Pledge: *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

1: Suppose that $\{A_n\}_{n\in\mathbb{N}}$ is a countable collection of countable sets.

(Is the union $\bigcup_{n\in\mathbb{N}} A_n = A_1 \cup A_2 \cup \ldots \cup A_n \cup \ldots$ countable? Try to prove or disprove your answer.)

Proof. Since we know that $\{A_n\}_{n\in\mathbb{N}}$ is the countable collection of countable sets, that means each set A_n where $n \in \mathbb{N}$ in the collection is countable. Because the union of two countable sets is still countable, then this means $A_1 \cup A_2$ is countable and forms its own countable set, suppose it is called A_12. Then doing $A_12 \cup A_3$ would also lead to a countable set. This patten continues for all $n \in \mathbb{N}$. □

(Is the product $\prod_{n\in\mathbb{N}} A_n = A_1 \times A_2 \times \ldots \times A_n \times \ldots$ countable? Again, try to prove or disprove your answer.)

Proof. Again, we know that $\{A_n\}_{n\in\mathbb{N}}$ is the countable collection of countable sets, that means each set A_n where $n \in \mathbb{N}$ in the collection is countable. Because the product of two countable sets is still countable, then this means $A_1 \times A_2$ creates its own countable set called A_12. Then doing $A_12 \times A_3$ would also lead to a countable set. This pattern continues for all $n \in \mathbb{N}$. □

2: Assuming the Schröder-Bernstein theorem (you may or may not find it helpful), see if you can explain whether the following sets have the same cardinality.

($A = (0, 1)$ and $B = [0, 1]$)

This doesn't have the same cardinality because the the set of numbers in A do not include 2 numbers, 0 and 1, which are included in B. This means the cardinality of A will always be 2 less than B.

($A = (0, 1)$ and $B = \mathbb{R}$)

Assuming the theorem we can show that (0,1) is a subset of $\mathbb{R}$ by simply taking a function $f : (0, 1) \to \mathbb{R}$ where any number from (0,1) will give itself. This shows that $\mid (0, 1) \mid \leq \mid \mathbb{R} \mid$. Now to show that $\mathbb{R} \geq (0, 1)$ we create a second function $g : \mathbb{R} \to (0, 1)$ where for any number $x \in \mathbb{R}$ if it is not negative you do $\frac{1}{2} + 2^{-x-1}$ and if it is negative then you get 2^{n-1}. This maps the function from $\mathbb{R}$ to (0,1) which shows that $\mid \mathbb{R} \mid \leq \mid (0, 1) \mid$, and this means that they have the same cardinality.

($A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 < 1\}$ and $B = [0, 1] \times [0, 1]$)

Trying to follow the same idea as before, I am stuck trying to create the first function showing that $\mid A \mid \leq \mid B \mid$. Mainly this is because of the set A where I can't visualize the set because with x being 0 then y can be up to but not including 1, and if y were to be 0, then x can be up to but not including 1 as well. So with this I would think the range would be (1, 1) however, trying to restrict x such that using a large enough y will satisfy $x^2 + y^2 < 1$ is something I cannot seem to figure out.

3: For any set A, we have $\mid A \mid < \mid 2^A \mid$.

An idea we can try to follow is the though of having a function that maps the elements from A to 2^A, and then try to show that not every element in A can map to an element in 2^A showing that 2^A has a larger cardinality.