Pledge: *I pledge my honor that I have abided by the Stevens Honor System.*

1: Find the moment estimator and maximum likelihood estimator for θ of the uniform distribution on (0, θ).

$$
\begin{aligned}
E(X) = \int_{-\infty}^{\infty} xf(x)dx &= \int_{0}^{\theta} x * \frac{1}{\theta}dx \\
&= \frac{x^2}{2\theta}\Big|_0^\theta \\
\overline{X} &= \frac{\theta}{2} \\
\hat{\theta} &= 2\overline{X}
\end{aligned}
$$

$$
\begin{aligned}
L(\theta, x) = \prod_{i=1}^{n} f(x_i, \theta) &= \prod_{i=1}^{n} \frac{1}{\theta} \\
&= \frac{1}{\theta^n} = \theta^{-n} \\
log(L(\theta, x) &= -nlog(\theta) \\
\frac{\partial logL(\theta, x)}{\partial \theta} &= 0 \\
\frac{-n}{\theta} &= 0
\end{aligned}
$$

Likelihood estimator for θ is decreasing, therefore, for it to be maximized, θ must be the largest value in the sample x.

2: Textbook Problems

(6.17) (a)

$$
\begin{aligned}
Margin\ of\ error = z_{.975} * \frac{\sigma}{\sqrt{n}} &= 1.96 * \frac{2.3}{\sqrt{340}} \\
&= 0.24
\end{aligned}
$$

$$
CI = \overline{X} \pm z_{.975} * \frac{\sigma}{\sqrt{n}} = (5.156, 5.644)
$$

(b)

$$
Margin\ of\ error = 0.321
$$

$$
CI = (5.079, 5.721)
$$

The interval is wider than that of part a. Due to the higher CI, we need to increase the interval this way our confidence on whether or not the true population mean is within that interval is higher.

(6.27) (a)

$$\begin{aligned} CI &= 11.5 \pm 1.96 * \frac{8.3}{\sqrt{1200}} \\ &= (11.02, 11.97) \end{aligned}$$

(b) No, because asking for individual times would not be appropriate for a CI because it is more so a range of values representing the average time spent.

(c) It will still be a good approximation because the sample size is relatively large.

(6.28) (a) $\overline{X} = 690$ and $\sigma = 498$

(b)

$$\begin{aligned} CI &= 690 \pm 1.96 * \frac{498}{\sqrt{1200}} \\ &= (661.82, 718.17) \end{aligned}$$

(c) Another way one could have directly calculated this interval from the previous exercise would be to multiply the old one by 60.

(6.58) (a) $H_a: \ \mu > \mu_0$

$P(Z > 1.77) = 0.038$

(b) $H_a: \ \mu < \mu_0$
$P(Z < 1.77) = 0.962$

(c) $H_a: \ \mu \neq \mu_0$
$P(|Z| \geq 1.77) = 0.079$

(6.59) (a) $H_a: \ \mu > \mu_0$

$P(Z > -1.69) = 0.955$

(b) $H_a: \ \mu < \mu_0$
$P(Z < -1.69) = 0.046$

(c) $H_a: \ \mu \neq \mu_0$
$P(|Z| \geq -1.69) = 0.091$

(6.71) (a)

$$\begin{aligned} z &= \frac{127.8 - 115}{\frac{30}{\sqrt{25}}} \\ &= 2.13 \\ P(Z > 2.13) &= 0.017 \end{aligned}$$

Because the p-value has a value less than α, we reject H_0 in that older students on average tend to have a higher SSHA score.

(b) The bigger of the two assumptions made in part a was that this was a simple and random sample (SRS), whereas the assumption that it is a normal distribution will not matter too much so long as there are not any outliers or skewness to the sample.

(6.73) (a) $H_0 : \mu = 0\ mpg$

$H_a : \mu \neq 0\ mpg$

(b)

$$\overline{x} = 2.73 \qquad \begin{aligned} z &= \frac{2.73 - 0}{\frac{3}{\sqrt{20}}} \\ &= 4.069 \end{aligned}$$

This results in an extremely small p-value (0.00005) and because of this, H_0 is rejected and so $\mu \neq 0$.

(6.99) (a)

$$\overline{x} = 2453.7 \qquad \begin{aligned} z &= \frac{2453.7 - 2403.7}{\frac{880}{\sqrt{100}}} \\ &= 0.57 \end{aligned}$$

$P(Z > 0.57) = 0.284$

(b)

$$\overline{x} = 2453.7 \qquad \begin{aligned} z &= \frac{2453.7 - 2403.7}{\frac{880}{\sqrt{500}}} \\ &= 1.27 \end{aligned}$$

$P(Z > 1.27) = 0.102$

(c)

$$\overline{x} = 2453.7 \qquad \begin{aligned} z &= \frac{2453.7 - 2403.7}{\frac{880}{\sqrt{2500}}} \\ &= 2.84 \end{aligned}$$

$P(Z > 2.84) = 0.002$

(6.120) (a)

$$\begin{aligned} P(Type\ I\ Error) &= P(x = 0 \cup x = 1 \cup x = 2) \\ &= 0.1 + 0.1 + 0.2 \\ &= 0.4 \end{aligned}$$

(b)

$$\begin{aligned} P(Type\ II\ Error) &= P(x = 3 \cup x = 4 \cup x = 5 \cup x = 6) \\ &= 0.4 \end{aligned}$$

(7.22) (a) $df = 15$

(b) $2.131 < t < 2.249$

(c) $0.02 < P < 0.025$

(d) 5%: Yes
1%: No

(e) $p - values = 0.0241$

(7.23) (a) $df = 26$

(b) $1.706 < t < 2.056$

(c) $0.05 < P < 0.1$

(d) 5%: No
1%: No

(e) $p - values = 0.0549$