CS352 HOMEWORK 1

Max sub array project

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Mathematical Analysis:

Algorithm 1 pseudocode:

```
def maxsubarray1(list):
    for each element in list
        for each element in list after first_element
            sum (first_element, next_element(s))
            If sum greater than max, save sum as max
            Repeat for each subsequent element
        set next element as first_element, repeat
```

This algorithm must pass through the list 2 times per element. So for every n elements it must pass through n-1 elements n-1 times. We are left with $n*(n-1)*(n-1) = n^3 – 2n^2 + n$. In the limit the n^3 term dominates, we are left with $O(n^3)$

Algorithm 2 pseudocode:

```
Def maxsubarray2(list):
    For each element in list
        sum(first_element)
        if sum greater thanmax, save sum as max
        set next element as first_element, repeat
```

This algorithm passes through the list once per element. For n elements we pass through the list n-1 times. The result is n^2-n, so in the limit it approaches an $O(n^2)$ running time.

Algorithm 3 pseudocode:

Def maxsubarray3(list):

If list has zero elements, return 0

If list has one element return the greater of 0 and the single element

Else, divide the list into two halves, left and right

Sum from the middle to the left.

If sum is larger than max_left, replace max_left with sum

Sum from the middle to the right.

If max_right is less than sum, replace max_right with sum.

Return the greater of (max(maxsubarray3(left_list),maxsubarray3(right_list)) and max_left + max_right)

This algorithm uses a divide and conquer strategy, the list is divided in half for each element. So $n*\log(n) - 2^{\log(n)} + 1$. The result is an O(n log n) running time.

Theoretical Correctness:

Using proof by induction, it is possible to prove the correctness of the recursive algorithm, maxsubarray3().

Since maxsubarray3() is a recursive algorithm it is best to prove with strong induction:

- Trivial case: list of zero elements, returns 0.
- The base case: list of one elements, returns the greater of 0 or the element listed.
    - Algorithm detects the largest possibility, either zero or the number.
    - Immediately returns this result, thus satisfying the requirement of finding the maximum of a single value.
- Inductive hypothesis: assume that maxsubarray3() on a region of k elements (where k >=1) returns the max array for that region.
    - k+1 > 1 by definition, executes neither base case, so the default code is executed.
    - There are three cases:
        - 1: Max array is fully enclosed in left half of the list
            - left_list > right_list && left_list > max_left + max_right
        - 2: Max array is fully enclosed in right half of the list
            - right_list > left_list && right_list > max_left + max_right
        - 3: Max array straddles the middle of the list
            - max_left + max_right > max_right && max_left + max_right > max_left
        - maxarray3() executes the same way for all these cases.
            - First it finds the greater of maxsubarray3(left_list) vs. maxsubarray3(right_list)
            - Since this is iterative, this will continue until we reach our base case of one element. So the maximums will be compared element by element.
            - Then it checks that max against the max of the middle list
            - It does this at each iteration step, ensuring that ONLY the highest value is cascaded forward.
            - So, on the unwind, the highest value, whether right, left or the middle, will be passed upward.
        - Thus, all cases are checked at every iteration step
- Hence the recursion returns the appropriate maximum, starting with the base case. It is proven by induction.

Testing:

I performed the test as specified in the document, it said that the resul was recorded.

Experimental Analysis:

The code to generate random numbers of the array size specified is attached as tester.py.

Results:

Algorithm 1, maxsubarray1():

Size	Run Time
100	0.000785804
200	0.0181916
300	0.141048503
400	0.633576894
500	2.139910388
600	5.79885602
700	13.63946042
800	29.4394686
900	56.0752856

As expected, this algorithm increases in runtime very quickly as the number of elements increase.

Algorithm 2, maxsubbarray2():

Size	Run Time
100	3.43E-05
200	0.000312591
300	0.00114181
400	0.003371
500	0.007214117
600	0.014108992
700	0.0253299
800	0.0407547
900	0.066061521
1000	0.097584081
2000	0.179565716
3000	0.362990689
4000	0.672370601
5000	1.228489614
6000	2.097222304
7000	3.481521106
8000	5.278179502
9000	7.971049905
10000	11.74170461

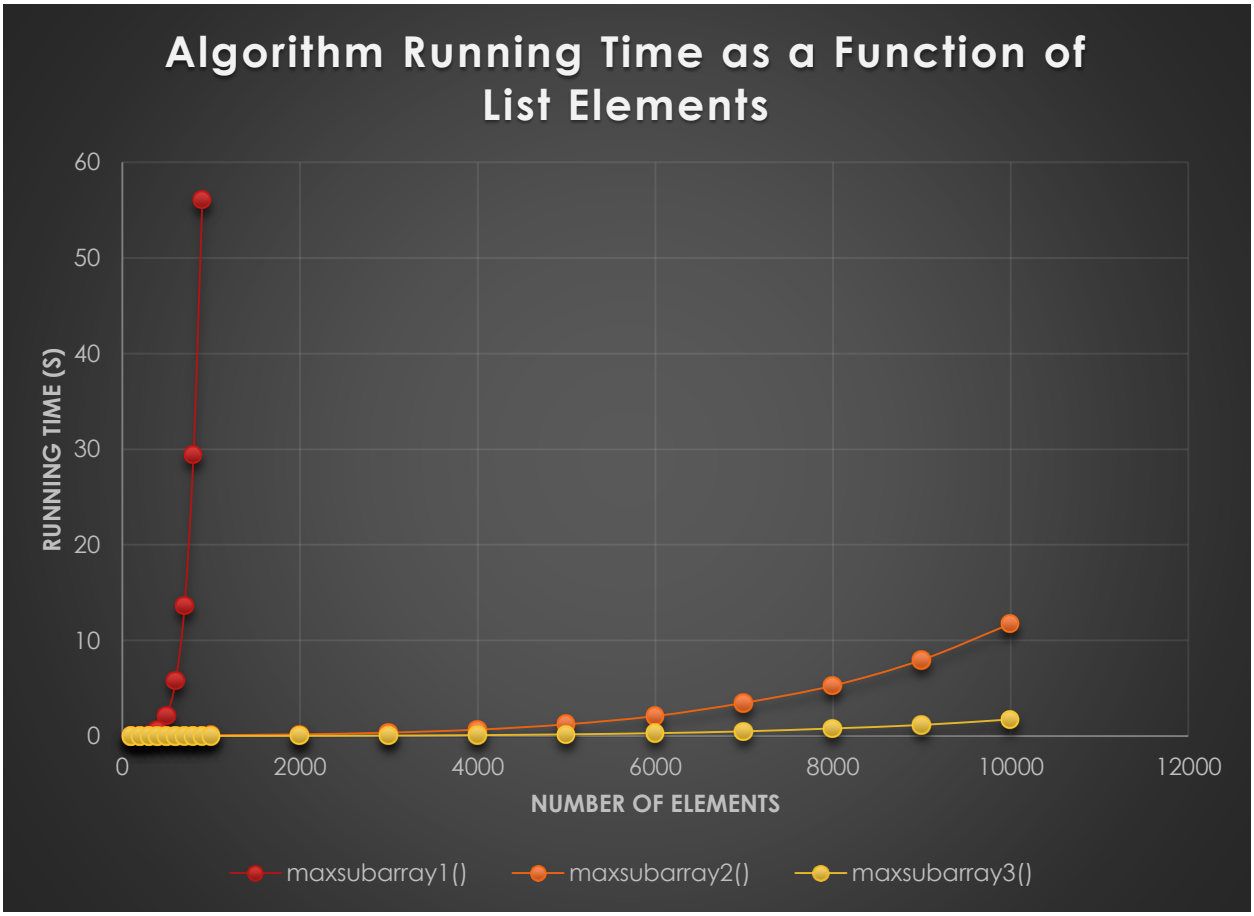
This algorithm performs much better than the previous, but is starting to become cumbersome around the final data points.

Algorithm 3, maxsubarray3():

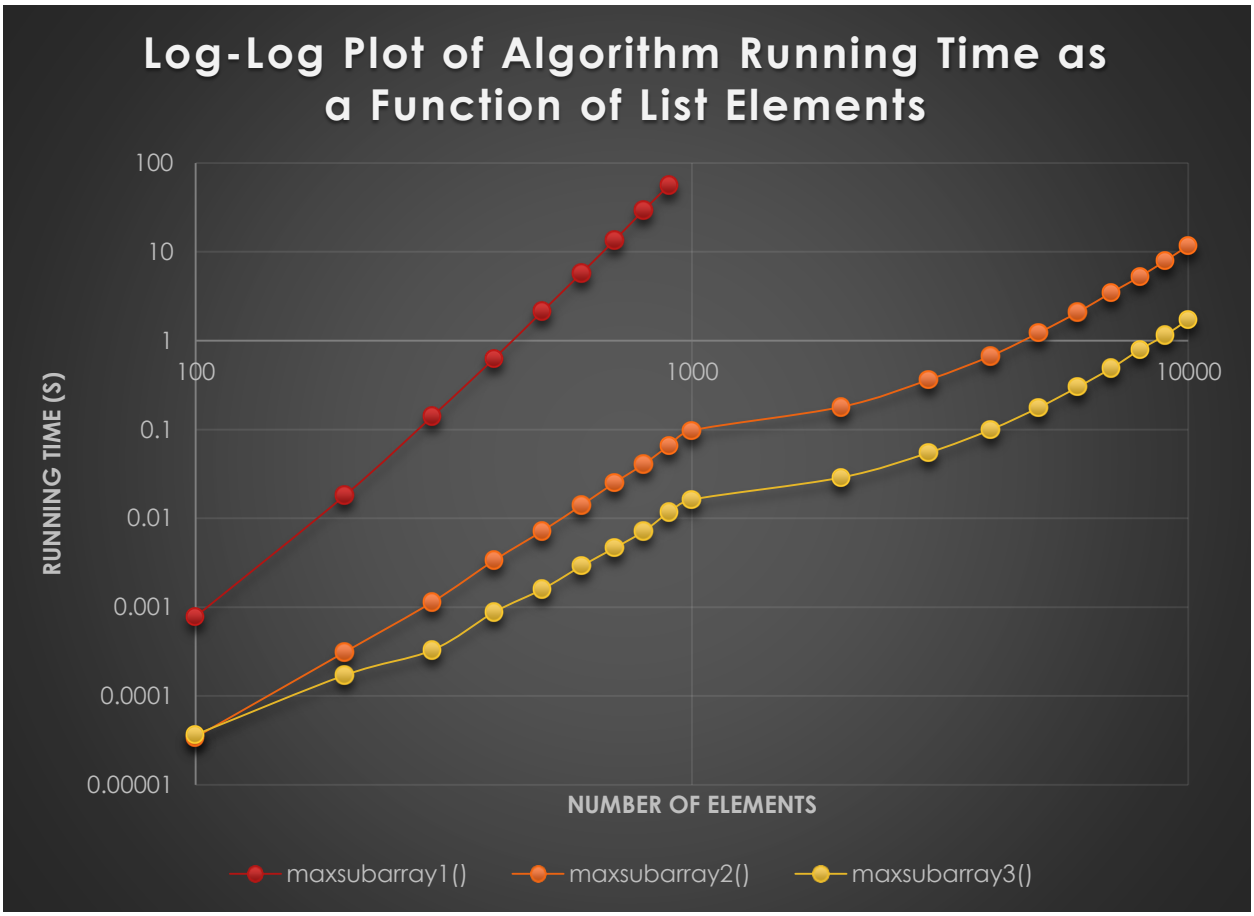
Size	Run Time
100	3.69E-05
200	0.000172
300	0.00033
400	0.000883
500	0.001599
600	0.002928
700	0.004678
800	0.007172
900	0.011804
1000	0.016281
2000	0.028871
3000	0.054926
4000	0.099858
5000	0.177135
6000	0.302171
7000	0.490979
8000	0.794477
9000	1.159381
10000	1.728725

This is far and away the most efficient method of the three presented here.

Plots



All three algorithms shown on a single plot. Graphically shows how much better the second process is compared to the first.



This log-log plot demonstrates the exponential nature of these algorithms.

Extrapolation and Interpretation:

Question 1:

By extrapolating the data experimentally gathered, it is possible to estimate the number of elements that any given algorithm could parse in one hour.

First we use the log-log plot to determine the slope of the lines. (Tabulated in question 2). Since the formula for the log-log line is of the form Time = constant * (elements)^(slope), we need to determine the constant to make our estimates. Using the slope from question 2 and the highest number of elements/time pair we can determine the constant.

- Constant of maxsubarray1(): c = $56.075/(900^{5.13}) = 3 \times 10^{-14}$
- Constant of maxsubarray2(): c = $11.7417/(10000^{2.22}) = 2 \times 10^{-8}$
- Constant of maxsubarray3(): c = $56.075/(10000^{1.60}) = 1 \times 10^{-6}$

So, maximum elements of each can be calculated using the following, 3600/(constant)^slope:

- Elements in one hour of maxsubarray1() = 2134
- Elements in one hour of maxsubarray2() = 97864
- Elements in one hour of maxsubarray3() = 939051

Question 2:

Log-log plot slope = log(y2/y1)/log(x2/x1)

Slope of maxsubarray1(): 5.13

Slope of maxsubarray2(): 2.22

Slope of maxsubarra3(): 1.60

There exist discrepancies between the values determined experimentally and the theoretical values above. This is due, at least in large part, to the simplification of Big O notation to focus on the dominating term.

In order to make the experiment less onerous we only ran values up to 900 elements on maxsubbary1 and 10000 elements on the other two functions. This means the dominating term hadn't really been given enough time to fully dominate. Thus each value is somewhat higher than the predicted value.