(1)

(a) $(1-0.2)(1-0.2)0.2 = 0.8\times 0.8\times 0.2 = 0.128$

(b) $C_6^2\times 0.2^2\times 0.8^4\times 0.2 = C_6^2\times 0.2^3\times 0.8^4 = 0.049152$

(c) Negative Binomial $(3, 0.2)$

$EX = r/p = 3/0.2 = 15 \quad Var[X] = r\cdot\frac{1-p}{p} = 3\cdot\frac{1-0.2}{0.2} = \frac{0.24}{0.2} = 1.2$

(2)

(a) $\mathbb{E}[X] = \sum x f(x) = \sum_{k=0}^{\infty} k\cdot\frac{e^{-\lambda}\lambda^k}{k!} = \lambda e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^{k-1}}{(k-1)!} = \lambda e^{-\lambda}\cdot e^{\lambda} = \lambda$

(b) $\mathbb{E}[X(X-1)] = \mathbb{E}[X^2-X] = \mathbb{E}[X^2]-\mathbb{E}[X]$.

$\sum_{k=0}^{\infty} k\cdot(k-1)\cdot\frac{e^{-\lambda}\lambda^k}{k!} = \lambda^2 e^{-\lambda}\sum_{k=0}^{\infty}\frac{\lambda^{k-2}}{(k-2)!} = \lambda^2 e^{-\lambda}e^{\lambda} = \lambda^2$

Thus, $\mathbb{E}[X^2]-\mathbb{E}[X] = \lambda^2 \rightarrow \mathbb{E}[X^2] = \lambda^2+\lambda$. Thus, $Var[X] = \mathbb{E}[X^2]-\mathbb{E}^2[X]$

$= \lambda^2+\lambda-\lambda^2 = \lambda$

(3)

(a) $\mathbb{E}[X] = \sum x f(x) = \sum_{k=1}^{n} k\cdot\frac{1}{H_n}\cdot\frac{1}{k} = \sum_{k=1}^{n}\frac{1}{H_n} = n\cdot\frac{1}{H_n} = \frac{n}{H_n}$

(b) $Var[X] = \cancel{\frac{n(n+1)}{2H_n}}\ \mathbb{E}[X^2]-\mathbb{E}^2[X] = \sum x^2 f(x) = \sum_{k=1}^{n} k^2\cdot\frac{1}{H_n}\cdot\frac{1}{k} - \left(\frac{n}{H_n}\right)^2$

$\sum_{k=1}^{n}\frac{k}{H_n} = \frac{1}{H_n}\cdot\frac{n(n+1)}{2} = \frac{n(n+1)}{2H_n}$

$Var[X] = \frac{n(n+1)}{2H_n} - \left(\frac{n}{H_n}\right)^2$

(4) (a) Standardize $Z = \frac{X-\mu}{6} = \frac{5-10}{6} = -\frac{5}{6}$

$P(X>5) = P(Z>-\frac{5}{6})$. Since $\Phi(0.83)\approx 0.7967$

$P(X>5)\approx 0.7967$

(b) Standardize $Z = \frac{X-\mu}{6} = \frac{4-10}{6} = -1 \quad \frac{16-10}{6} = 1$

$P(4<X<16) = P(-1<Z<1) = \Phi(1)-\Phi(-1)$

$\Phi(1)\approx 0.84134 \quad \Phi(-1)\approx 0.15866$. ~~P(X<~~ $P(4<X<16)\approx 0.6827$

(c) $Z = \frac{8-10}{6} = -\frac{1}{3}$. $P(X<8) = P(Z<-\frac{1}{3})\approx 0.3707$.

(d) $Z = \frac{20-10}{6} = \frac{10}{6} = \frac{5}{3} \quad P(X<20) = P(Z<\frac{5}{3})$

Since $\Phi(\frac{5}{3})\approx 0.9525$. ~~P(X~~ $P(X<20)\approx 0.9525$.

(e) $\frac{16-10}{6} = 1 \quad P(X>16)\approx 1-0.8413 = 0.1587$

(5) standardize $\frac{9-5}{\sigma} = \frac{4}{\sigma}$

$P(X>9) = P(Z > \frac{4}{\sigma}) = 0.2$ This means $\Phi(z) = 1-0.2 = 0.8 \Rightarrow z \approx 0.84$

Thus, $\frac{4}{\sigma} \approx 0.84 \to \sigma \approx 4.76$ $Var(X) = \sigma^2$ is approximately $(4.7619)^2$
≈ 22.68

(6) After 1000 periods, the price is $S_0 \cdot u^k \cdot d^{1000-k}$

$k \sim Binomial(1000, 0.52)$ is the number of up moves.

We want $u^k \cdot d^{1000-k} \geq 1.3$

$\ln u^k + \ln d^{1000-k} \geq \ln(1.3) \to k\ln u + (1000-k)\ln d \geq \ln(1.3)$

$k\ln u + 1000\ln d - k\ln d \geq \ln(1.3) \to k \geq \frac{\ln(1.3) - 1000\ln d}{\ln u - \ln d}$

Since $u = 1.012$, $d = 0.99$, $k_{min} \approx 469.21$. we need $k = 470$.

$P(k \geq 470)$ where $k \sim Bin(1000, 0.52)$

$\mu = np = 1000 \times 0.52 = 520$. $\sigma = \sqrt{np(1-p)} \approx 15.7987$.

$z = \frac{470-520}{15.7987} \approx -3.16$. $P(k \geq 470) = 1 - \Phi(-3.16) \approx 0.999$.

(7)

(a) $P(T>2) = 1 - \lambda t = 1 - (1/2) \cdot 2 = 1 - 1 = 0$.

(b) $P(T \geq 10 \mid T > 9) = \cancel{\frac{P(T\geq 10)}{P(T>9)}} = P(T>10) - P(T>9)$

$= (1 - 1/2 \cdot 10) - (1 - 1/2 \cdot 9) = (1-5) - (1-4.5) = -0.5$.