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MAT 208 Quiz 2.1 – 2.4　　　　Score: ____________

Directions: Answer the following questions; be sure to include all relevant and supporting work.

Let *A, B, C, D,* and *E* be matrices with size shown below.

A: 3 X 4　　*B*: 3 x 4　　*C*: 4 x 2　　*D*: 4 x 2　　*E*: 4 x 3

If defined, give the size of the following matrices. If not, explain why.

1.) $A + B$ = matrix of size 3x4. Sizes must be the same for both matrices A and B. Resulting matrix takes the same size.

2.) $E - 2A$ = The resulting matrix is not defined. After doing scalar multiplication on A, the size is still (3x4) and (3x4) ≠ (4x3)

(4x3) - (3x4)

3.) AC = The resulting size is 3x2 because since the columns of A are the same as rows in C, the resulting size is the row of A by the column of C giving (3x2).

4.) $2D + C$ = After scalar multiplication to D, the size is still (4x2) and C's size is (4x2), therefore, since size of D = size of C, the resulting matrix size is also (4x2).

5.) Given $A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}$. Find $A^T A$ and AA^T; show that each of these are symmetric.

A^TA:

$A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & 0 \\ 2 & 2 \\ 1 & -1 \end{bmatrix}$

$A^TA = \underset{3\times 2}{\begin{bmatrix} 4 & 0 \\ 2 & 2 \\ 1 & -1 \end{bmatrix}} \underset{2\times 3}{\begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}} = \begin{bmatrix} 4(4)+0(0) & 4(2)+0(2) & 4(1)+0(-1) \\ 2(4)+2(0) & 2(2)+2(2) & 2(1)+2(-1) \\ 1(4)+-1(0) & 1(2)+-1(2) & 1(1)+-1(-1) \end{bmatrix}$

✓

$= \begin{bmatrix} 16 & 8 & 4 \\ 8 & 8 & 0 \\ 4 & 0 & 2 \end{bmatrix} = B$

$B^T = \begin{bmatrix} 16 & 8 & 4 \\ 8 & 8 & 0 \\ 4 & 0 & 2 \end{bmatrix} = B$

$\boxed{\therefore A^TA \text{ is symmetric}}$

AA^T:

$A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}$ $A^T = \begin{bmatrix} 4 & 0 \\ 2 & 2 \\ 1 & -1 \end{bmatrix}$

$AA^T = \underset{2\times 3}{\begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & -1 \end{bmatrix}} \underset{3\times 2}{\begin{bmatrix} 4 & 0 \\ 2 & 2 \\ 1 & -1 \end{bmatrix}}$

✓

$= \begin{bmatrix} 4(4)+2(2)+1(1) & 4(0)+2(2)+1(-1) \\ 0(4)+2(2)+(-1)(1) & 0(0)+2(2)+(-1)(-1) \end{bmatrix}$

$= \begin{bmatrix} 21 & 3 \\ 3 & 5 \end{bmatrix} = B$

$B^T = \begin{bmatrix} 21 & 3 \\ 3 & 5 \end{bmatrix} = B$

$\boxed{\therefore AA^T \text{ is symmetric}}$

6.) Solve the system of equations using an inverse matrix. You may use your calculator to find the inverse and to do the multiplication.

$$x_1 + 2x_2 + x_3 = 2$$
$$x_1 + 2x_2 - x_3 = 4$$
$$x_1 - 2x_2 + x_3 = -2$$

$Ax = B$, $x = A^{-1}B$

$A = \begin{bmatrix} 1 & 2 & 1 \\ 1 & 2 & -1 \\ 1 & -2 & 1 \end{bmatrix}$ $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ $B = \begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}$

Find A^{-1}:

$A^{-1} = \begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 0 & -1/4 \\ 1/2 & -1/2 & 0 \end{bmatrix}$

$A^{-1}B = \underset{3\times3}{\begin{bmatrix} 0 & 1/2 & 1/2 \\ 1/4 & 0 & -1/4 \\ 1/2 & -1/2 & 0 \end{bmatrix}} \underset{3\times1}{\begin{bmatrix} 2 \\ 4 \\ -2 \end{bmatrix}} = \begin{bmatrix} 0(2) + \frac{1}{2}(4) + \frac{1}{2}(-2) \\ \frac{1}{4}(2) + 0 + (-\frac{1}{4})(-2) \\ \frac{1}{2}(2) + (-\frac{1}{2})(4) + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} = X$

$x_1 = 1$, $x_2 = 1$, $x_3 = -1$

$\boxed{\therefore \text{ solution is } (1, 1, -1)}$

7.) Find a sequence of elementary matrices whose product is the given nonsingular matrix.

$$A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$\downarrow R_1 + R_2$

$\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $-R1+R2$

$\downarrow 2R2+R1$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ RREF ü $E_2 = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $E_2^{-1} = \begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ $-2R2+R1$

B

$B = E_2 E_1 A$

$E_2^{-1} B = E_1 A$

$\underbrace{E_1^{-1} E_2^{-1}}\ B = A$

$\underset{E_1^{-1}}{\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} \underset{E_2^{-1}}{\begin{bmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}} = \begin{bmatrix} 1(1)+0(0)+0 & 1(-2)+0+0 & 0+0+0 \\ -1(1)+0+0 & -1(-2)+1+0 & 0+0+0 \\ 0(1)+0+0 & 0+0+0 & 0+0+1 \end{bmatrix} = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} = A$ ✓

$\boxed{\therefore \text{ sequence is } E_1^{-1} E_2^{-1} = A}$