Question 1. (5 points)

Classify the systems as either being discrete or continuous:

a) Elevator system (You are interested in modeling the number of people waiting on each floor and traveling within the elevators.)

b) Judicial system (You are interested in modeling the number of cases waiting for trial.)

c) The in-air flight path of an airplane as it moves from an origin to a destination.

a) discrete ~ counting # of ppl (finite, integer val)

b) discrete - counting # of ppl (finite, integer val)

c) continuous - measurement (can be non integer value)

Question 2. (3 points) *True* or *False* (If it is false you need to provide the true statement)
The simulation clock for a discrete event dynamic stochastic model jumps in equal increments of time in the defined time units. For example, 1 second, 2 seconds, 3 seconds, etc.

random

The simulation clock for a discrete dynamic stochastic model jumps in increments depending on the interarrival and/or service times (departure) which are not always equal increments

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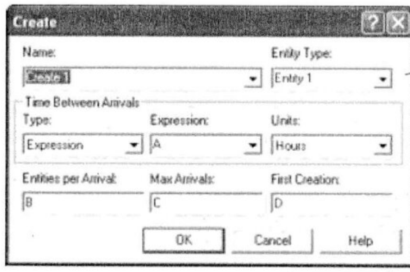
Q3 6.65

Question 3 (7 points)
Groups of customers arrive to a Blues, Bikes, and BBQ T-Shirt Concession Stand according to a Poisson process with a mean rate of 8 per hour. There is a 25% chance that a family of 4 will want T-Shirts, a 35% chance that a family of 3 will want T-Shirts, a 15% chance that a couple will want matching T-Shirts, and a 25% chance that an individual person will want a T-Shirt.

IAT = 8

0.25 – 4
0.35 – 3
0.15 – 2
0.25 – 1



Specify expressions for A, B, C, and D in the above CREATE module to properly generate customers for the T-Shirt Stand.

- A: EXPO(8)
- B: 1
- C: Infinite
- D: ∅

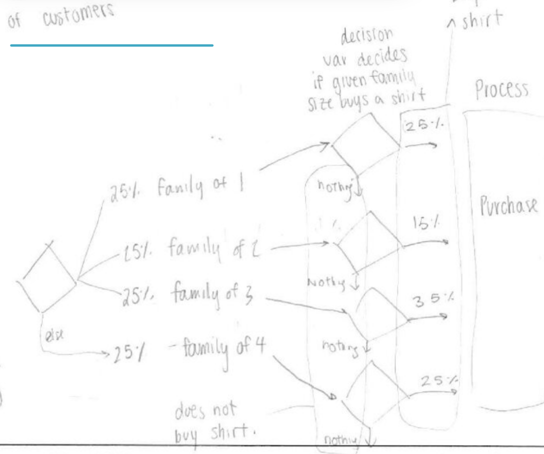
not specified in question

start at time ∅

part a - incorrect rate -0.35

arrival of customers

decision variable n-equal to decide family size (assume equal prob)



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Question 4. (55 points)

A Tim Hortons branch has one drive thru server and room for 2 additional cars to wait in line. Arriving customers to the drive thru service when the queue is full, leave without any order. The time between arrivals follows an exponential distribution with the mean time of 3 minutes and the service time is based on the following distribution:

Service time (Minutes)	1	2	3	4
Probability	0.22	0.48	0.2	0.1

We are interested to simulate the system to estimate the probability of balking because of full queue.

a. **(20 points)** Formulate the simulation model to be implemented in Excel. To do so, define all notations and formulas needed with all details to for Excel implementation.

b. **(15 points)** Now assume that we want to formulate the system in terms of discrete event simulation (DES). Provide the static and dynamic models of the simulation model.

c. **(20 points)** Now assume we want to simulate the system manually with the approach of DES. Start the simulation with one customer being served, leaving at time 3, and one in the queue. Process the manual simulation for 6 ~~new customers~~ time epochs (rows) in a table with the following format:

Clock (t)	Event type	System's state		IAT	Service time	FEL	Statistics	
		# in line	server's status				Cumulative number of balking	Server's up time

exp (3) For running the simulation consider the following random numbers in order as many as needed:

~~0.18~~, ~~0.45~~, ~~0.96~~, ~~0.78~~, ~~0.35~~, 0.48, 0.15, 0.08, 0.86, 0.65, 0.58, 0.38, 0.82, 0.25, 0.42

At the end calculate the following performance measures:

- Time-weighted average number of customers in system
- Server's utilization rate
- Estimated probability of balking

IAT = 3 min, expo.

Q S

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Question 4. (Continued)

a)

LO = # of lost customers

$LS(t)$ – # of customers in service (0, 1) (idle, in service)

$LQ(t)$ – # of customers in queue (0, 1, 2)

a – interarrival time = EXPON.DIST(rand(), 3, F)

S – service time

L = IF(rand() ≤ 0.22, 1, IF(rand() ≤ 0.70, 2, IF(rand() ≤ 0.9, 3, 4)))

Origin:
$LS(t)=0$
$LO=0$
$LQ(t)=0$

Events: arrival, departure
(A) (D)

Arrival
① Generate arrival event (A, t + a)
② If(LS(t) = 1,
go to ③, and
else LS(t) = 1 & go to ④ & schedule departure event (D, t + s) & go to ④)
③ If (LQ(t) < 2,
LQ(t) = LQ(t) + 1 & go to ④,
else LO = LO + 1 (customer exits system) & go to ④)
④ Collect stats & update counters

Departure
① Generate a departure event (D, t + s)
② LS(t) = LS(t) − 1
③ If (LQ(t) > 0,
LQ(t) = LQ(t) − 1 & LS(t) = 1 & schedule departure event (D, t+s) & go to ④,
do nothing (no queue) & go to ④)
④ Compute stats & update counters

STATS

avg # customers $= \int_0^{t_E} \frac{LS(t)}{t_E}\,dt$

avg queue length $= \int_0^{t_E} \frac{LQ(t)}{t_E}\,dt$

Question 4. (Continued)

STATIC

b) System State
LS(t) – # of customers in service at t   LO(t) = # of lost customers at t
LQ(t) – # of customers in queue at t

Entities: customers/cars, servers
$\underbrace{\text{customers/cars}}_{\text{entity}}$

Events: arrival (A), departure (D), End of simulation (E)
Event Notice: (A, t+a), (D, t+s), (E, ∞)
Attributes: N/A
Activities: interarrival time (a), service time (s)
FEL: { (A, t+a), (D, t+s), (E, ∞)
Lists: N/A
Delays - wait time in queue

define s = ① Let r = rand()   a = EXPO(3)
② If r ≤ 0.22, then s = 1 else go to ③   LS(t) = 0
③ If r ≤ 0.70, then s = 2 else go to ④   LQ(t) = 0
④ If r ≤ 0.90, then s = 3 else go to ⑤   LO(t) = 0
⑤ If r ≤ 1, then s = 4

Dynamic

Arrival
① Generate arrival event (A, t+a)
② If LS(t) = 1
then go to ③
else LS(t) = 1 and schedule departure event (D, t+s) and go to ④
③ If LQ(t) = 2
then LO(t) = LO(t) + 1 and go to ④
else LQ(t) = LQ(t) + 1 and go to ④
④ Collect stats & update counters and go to FEL

Question 4. (Continued)

DYNAMIC CONT'D

① Generate departure event (D, t+s)

② $LS(t) = LS(t) - 1$

③ If $LQ(t) > 0$

then $LQ(t) = LQ(t) - 1$ AND $LS(t) = 1$ AND go to ④

schedule a departure event (D, t+s)

else do nothing (no one in queue) → go to ④

④ Compute statistics and update counters and go to FEL

STATISTICS

avg # of customers in system $= \int_0^{t_E} \frac{LS(t)}{t_E} dt$

avg # of customers in queue $= \int_0^{t_E} \frac{LQ(t)}{t_E} dt$

# of lost customers = avg (LO(t)

max # of customers in system = max (LS(t))

max # of customers in queue = max (LQ(t))

c)

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Calculations are not correct -3

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Q5 18

M/M

Question 5. (20 points) The manager of a bank must determine how many tellers should be available. For every minute a customer stands in line, the manager believes that a delay cost of 5 cents is incurred. An average of 15 customers per hour arrive at the bank. On the average, it takes a teller 6 minutes to complete the customer's transaction. It costs the bank \$9 per hour to have a teller available. Inter-arrival and service times can be assumed to be exponentially distributed.

a) What is the minimum number of tellers that should be available in order for the system to be stable (i.e. not have an infinite queue)? M/M/c L_Q

b) If the system has 3 tellers, what is the probability that there will be no one in the bank? M/M/3 $P_0 = ?$

c) What is the expected total cost of the system per hour, when there are 2 tellers? M/M/2 W_Q

$\lambda = \frac{1}{15}$

$\mu = \frac{1}{\frac{60}{6}} = \frac{1}{10}$

\$9 /teller

\$0.05 / minute of delay

$\sigma = \sqrt{\frac{1}{(\frac{1}{15})^2}} = 15$

a) L_Q is stable when denominator $\neq 0$ or negative

for $c = 1$ (M/M/1)

$L_Q = \frac{(\frac{1}{15})^2}{\frac{1}{10}(\frac{1}{10} - \frac{1}{15})} = \frac{4}{3}$ each hour

not infinite

for $c = 2$ (M/M/2)

trial (for check)

for M/M/2

↳ from part c)

$L_Q = \frac{1}{12}$ (also stable) each hour

∴ the number of min tellers that should be available is 1

b) and c) on next page

any part - rate parameter is not calculated correctly -1

$\lambda = \frac{1}{15}$ $\mu = \frac{1}{10}$

Question 5. (Continued)

b) M/M/3 $P_0 = ?$ $c = 3$ $\rho = \frac{\lambda}{c\mu} = \frac{\frac{1}{15}}{3(1/10)} = \frac{2}{9}$

$$P_0 = \left\{ \left[\sum_{n=0}^{2} \frac{(c\rho)^n}{n!} \right] + \left[\frac{(c\rho)^c}{c!\,(1-\rho)} \right] \right\}^{-1}$$

$$= \left\{ \left[\frac{(3\rho)^0}{0!} + \frac{(3\rho)^1}{1!} + \frac{(3\rho)^2}{2!} \right] + \left[\frac{(3\rho)^3}{3!\,(1-\rho)} \right] \right\}^{-1}$$

$$= \left\{ 1 + \frac{2}{3} + \frac{2}{9} + \frac{4}{63} \right\}^{-1}$$

$$= \frac{21}{41} = 0.512$$

∴ the prob there is no one in the system is 0.51

c) M/M/2 $\rho = \frac{\frac{1}{15}}{2(1/10)} = \frac{1}{3}$

$W_Q = W - 1/\mu$

$= \frac{45}{4} - \frac{1}{\frac{1}{10}}$

$= 1.25$

$W = \frac{L}{\lambda}$

$= \frac{\left(\frac{3}{4}\right)}{\frac{1}{15}}$

$= \frac{45}{4}$

$L = 2(1/3) + \frac{\frac{1}{3}(1/6)}{2/3}$

$= \frac{3}{4}$

$P_0 = \left\{ \left[\sum_{n=0}^{1} \frac{(2(1/3))^n}{n!} \right] + \left[\frac{(2(1/3))^2}{2!\,(2/3)} \right] \right\}^{-1}$

$= \left\{ \frac{1}{1} + \frac{2}{3} + \frac{1}{3} \right\}^{-1}$

$= 2^{-1}$

$= \frac{1}{2}$

$= 0.5$

$P(L(n>2)) = \frac{(2(1/3))^2 (0.5)}{2!\,(2/3)}$

$= \frac{1}{6}$

$L_Q = \lambda W_Q$

$= \frac{1}{15}(1.25)$

$= 1/12$

$E(x) = 2(9) + 1.25(0.05)\left(\frac{1}{12}\right)$
$= 18.005$
$= \$18.01$

∴ the expected cost of the ... per hour is \$18.01 when there are 2 tellers

part c - time unit is not taken into account -1

f(x)
F(x)

| 3 | 2 | | 1 |

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c)

	Clock (t)	Event type	system state LQ(t)	LS(t)	IAT	Service time (min)	FEL	stats Cumulative # of balking	server up time	
①	0	A	1	1	r=0.18 0.066	0	(D, 3, c_1) (A, 0.066, C_3)	0	0	| 2 | | 1 |
②	0.066	A	2	1	r=0.45 0.20	0	(D, 3, c_1) (A, 0.266, C_4)	0	0	| 3 | 2 | | 1 |
③	0.266	A	2	1	r=0.96 1.07	0	(D, 3, c_1) (A, 1.336, C_5) 0.266 +1.07	1	0	4 | 3 | 2 | | 1 | ↳ leaves
④	1.336	A	2	1	r=0.78 0.50	0	(D, 3, C_1) (A, 1.836, C_6)	2	0	5 | 3 | 2 | | 1 | ↳ leaves
⑤	1.836	A	2	1	r=0.35 0.14	0	(D, 3, C_1) (A, 1.97, C_7)	3	0	6 | 3 | 2 | | 1 | ↳ leaves
⑥	1.97	A	2	1	r=0.48 0.22	∅	(D, 3, C_1) (A, 2.19, C_8)	4	0	7 | 3 | 2 | | 1 |

exponential equation

$$\text{Expo}(r) = \frac{-\ln(1-r)}{\lambda}$$

$\lambda = 3$

① IAT = Expo(0.45) = 0.19

server utilization rate $= \dfrac{\sum_0^{1.97}(\text{server up time})}{1.97} = \phi$

time weighted avg # of cust in system $= \dfrac{\sum_0^{1.97}(LQ(t) + LS(t))}{1.97} = 3$

probability of balking $= \dfrac{\text{\# of balking (cumulative)}}{6} = \dfrac{4}{6} = 0.67$

↳ # of epochs