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1. $\{a^i b^k c^i d^k : i, k \geq 0\}$

We assume the above CFL is context free

Take string $s = a^p b^p c^p d^p$ For a pumping length p.

Since $s \geq p$, we can split s into the following substrings, $s = uvxyz$, and it should meet these 3 conditions:

① $uv^ixy^iz \in A$, $\forall i \geq 0$

② $|vy| > 0$

③ $|vxy| \leq p$

$$s = \underbrace{a \ldots a}_{p}\underbrace{b \ldots b}_{p}\underbrace{c \ldots c}_{p}\underbrace{d \ldots d}_{p}$$

Trying to find windows in the string (vxy) that are at most of length p:

- choosing all a's: This won't work as the proportion of a's to c's is off, resulting string is not in the lang.
- choosing combo of a's and b's: This won't work because the proportion of both a to c and b to d is broken, resulting string is not in lang.
- choosing all b's: won't work as too many b's now # of b's ≠ # of d's. resulting string is not in lang.
- choosing combo of b's and c's: won't work as # a's ≠ # c's and # b's ≠ # d's resulting string not in lang.
- choosing all c's: # a's ≠ # c's, resulting string is not in lang.
- choosing combo of c's and d's: # a's ≠ # c's and # b's ≠ # d's, resulting string not in lang.
- choosing all d's: # b's ≠ # d's, resulting string not in lang.

Since none of all possible windows satisfy the 3 conditions, and the window cannot encompass the whole string due to condition ③, the language is not context-free.

2. $L = \{a^i b^k c^i d^k : i, k \geq 0\}$

For a 2-stack PDA, we will refer to the first stack as "stack 1" and the other as "stack 2".

The following description of a 2-stack PDA will accept all strings in the lang.:

Push a's onto stack 1

when the first b is read:

change states

push b's onto stack 2

when the first c is read:

change state

pop stack 1

Loop: For every c read, pop stack 1

when the first d is read:

change state

pop stack 2

Loop: For every d read, pop stack 2

Enter the accept state when both stack 1 and stack 2 are empty

3. $L_{add} = \{a^i b^{i+j} c^j : i, j \geq 0\}$

You can think of $a^i b^{i+j} c^j : i, j \geq 0$ as $a^i b^i b^j c^j : i, j \geq 0$ and w/ this we can construct a CFG for it:

$S \rightarrow TU$

$T \rightarrow aTb \mid \varepsilon$

$U \rightarrow bUc \mid \varepsilon$

This is accomplished by breaking down the above language into 2 parts, then using the closed operation - concatenation.

$L_1 = \{a^i b^i : i \geq 0\}$, the following is its respective CFG:

$S_1 \rightarrow aS_1b \mid \varepsilon$

$L_2 = \{b^j c^j : j \geq 0\}$, the following is its respective CFG:

$S_2 \rightarrow bS_2c \mid \varepsilon$

Using concatenation $(\{a^i b^i : i \geq 0\} \circ \{b^j c^j : j \geq 0\})$, this produces the CFG for L, shown above, proving that L is context-free.

3. $L_{mult} = \{a^i b^{ij} c^j : i,j \geq 0\}$

To prove that this lang. is not context-free, we first assume it is context-free in order to use the pumping Lemma:

Take string $s = a^p b^{p^2} c^p$ for pumping length p

Since $s \geq p$, we can split s into the following substrings, $s = uvxyz$, and it should satisfy these 3 conditions:

① $uv^ixy^iz \in A, \forall i \geq 0$

② $|vy| > 0$

③ $|vxy| \leq p$

$s = \underbrace{a \ldots a}_{p}\underbrace{b \ldots b}_{p^2}\underbrace{c \ldots c}_{p}$

Trying to find windows in the string (vxy) that are of length at most p:

- choosing all a's: # of a's will be disproportional to rest of string and # of b's ≠ # of a's × # of c's, and resulting string will not be in lang.
- choosing b's: # of b's will be greater than product of # of a's and # of c's. resulting string is not in lang.
- choosing all c's: # of b's will be less than the product of the # of a's and # of c's as the # of c's will have increased. Resulting string will not be in lang.
- choosing combo of a's and b's: for this window, say we add an additional a to the string resulting in $a^{p+1} b^{(p+1)p} c^p = a^{p+1} b^{p^2+p} c^p$, with this we would have to cover an additional p-amount of b's in our window for every a added and $|uvy| = p+1$ but $p+1 \nleq p$, therefore, resulting string does not satisfy condition ③
- choosing combo of b's and c's: for this, it is similar as above. add an additional c to the string: $a^p b^{p(p+1)} c^{p+1} = a^p b^{p^2+p} c^{p+1}$ which means we need to cover an additional p-amount of b's for every c added which means $|uvy| = p+1$ and $p+1 \nleq p$ resulting in a contradiction w/ condition ③.

3. For every potential window of vxy, it does not satisfy all 3 conditions, therefore, the language is not context-free.

4. L = compliment of $\{a^n b^n \mid n \geq 0\}$

Break down L into $L_1 \cup L_2$ where $L_1 = \{a^n b^m \mid m \neq n\}$ and

L_2 = lang. of b followed by an a.

The CFG for L_1:

$S_1 \to T_1 \mid U_1 \mid aS_1b$

$T_1 \to aT_1 \mid a$

$U_1 \to U_1b \mid b$

The CFG for L_2:

$S_2 \to T_2\, ba\, T_2$

$T_2 \to T_2T_2 \mid a \mid b \mid \varepsilon$

We then union the two CFG's to create the CFG for L.

$S \to S_1 \mid S_2$

$S_1 \to T_1 \mid U_1 \mid aS_1b$

$S_2 \to T_2\, ba\, T_2$

$T_1 \to aT_1 \mid a$

$T_2 \to T_2T_2 \mid a \mid b \mid \varepsilon$

$U_1 \to U_1b \mid b$

$\{x_1 \# x_2 \# \ldots \# x_k \mid k \geq 1$ each $x_i \in \{a,b\}^*$ and for some i, j $x_i = x_j^R\}$

Push $ on stack to indicate the last element

Read first letter of x_i nondeterministically and push to the stack.

Repeat for every symbol in x_i.

Stop pushing to stack once a # is read.

In some x_i, read the random symbols of a's and b's

Once the reverse of x_i, by recognizing nondeterministically, read and pop stack until $ is reached.

Stop when $ is read. as stack is now empty, enter accept state.

Optional problem 1:

With C as a context-free language and R as a regular lang. with PDA P that recognizes C and D be the DFA that recognizes R.

We construct a PDA P' that recognizes $C \cap R$ with the set of states being $Q \times Q'$ where Q is the set of states from P and Q' is the set of states from C.

P' will do what P does but now it keeps track of the states from D.

It accepts string w IFF it stops at a state $q \in F_P \times F_D$ where F_P is the set of accept states in P and F_D is set of accept states of D.

Now, since $C \cap R$ is recognized by P', it is context-free.

Optional problem 2:

I pledge my honor that I have abided by the Stevens Honor System. - [signature]