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## ## Problem Set 5 ##



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CS-334

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1. $\{ 0^{2^i} : i \geq 0 \}$

Consider a string s in the language $\{ 0^{2^i} : i \geq 0 \}$ with pumping length p.

$S = 0^{2^p}$

Lets say the language is regular.

Since Lang is regular, we can divide s into 3 parts, $S = xyz$ and must satisfy:

① $|y| > 0$

② $|xy| \leq p$

③ $xy^iz \in L \quad \forall i \geq 0$

Picking i to be 2, the second condition says $|xy| \leq p$, and we know $p < 2^p$ so $|y| < 2^p$

Therefore, $|xy^2z| = |xyz| + |y| < 2^p + 2^p = |xyz| + |y| < 2^{p+1}$

The first condition says $|y| > 0$, so now $2^p < |xyyz| < 2^{p+1}$, but doing this means that $|xyyz|$ isn't a power of 2, therefore, $|xy^2z| \notin L$ making it not regular.

2. Language $B = \{ 0^i 1^j : i \neq j \}$ and Language $C = \{ 0^i 1^i : i \geq 0 \}$. Assume B is regular.

The compliment of language B would be C unioned w/ all other possible strings not in B or C. Since B is regular, the compliment should be as well. We know that language C is not regular, therefore, when it is unioned w/ any other language, the result is a language that isn't regular. Because of this, B is not regular.

3. ① $0001^*$ Minimum pumping length is 4.

Following the same methodology as the example, a string of length 3 cannot be pumped as it would not be in the language. Any string of length 4 can however. by choosing $x = 000$, $y = 1$, and z to be the rest of the string.

③ $0^*1^*0^*1^* \cup 10^*1^*$ Minimum pumping length is 1

Since you can choose either $0^*1^*0^*1^*$ or $10^*1^*$ thanks to the union, we (2) will work with $0^*1^*0^*1^*$. $P \neq 0$ because $\varepsilon$ can't be pumped. So if $p = 1$, then it is either a 0 or 1, and the two of them can be pumped.

(3) $0^*1^*$ Minimum pumping length is 1

Same reasoning for above, it can't be 0 as the empty string, $\varepsilon$, can't be pumped. However, w/ a length of 1, 0 or 1 can both be pumped.

(4) $(01)^*$ Minimum pumping length is 1.

The pumping lemma states that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or greater. Since $(01)^*$ always has an even string length $\geq 2$, besides $\varepsilon$ but this can't be pumped, $p=1$ is valid because $2 \geq 1$.

(5) $1^*01^*01^*$ Minimum pumping length is 3.

P cannot be 2 because the only string that allows for this is 00 and it cannot be pumped. as it is not in the language. If $p = 3$, then it can be pumped with $x = 00$, $y = 1$, and $z =$ the rest of the string

4. (1) $\{w : w = w^R\}$, the language of palindromes.

$S \rightarrow 1S1 \mid 0S0 \mid 1 \mid 0 \mid \varepsilon$

(2) $\{w : w$ starts and ends w/ the same symbol $\}$

$S \rightarrow \varepsilon \mid 0 \mid 1 \mid 0T0 \mid 1T1$

$T \rightarrow 1T \mid 0T \mid \varepsilon$

(3) $\{w : w$ contains more 0's than 1's $\}$

$S \rightarrow 0T \mid T0 \mid T0T$

$T \rightarrow 0T1 \mid 1T0 \mid 0T \mid T0 \mid \varepsilon$