Eric Rouse

Individual Assignments #58

Assignment: *Section 1.6: 2, 4,6,14,16 (use the contrapositive), 18a*

Q2

If a is even and b is even then a+b is even.

P = a is even $\wedge$ b is even

Q = a + b is even

Assume P is true, a is even $\wedge$ b is even.

$\Longrightarrow$ a = 2k; b = 2m

$\Longrightarrow$ a + b $\equiv$ 2k + 2m $\equiv$ 2 (k+m) ◻
(because k+m is an integer, it follows the accepted definition of even numbers, a + b is even)

Q4

If a is even then –a is even

P = a is even

Q = -a is even

Assume P is true, a is even.

$\Longrightarrow$ a = 2k

$\Longrightarrow$ -a $\equiv$ -(2k) $\equiv$ 2(-k) ◻
(because –k is an integer, it follows the accepted definition of even numbers, -a is even)

Q6

If a and b are odd then ab is odd

P = a is odd $\wedge$ b is odd

Q = ab is odd

Assume P is true, a and b are odd.

$\Longrightarrow$ a = 2k + 1; b = 2m + 1

$\Longrightarrow ab \equiv (2k + 1)*(2m+1) \equiv 4km + 2(k+m) + 1 \equiv 2*(2km + k + m) + 1$ □
(because (2km + k + m) is an integer, it follows the accepted definition of odd numbers, ab is odd)

Q14

If x is rational and x != 0 then 1/x is rational

P = x is rational and x!= 0

Q = 1/x is rational

Assume P is true.

$\Longrightarrow x = a/b$ (rational numbers can be expressed as a ratio of integers)

$\Longrightarrow 1/x \equiv 1/(a/b) \equiv b/a$ □
(b/a is a ratio of integers, hence 1/x is rational by the definition of rational numbers)

Q16

If mn is even then m is even or n is even for all integers

P = mn is even

Q = m is even ⋁ n is even

Contrapositive: ¬Q → ¬P

¬Q ≡ ¬(m is even ⋁ n is even) ≡ ¬m is even ⋀ ¬n is even ≡ m is odd ⋀ n is odd; assumed to be true, so that m = 2a +1 and n = 2b + 1

¬P ≡ ¬(mn is even) ≡ mn is odd

$\Longrightarrow mn \equiv (2a+1)*(2b+1) \equiv 4ab + 2(a+b) + 1 \equiv 2*(2ab + a + b) + 1$ □
(by the definition of odd numbers; mn is odd)

Q18a

If 3n+2 is even then n is even for all integers.

P = 3n + 2 is even

Q = n is even

Contrapositive: ¬Q → ¬P

¬Q ≡ ¬(n is even) ≡ n is odd

$\neg P \equiv \neg(3n + 2 \text{ is even}) \equiv 3n + 2 \text{ is odd}$

Substitute:

$\Longrightarrow 3n + 2 \equiv 3(2a + 1) + 2 \equiv 6a + 4 + 1 \equiv 2(3a + 2) + 1$ □
(3n + 2 is odd by definition of odd numbers)