Pledge: *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

---

1: Try your hand at the following lemma.

Lemma 1. *Let R be an equivalence relation on a nonempty set A, and let $a, b \in A$. Then $[a] = [b]$ if and only if $(a, b) \in R$.*

Proof. ($\Rightarrow$) Assume $[a] = [b]$. Since we know that R is an equivalence relation, it is reflexive. Therefore, $[a] = [a]$ and from our original assumption we say that $[a] = [b]$, so it shows that $a, b \in R$.
$\Leftarrow$ Assume that $(a, b) \in R$. Let $x \in [a]$, and since $(a, b) \in R$, due to transitivity $(x, b) \in R$ so $x \in [b]$. Let $y \in [b]$, and since $(a, b) \in R$, due to symmetry we have $(y, a) \in R$ so $y \in [a]$. Therefore, it is proven. $\square$

2: Recall that a partition of a set A is a collection of nonempty pairwise-disjoint subsets of A whose union is all of A. Now try your hand at proving the following theorem.

Theorem 2. *If R is an equivalence relation on a nonempty set A, then the set $\Pi = \{[a]\}_{a \in A}$ of equivalence classes of elements of A is partition of A.*

I'm not sure where to really begin when trying to prove this and cannot form it into a formal proof, but loosely the logic I would try to follow would be to show that because Π is set of equivalence classes, we can use it to show that they form sets whose union will form A.

3: Given integers $a, b \in \mathbb{Z}$ and an integer $n > 1$, we say that A and B are congruent modulo n written $a \equiv b \pmod{n}$ if n divides $a - b$.

(Prove that for any $n > 1$, being congruent modulo n is an equivalence relation.)

Proof. To show that for any $n > 1$, being congruent modulo n is an equivalence relation we need to show that congruency is both reflexive, symmetric, and transitive.
(Reflexive) We can say $a - a = 0x$ where x is some integer. Then it is reasonable to say that $a \equiv a(\text{mod } n)$.
(Symmetric) If we assume that $a \equiv b \pmod{n}$, then it stands that $a - b = nx$ where x is some integer. Then we can multiply both sides by -1 and obtain $b - a = n(-x)$ which is the same as saying $b \equiv a \pmod{n}$,
(Transitive) Assume we have $a \equiv b \pmod{n}$ and $b \equiv c \pmod{n}$. Then we have the two equations $a - b = nx$ for some integer x and $b - c = ny$ for some integer y. When combining the equations together we get $(a - b) + (b - c) = nx + ny \Rightarrow a - c = n(x + y)$ and since $x, y \in \mathbb{Z}$ we can say that $a \equiv c \pmod{n}$. $\square$

(Describe the equivalence classes of congruence modulo 3.)

Since we are going up to 3 for this modulo there are only three sets of integers that satisfy this congruence. The first is the set of all integers which are congruent to 1 mod 3. The second is the set of all integers which are congruent to 2 mod 3. Finally, the third is the set of all integers which are congruent to 3 mod 3.