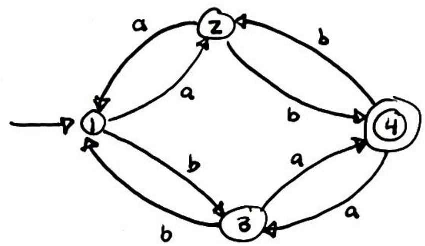
Eric Altenburg ## Problem Set 4 ##

9/24/19 CS-334

I pledge my honor that I have obided by the Stevens Honor System. - Eric Altenburg

1.

	1	2	3	4	5	6	7	8	9
1	-	✓	✓	✓		✓		✓	
2	-	-	✓	✓	✓	✓	✓		✓
3	-	-	-	✓	✓		✓	✓	✓
4	-	-	-	-	✓	✓	✓	✓	✓
5	-	-	-	-	-	✓		✓	
6	-	-	-	-	-	-	✓	✓	✓
7	-	-	-	-	-	-	-	✓	
8	-	-	-	-	-	-	-	-	✓
9	-	-	-	-	-	-	-	-	-



2. a) Prove: if A is a reg. lang., then so is $\bar{A}$, the compliment of A.

The compliment of A, $\bar{A}$, is defined as $\Sigma^* - A$.

Take a DFA that accepts language A, call it $M = \{Q, \Sigma, \delta, q_0, F\}$

In order to make $\bar{A}$ a reg. lang. a DFA must recognize it. To do so, make a new DFA called $M_2 = \{Q_2, \Sigma, \delta_2, q_{02}, F_2\}$ where $F_2 = Q - F$. In this DFA, we use the same structure as M, but turn all accept states to non-accept states. Then turn all old non-accept states in M and make them accept states in M_2. This DFA will recognize $\bar{A}$ as it accepts the things M doesn't, which is A. Since a DFA recognizes $\bar{A}$, $\bar{A}$ is now regular.

b) Prove that if A, B are reg., then so is $A-B$, the diff. of A and B.

The difference between regular languages can be rewritten as $A \cap \bar{B}$. We now know that the union of two languages is closed therefore allowing it to be regular. Also, the compliment of a language is considered to be regular if the original language is regular as well. Therefore, by using this knowledge, $A \cup \bar{B}$ is regular which means $A-B$ is as well.

2. c) $L = \{a^i b^j c^k : i, j, k \geq 0 \text{ and } i = 1 \Rightarrow j = k\}$

① $\forall i \geq 0$, $xy^iz \in A$

② $|y| > 0$

③ $|xy| \leq p$ where p is pumping threshold

Hint : set $p = 2$. Let s be a string in the lang such that $|s| \geq 2$.

Then we say that s can be divided into 3 substrings x, y, z. $(s = xyz)$

Now consider any string $a^i b^j c^k$ that is in the language.

In the case where $i = 1$ or $i > 2$, set $x = \varepsilon$ and $y = a$.

if $i = 1$, then $j = k$, so xyz will still belong to L

if $i > 2$, then through pumping y, all strings will have 2 or more a's.

meaning it is still in L.

$|y| = 1$ which is > 0 $(|y| = 1 > 0)$ and $|xy| \leq p$.

If $i = 2$, let $x = \varepsilon$ and $y = aa$, $|y| > 0$ and $|xy| \leq p$,

xy^iz will receive an even # of a's w/ b^*c^* coming after it, therefore, $xy^iz \in L$.

If $i = 0$, the $x = \varepsilon$, and $y = b$ as the first two symbols are either bb or bc,

$|y| > 0$, $|xy| \leq p$, xy^iz will just be y repeated several times, therefore, it is still in L.

in the last case of $i = 0$ and $j = 0$, $y = c$, because cc are the first 2 symbols in s,

$|y| > 0$, $|xy| \leq p$, xy^iz (which is now $b^i c^k$) is also in L, the 3 conditions

are met here as well.

Therefore, it is evident that in all cases, the 3 conditions of the pumping lemma

are met.

d) Prove that L is not regular. Use part (b). $L = b^*c^* \cup aaa^*b^*c^* \cup \{ab^ic^i : i \geq 0\}$

We can prove this with a contradiction. Let's say the language

$L = b^*c^* \cup aaa^*b^*c^* \cup \{ab^ic^i : i \geq 0\}$ is regular, then we can rewrite

it as:

$L - b^*c^* - aaa^*b^*c^* = \{ab^ic^i : i \geq 0\}$

If this language truly is regular, then both the left-hand and right-hand side

should be regular.

Now, using the pumping lemma :

pumping length p

say string s is in the language is $s = ab^pc^p$

say $p=4$, then s will become $s = \underbrace{a\,bbbb\,cccc}_{\text{split this up into 3 substrings } xyz} = ab^4c^4$

The pumping lemma says that these three conditions must be satisfied:

① $\forall i \geq 0: xy^iz \in L$

② $|y| > 0$

③ $|xy| \leq p$

So splitting up string s we get:

case 1: y is in the b part $\underbrace{a}_{x}\,\underbrace{bbbb}_{y}\,\underbrace{cccc}_{z}$

$xy^iz = xy^2z$, choose $i=2$ for example.

then we have ab bbbbbb cccc $= a^1b^7c^4$

$xy^2z \notin L$, $|xy| \leq p$ $p=4$ $8 \nleq 4$

case 2: y is in c part $\underbrace{abbbb}_{x}\,\underbrace{ccc}_{y}\,\underbrace{c}_{z}$

$i=2$, abbbb cccccc c $= ab^4c^7$

$xy^2z \notin L$, $11 \nleq 4$

case 3: y is in bc part $\underbrace{abb}_{x}\,\underbrace{bbcc}_{y}\,\underbrace{cc}_{z}$

$i=2$, abb bbccbbcc cc $= ab^4c^2b^2c^4 \neq ab^ic^i$

$xy^2z \notin L$, $11 \nleq 4$

In all of the above possible substrings, the 3 conditions of the pumping lemma are not satisfied at the same time. Therefore, $\{ab^ic^i : i \geq 0\}$ is not regular.

Therefore, since the right-hand side is not regular, the language as a whole is not regular.

e) C and D do not contradict the pumping lemma because it states that all regular langs must satisfy the 3 conditions above, but it is not necessary that any random language that satisfies the 3 conditions is a reg. lang. In other words, in C the lemma was not used to prove a language is not regular, it was the opposite, which is not the proper use of it.