Pledge: *I pledge my honor that I have abided by the Stevens Honor System.* -Eric Altenburg

1: Prove that if two sets A and B have the same cardinality, then for any set C, the sets $A \times C$ and $B \times C$ have the same cardinality as well.

Proof. Let $n =| A |$, and since $| A |=| B |$, then $n =| A |=| B |$. Additionally, let $m =| C |$. To determine the cardinality of the Cartesian product, you multiply the cardinality of the two sets (i.e. $| X \times Y |=| X | \cdot | Y |$). Therefore,

$$\begin{gathered} | A \times C | =| B \times C | \\ | A | \cdot | C | =| B | \cdot | C | \\ n \cdot m = n \cdot m. \end{gathered}$$

This shows that the cardinality of the two Cartesian products are always equal if $| A |=| B |$ which completes the proof. $\square$

2: Is it true that a set A is countable if and only if there exists a surjection $f : \mathbb{N} \to A$? (Recall that a surjection is an onto function.) If so, prove it. If not, state and prove a closely related theorem.

3: Prove that the infinite strip $S = \{(x, y) \in \mathbb{R}^2 \mid x \in \mathbb{R} \text{ and } 0 \leq y \leq 1\}$ and the Cartesian plane $\mathbb{R}^2$ have the same cardinality. *Hint:* You may assume the Schröder-Bernstein theorem.

4: Prove the theorem whose proof we didn't complete in class: For any set A, we have $| A |<| 2^A |$.