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Individual Assignments #58

Assignment: *Assignment: Section 2.2: 2, 4, 12, 16, 18, 20*

Q2

a) A∩B
b) A∩$\overline{B}$
c) A∪B
d) $\bar{A} \cup \bar{B}$

Q4

a) $A \cup B \ = \{a, b, d, e, f, g, h\}$
b) $A \cap B = \{a, b, c, d, e\}$
c) $A - B = \emptyset$
d) $B - A = \{f, g, h\}$

Q12

If $x \ \in \big(A \cup (A \cup B)\big)$ then $x \ \in A \ \vee x \in A \cap B$ by definition of union.

Since $x \in A \cap B \ \Longrightarrow x \ \in A$(by definition of intersection) that implies that $A \cup (A \cup B) \subseteq A$

And $x \ \in A$ by definition for A so $A \subseteq A \cup (A \cup B)$

Each is a subset of the other □

Q16

a) Implies: $x \in A \ \wedge x \in B \ \subseteq x \in A$; since x is an element of A one each side and it is joined conjunctively on the left it is a subset of the right.
b) Implies: $x \in A \subseteq x \in A \ \vee x \in B$ since x is an element of A on both sides and it is the only case on the left side, it is a subset of the right.
c) Implies: $x \in A \ \wedge x \notin B \ \subseteq x \in A$; since x is an element of A one each side and it is joined conjunctively on the left it is a subset of the right.
d) Implies: $(x \in A \wedge ((x \in B) \wedge (x \notin A))$ which expands to $(x \in A \wedge x \in B) \wedge (x \in A \wedge x \notin A)$. $x \in A \wedge x \notin A = \ \emptyset$ by complement law; $(x \in A \wedge x \in B) \wedge \emptyset = \ \emptyset$ by the complement law.
e) Implies: $x \in A \cup (B - A) iff x \in A \cup B$
Focusing on the right side: $x \in A \cup (B - A) iff x \in A \vee (x \in B \wedge x \notin A) iff (x \in A \vee x \in B) \wedge (x \in A \vee x \notin A)$
$(A \cup B) \cap (A \cup \overline{A)}$ by complememnt law and $(A \cup B) \cap U$ by identity law
thus $(A \cup B)$ □

Q18

a) Implies: $x \in A \ \vee x \in B \ \subseteq x \in A \ \vee x \in B \ \vee x \in C$ it is clear that the left side is a subset of the right as $x \in A \ \vee x \in B$ appears on both sides and is disjunctively joined on the right.

b) Implies: $x \in A \ \wedge x \in B \ \wedge x \in C \subseteq : x \in A \ \wedge x \in B$. Since the left shows that x must be an element of A and of B and also C it is clear that it is a subset of the right which is all A and B.

c) Implies: $if\ x \in (A - B) - C\ then\ x \ \in A - C$. The left becomes $(x \in A \wedge x \notin C) \wedge (x \notin B \vee x \notin C)$ and the right becomes $(x \in A \wedge x \notin C)$ by definition. Thus the left is a subset of the right again because of conjunction.

d) Implies: $(x \in A \wedge x \notin C) \wedge (x \in C \wedge x \notin B)$, which expands to:$(x \in A \wedge x \in C) \wedge (x \in A \wedge x \notin B) \wedge (x \notin C \wedge x \in C) \wedge (x \in C \wedge x \notin B)$, focusing on $(x \notin C \wedge x \in C)$ gives $C \cup \bar{C} \ = \emptyset$ by complement law, and ther rest is $\cap \ \emptyset$ so it is $\emptyset$ by domination law.

e) If x is an element of B and not A OR C and not A then x is an element of B or C by not A, Implies:

f) $(x \in B \wedge x \notin A) \vee (x \in C \wedge x \notin A) iff\ (x \in B \vee x \in C) \wedge x \notin A$, expand the left: $(x \in B \vee x \in C) \wedge (x \in B \vee x \notin A) \wedge (x \notin A \vee x \in C) \wedge (x \notin A \vee x \notin A)$.

   The whole middle section is redundant, it is literally the same terms as the first so, removing $(x \in B \vee x \notin A) \wedge (x \notin A \vee x \in C)$ and simplifiying$(x \notin A \vee x \notin A)$to $(x \notin A)$ leaves us with $(x \in B \vee x \in C) \wedge (x \notin A)$ which is equivalent to the right side.

Q20

Implies: $x \in (A \cap B) \cup (A \cap \overline{B})\ iff\ x \in A$

By the distributive law: $A \cap (B \cup \overline{B})$

By the complement law: $A \cap U$

By identity law: A

Thus $A = A$