CS225 Assignment 8 Solution Set

EECS, Oregon State University

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5.3 Permutations and Combinations

Textbook 5.3

6. Find the value of each of these equations
Solution:

a). $C(5,1) = \dfrac{5!}{1! \cdot 4!} = \dfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 5.$

b). $C(5,3) = \dfrac{5!}{3! \cdot 2!} = \dfrac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1 \cdot 2 \cdot 1} = 10.$

c). $C(8,4) = \dfrac{8!}{4! \cdot 4!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 70.$

d). $C(8,8) = \dfrac{8!}{8! \cdot 0!} = \dfrac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 1} = 1.$

e). $C(8,0) = \dfrac{8!}{0! \cdot 8!} = \dfrac{1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 1.$

f). $C(12,6) = \dfrac{12!}{6! \cdot 6!} = \dfrac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = 924.$

10. There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

Solution:

The number of different orders that the names of candidates be printed is the number of 6-permutations of a set of the six different names of the candidates. Consequently, the answer is $P(6,6) = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$.

12. How many bit strings of length 12 contain

a). exactly three 1s?
Solution:
To satisfy that exactly three 1s appear in the bit string, we need to choose 3 among the 12 bits and set them to 1 and the rest 0. Hence, the answer is $C(12,3) = \dfrac{12!}{3! \cdot 9!} = 220$.

b). at most three 1s?
Solution:
There are four possible cases that satisfy this condition: Among the 12 bits, there are 0, 1, 2, or 3 1s. Hence, the answer is

$$\begin{aligned} C(12,0)+C(12,3)+C(12,2)+C(12,1) &= \frac{12!}{0!\cdot 12!}+\frac{12!}{1!\cdot 11!}+\frac{12!}{2!\cdot 10!}+\frac{12!}{3!\cdot 9!} \\ &= 1+12+66+220 \\ &= 299. \end{aligned}$$

c). at least three 1s?
Solution:
In total there are 2^{12} different strings of length 12. Among these strings, there are $C(12,0)+C(12,1)+C(12,2)$ that have less than three 1s. Therefore, the number of strings that have at least three 1s is $2^{12}-[C(12,0)+C(12,1)+C(12,2)] = 4096-(1+12+66) = 4017$.

d). an equal number of 0s and 1s?
Solution:
$C(12,6) = \dfrac{12!}{6!\cdot 6!} = 924.$

24. How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other?[*Hint:* First position the women and then consider possible positions for the men.]

Solution:
First there are $P(10,10)$ possible ways to position the 10 women. For a fixed line formed by the 10 women, men will be positioned between pairs of women or on the outside of the line, so that no two men stand next to each other. Since there are 9 positions between pairs of women and 2 positions outside the line, in total there are 11 such positions that the 6 men can be positioned. Hence, there would be $P(11,6)$ possible ways to position the 6 men. Hence, the number of ways is

$$P(10,10)\cdot P(11,6) = 10!\cdot\frac{11!}{5!} = 1,207,084,032,000.$$

26. Thirteen people on a softball team show up for a game.

a). How many ways are there to choose 10 players to take the field?
Solution:
$C(13,10) = \dfrac{13!}{10!\cdot 3!} = 286.$

b). How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
Solution:
Since the position of the players matters, the number of ways to select the players should be the 10-permutations of the set of the team. Consequently, the answer is $P(13,10) = \dfrac{13!}{3!} = 1,037,836,800.$

c). Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?
Solution:
In total there are $C(13, 10)$ ways to choose 10 players out of 13. Among these, there is only one way that does not have a women (i.e., all the selected 10 players are men). Hence, the number of ways to select the players such that at least one of them must be a woman is $C(13, 10) - 1 = 286 - 1 = 285$.

Note that one can also work out the result be considering all possible cases of the numbers of women and men. There are three cases to be considered:

a) There are 1 women and 9 men among the 10 players. The number of ways in this case is: First, there are $C(3, 1)$ ways to choose a woman among the 3 women. Then there are $C(10, 9)$ ways to choose 9 men among the 10 men. Hence, the number of possible ways is $C(3, 1) \cdot C(10, 9) = 3 \cdot 10 = 30$.

b) There are 2 women and 8 men among the 10 players. The number of ways in this case is $C(3, 2) \cdot C(10, 8) = 3 \cdot 45 = 135$.

c) There are 3 women and 7 men among the 10 players. The number of ways in this case is $C(3, 3) \cdot C(10, 7) = 1 \cdot 120 = 120$.

Hence, the total number of ways to choose 10 players such that at least one of these players must be a women is $30 + 135 + 120 = 285$.

5.4 Permutations and Combinations with Repetition

Textbook 5.5

6. How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?
Solution:
Suppose that a box with three compartments, containing the three elements, which are separated by two dividers. The number of ways to select five unordered elements from a set with three elements corresponds to the number of ways to arrange 2 separators and 5 elements. Consequently, the number of ways to select the five elements is the number of ways to select the positions of the five elements from 7 possible positions, which can be done in $C(7, 5) = 21$ ways(Theorem 2).

8. How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?
Solution:
If repetition is allowed when choosing a dozen donuts from 21 different kinds, according to Theorem 2, there are $C(12 + 21 - 1, 12) = C(31, 12) = 141120525$ different ways to select. If repetition is not allowed when choosing a donuts from the 21 varieties, there are $C(21, 12) = 293930$ different ways to select.

12. How many different combinations of pennies, nickles, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?
Solution:

The question can be viewed as to select 20 unordered elements (20 coins) from a set with 5 elements when repetition is allowed. According to Theorem 2, there are $C(20+5-1, 20) = C(24, 20) = C(24, 4) = 10626$ different combinations.

14. How many solutions are there to the equation

$$x_1 + x_2 + x_3 + x_4 = 17$$

where x_1, x_2, x_3 and x_4 are nonnegative integers?
Solution:

To count the number of solutions, we note that a solution corresponds to the ways of selecting 17 items from a set with four elements so that x_1 items of type one, x_2 items of type two, x_3 items of type three and x_4 items of type four. Hence, the number of solutions is equal to the number of 17-combinations with repetitions allowed from a set with four elements. From Theorem 2, the equation has

$$C(4 + 17 - 1, 17) = C(20, 17) = C(20, 3) = 1140 \text{ solutions.}$$