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MAT 208 Quiz 1.1 – 1.3　　　　Score: ______

Directions: Answer the following questions; be sure to include all relevant and supporting work.

1.) Solve the following system of equations.

$$\begin{aligned} 2x + y - 3z &= 4 \\ 4x \quad + \quad 2z &= 10 \\ -2x + 3y - 13z &= -8 \end{aligned}$$

$$\begin{cases} 2x + y - 3z = 4 \\ 4x \quad + 2z = 10 \\ -2x + 3y - 13z = -8 \end{cases}$$

interchange EQ2, EQ3

$$\begin{cases} 2x + y - 3z = 4 \\ -2x + 3y - 13z = -8 \\ 4x \quad + 2z = 10 \end{cases}$$

E1+EQ2

$$\begin{cases} 2x + y - 3z = 4 \\ 4y - 16z = -4 \\ 4x \quad + 2z = 10 \end{cases}$$

-2EQ1+EQ3

$$\begin{cases} 2x + y - 3z = 4 \\ 4y - 16z = -4 \\ -8y - 34z = 18 \end{cases}$$

$$\begin{cases} 2x + y - 3z = 4 \\ 4y - 16z = -4 \\ -8y - 34z = 18 \end{cases}$$

2EQ2+EQ3

$$\begin{cases} 2x + y - 3z = 4 \\ 4y - 16z = -4 \\ -66z = 10 \end{cases}$$

$-\frac{1}{66}$ EQ3

$$\begin{cases} 2x + y - 3z = 4 \\ 4y - 16z = -4 \\ z = -10/66 \end{cases}$$

Backsolve:

$z = -\frac{5}{33}$

$4y - 16\left(\frac{-5}{33}\right) = -4$

$y = \frac{-53}{33}$

$2x + \left(-\frac{53}{33}\right) - 3\left(-\frac{5}{33}\right) = 4$

$x = \frac{85}{33}$

$$\boxed{\text{solution: } \left(\frac{85}{33}, \frac{-53}{33}, \frac{-5}{33}\right)}$$

2.) Solve the following system of equations using either Gaussian elimination with back solving (REF) or Gauss-Jordan (RREF).

$$\begin{aligned} x \quad - \quad 3z &= -2 \\ 3x + y - 2z &= 5 \\ 2x + 2y + z &= 4 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 3 & 1 & -2 & 5 \\ 2 & 2 & 1 & 4 \end{bmatrix}$$

-3R1+R2
-2R1+R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 2 & 7 & 8 \end{bmatrix}$$

-2R2+R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & -7 & -14 \end{bmatrix}$$

-7R3

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -3 & -2 \\ 0 & 1 & 7 & 11 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

3R3+R1
-7R3+R2

$$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 1 & 2 \end{bmatrix} \leftarrow \text{RREF} ☺$$

$$\boxed{\text{solution: } (4, -3, 2)}$$

3.) Solve the homogenous linear system corresponding to the given coefficient matrix. Please note that the final column is NOT the constant column.

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{aligned} x \quad\quad\quad &= 0 \\ y + z &= 0 \\ 0 &= 0 \end{aligned}$$

$\Downarrow$

inf. sols.

Make final var. free var.

$z = t$

$y + z = 0$

$y + t = 0$

$y = -t$

$x = 0$

$\boxed{\text{solution: } (0, -t, t)}$

4.) Find the quadratic equation ($f(x) = ax^2 + bx + c$) that contains the three following points.

$(2,5), (3,0), (4,20)$ 2 degrees

$p(1) = a_0 + a_1(2) + a_2(2)^2 = 5$

$= a_0 + 2a_1 + 4a_2 = 5$

$p(2) = a_0 + a_1(3) + a_2(3)^2 = 0$

$= a_0 + 3a_1 + 9a_2 = 0$

$p(3) = a_0 + a_1(4) + a_2(4)^2 = 20$

$= a_0 + 4a_1 + 16a_2 = 20$

$$\begin{bmatrix} 1 & 2 & 4 & 5 \\ 1 & 3 & 9 & 0 \\ 1 & 4 & 16 & 20 \end{bmatrix} \xrightarrow{\substack{-R1+R2 \\ -R1+R3}} \begin{bmatrix} 1 & 2 & 4 & 5 \\ 0 & 1 & 5 & -5 \\ 0 & 2 & 12 & 15 \end{bmatrix} \xrightarrow{\substack{-2R2+R1 \\ -2R2+R3}} \begin{bmatrix} 1 & 0 & -6 & 15 \\ 0 & 1 & 5 & -5 \\ 0 & 0 & 2 & 25 \end{bmatrix} \xrightarrow{\frac{1}{2}R3} \begin{bmatrix} 1 & 0 & -6 & 15 \\ 0 & 1 & 5 & -5 \\ 0 & 0 & 1 & 25/2 \end{bmatrix} \xrightarrow{\substack{6R3+R1 \\ -5R3+R2}} \begin{bmatrix} 1 & 0 & 0 & 90 \\ 0 & 1 & 0 & -135/2 \\ 0 & 0 & 1 & 25/2 \end{bmatrix}$$

$\leftarrow$ RREF :)

$$\boxed{\text{solution: polynomial is } p(x) = \left(\tfrac{25}{2}\right)x^2 - \left(\tfrac{135}{2}\right)x + 90}$$

I was going to do this quiz in LaTeX, but I soon realized writing it out was faster/easier in the long run.