

ANSC 446 Formula sheet for Exam 1

R = net replacement rate = N_{t+1} / N_t
 $N_t = R^t N_0$ = population size in generation t

Binomial probability: $\Pr(i) = ((N!/(i!j!)) p^i q^j$
 Where $p = P(A)$ = probability of event A, $q = P(B)$
 = probability of event B, i = number of occurrences of event A, j = number of occurrences of event B

Multinomial probability: $\Pr(i,j) = (N!/(i!j!k!)) P^i H^j Q^k$

Independent events: $P(A \text{ or } B) = P(A) + P(B) - P(AB)$
 Mutually exclusive events: $P(A \text{ or } B) = P(A) + P(B)$

Bayes' theorem: event A, outcome B; A^C is complement of A

$$P(A|B) = \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^C)P(A^C)}$$

Hardy Weinberg principle for two alleles A_1 with frequency p, and A_2 with frequency q, genotypes A_1A_1 , A_1A_2 , A_2A_2 have frequencies p^2 , $2pq$, q^2 respectively, and $p^2 + 2pq + q^2 = 1$

Testing for Hardy-Weinberg equilibrium:

O: Observed

E: Expected

$$\chi^2 = \sum_{i=1}^k \frac{(O - E)^2}{E}$$

Genotype frequencies for A_1A_1 , A_1A_2 , A_2A_2 are designated, respectively, P, H, and Q

Relationship between allele and genotype frequencies: $p = P + \frac{1}{2}H$ $q = Q + \frac{1}{2}H$

For multiple alleles, $p_i = P_{ii} + \frac{1}{2} \sum P_{ij}$ where $j \neq i$

Expected heterozygosity: $H_E = 1 - \sum p_i^2$

Estimated heterozygosity should be corrected for small sample sizes: multiply by $2N/(2N-1)$

Maximum $H_E = n-1 / n$ where n is no. alleles

Observed heterozygosity, $H_O = \sum N_{ij}/N$ where $i \neq j$

Possible pairwise comparisons: $S(S-1)/2$

Effective no. of alleles: $A_E = 1/(1-H)$

Bonferroni correction: $1/n$ where n is number of hypotheses tested.

p distance: s/n where s is segregating sites; n is sequence length

nucleotide diversity π -hat = $(N/(N-1)) \sum (p_i p_j) \pi_{ij}$
 where N is sample size; p is frequency and π is proportion of differences (draw unit square)

Heterozygosity estimates across alleles and loci:

$$\hat{H} = \frac{1}{Nm} \sum_{i=1}^N \sum_{j=1}^m H_{ij}$$

Allele frequency estimates (draw unit square):

(2) Codominance	$\hat{q} = \frac{\frac{1}{2}N_{12} + N_{22}}{N}$
(3) Dominance	$\hat{q} = \left(\frac{N_{22}}{N} \right)^{1/2}$
(4) Codominance, X-linked or haplo-diploid	$\hat{q}_f = \frac{\frac{1}{2}N_{12} + N_{22}}{N_f}$
	$\hat{q}_m = \frac{N_2}{N_m}$

Dominant series, 3 alleles:

$$\begin{aligned} \hat{p}_1 &= 1 - \left(\frac{N_{22} + N_{23} + N_{33}}{N} \right)^{1/2} \\ \hat{p}_2 &= \left(\frac{N_{22} + N_{23} + N_{33}}{N} \right)^{1/2} - \left(\frac{N_{33}}{N} \right)^{1/2} \\ \hat{p}_3 &= \left(\frac{N_{33}}{N} \right)^{1/2} \end{aligned}$$

Normal distribution: 68% within ± 1 sd of mean;

95% within ± 1.96 sd of mean. $sd = \sqrt{V_x}$

V_x = variance = $(1/n-1) \sum (x_i - \bar{x})^2$

Are 2 allele frequencies different in 2 populations:

$$\chi^2 = \frac{2N V(\hat{p})}{\bar{p}\bar{q}}$$

$$V(\hat{p}) = \sum \frac{N_j}{N} \hat{p}_j^2 - \bar{p}^2$$

Nei's standard genetic distance, $D = -\ln(I)$ where I is genetic identity:

$$J_{xy} = \sum_{i=1}^n p_{i \cdot x} p_{i \cdot y}, \quad J_x = \sum_{i=1}^n p_{i \cdot x}^2, \quad J_y = \sum_{i=1}^n p_{i \cdot y}^2, \quad I = \frac{J_{xy}}{(J_x J_y)^{1/2}}$$