

**Question 2.20 in DPV** Write succinct pseudocode for your method.

**Answer:** The following pseudocode counts the number of entries in  $x$  with a given value. We input  $M_{\min} = \min_i x_i$  and  $M_{\max} = \max_i x_i$ . Indices start at 0.

```
SORT( $M_{\min}, M_{\max}, x$ )
  initialize an all-zero array  $S$  with  $M_{\max} - M_{\min} + 1$  entries
  initialize an array  $y$  the same size as  $x$ 
  for  $i=0 \dots n-1$ 
    increase  $S[x(i) - M_{\min}]$  by one
  for  $j=0 \dots M-1$ 
    if  $S[j] > 0$ 
      for  $k=1 \dots S[j]$ 
         $y[j+k-1] = j + M_{\min}$ 
  return  $y$ 
```

Since the algorithm has one loop with  $n$  iterations and one loop with  $M$  iterations (writing to an array of length  $n$ ), the running time is  $O(n+M)$ .

The sorting lower bound does not hold because comparisons are not used in this sorting algorithm. Only simple array accesses are used.

**discussion forum questions** Show that the following problems are in NP (and give the poly-time verifiers):

**COMPOSITE** Is the integer  $x$  the product of two other integers?

**SET COVER** Given a collection of sets  $S_1, \dots, S_m$  of a ground set and an integer  $k$ , determine if there exists a subcollection of size  $k$  such that the union of the sets in the subcollection is the same as the union of the sets in the collection.

**SAT** (See DPV for a definition)

**ROOT** Given a polynomial  $P(x)$  (e.g.  $P(x) = x^3 - 3x + 1$ ) and an interval  $[a, b]$ , does  $P$  have a root in  $[a, b]$ , i.e. does there exist an integer  $x$  s.t.  $a \leq x \leq b$  s.t.  $P(x) = 0$  (What if  $x$  is not required to be an integer?)

*answer should be easy*