64) + (3/64) = \mathbf{12/64}$$

$$d. B b B, B B b, b B B = \mathbf{3}$$

$$e. \frac{3!}{2!1!} (3/4)^2 (1/4)^1 = \mathbf{27/64}$$

3. For this problem, consider that blond and brown hair are controlled by a single locus. In the United States, the probability is 3/7 that a brown haired individual is homozygous. A recessive blonde haired mother (bb) is mated with a brown haired father (B_) and they have three children with brown hair. What is the probability that the father is homozygous?

$$\frac{P(3B \text{ children} | bb \times BB)P(bb \times BB)}{P(3B \text{ children} | bb \times BB)P(bb \times BB) + P(3B \text{ children} | bb \times Bb)P(bb \times Bb)} =$$

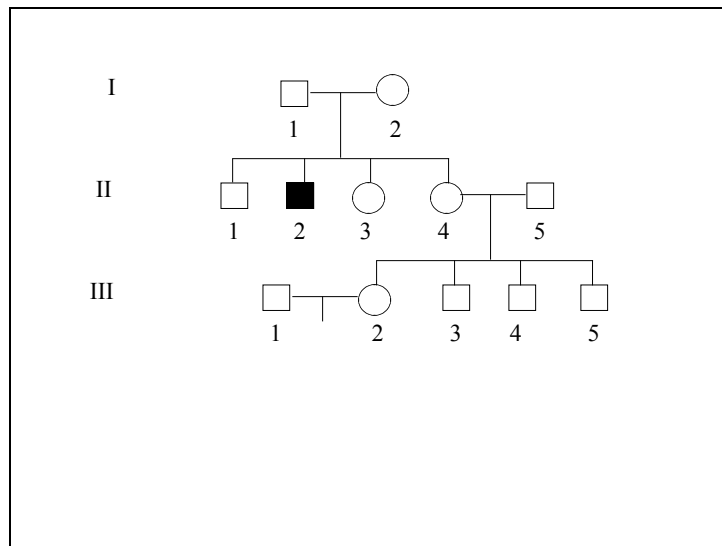
$$\frac{(3/7)(1)}{(1)(3/7) + (1/2)^3(4/7)} = .857$$

4. In cats, the polydactyl gene creates extra toes on the cats' paws. A breeder bred a polydactyl Tom Cat (ee) to a phenotypically normal queen (E_), hoping to have some polydactyl kittens. However, in a litter of three kittens none were polydactyl. Among U.S. cats that are not polydactyl, the probability that a cat is heterozygous for polydactyl is 3/10. What is the probability that the queen is homozygous?

$$P_{EE} = \frac{P(3E \text{ kittens} | ee \times EE)P(ee \times EE)}{P(3E \text{ kittens} | ee \times EE)P(ee \times EE) + P(3E \text{ kittens} | ee \times Ee)P(ee \times Ee)} =$$

$$P_{EE} = \frac{(1)(7/10)}{(7/10)(1) + (3/10)(1/2)^3} = .95$$

5. Pedigree



The above pedigree is for the sex-linked trait of red-green color blindness. Use the information from the pedigree to answer the following questions.

a. What is the probability that I-2 is heterozygous?

- b. Based on ancestors, what is the probability that II-4 is heterozygous?
- c. Based on ancestors, what is the probability that III-2 is heterozygous?
- d. Using ancestors and descendants, what is the probability that II-4 is heterozygous?
- e. Using ancestors and relatives, what is the probability that III-2 is heterozygous?
- f. If III-2 has a son, what are his chances of being color blind?

a. $P(\text{heterozygous}) = 1$

b. $P(\text{II-4 heterozygous}) = .5$

c. $P(\text{III-2 heterozygous}) = P(\text{III-2 is } X^R X^r | \text{II-4 is } X^R X^r) P(\text{II-4 heterozygous}) = (.5)(.5) = .25$

d.

$$P(A) = P(\text{II-4 is } X^R X^r * X^R Y) = \frac{1}{2}$$

$$P(\text{notA}) = P(\text{II-4 } X^R X^R * X^R Y) = \frac{1}{2}$$

$$P(B|A) = P(3 \text{ normal sons} | X^R X^r * X^R Y) = \left(\frac{1}{2}\right)^3$$

$$P(B|\text{notA}) = P(3 \text{ normal sons} | X^R X^R * X^R Y) = (1)^3$$

$$\begin{aligned} P(3 \text{ normal sons}) &= P(B|A)P(A) + P(B|\text{notA})P(\text{notA}) \\ &= \left(\frac{1}{2}\right)^3 \left(\frac{1}{2}\right) + (1)^3 \left(\frac{1}{2}\right) = .5625 \end{aligned}$$

$$\begin{aligned} P(X^R X^r * X^R Y / 3 \text{ normal sons}) &= \frac{P(3 \text{ normal} | X^R X^r * X^R Y) P(X^R X^r * X^R Y)}{P(3 \text{ normal sons})} \\ &= \frac{\left(\frac{1}{2}\right)^3 * \left(\frac{1}{2}\right)}{(.5625)} = .1111 \end{aligned}$$

e. $P(\text{III-2 is } X^R X^r) = P(\text{III-2 is } X^R X^r | \text{II-4 is } X^R X^r) P(\text{II-4 is } X^R X^r)$
 $= \left(\frac{1}{2}\right) * (.1111) = .05556$

f. $P(\text{son of III-2 is } X^r Y) = P(\text{son III-2 is } X^r Y | \text{III-2 is } X^R X^r) P(\text{III-2 is } X^R X^r)$
 $= \left(\frac{1}{2}\right) * (.05556) = .02778$

6. Coat Color in horses is determined by multiple alleles. A complete black horse (C1 black horse with black mane and tail) is dominant to a bay horse (C2 brown horse with black legs, mane and tail) and a mahogany bay. A Bay is dominant to a mahogany bay (C3 brown horse with black roots, legs, mane, and tail). Given a sample of 7000 horses (Black, bay, and mahogany bay), what is the allele frequencies of the three phenotypes.

Color	Observed Number	Observed Frequency
Black	2527	.3610
Bay	2550	.3643
Mahogany Bay	1923	.2747

$$P3 = \sqrt{.2747} = .5241$$

$$P2 = \sqrt{(.3643 + .2747)} - \sqrt{.2747} = .2753$$

$$P1 = 1 - .5241 - .2753 = .2006$$

Hint: draw a unit square

7. If [x] cattle are red (bb) and [y] are black (BB) and [z] are roan (Bb), what is the gene frequency of the b allele?

$$((x)^2 + z) / (2 * (x + y + z))$$

8. a. Three genotypes for Citrullinemia have been identified in cattle. Among 13 offspring born from matings between heterozygote individuals, what is the probability that 6 offspring are normal, 5 offspring are carriers and 2 offspring are affected?

b. If the offspring were not observed until one month of life, only normal and carrier offspring would have survived to then. Among the 11 surviving offspring, what is the probability that 6 offspring are normal and 5 offspring are carriers?

a. $Nn \times Nn$ yields $\frac{1}{4} NN$, $\frac{1}{2} Nn$ and $\frac{1}{4} nn$ offspring at birth. The probability of this particular combination of offspring would be estimated from the multinomial expansion.

$$\{13! / [6! * 5! * 2!]\} * \{[(1/4)^6] * [(1/2)^5] * [(1/4)^2]\} = \{36036\} * \{.000000477...\} = .01718 \sim .017$$

b. At one month of age, $Nn \times Nn$ yields $1/3 NN$ and $2/3 Nn$ and zero nn offspring. The binomial expansion can be used to answer this question.

$$\{11! / [6! * 5!]\} * \{[(1/3)^6] * [(2/3)^5]\} = \{462\} * \{.00018064...\} = .08345611 \sim .083$$

The multinomial expansion can also be used to answer this question.

$$\{11! / [6! * 5! * 0!]\} * \{[(1/3)^6] * [(2/3)^5] * [0^0]\} = \{462\} * \{.00018064...\} = .08345611 \sim .083$$

9. If each pair of parents only has 2 offspring and the current population size is 500, what is the expected population size in 4 generations?

500

10. If each pair of parents only has 2 offspring and the current population size is 500, what is the expected population size in 4 years?

Population size is constant at 500.

11. a. If each parent only has 2 offspring and the current population size is 500, what is the expected population size in 4 generations?

b. Is this a realistic answer?

a. $x_t = b^t x_0$ or $x_t = 2^4 (500) = 8000$.

b. What do you think? Will area to support the population be sufficiently large to maintain this amount of growth?

12. If each parent only has 2 offspring and the current population size is 500, what is the expected population size in 4 years?

This question cannot be answered because the generation interval is unknown. If the generation interval was one year, the answer would be the same as question 19a. If the generation interval was 4 years, the new population size would be 1000 from $[2^1(500)]$.