Pledge: *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

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1: Let $x, y \in \mathbb{Z}$. Prove that if xy is odd, then x is odd an y is odd.

Proof. To prove this claim, we consider the possible combinations of x and y being odd and even.

Case: x is odd and y is even.

Let $x = 2j + 1$ and $y = 2k$ where $j, k \in \mathbb{Z}$.

$$\begin{aligned} xy &= (2j+1)(2k) \\ &= 4jk + 2k \\ &= 2(2jk + k) \end{aligned}$$

This is an even number.

Case: x is odd and y is odd.

Let $x = 2j + 1$ and $y = 2k + 1$ where $j, k \in \mathbb{Z}$.

$$\begin{aligned} xy &= (2j+1)(2k+1) \\ &= 4jk + 2j + 2k + 1 \\ &= 2(2jk + j + k) + 1 \end{aligned}$$

This is an odd number.

Case: x is even and y is even.

Let $x = 2j$ and $y = 2k$ where $j, k \in \mathbb{Z}$.

$$\begin{aligned} xy &= (2j)(2k) \\ &= 4jk \\ &= 2(2jk) \end{aligned}$$

This is an even number.

Case: x is even and y is odd.

Let $x = 2j$ and $y = 2k + 1$ where $j, k \in \mathbb{Z}$.

$$\begin{aligned} xy &= (2j)(2k+1) \\ &= 4jk + 2j \\ &= 2(jk + j) \end{aligned}$$

This is an even number.

The only odd solution is for both x and y to be odd. $\square$

2: Prove that if n is a nonnegative integer, then $2^n + 6^n$ is an integer.

Proof. To prove this claim, we consider cases of n being 0 and a nonnegative integer.

Case: $n = 0$

$$\begin{aligned} 2^n + 6^n &= 2^0 + 6^0 \\ &= 1 + 1 \\ &= 2 \end{aligned}$$

2 is of course even.

Case: $n \geq 0$

$$\begin{aligned} 2^n + 6^n &= 2^n + (2 \cdot 3)^n \\ &= 2^n + 2^n 3^n \\ &= (2 \cdot 2^{n-1}) + (2 \cdot 2^{n-1} \cdot 3^n) \\ &= 2(2^{n-1} + 2^{n-1} 3^n) \end{aligned}$$

This is the definition of an even number. In both cases, the end result is always an even number. □

3: The *triangle inequality* asserts that for any $x, y, z \in \mathbb{R}$, we have $|x - y| \leq |x - z| + |y - z|$.

(What's the intuitive idea behind this inequality? What is it trying to say, and where do you think it gets its name from?)

Well based on the name alone, I believe it is referring to the sides of a triangle. The LHS takes the difference of any two sides and in every case it must be $\leq$ to the difference of those two sides subtracted from the third which are then added together. It seems that it has to do with the length of the third side so it does not exceed the difference of the other two.

(Suppose you want to prove the triangle inequality by cases. List the different cases you would need to prove.)

Based on the idea that the theorem seems to rely on the difference of two sides being less than the differences of them with the third side, there would be four cases.

Case 1: $x > y, z$

Case 2: $y > x, z$

Case 3: $z > x, y$

Case 4: $x = y = z$

I feel like this would adequately cover all possible combinations of triangles that can be formed.

(Try to prove at least some of the cases you listed in the previous step.)

I am not sure how to prove the first three cases; I seem to get hung up on trying to work with the inequalities with the absolute values of the differences of the sides.

Though with case 4:

Since $x = y = z$

$$\begin{aligned} |x-y| \leq |x-z| + |y-z| &= |x-x| \leq |x-x| + |x-x \\ &= 0 \leq 0 + 0 \\ &= 0 \leq 0 \end{aligned}$$

Therefore, with all the values of x, y, z being the same, an equilateral triangle would follow the triangle inequality theorem. $\square$