Eric Altenburg   Midterm Work

7/25/20   Linear

1

① $\begin{bmatrix} 2 & -4 \\ 1 & 0 \end{bmatrix} \leftarrow A$ $\quad$ 1 0 / 0 1

$\xrightarrow{\frac{1}{2}R1}$

$\begin{bmatrix} 1 & -2 \\ 1 & 0 \end{bmatrix}$ $\quad E_1 = \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$ $\quad E_1^{-1} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$

$\xrightarrow{-R1+R2}$

$\begin{bmatrix} 1 & -2 \\ 0 & 2 \end{bmatrix}$ $\quad E_2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ $\quad E_2^{-1} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$

$\xrightarrow{\frac{1}{2}R2}$

$\begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$ $\quad E_3 = \begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$ $\quad E_3^{-1} = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$

$\xrightarrow{2R2+R1}$

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad E_4 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$ $\quad E_4^{-1} = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$

B

$$B = E_4 E_3 E_2 E_1 \cdot A$$

$$E_4^{-1} \cdot B = E_3 E_2 E_1 A$$

$$E_3^{-1} E_4^{-1} B = E_2 E_1 A$$

$$E_2^{-1} E_3^{-1} E_4^{-1} B = E_1 A$$

$$E_1^{-1} E_2^{-1} E_3^{-1} E_4^{-1} B = A$$

$$\begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -4 \\ 1 & 0 \end{bmatrix}$$

2. $(2,1,3) = a(1,2,3) + b(2,3,1)$

$\quad = (a, 2a, 3a) + (2b, 3b, b)$

$a + 2b = 2$ $\quad$ 1 2 2

$2a + 3b = 1$ $\quad$ 2 3 1

$3a + b = 3$ $\quad$ 3 1 3

$\rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \rightarrow$ $\quad a = 0$, $\quad b = 0$, $\quad 0 = 1$ ✗

no solutions

∴ no solution to system of equations.

v(2,1,3) cannot be written as a linear combination with (1,2,3) (2,3,1)

3.

$$\begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 0 & 2 & -4 \\ -2 & -4 & 1 & -2 \end{bmatrix}$$

swap R2, R3

$$\begin{bmatrix} 1 & 2 & 1 & -2 \\ -2 & -4 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

2R1+R2

$$\begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 0 & 3 & -6 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

$\frac{1}{3}$R2

$$\begin{bmatrix} 1 & 2 & 1 & -2 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

-R2+R1

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 2 & -4 \end{bmatrix}$$

-2R2+R3

$$\begin{bmatrix} 1 & 2 & 0 & 0 \\ 0 & 0 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

RREF :)

$a + 2b = 0$

$c - 2d = 0$

$0 = 0$

inf sol., make last var free

let $d = t$

$c - 2(t) = 0$

$-2t = -c$

$2t = c$

let $b = s$

$a + 2s = 0$

$2s = -a$

$-2s = a$

$\therefore (-2s, s, 2t, t)$

(1, 2)

-2, 1

(1, 2)

d 2

(4) $2x + 3y + 3z = 3$

$6x + 6y + 12z = 13$

$12x + 9y - z = 2$

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 6 & 6 & 12 \\ 12 & 9 & -1 \end{bmatrix} \qquad |A| = \begin{vmatrix} 2 & 3 & 3 & 2 & 3 \\ 6 & 6 & 12 & 6 & 6 \\ 12 & 9 & -1 & 12 & 9 \end{vmatrix}$$

$$= (-12 + 432 + 162) - (-18 + 216 + 216)$$

$$= 168$$

$$|A_x| = \begin{vmatrix} 3 & 3 & 3 & 3 & 3 \\ 13 & 6 & 12 & 13 & 6 \\ 2 & 9 & -1 & 2 & 9 \end{vmatrix} = (-18 + 72 + 351) - (-39 + 324 + 36) = 84$$

$$x = \frac{84}{168} = \frac{1}{2}$$

$$|A_y| = \begin{vmatrix} 2 & 3 & 3 & 2 & 3 \\ 6 & 13 & 12 & 6 & 13 \\ 12 & 2 & -1 & 12 & 2 \end{vmatrix} = (-26 + 432 + 36) - (-18 + 48 + 468) = -56$$

$$y = \frac{-56}{168} = -\frac{1}{3}$$

$$|A_z| = \begin{vmatrix} 2 & 3 & 3 & 2 & 3 \\ 6 & 6 & 13 & 6 & 6 \\ 12 & 9 & 2 & 12 & 9 \end{vmatrix} = (24 + 468 + 162) - (36 + 234 + 216)$$

$$= 168$$

$$z = \frac{168}{168} = 1$$

$\therefore$ sol. is $\left(\frac{1}{2}, -\frac{1}{3}, 1\right)$

5. $\left[\begin{array}{ccc|ccc} 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & -7 & 0 & 1 & 0 \\ -16 & 11 & 1 & 0 & 0 & 1 \end{array}\right] \rightarrow 4R1+R3$

$\left[\begin{array}{ccc|ccc} 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & -7 & 0 & 1 & 0 \\ 0 & 15 & 1 & 4 & 0 & 1 \end{array}\right] \rightarrow$ $\quad E_1^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix}$

$-5R2+R3 \quad E_2^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix}$

$\left[\begin{array}{ccc|ccc} 4 & 1 & 0 & 1 & 0 & 0 \\ 0 & 3 & -7 & 0 & 1 & 0 \\ 0 & 0 & 36 & 0 & -5 & 1 \end{array}\right]$ ✓

$U = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & -7 \\ 0 & 0 & 36 \end{bmatrix} \quad L = E_1^{-1} \cdot E_2^{-1}$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 5 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 5 & 1 \end{bmatrix}$

$L \cdot U = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -4 & 5 & 1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & -7 \\ 0 & 0 & 36 \end{bmatrix} = \begin{bmatrix} 4 & 1 & 0 \\ 0 & 3 & -7 \\ -16 & 11 & 1 \end{bmatrix}$

6. $S = \{(1,1,0), (0,1,1), (1,1,1)\}$ R^3

$$\begin{bmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{bmatrix} \xrightarrow{RREF} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$ ✓ $\therefore$ S is linearly independent

$$\begin{vmatrix} 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \end{vmatrix} = (1+0+1)-(0+1+0) = 2-1 = \underline{\underline{1}} \neq 0$$

$\therefore$ S spans R^3

Because the set S spans R^3 and is linearly independent, it is a basis.

$(1,2,3)$

$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \\ 0 & 1 & 1 & 3 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$\therefore (1,2,3) = -1(1,1,0) + 1(0,1,1) + 2(1,1,1)$

7. $|A| = 4$ $|B| = 2$

a) $|BA| = 2 \cdot 4 = 8$

b) $|B^2| = 2^2 = 4$

c) $|2A| = 2^4 |A| = 16 \cdot 4 = 64$

d) $|(AB)^T| = |B^T A^T| = |B^T| \cdot |A^T| = 4 \cdot 2 = 8$

e) $|B^{-1}| = \frac{1}{|B|} = \frac{1}{2}$

8. let $z = t$

$y + 2(t) = 0$

$y = -2t$

$x - 2z = 0$

$x - 2(t) = 0$

$x = 2t$

$w + 3t = 0$

$w = -3t$ $\therefore$ solution is $(-3t, 2t, -2t, t)$