CS225 Assignment 1 Solution Set

EECS, Oregon State University

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1.1 Propositions; Simple Formulas

2. Which of these are propositions? What are the truth values of those that are propositions?

   a) Do Not pass go.
   Not a proposition.

   b) What time is it?
   Not a proposition.

   c) There are not black flies in Maine.
   Proposition. Truth value: False.

   d) $4 + x = 5$
   Not a proposition.

   e) The moon is made of green cheese.
   Proposition. Truth value: False.

   f) $2^n \geq 100$
   Not a proposition.

6. Let p and q be the propositions “The election is decided” and “ the votes have been counted”, respectively. Express each of these compound propositions as an English sentence.

   a) $\neg p$
   The election is not decided.

   b) $p \vee q$
   The election is decided or the votes have been counted.

   c) $\neg p \wedge q$
   The election is not decided and the votes have been counted.

   d) $q \rightarrow p$
   Many solutions. Some examples are

      1. If the votes have been counted, then the elections is decided.

2. The votes have been counted only if the elections is decided.
3. The election is decided unless the votes have not been counted.

e) $\neg q \rightarrow \neg p$
Many solutions. Some examples are

1. If the votes have not been counted, then the election is not decided.
2. The votes have not been counted only if the election is not decided.
3. The election is not decided unless the votes have been counted.

f) $\neg p \rightarrow \neg q$
Many solutions. Some examples are

1. If the election is not decided, then the votes have not been counted.
1. The election is not decided only if the votes have not been counted.
2. The votes have not been counted unless the election is decided.

g) $p \leftrightarrow q$
The election is decided if and only if the votes have been counted.

h) $\neg q \vee (\neg p \wedge q)$
Either the votes have not been counted or the election is not decided and the votes have been counted.

10. Write propositions using p, q and r and logical connectives.

a) $r \wedge \neg q$

b) $p \wedge q \wedge r$

c) $r \rightarrow p$

d) $p \wedge \neg q \wedge r$

e) $(p \wedge q) \rightarrow r$

f) $r \leftrightarrow (p \vee q)$

20. Write each of these statements in the form of "if p, then q" in English.

a) If I remember to send you the address, then you have sent me an e-mail message.

b) If you were born in the United States, then you are a citizen of this country.

c) If you keep you textbook, then it will be a useful reference in your future courses.

d) If the Red Wing's goalie plays well, then they will win the Stanley Cup.

e) If you get the job, then you had the best credentials.

f) If there is a storm, then the beach erodes.

g) If you can log on to the server, then you have a valid password.

h) If you do not start your climb too late, then you will reach the summit.

28. Construct a truth table for each of these compound propositions.

a) $p \to \neg p$

p	$\neg p$	$p \to \neg p$
T	F	F
F	T	T

b) $p \leftrightarrow \neg p$

p	$\neg p$	$p \leftrightarrow \neg p$
T	F	F
F	T	F

c) $p \oplus (p \vee q)$

p	q	$p \vee q$	$p \oplus (p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	T
F	F	F	F

d) $(p \wedge q) \to (p \vee q)$

p	q	$p \wedge q$	$p \vee q$	$(p \wedge q) \to (p \vee q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	T	T
F	F	F	F	T

e) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

p	q	$\neg p$	$q \to \neg p$	$p \leftrightarrow q$	$(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$
T	T	F	F	T	F
T	F	F	T	F	F
F	T	T	T	F	F
F	F	T	T	T	T

f) $(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	$p \leftrightarrow q$	$\neg q$	$p \leftrightarrow \neg q$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	T	F	F	T
T	F	F	T	T	T
F	T	F	F	T	T
F	F	T	T	F	T

32. Construct a truth table for each of these compound propositions.

a) $(p \vee q) \vee r$

p	q	r	$p \vee q$	$(p \vee q) \vee r$
T	T	T	T	T
F	T	T	T	T
T	F	T	T	T
T	T	F	T	T
F	F	T	F	T
F	T	F	T	T
T	F	F	T	T
F	F	F	F	F

b) $(p \vee q) \wedge r$

p	q	r	$p \vee q$	$(p \vee q) \wedge r$
T	T	T	T	T
F	T	T	T	T
T	F	T	T	T
T	T	F	T	F
F	F	T	F	F
F	T	F	T	F
T	F	F	T	F
F	F	F	F	F

c) $(p \wedge q) \vee r$

p	q	r	$p \wedge q$	$(p \wedge q) \vee r$
T	T	T	T	T
F	T	T	F	T
T	F	T	F	T
T	T	F	T	T
F	F	T	F	T
F	T	F	F	F
T	F	F	F	F
F	F	F	F	F

d) $(p \wedge q) \wedge r$

p	q	r	$p \wedge q$	$(p \wedge q) \wedge r$
T	T	T	T	T
F	T	T	F	F
T	F	T	F	F
T	T	F	T	F
F	F	T	F	F
F	T	F	F	F
T	F	F	F	F
F	F	F	F	F

e) $(p \vee q) \wedge \neg r$

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
T	T	T	T	F	F
F	T	T	T	F	F
T	F	T	T	F	F
T	T	F	T	T	T
F	F	T	F	F	F
F	T	F	T	T	T
T	F	F	T	T	T
F	F	F	F	T	F

f) $(p \wedge q) \vee \neg r$

p	q	r	$p \wedge q$	$\neg r$	$(p \wedge q) \vee \neg r$
T	T	T	T	F	T
F	T	T	F	F	F
T	F	T	F	F	F
T	T	F	T	T	T
F	F	T	F	F	F
F	T	F	F	T	T
T	F	F	F	T	T
F	F	F	F	T	T

48.

a) "The user has paid the subscription fee, but does not enter a valid password."
$r \wedge \neg p$

b) "Access is granted whenever the user has paid the subscription fee and enters a valid password."
$(r \wedge p) \rightarrow q$

c) "Access is denied if the user has not paid the subscription fee."
$\neg r \rightarrow \neg q$

d) "If the user has not entered a valid password but has paid the subscription fee, then access is granted."
$(\neg p \wedge r) \rightarrow q$

1.2 Logical Equivalence

8. Use De Morgan's laws to find the negation of each of the following statements.

a) "Kwame will take a job in industry or go to graduate school."

Solution: Let p be "Kwame will take a job in industry" and q be "Kwame will go to graduate school". Then "Kwame will take a job in industry or go to graduate school" can be represented as $p \vee q$. Applying the second De Morgan's law, $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$, which can be interpreted as "Kwame will neither take a job in industry nor go to graduate school".

b) "Yoshiko knows Java and calculus".

Solution: Let p be "Yoshiko knows Java" and q be "Yoshiko knows calculus". Then "Yoshiko knows Java and calculus" can be represented as $p \wedge q$. Applying the first De Morgan's law, $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$, which can be interpreted as "Yoshiko does not know Java or he does not know calculus".

c) "James is young and strong".

Solution: Let p be "James is young" and q be "James is strong". Then "James is young and strong" can be represented as $p \wedge q$. Applying the first De Morgan's law, $\neg(p \wedge q)$ is equivalent to $\neg p \vee \neg q$, which can be interpreted as "James is not young or he is not strong".

d) "Rita will move to Oregon or Washington".

Solution: Let p be "Rita will move to Oregon" and q be "Rita will move to Washington". Then "Rita will move to Oregon or Washington" can be represented as $p \vee q$. Applying the second De Morgan's law, $\neg(p \vee q)$ is equivalent to $\neg p \wedge \neg q$, which can be interpreted as "Rita will neither move to Oregon nor Washington".

14. Determine whether $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is a tautology.

There are different directions to solve this problem. One way to solve it is to construct the truth table and see if all the truth assignments evaluate to true. Another way to do it is to simplify the formula using equivalences and then see if the truth value of the simplified formula to be true. One example solution of the later one is given below.

$$
\begin{array}{lll}
(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q & & \\
\equiv & (\neg p \wedge (\neg p \vee q)) \rightarrow \neg q & \text{by Example 3} \\
\equiv & \neg(\neg p \wedge (\neg p \vee q)) \vee \neg q & \text{by Example 3} \\
\equiv & (\neg(\neg p) \vee \neg(\neg p \vee q)) \vee \neg q & \text{by the first De Morgan Law} \\
\equiv & (p \vee \neg(\neg p \vee q)) \vee \neg q & \text{by Double negation Law} \\
\equiv & (p \vee (\neg(\neg p) \wedge \neg q)) \vee \neg q & \text{by the second De Morgan Law} \\
\equiv & (p \vee (p \wedge \neg q)) \vee \neg q & \text{by Double negation Law} \\
\equiv & p \vee \neg q & \text{by the first Absorption Law}
\end{array}
$$

The formula above cannot be simplified further. Because the true value of $p \vee \neg q$ can be either **T** (e.g. $p = T, q = T$) or **F** (e.g. $p = F, q = T$), $(\neg p \wedge (p \rightarrow q)) \rightarrow \neg q$ is not tautological.
(NOTE: There exist multiple sequences to derive the final formula.)

16. Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\neg p \wedge \neg q)$ are equivalent.

Similar to 14, different directions exist to solve the problem. One way to prove the two formulas are equivalent is to create a truth table for each formula and see if they match. Another way is to use a series of equivalences to derive one expression from the other. An example solution for the later method is given below.

The second formula can be represented as:

$$\begin{aligned}
&(p \wedge q) \vee (\neg p \wedge \neg q) \\
&\qquad \equiv (p \vee (\neg p \wedge \neg q)) \wedge (q \vee (\neg p \wedge \neg q)) && \text{by Distributive Law} \\
&\qquad \equiv ((p \vee \neg p) \wedge (p \vee \neg q)) \wedge ((q \vee \neg p) \wedge (q \vee \neg p)) && \text{by Distributive Law} \\
&\qquad \equiv (\mathbf{T} \wedge (p \vee \neg q)) \wedge (\mathbf{T} \wedge (q \vee \neg p)) && \text{by Negation Law} \\
&\qquad \equiv (p \vee \neg q) \wedge (q \vee \neg p && \text{by Identity Law} \\
&\qquad \equiv (q \rightarrow p) \wedge (p \rightarrow q) && \text{by Example 3} \\
&\qquad \equiv p \leftrightarrow q
\end{aligned}$$

which equals to the first formula.

20. Show that $\neg(p \oplus q)$ and $p \leftrightarrow q$ are equivalent.

p q	$(p \oplus q)$	$\neg(p \oplus q)$
T T	F	T
T F	T	F
F T	T	F
F F	F	T

(a).Truth Table for $\neg(p \oplus q)$

p q	$p \rightarrow q$	$q \rightarrow p$	$p \leftrightarrow q$
T T	T	T	T
T F	F	T	F
F T	T	F	F
F F	T	T	T

(b). Truth Table for $p \leftrightarrow q$

It's obvious that $\neg(p \oplus q)$ and $p \leftrightarrow q$ share the same truth table, thus the two are equivalent.

32. Show that $(p \wedge q) \rightarrow r$ and $(p \rightarrow r) \wedge (q \rightarrow r)$ are not equivalent.

One way to show that two formulas are not equivalent is to create a truth table for each of the formula and show that they do not match. Another way to do so is to simplify the two formulas to two obviously inequivalent formulas. An example solution is shown below.

The first formula can be simplified as:

$$\begin{aligned}
(p \wedge q) \rightarrow r &\equiv \neg(p \wedge q) \vee r && \text{by Example 3} \\
&\equiv r \vee \neg(p \wedge q) && \text{Communicative Law} \\
&\equiv r \vee (\neg p \vee \neg q) && \text{De Morgan Law}
\end{aligned}$$

The second formula can be simplified as:

$$\begin{aligned}
(p \rightarrow r) \wedge (q \rightarrow r) &\equiv (\neg p \vee r) \wedge (\neg q \vee r) && \text{by Example 3} \\
&\equiv r \vee (\neg p \wedge \neg q) && \text{Distributive Law}
\end{aligned}$$

Because $\neg(p \wedge q)$ is not equivalent to $\neg(p \vee q)$, the two formulas are not equivalent.