Homework 3

MS&E 220 (Probabilistic Analysis)

October 17, 2025

Due: October 31 at 10:00 PM

Show your work and justify each step.
Answers submitted without explanation will not receive full credit.

1. [**8 points**] An oil company conducts a geological study indicating that an exploratory oil well has a 20% chance of striking oil. Assume independence across wells.

   (a) [**2 points**] What is the probability that the first strike occurs on the third well drilled?

   (b) [**2 points**] What is the probability that the third strike occurs on the seventh well drilled?

   (c) [**4 points**] If the company wants to establish three producing wells, compute the mean and variance of the total number of wells that must be drilled.

2. [**6 points**] Suppose X is a discrete random variable that follows a Poisson distribution, i.e., with the following probability mass function:

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}, \quad \text{for } k = 0, 1, 2, \ldots$$

   (a) [**3 points**] Give a proof of the value of $E[X]$;

   (b) [**3 points**] What is $\mathrm{Var}[X]$?

3. [**6 points**] A random variable X has the following probability mass function:

$$P(X = k) = \frac{1}{H_n} \cdot \frac{1}{k}, \quad \text{for } k = 1, 2, \ldots, n,$$

   where $H_n = \sum_{k=1}^{n} \frac{1}{k}$ is the n-th harmonic number.
   Compute:

   (a) [**3 points**] $E[X]$ in terms of n;

   (b) [**3 points**] $\mathrm{Var}[X]$ in terms of n.

4. [**10 points**] The normal random variable X is with parameters $\mu = 10$ and $\sigma^2 = 36$. Compute:

   (a) [**2 points**] $\mathbb{P}(X > 5)$;

   (b) [**2 points**] $\mathbb{P}(4 < X < 16)$;

   (c) [**2 points**] $\mathbb{P}(X < 8)$;

   (d) [**2 points**] $\mathbb{P}(X < 20)$;

   (e) [**2 points**] $\mathbb{P}(X > 16)$.

5. [**6 points**] Suppose $X \sim \mathscr{N}(5, \sigma^2)$ and it is known that $\mathbb{P}(X > 9) = 0.2$. Determine an *approximate* value of $\mathrm{Var}(X) = \sigma^2$. Show your standardization step and identify the z-value you use from the table.

6. [**6 points**] A model for the movement of a stock supposes that if the present price of the stock is s, then, after one period, it will be either us with probability p or ds with probability $1 - p$. Assuming that successive movements are independent, approximate the probability that the stock's price will be up at least 30 percent after the next 1000 periods if $u = 1.012$, $d = 0.990$, and $p = 0.52$.

7. [**8 points**] Let the repair time T (hours) be $\mathrm{Exp}(\lambda = 1/2)$, i.e., with density $f_T(t) = \lambda e^{-\lambda t}$ for $t \geq 0$. Compute:

   (a) [**4 points**] the probability that a repair time exceeds 2 hours;

   (b) [**4 points**] the conditional probability that a repair takes at least 10 hours, given that its duration exceeds 9 hours.