

## Lecture 3 - Perfect Secrecy

1. Perfect correctness
  - For any  $k \in K$ ,  $m \in M$  and any ciphertext  $c$  output of  $Enc_k(m)$ , it holds that  $Pr[Dec_k(c) = m] = 1$
2. Towards defining perfect security
  - Defining security for an encryption scheme is not trivial
  - 3 sources of randomness:
    - generating message
    - choosing key
    - whatever randomness is being used in the probabilistic encryption
3. Attempt 1: Protect the key  $k$ !
  - Security means that
    - the adversary should **not** be able to **compute the key  $k$**
  - Wrong: maybe  $C$  (ciphertext) isn't secure and if they can get a hold of  $C$ , then they can get the message
    - what if  $Enc_k(m) := m$
4. Attempt 2: Don't learn  $m$ !
  - Security means that
    - the adversary should not be able to compute the message  $m$
  - Wrong: difference between knowing  $m$  (message) verbatim and inferring part of  $m$  (they can still get an idea of what the message was)
    - If the scheme protects part of  $m$ , the part that's not protected could leak and ruin  $m$
5. Attempt 3: Learn nothing!
  - Security means that
    - the adversary should not be able to learn any information about  $m$
  - Problem with this is that it ignores the adversary's prior knowledge on  $m$ 
    - Attacker may know something about  $m$ , therefore, it's nearly impossible for them to not learn **any** information about  $m$
6. Attempt 4: Learn nothing more!
  - Security means that
    - the adversary should not be able to learn any additional information on  $m$
  - Even if they know something about  $m$ , nothing additional from that point on is learned about  $m$  through  $C$
  - INCLUDE
  - Nothing in Eve's knowledge of  $M$  changed before or after the message was encrypted
7. Two equivalent views of perfect secrecy
  - a posteriori = a priori
    - For every  $D_M$ ,  $m \in M$  and  $c \in C$ , for which  $Pr[C = c] > 0$ , it holds that  $Pr[M = m|C = c] = Pr[M = m]$
  - $C$  is independent of  $M$ 
    - For every  $m, m' \in M$  and  $c \in C$ , it holds that  $Pr[Enc_k(m) = c] = Pr[Enc_k(m') = c]$ 
      - \* Equally likely that  $c$  hides  $m$  but also likely that  $c$  hides another message  $m'$  that is different from  $m$
  - Both are basically the same at capturing the idea of perfect secrecy

## Lecture 3.1 - The one-time pad

1. A perfect cipher
  - A type of “substitution” cipher that is “absolutely unbreakable”
    - substitution cipher
      - \* individually replace plaintext characters with shifted ciphertext characters
      - \* independently shift each message character in a random number
        - to encrypt a plaintext of length  $n$ , use  $n$  uniformly random keys  $k_1, \dots, k_n$