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I pledge my honor that I have abided by the Stevens Honor System. - *Eric Attenburg* *Hamzah Nizami*

- 1) let $F = \text{proc } (x: \text{int}) \{ \text{proc } (y: \text{int}) \{ x - y \} \}$
in $(F(3-4))$

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \Gamma(x) = \text{int} \\ \hline \Gamma \vdash x : \text{int} \end{array} \quad \begin{array}{c} \checkmark \\ \Gamma(y) = \text{int} \\ \hline \Gamma \vdash y : \text{int} \end{array} \\
 \hline \Gamma(x: \text{int}, y: \text{int}) \vdash x - y : \text{int} \quad \text{Tdiff} \\
 \hline x: \text{int} \vdash \text{proc } (y: \text{int}) \{ x - y \} : \text{int} \rightarrow \text{int} \quad \text{Tproc} \\
 \hline t \vdash \text{proc } (x: \text{int}) \{ \text{proc } (y: \text{int}) \{ x - y \} \} : \text{int} \rightarrow \text{int} \rightarrow \text{int} \quad \text{Tproc} \\
 \hline t \vdash \text{let } \underbrace{F}_{\text{id}} = \underbrace{\text{proc } (x: \text{int}) \{ \text{proc } (y: \text{int}) \{ x - y \} \}}_{e_1} \text{ in } \underbrace{(F(3-4))}_{e_2} : \text{int} \quad \text{Tlet}
 \end{array}$$

Because of the highlighted section, there is no type environment Γ and type t that will allow $\Delta \vdash (F(3-4)) : t$ to be derivable. $\text{int} \rightarrow \text{int} \neq \text{int}$.

- 2) if $\text{zero?}(\text{zero?}(0))$ then 1 else 2 is not typable because for any type-environment Γ with type t , the above expression is not derivable.

$$\begin{array}{c}
 \begin{array}{c} \times \\ \Gamma \vdash \text{zero?}(0) : t \end{array} \quad \begin{array}{c} \checkmark \\ \Gamma \vdash 1 : t \end{array} \quad \begin{array}{c} \checkmark \\ \Gamma \vdash 2 : t \end{array} \\
 \hline \Gamma \vdash \text{if } \text{zero?}(\text{zero?}(0)) \text{ then } 1 \text{ else } 2 : t \quad \text{Tif}
 \end{array}$$

- 3) ① $\text{bool} \rightarrow \text{int} : \text{proc } (x: \text{bool}) \{ \text{if } x \text{ then } 1 \text{ else } 2 \}$

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \Gamma(x) : \text{bool} \\ \hline \Gamma \vdash x : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \Gamma \vdash 1 : \text{int} \end{array} \quad \begin{array}{c} \checkmark \\ \Gamma \vdash 2 : \text{int} \end{array} \\
 \hline \Gamma[x: \text{bool}] \vdash \text{if } x \text{ then } 1 \text{ else } 2 : \text{int} \quad \text{Tif} \\
 \hline t \vdash \text{proc } (x: \text{bool}) \{ \text{if } x \text{ then } 1 \text{ else } 2 \} : \text{bool} \rightarrow \text{int} \quad \text{Tproc}
 \end{array}$$

The above type judgement is derivable, therefore, the expression, $\text{proc } (x: \text{bool}) \{ \text{if } x \text{ then } 1 \text{ else } 2 \}$ is typable.

② $(\text{bool} \rightarrow \text{int}) \rightarrow \text{int} : \text{proc } (x:\text{bool}) \{ \text{proc } (y:\text{int}) \{ \text{if } x \text{ then } y \text{ else } 99 \} \}$

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \hline \Gamma(x) = \text{bool} \\ \hline \Gamma \vdash x : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Gamma(y) = \text{int} \\ \hline \Gamma \vdash y : \text{int} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Gamma 99 = \text{int} \\ \hline \Gamma \vdash 99 : \text{int} \end{array} \\
 \hline \Gamma[x:\text{bool}, y:\text{int}] \vdash \text{if } x \text{ then } y \text{ else } 99 : \text{int} \quad \text{Tif} \\
 \hline x:\text{bool} \vdash \text{proc } (y:\text{int}) \{ \text{if } x \text{ then } y \text{ else } 99 \} : \text{int} \rightarrow \text{int} \quad \text{Tproc} \\
 \hline \vdash \text{proc } (x:\text{bool}) \{ \text{proc } (y:\text{int}) \{ \text{if } x \text{ then } y \text{ else } 99 \} \} : (\text{bool} \rightarrow \text{int}) \rightarrow \text{int} \quad \text{Tproc}
 \end{array}$$

The above type judgement is derivable, therefore, the expression $\text{proc } (x:\text{bool}) \{ \text{proc } (y:\text{int}) \{ \text{if } x \text{ then } y \text{ else } 99 \} \}$ is typable.

③ $\text{bool} \rightarrow (\text{bool} \rightarrow \text{bool}) :$
 $\text{proc } (a:\text{bool}) \{ \text{if } a$
 then $\text{proc } (b:\text{bool}) \{ \text{if } b \text{ then } \text{zero?}(a) \text{ else } \text{zero?}(1) \}$
 else $\text{proc } (c:\text{bool}) \{ \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(a) \} \}$

$$\begin{array}{c}
 \begin{array}{c} \checkmark \\ \hline \Delta(b) = \text{bool} \\ \hline \Delta \vdash b : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Delta \vdash 0 : \text{int} \\ \hline \Delta \vdash \text{zero?}(0) : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Delta \vdash 1 : \text{int} \\ \hline \Delta \vdash \text{zero?}(1) : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Gamma(c) = \text{bool} \\ \hline \Gamma \vdash c : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Gamma \vdash 1 : \text{int} \\ \hline \Gamma \vdash \text{zero?}(1) : \text{bool} \end{array} \quad \begin{array}{c} \checkmark \\ \hline \Gamma \vdash 0 : \text{int} \\ \hline \Gamma \vdash \text{zero?}(0) : \text{bool} \end{array} \\
 \hline \Delta \vdash b : \text{bool} \quad \Delta \vdash \text{zero?}(0) : \text{bool} \quad \Delta \vdash \text{zero?}(1) : \text{bool} \quad \Gamma \vdash c : \text{bool} \quad \Gamma \vdash \text{zero?}(1) : \text{bool} \quad \Gamma \vdash \text{zero?}(0) : \text{bool} \\
 \hline \Delta[b:\text{bool}] \vdash \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) : \text{bool} \quad \Gamma[c:\text{bool}] \vdash \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) : \text{bool} \\
 \hline \Delta[b:\text{bool}] \vdash \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) : \text{bool} \quad \Gamma[c:\text{bool}] \vdash \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) : \text{bool} \\
 \hline \Sigma(a) = \text{bool} \quad \text{proc } (b:\text{bool}) \{ \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) \} \quad \text{proc } (c:\text{bool}) \{ \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) \} \\
 \hline \Sigma[a:\text{bool}] \vdash a : \text{bool} \quad \text{else } \text{zero?}(1) : \text{bool} \rightarrow \text{bool} \quad \text{else } \text{zero?}(0) : \text{bool} \rightarrow \text{bool} \\
 \hline a:\text{bool} \vdash \text{if } a \\
 \hline \text{then } \text{proc } (b:\text{bool}) \{ \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) \} \\
 \hline \text{else } \text{proc } (c:\text{bool}) \{ \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) \} : \text{bool} \rightarrow \text{bool} \\
 \hline a:\text{bool} \vdash \text{if } a \\
 \hline \text{then } \text{proc } (b:\text{bool}) \{ \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) \} \\
 \hline \text{else } \text{proc } (c:\text{bool}) \{ \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) \} : \text{bool} \rightarrow \text{bool}
 \end{array}$$

$\vdash \text{proc } (a:\text{bool}) \{ \text{if } a$
 then $\text{proc } (b:\text{bool}) \{ \text{if } b \text{ then } \text{zero?}(0) \text{ else } \text{zero?}(1) \}$
 else $\text{proc } (c:\text{bool}) \{ \text{if } c \text{ then } \text{zero?}(1) \text{ else } \text{zero?}(0) \} \}$
 $\text{bool} \rightarrow (\text{bool} \rightarrow \text{bool})$

The above type judgement is derivable, moreover, the expression

$\text{proc } (a:\text{bool}) \{ \text{if } a$

$\text{then proc } (b:\text{bool}) \{ \text{if } b \text{ then zero?}(0) \text{ else zero?}(1) \}$

$\text{else proc } (c:\text{bool}) \{ \text{if } c \text{ then zero?}(1) \text{ else zero?}(0) \} \}$

is typable.