CS225 Assignment 2 Solution Set

EECS, Oregon State University

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1.3 Predicates and Quantifiers

6. Let N(x) be the statement "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a) $\exists x N(x)$
There is a student in my school who has visited North Dakota.

b) $\forall x N(x)$
Every student in my school has visited North Dakota.

c) $\neg \exists x N(x)$
No student in my school has visited North Dakota.

d) $\exists x \neg N(x)$
There is a student in my school who has not visited North Dakota.

e) $\neg \forall x N(x)$
Not every student in my school has visited North Dakota.

f) $\forall x \neg N(x)$
No student in my school has visited North Dakota.

8. Translate these statements into English, where R(x) is "x is a rabbit" and H(x) is "x hops" and the domain consists of all animals.

a) $\forall x(R(x) \rightarrow H(x))$
If an animal is a rabbit, then it hops.

b) $\forall x(R(x) \wedge H(x))$
Every animal is a hopping rabbit.

c) $\exists x(R(x) \rightarrow H(x))$
There is an animal such that if it is a rabbit, then it hops.
(Note that this is a very strange sentence that would rarely be used in English. Also note that the statement is true if there is just one animal that is not a rabbit, since it would make the implication true for that particular animal.)

d) $\exists x(R(x) \wedge H(x))$
There is an animal that is a hopping rabbit.

10. Let C(x) be the statement "x has a cat", let D(x) be the statement "x has a dog", and let F(x) be the statement "x has a ferret". Express each of these statements in terms of C(x), D(x), F(x), quantifiers, and logical connectives. Let the domain consist of all students in your class.

a) A student in your class has a cat, a dog and ferret.
$\exists x(C(x) \wedge D(x) \wedge F(x))$

b All students in your class have a cat a, a dog, or a ferret.
$\forall x(C(x) \vee D(x) \vee F(x))$

c) Some student in your class has a cat and a ferret, but not a dog.
$\exists x(C(x) \wedge \neg D(x) \wedge F(x))$

d) No student in your class has a cat, a dog and a ferret.
$\neg\exists x(C(x) \wedge D(x) \wedge F(x))$
(or equivalently, $\forall x \neg(C(x) \wedge D(x) \wedge F(x))$)

e) For each of the three animals, cats, dogs, and ferrets, there is a student in your class who has one of these animals as a pet.
$\exists x C(x) \wedge \exists x D(x) \wedge \exists x F(x)$
(Note: This is equivalent to $\exists x C(x) \wedge \exists y D(y) \wedge \exists z F(z)$, where the domain of y and z also consists of all students in your class.)

12. Let Q(x) be the statement "$x + 1 > 2x$". If the domain consists of all integers, what are these truth values?

a) Q(0): T

b) Q(-1): T

c) Q(1): F

d) $\exists x Q(x)$: T (e.g. $x = 0$)

e) $\forall x Q(x)$: F (e.g. $x = 2$)

f) $\exists x \neg Q(x)$: T (e.g. $x = 2$)

g) $\forall x \neg Q(x)$: F (e.g. $x = 0$)

16. Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a) $\exists x(x^2 = 2)$: T

b) $\exists x(x^2 = -1)$: F

c) $\forall x(x^2+2 \geq 1)$: T

d) $\forall x(x^2 \neq x)$: F

24. Translate in two ways each of these statements into logical expressions using predicates, quantifiers, and logical connectives,. First, let the domain consists of the students in your class and second, let it consist of all people.

a) Everyone in your class has a cellular phone.
Let I(x) be propositional function "x is in your class" and C(x) be propositional function "x has a cellular phone".
1). $\forall x C(x)$.
2). $\forall x(I(x) \rightarrow C(x))$.

b) Somebody in your class has seen a foreign movie.
Let I(x) be propositional function "x is in your class" and M(x) be propositional function "x has seen a foreign movie".
1). $\exists x M(x)$.
2). $\exists x(I(x) \wedge M(x))$.

c) There is a person in your class who cannot swim.
Let I(x) be propositional function "x is in your class" and S(x) be propositional function "x can swim".
1). $\exists x \neg S(x)$.
2). $\exists x(I(x) \wedge \neg S(x))$

d) All students in your class can solve quadratic equations.
Let I(x) be propositional function "x is in your class" and E(x) be propositional function "x can solve quadratic equations".
1). $\forall x E(x)$.
2). $\forall x(I(x) \rightarrow E(x))$.

e) Some student in your class does not want to be rich.
Let I(x) be propositional function "x is in your class" and R(x) be propositional function "x wants to be rich".
1). $\exists x \neg R(x)$ (or equivalently, $\neg \forall x R(x)$).
2). $\exists x(I(x) \wedge \neg R(x))$ (or equivalently, $\exists x \neg(I(x) \rightarrow R(x))$ or $\neg \forall x(I(x) \rightarrow R(x))$.

1.6 Introduction to Proofs

2. Use a direct proof to show that the sum of two even integers is even.

Proof: Let n_1 and n_2 be two even integers. By the definition of even numbers, we denote $n_1 = 2k_1$ and $n_2 = 2k_2$, where k_1 and k_2 are both integers. The summation of n_1 and n_2 can be denoted by $n_1 + n_2 = 2 * (k_1 + k_2)$. By the definition of an even number, we can conclude that $n_1 + n_2$ is also an even number, which completes our proof that the sum of two even integers is even.

4. Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

Proof: Let n be an even number. By the definition of even numbers, we denote $n = 2k$ where k is an integer. The additive inverse of integer n is its negative and can be denoted by $-n = -2k = 2 * (-k)$ where $-k$ is also an integer. By definition of an even number, we can conclude that $-n$ is also an even number, which completes our proof.

6. Use a direct proof to show that the product of two odd numbers is odd.

Proof: Let n_1 and n_2 be two odd numbers. By the definition of odd numbers, we denote $n_1 = 2k_1+1$ and $n_2 = 2k_2+1$ where k_1 and k_2 are both integers. The product of of n_1 and n_2 can be denoted by $n_1*n_2 = (2k_1+1)*(2k_2+1) = 4k_1k_2+2k_1+2k_2+1 = 2(2k_1k_2+k_1+k_2)+1$. By the definition of an odd number, we can conclude that $n_1 * n_2$ is an odd number.

14. Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.

Proof: We give a direct proof. Given that x is rational, by the definition of rational number, there exist integers p and q, with $q \neq 0$, such that $x = p/q$. Because $x \neq 0$, it is obvious that $p \neq 0$ (Why?). So we have $1/x = q/p$, where p and q are integers and $p \neq 0$. By the definition of rational number, $1/x$ is rational.

16. Prove that if m and n are integers and mn is even, then m is even or n is even. (Use the contrapositive)

Proof: Let p be the proposition that "m and n are integers and mn is even", and q is the statement that "m is even or n is even". The statement can be expressed by $p \rightarrow q$. The contrapositive of the implication is "If neither m is even nor n is even, then mn is not even." This is the same as "If m is odd and n is odd, then mn is odd." We already proved that this was true in problem 6. Since the contrapositve of the original implication is true, we proved that the original implication is true.

18(a). Prove that if n is an integer and $3n + 2$ is even, then n is even using a proof by contraposition.

Proof: Let p be the proposition that "n is an integer and $3n + 2$ is even", and q is the proposition that n is even. Assume the conclusion of the conditional statement "If n is an integer and $3n+2$ is even, then n is even" is false; namely, assume that n is not even (i.e., n is odd). Then, by the definition of odd integer, $n = 2k + 1$ for some k. Substituting $2k + 1$ for n, we find that $3n+2 = 3(2k+1)+2 = 6k+5 = (6k+4)+1 = 2(3k+2)+1 = 2k'+1$. where $k' = 3k+2$ and k' is an integer. By the definition of an odd number, we can conclude that $3n+2$ is an odd number and hence is not even. This is a negation of the hypothesis of the theorem.

Because the negation of the conclusion of the conditional statement implies that the hypothesis is false, the original conditional statement is true. This completes our proof by contraposition.