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Individual Assignments #58

Assignment: *4.3: 12, 26c ("5 | a+b" meas that 5 divides a+b), 43, 44; pg. 332 problem 58*

Q12

By Definition of Fibonacci:

$f_0 = 0$

$f_1 = 1$

$f_{n+1} = f_n + f_{n-1}$

BASE: n=1;

$f_1^2 = f_1 * f_0$

$1 = 1(f_1 + f_0) = 1*(1+1) = 1$

1 = 1; base case checks out

INDUCTIVE:

Assume m and m-1 holds, show that m+1 holds true also.

Trying to prove that $\sum_{i=1}^{n} f_n^2 = f_n * f_{n+1}$.

Find the sum of m+1:

$$\sum_{i=1}^{m+1} f_n^2 = \sum_{i=1}^{m} f_n^2 + f_{m+1}^2 = f_m * f_{m+1} + f_{m+1}^2$$

Substitute m+1 into right side of equation to prove:

$f_n * f_{n+1} = f_{m+1} * f_{m+2}$

Substitute f_{m+2} by definition of Fibonacci sequence.

$f_{m+1} * (f_m + f_{m+1})$

Expand:

$f_m * f_{m+1} + f_{m+1}^2$

Which is the same as what we assumed the sum of m+1 would be. We have shown the base and inductive cases, thus by induction it is proven.

Q26c

BASE: (0+0)/5 = 0; check.

INDUCTIVE: Since we add 2 to one member of the pair and 3 to the other member in all cases at each recursive step we are in essence adding five to the sum a+b. Thus if a+b starts out divisible by five (base case) then it will always be divisible by 5.

Q43

Definitions of set T binary trees:
BASE: single node ∊T
RECURSIVE: If T1 and T2 are full binary trees then T=T1*T2 has a height h(T)=1+max(h(T1),h(T2))

Prove h(T) ≥ 2h(T) + 1

BASE: h(T) = 0
1≥2*0+1
1≥1 check!

INDUCTIVE:
Assume h(T1)≥2(h(T1))+1 and h(T2)≥2(h(T2))+1 where T1 and T2 are binary trees.
n(T) = 1 + n(T1) + n(T2) and h(T) = 1 + max(h(T1),h(T2))
n(T) ≥ 1 + (2(h(T1))+1)+(2(h(T2))+1)
n(T) ≥ 3 + 2(h(T1)+h(T2)) = 3 +2(max(h(T1),h(T2)))
n(T) ≥ 3 + 2(h(T)-1) = 3+2h(T)-2
n(T) ≥ 2h(T)+1 check!

Q44

BASE: For any connected node with leaves there are two leaves and a single vertices.
RECURESIVE: if T1 and T2 are full binary rees then T=T1*T2 has n(T)=1+n(T1)+n(T2) nodes and n(T) = i(T) + l(T) and i(T) = 1+i(T1)+i(T2) and l(T) = l(T1) + l(T2)

Show that l(T) = i(T) + 1

BASE: single node, leaves = 1, vertices = 0. Check!

RECURSIVE:
n(T) = 1 + n(T1) + n(T2)
n(T) = 1 + i(T1)+l(T1)+i(T2)+l(T2)
substitute l(T) = i(T) +1 to see if it equals 1 + n(T1) + n(T2)
n(T) = 1+i(T1)+i(T1)+1+i(T2)+i(T2)+1
simplify, recognize that n(T1) = i(T1) + l(T1):
n(T) = 1 + n(T1) + n(T2) Check!

Thus it is shown by induction that the number of leaves equals the number of internal vertices plus 1.

P332Q58

BASE: λ is empty which means there are zero (and zero). Check!

RECURSIVE: Assume x∊B equal to (,). Adding (() and ()) to the string results in an equal amount of parenthesis. Thus it is inductively proven.