Eric Rouse

Individual Assignments #58

Assignment: *Section 1.6: 8 (use contradiction), 22, 24, 26, 30; Section 1.7: 6, 28, 34*

S1.6 Q8

If n is a perfect square then n+2 is not a perfect square.

P = a is perfect square

Q = n + 2 is not a perfect square

Assume for sake for sake of contradiction P →¬Q.

⟹ n+2 is a perfect square

The difference between one square and the next is calculable. Assume x^2 for a perfect square and $(x+1)^2$ for the next perfect square after x.

The difference: $(x+1)^2 - x^2 = x^2 + 2x + 1 - x^2 = 2x + 1$.

So, the smallest distance between perfect squares is when x = 0 ⟹ difference of 1.

The next jump is when x = 1 ⟹ difference of 3.

Thus n + 1 and n + 3 are possible. But n + 2 is not.

Proven by contradiction. □

S1.6 Q22

P = you pick 3 socks out of a drawer of only blue and black socks

Q = 2 socks are blue ⋁ 2 socks are black.

Since there are only four possible combinations of socks as shown in the table, an exhaustive proof is effective.

Sock 1	Sock 2	Sock 3
black	black	black
black	black	blue
black	blue	blue
blue	blue	blue

Thus, any combination has either 2 black socks or 2 blue socks. □

S1.6 Q24

P = at least 3 of any 25 days are in the same month of the year. Assume for contradiction that 2 or less days are in any given month. Since there are 12 months in a year we are left with only 24 days chosen. This is shows we need at least three in a month because we need to choose a total of 25 days.

Proven by contradiction. □

S1.6 Q30

Show that a < b ≡ (a+b)/2 > a ≡ (a+b)/2 < b.

Case a<b:

⟹a - b < 0

Case (a+b)/2 > a:

⟹a + b – 2a > a

⟹ - (b - a > 0)

⟹a – b < 0

Case (a+b)/2 < b:

⟹a + b – 2b < 0

⟹a – b < 0

Thus all three cases evaluate to the same thing.

a - b < 0 ≡ a - b < 0 ≡ a - b < 0 □

S1.7 Q6

Prove that ∃xP(x), P: $n_i = n_1 + n_2 \ldots n_{i-1}$

Constructive proof: 1 + 2 = 3□

S1.7 Q28

P = no solutions for x & y for $2x^2 + 5y^2 = 14$ if x and y are integers.

First, solve for y: $y = \pm\sqrt{\frac{14-2x^2}{5}}$. When x is 0 or 1 or 2 the equation evaluates to a non integer. When x is three or greater the result is an imaginary number, hence no integer can be used for x and y to solve the equation. It also holds for the negative integers, because x^2 of any negative integer is positive. □

In the interest of completeness, if you do the same for x: : $x = \pm\sqrt{\frac{14-5y^2}{2}}$. When y is 0 or 1 the equation evaluates to a non integer. When y is 2 or greater the result is an imaginary number, hence no integer can be used for x and y to solve the equation. It also holds for the negative integers, because y^2 of any negative integer is positive. □

S1.7 Q34

P = between every rational number and every irrational number there is an irrational number.

A rational number by definition is a quotient of two integers, $\frac{a}{b}$. It is less than $\frac{a+i}{b}$, where i is some irrational number. By dividing the i by two we get a smaller irrational number, because $0 < \frac{i}{2} < i$.

So $\frac{a+\frac{i}{2}}{b}$ is between $\frac{a}{b}$ and $\frac{a+i}{b}$. This term can always be reduced by half, hence there will always be an irrational number between any rational number and irrational number.