

**Pledge:** *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

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**1:** Prove that each of the following relations is an equivalence relation. Then describe the corresponding equivalence classes, e.g. by giving a geometric description.

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(The relation  $R$  defined on  $\mathbb{R}^2$  by  $((a, b), (c, d)) \in R$  if  $|a| + |b| = |c| + |d|$ .)

(The relation  $S$  defined on the set of positive rational numbers  $\mathbb{Q}_{>0}$  by  $(a, b) \in S$  if  $\frac{a}{b} = 2^n$  for some  $n \in \mathbb{Z}$ .)

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**2:** Let  $R$  and  $S$  be equivalence relations on a set  $A$ . Prove or disprove the following statements.

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(The relation  $R \cup S$  is an equivalence relation on  $A$ .)

(The relation  $R \cap S$  is an equivalence relation on  $A$ .)

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**3:** The set of integers *modulo*  $n$ , where  $n > 1$  is a natural number, is denoted  $\mathbb{Z}/n\mathbb{Z}$  and is defined as the set of equivalence classes under the equivalence relation on  $\mathbb{Z}$  of being congruent modulo  $n$ . Prove that it is possible to define addition and multiplication operations on  $\mathbb{Z}/n\mathbb{Z}$  via the formulas  $[a] + [b] = [a + b]$  and  $[a] \cdot [b] = [a \cdot b]$ , respectively.

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