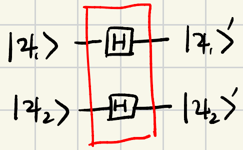


Q: How to write matrix form of this operation in 2qubit basis?

A: Step 1: Write out state vector on left hand side.

We have 2 general qubit states:

$|\psi_1\rangle = a_1|0\rangle + b_1|1\rangle$ and $|\psi_2\rangle = a_2|0\rangle + b_2|1\rangle$

and their tensor product state is

$$|\psi\rangle_{LHS} = |\psi_1\rangle \otimes |\psi_2\rangle = a_1a_2|00\rangle + a_1b_2|01\rangle + b_1a_2|10\rangle + b_1b_2|11\rangle$$

Writing in vector form:

$$|\psi\rangle_{LHS} = \begin{array}{c}\begin{bmatrix} a_1a_2 \\ a_1b_2 \\ b_1a_2 \\ b_1b_2 \end{bmatrix}\end{array}\begin{array}{l}|00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle\end{array}$$

Step 2: Write out state vector on right hand side.

On the right hand side, we have the Hadamard gate applied to both qubits:

$|\psi_1\rangle' = H_1|\psi_1\rangle = \frac{1}{\sqrt{2}}\left[a_1(|0\rangle+|1\rangle)+b_1(|0\rangle-|1\rangle)\right]$ and $|\psi_2\rangle' = H_2|\psi_2\rangle = \frac{1}{\sqrt{2}}\left[a_2(|0\rangle+|1\rangle)+b_2(|0\rangle-|1\rangle)\right]$

and the new tensor product state is

$$|\psi\rangle_{RHS} = |\psi_1\rangle' \otimes |\psi_2\rangle' = \frac{1}{2}\Big[a_1a_2(|00\rangle+|01\rangle+|10\rangle+|11\rangle) + a_1b_2(|00\rangle-|01\rangle+|10\rangle-|11\rangle)$$
$$+ b_1a_2(|00\rangle-|01\rangle-|10\rangle-|11\rangle) + b_1b_2(|00\rangle-|10\rangle-|01\rangle+|11\rangle)\Big]$$

In vector form, this becomes:

$$|\psi\rangle_{RHS} = \frac{1}{2}\cdot \begin{bmatrix} a_1a_2 + a_1b_2 + b_1a_2 + b_1b_2 \\ a_1a_2 - a_1b_2 + b_1a_2 - b_1b_2 \\ a_1a_2 + a_1b_2 - b_1a_2 - b_1b_2 \\ a_1a_2 - a_1b_2 - b_1a_2 + b_1b_2 \end{bmatrix}\begin{array}{l}|00\rangle \\ |01\rangle \\ |10\rangle \\ |11\rangle\end{array}$$

Step 3: Write out the form $Ax=b$

We're looking for the matrix that turns $|\psi\rangle_{LHS} \to |\psi\rangle_{RHS}$:

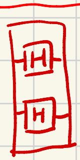
$$\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \\ x_{41} & x_{42} & x_{43} & x_{44} \end{bmatrix}\begin{bmatrix} a_1a_2 \\ a_1b_2 \\ b_1a_2 \\ b_1b_2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} a_1a_2 + a_1b_2 + b_1a_2 + b_1b_2 \\ a_1a_2 - a_1b_2 + b_1a_2 - b_1b_2 \\ a_1a_2 + a_1b_2 - b_1a_2 - b_1b_2 \\ a_1a_2 - a_1b_2 - b_1a_2 + b_1b_2 \end{bmatrix}$$

Solving top row example:

$$x_{11}a_1a_2 + x_{12}a_1b_2 + x_{13}b_1a_2 + x_{14}b_1b_2 = \frac{1}{2}(a_1a_2 + a_1b_2 + b_1a_2 + b_1b_2)$$

$$\Rightarrow x_{11} = \frac{1}{2},\ x_{12} = \frac{1}{2},\ x_{13} = \frac{1}{2},\ x_{14} = \frac{1}{2}$$

Extending to rest of rows we have final form of our matrix:



$$= \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

You can follow same steps to solve for any arbitrary operation (like the CNOT, etc).

In this case, since we have 2 separable operations, we could have taken a vast shortcut:

$$|H\rangle \otimes |H\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}_1 \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}_2$$
$$= \frac{1}{2}\begin{bmatrix} 1\cdot\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & 1\cdot\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ 1\cdot\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & -1\cdot\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \end{bmatrix}$$