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Individual Assignments #58

Assignment: *Section 4.2: 2, 4, 12, 30*

Q2

We are told that the first, second and third domino all fall down. We are also told that when a domino falls the domino 3 farther down the line also falls down. Hence the base case is satisified.

For the inductive step we assume that all k dominos fall down. As long as k > 3 the k+1 domino will fall because the k-2 domino fell.

Q4

a) P(18) = 2 each of 7 cent stamps and 1 each 4 cent stamp for a total of 18 cents postage.
   P(19) = 3 each of 4 cent stamps and 1 each 7 cent stamp for a total of 19 cents postage.
   P(20) = 5 each of 4 cent stamps for a total of 20 cents postage.
   P(21) = 3 each of 7 cent stamps for a total of 21 cents postage.
b) If $n \geq 18$ all P(k) postages are validly made from 4 cent and 7 cent stamps where $k \geq 21$.
c) All P(k+1) postages to be valid using 4 cent and 7 cent stamps.
d) Because $k \geq 21$we know that P(k-3) is true, postage of k-3 is proven. Add a single 4 cent stamp and k+1 is valid.
e) Since the basis and inductive step are done it si shown that the statement is true for any integer $n \geq 18$ by strong induction.

Q12

Prove all positive integers of n can be written as a summation of powers of 2.

Even numbers

$P(2) = 2^1$

$P(4) = 2^2$

$P(6) = 2^1+2^2$

…

$P(k) = 2^m + 2^n$ …

For P(k+1) to be even P(k) must be odd.

For the even case if $k \geq 3$ the $k+1$ can be represented as powers of two because $k-1$ is shown already to be made up of sums of powers of two and adding 2^1 gets us to $k+1$.

Odd numbers

Assum k is even so that $k+1$ is odd. Since k is even it is shown above to be made up of sums of powers of two. Adding 2^0 to k yields $k+1$.

Q30

If we assume $a^k = 1$ as the "proof" assumes then $a^{k-1} \neq 1$ as it states. $a^k = 1 \; and \; a^{-1} = 1/a$. Thus, by using the logic of the proof, the expression reduces:

$$a^{k+1} = \frac{a^k * a^k}{a^{k-1}} = \frac{1 * 1 * a}{1} = a$$

Since $a^{k+1} = a$ P(k) → P(k+1) is false and the proof is invalid.