Pledge: *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

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1: Write a proof for the following theorem:

Theorem 1. *For any natural number n, the number $n^2 - n$ is even.*

Proof. Case 1: Let n be an even natural number.

Because n is even, it can be written as $n = 2k$ where $k \in \mathbb{N}$. Then:

$$\begin{aligned} n^2 - n &= (2k)^2 - (2k) \\ &= 4k^2 - 2k \\ &= 2(2k^2 - k) \end{aligned}$$

This is the definition of an even number.

Case 2: Let n be an odd natural number.

Because n is odd, it can be rewritten as $n = 2k + 1$ where $k \in \mathbb{N}$. Then:

$$\begin{aligned} n^2 - n &= (2k+1)^2 - (2k+1) \\ &= 4k^2 + 4k + 1 - 2k - 1 \\ &= 4k^2 + 2k \\ &= 2(2k^2 + k) \end{aligned}$$

This is the definition of an even number.

Since these two cases cover all possible values of n, it follows that $n^2 - n$ always results to being even. $\square$