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MAT 208 Quiz 3.1 – 3.3 Score:

Directions: Answer the following questions; be sure to include all relevant and supporting work.

1.) Find the value of λ that makes the determinant 0.

$$\begin{vmatrix} \lambda+2 & 2 \\ 1 & \lambda \end{vmatrix}$$

$$\begin{vmatrix} \lambda+2 & 2 \\ 1 & \lambda \end{vmatrix} = [(\lambda+2)(\lambda)] - 2 = 0$$

$$\lambda^2 + 2\lambda - 2 = 0$$

$$\lambda_1 = -1 - \sqrt{3}$$

$$\lambda_2 = -1 + \sqrt{3}$$

$$\lambda_1: \begin{vmatrix} (-1-\sqrt{3})+2 & 2 \\ 1 & (-1-\sqrt{3}) \end{vmatrix} = \begin{vmatrix} 1-\sqrt{3} & 2 \\ 1 & (-1-\sqrt{3}) \end{vmatrix} = (1-\sqrt{3})(-1-\sqrt{3}) - 2$$
$$= -1 - \cancel{\sqrt{3}} + \cancel{\sqrt{3}} + 3 - 2 = 2 - 2 = 0 \checkmark$$

$$\lambda_2: \begin{vmatrix} -1+\sqrt{3}+2 & 2 \\ 1 & -1+\sqrt{3} \end{vmatrix} = \begin{vmatrix} 1+\sqrt{3} & 2 \\ 1 & -1+\sqrt{3} \end{vmatrix} = (1+\sqrt{3})(-1+\sqrt{3}) - 2$$
$$= -1 + \cancel{\sqrt{3}} - \cancel{\sqrt{3}} + 3 - 2 = 2 - 2 = 0 \checkmark$$

$$\boxed{\therefore \lambda = -1-\sqrt{3},\ -1+\sqrt{3}}$$

2.) Use ***elementary row operations*** to evaluate the given determinant.

$$\begin{vmatrix} 1 & 7 & -3 \\ 1 & 3 & 1 \\ 4 & 8 & 1 \end{vmatrix}$$

$$A = \begin{vmatrix} 1 & 7 & -3 \\ 1 & 3 & 1 \\ 4 & 8 & 1 \end{vmatrix}$$

$\downarrow -R1+R2$

$$A' = \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 4 & 8 & 1 \end{vmatrix}$$

$\downarrow -4R1+R3$

$$A'' = \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 0 & -20 & 13 \end{vmatrix}$$

$\downarrow -5R2+R3$

$$A''' = \begin{vmatrix} 1 & 7 & -3 \\ 0 & -4 & 4 \\ 0 & 0 & -7 \end{vmatrix} \leftarrow \text{Triangular}$$

$$|A'''| = (1)(-4)(-7) = \boxed{28}$$

3.) Given A and B are square matrices of order 3 such that $|A| = 4$ and $|B| = 5$. Find the following:

a.) $|AB| = 4 \cdot 5 = \boxed{20}$

b.) $|2A| = 2^3 |A| = 8(4) = \boxed{32}$

c.) Are A and B singular or nonsingular? Explain.

nonsingular = invertible

A & B are nonsingular (or invertible) because each of their determinants do not equal 0.

$\boxed{\because |A| \neq 0 \ \& \ |B| \neq 0, \therefore \text{A and B are nonsingular}}$

d.) If A and B are nonsingular, find $|A^{-1}|$ and $|B^{-1}|$.

$|A^{-1}| = \frac{1}{|A|} = \boxed{\frac{1}{4}}$

$|B^{-1}| = \frac{1}{|B|} = \boxed{\frac{1}{5}}$

Bonus: Find $|(AB)^T|$.

$|(AB)^T| = |B^T A^T| = |B^T| \cdot |A^T|$

$= 5 \cdot 4 = \boxed{20}$