Kitty's Calculations on a Tree

Eric Altenburg, Hamzah Nizami, and Connie Xu

Outline

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What is Kitty's Calculations on a Tree?

Link to problem: here

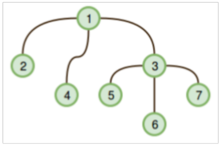
- You are given a tree and a set of subsets.
- The goal of this is to calculate the below expression on each of the sets.

$$(\sum_{u,v} u * v * dist(u, v)) \mod (10^9 + 7)$$

Kitty's Calculations on a Tree Continued

What goes in:

Kitty's Tree **n = 7**



Alex's set **q = 3**

{5}, {2,4,5}, {2,4}

What comes out:

{2,4} Only pair that can be formed is (2,4)

$$(2 \cdot 4 \cdot dist(2,4)) \bmod (10^9+7)$$
$$\Rightarrow (2 \cdot 4 \cdot 2) \bmod (10^9+7)$$
$$\Rightarrow 16$$

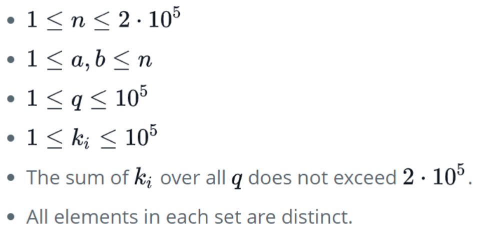
{5} No pairs can be formed because we do not have at least two elements.

{2,4,5} Pairs that can be formed: (2,4), (2,5), and (4,5)

$$(2 \cdot 4 \cdot dist(2,4) + 2 \cdot 5 \cdot dist(2,5) + 4 \cdot 5 \cdot dist(4,5)) \bmod (10^9+7)$$
$$\Rightarrow (2 \cdot 4 \cdot 2 + 2 \cdot 5 \cdot 3 + 4 \cdot 5 \cdot 3) \bmod (10^9+7)$$
$$\Rightarrow 106$$

Figure 1: Shows an example input and expected output

Constraints of the problem



- $1 \le n \le 2 \cdot 10^5$
- $1 \le a, b \le n$
- $1 \le q \le 10^5$
- $1 \le k_i \le 10^5$
- The sum of k_i over all q does not exceed $2 \cdot 10^5$.
- All elements in each set are distinct.

n is the number of nodes on a tree, a,b are the subset pairs in the tree, q is the number of sets, and k is the number of distinct nodes

Test Cases and Descriptions

The first test case is shown in Figure 1 in our presentation. This test case is a simple test case to ensure that our code runs properly.

- The first number on line 1 denotes how many nodes are in the tree.
- The second number on line 1 denotes the number of sets.
- The following n-1 lines will show the pair of integers that are connected in the tree.
- Then, the subsequent lines will first denote the number of elements in the set, and then the next line will be which numbers are in each set.

Test Cases and Descriptions

The syntax for the input is the following:

```
7 3
1 2
1 3
1 4
3 5
3 6
3 7
2
2 4
1
5
3
2 4 5
```

Test Cases and Descriptions

The output of the above test case should yield for each set, the summation of each. Hence, this should return 16, 0, and 106, respectively (as the first set 2,4, the second set is 5 and the third set is 2,4,5).
To find all of the test cases, just click here.

Time to Trace!

Now, we shall trace!

Failed Attempts

The different approaches we took before we arrived at our current solution are the following:

- Brute Force-ish
    - We attempted to implement a tree and we traversed it with the distance and did what the problem described to do, but it was too slow.
- Floyd's Algorithm
    - This did not work because we quickly realized that this would likely exceed the time limit. This algorithm also proved to be tough to implement with this problem so we moved on and tried a different approach.
- Intermediate vertices
    - Looking at the intermediate vertices proved futile as we attempted this problem. It did not provide helpful information on its own.
- Depth first search
    - DFS on its own was not the solution. If the tree was too deep, this algorithm would have exceeded the time limit on its own.

Failed Attempts

- Formula manipulation
  - We thought messing with the formula may be effective as with previous homework from 370, but this information alone was not quite helpful.
- Lowest Common Ancestor
  - Alone, this did not work.

With most of the attempts to find a solution, we found ourselves failing due to time constraints, space constraints, or incorrect implementation.
Ultimately, we found that a combination of lowest common ancestor and formula manipulation was the way to go and this is what we ended up implementing. There was a lot of tracing that went into this.