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MAT 208 Quiz 5.1 – 5.3

1.) Determine whether or not $S = \{(x, 2x) | x \text{ is a real number}\}$ is a vector space or not.

yes, $u = (u, 2u)$
$v = (v, 2v)$

$u + v = \langle u+v, 2u + 2v \rangle = \langle u+v, 2(u+v) \rangle$ ✓ closed under addition

$cu = \langle cu, 2cu \rangle = \langle cu, c(2u) \rangle$ ✓ closed under scalar multiplication

2.) Find an orthonormal basis for $S = \{(1, 1, -2), (1, 2, -3), (0, 1, 1)\}$.

Let $v_1 = (1, 1, -2)$ $v_2 = (1, 2, -3)$ $v_3 = (0, 1, 1)$

$w_1 = v_1 = (1, 1, -2)$

$w_2 = v_2 - \frac{\langle v_2, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 = (1, 2, -3) - \frac{\langle (1, 2, -3), (1, 1, -2) \rangle}{\langle (1, 1, -2), (1, 1, -2) \rangle} (1, 1, -2)$

$= (1, 2, -3) - \frac{3}{2}(1, 1, -2) = (1, 2, -3) - (\frac{3}{2}, \frac{3}{2}, -3) = (-\frac{1}{2}, \frac{1}{2}, 0)$

$w_3 = v_3 - \frac{\langle v_3, w_1 \rangle}{\langle w_1, w_1 \rangle} w_1 - \frac{\langle v_3, w_2 \rangle}{\langle w_2, w_2 \rangle} w_2$

$= (0, 1, 1) - \frac{\langle (0,1,1), (1,1,-2) \rangle}{\langle (1,1,-2), (1,1,-2) \rangle} (1, 1, -2) - \frac{\langle (0,1,1), (-1/2, 1/2, 0) \rangle}{\langle (-1/2, 1/2, 0), (-1/2, 1/2, 0) \rangle} (-1/2, 1/2, 0)$

$= (0, 1, 1) + (\frac{1}{6}, \frac{1}{6}, -\frac{1}{3}) - (-1/2, 1/2, 0) = (2/3, 2/3, 2/3)$

$S' = \{(1, 1, -2), (-\frac{1}{2}, \frac{1}{2}, 0), (\frac{2}{3}, \frac{2}{3}, \frac{2}{3})\}$

$S'' = \{\frac{1}{\sqrt{6}}(1, 1, -2), \sqrt{2}(-\frac{1}{2}, \frac{1}{2}, 0), \frac{\sqrt{3}}{2}(\frac{2}{3}, \frac{2}{3}, \frac{2}{3})\} = \{(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}), (-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}, 0), (\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3})\}$

$\boxed{= \{(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\sqrt{\frac{2}{3}}), (-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0), (\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})\}}$

$\frac{-\sqrt{2} \cdot \cancel{\sqrt{2}}}{\sqrt{3} \cdot \cancel{\sqrt{2}}}$ $\frac{-\cancel{\sqrt{2}}}{\sqrt{2} \cdot \cancel{\sqrt{2}}}$ $\frac{\cancel{\sqrt{3}}}{\sqrt{3} \cdot \cancel{\sqrt{3}}}$

Bonus: Find a unit vector that is orthogonal to $v = [1, 3, 4]$ and $u = [2, -6, -5]$

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Cross product of v & u is orthogonal vector:

$$v \times u = \begin{array}{ccc} + & - & + \end{array} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ 2 & -6 & -5 \end{vmatrix} = \hat{i}\begin{vmatrix} 3 & 4 \\ -6 & -5 \end{vmatrix} - \hat{j}\begin{vmatrix} 1 & 4 \\ 2 & -5 \end{vmatrix} + \hat{k}\begin{vmatrix} 1 & 3 \\ 2 & -6 \end{vmatrix}$$

$$= \hat{i}(-15 - (-24)) - \hat{j}(-5 - 8) + \hat{k}(-6 - 6)$$

$$= \hat{i}(9) - \hat{j}(-13) + \hat{k}(-12)$$

$$= 9\hat{i} + 13\hat{j} - 12\hat{k}$$

$$\sqrt{9^2 + 13^2 + (-12)^2} = \sqrt{394}$$

$$\frac{9\hat{i} + 13\hat{j} - 12\hat{k}}{\sqrt{394}} = \frac{1}{\sqrt{394}}(9, 13, -12) = \left(\frac{9}{\sqrt{394}}, \frac{13}{\sqrt{394}}, \frac{-12}{\sqrt{394}}\right) \quad , \frac{-6 \cdot \sqrt{2} \cdot \cancel{\sqrt{2}}}{\sqrt{197} \cdot \cancel{\sqrt{2}}}$$

$$\boxed{= \left(\frac{9}{\sqrt{394}}, \frac{13}{\sqrt{394}}, \frac{-6\sqrt{2}}{\sqrt{197}}\right)}$$